NEUTROSOPHIC DUPLET STRUCTURES

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Abstract.

The Neutrosophic Duplets and the Neutrosophic Duplet Algebraic Structures were introduced by Florentin Smarandache in 2016:
http://fs.unm.edu/NeutrosophicDuplets.htm

Let *U* be a universe of discourse, and a set *D* included in *U*, endowed with a well-defined law #.

1. Definiton of the Neutrosophic Duplet (ND).

We say that <*a*, *neut(a)*>, where *a*, and its neutral *neut(a)* belong to D, is a neutrosophic duplet if:

1) *neut(a)* is different from the unitary element of *D* with respect to the law # (if any);

2) *a#neut(a)* = *neut(a)#a* = *a*;

3) there is no opposite *anti(a)* belonging to *D* for which *a#anti(a)* = *anti(a)#a* = *neut(a)*.

2. Example of Neutrosophic Duplets.

In $(Z_8, \#)$, the set of integers with respect to the regular multiplication modulo 8, one has the following neutrosophic duplets:

<2, 5 >, <4, 3>, <4, 5>, <4, 7>, and <6, 5>.

Proof:

Let $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$, having the unitary element 1 with respect to the multiplication $\#$ *modulo 8*.

$2 \# 5 = 5 \# 2 = 10 = 2 \pmod 8$,

so $neut(2) = 5 \neq 1$.

There is no $anti(2) \in Z_8$, because:

$2 \# anti(2) = 5 \pmod 8$,

or $2y = 5 \pmod 8$ by denoting $anti(2) = y$, is equivalent to:

$2y - 5 = M_8$ {multiple of 8}, or $2y - 5 = 8k$, where k is an integer, or

$2(y - 4k) = 5$, where both y and k are integers, or:

e*ven number* $=$ *odd number*, which is impossible.

Therefore, we proved that *<2, 5>* is a neutrosophic duplet.

Similarly for *<4, 5>, <4, 3>, <4, 7>*, and *<6, 5>*.

A counter-example: <*0, 0*> is not a neutrosophic duplet, because it is a neutrosophic triplet: <*0, 0, 0*>, where there exists an *anti(0) = 0*.

3. Definiton of the Neutrosophic Extended Duplet (NED).

Let *U* be a universe of discourse, and a set *D* included in *U*, endowed with a well-defined law #.

We say that <*a*, $_e$*neut(a)*>, where *a*, and its extended neutral $_e$*neut(a)* belong to *D*, such that:

1) $_e$*neut(a)* may be equal or different from the unitary element of *D* with respect to the law # (if any);

2) $a\#_e neut(a) = {}_e neut(a)\#a = a$;

3) there is no extended opposite $_e$*anti(a)* belonging to *D* for which $a\#_e anti(a) = {}_e anti(a)\#a = {}_e neut(a)$.

4. Definition of Neutrosophic Duplet Strong Set (NDSS).

A neutrosophic Duplet Strong Set is a set *D*, such that for any $x \in D$ there is a *neut(x)* ∈ *D* and *no anti(x)* ∈ *D.*

5. Definition of Neutrosophic Duplet Weak Set (NDWS).

A neutrosophic Duplet Weak Set is a set *D*, such that for any $x \in D$ there is a neutrosophic duplet <*y*, *neut(y*>

included in *D*, such that $x = y$ or $x = neut(y)$.

6. Definition of Neutrosophic Extended Duplet Strong Set (NEDSS).

A Neutrosophic Extended Duplet Strong Set is a set *D*, such that for any $x \in D$ there is an $_{e}neut(x) \in D$ and *no* $_{e}anti(x) \in D$.

7. Definition of Neutrosophic Extended Duplet Weak Set (NEDWS).

A Neutrosophic Extended Duplet Weak Set is a set *D*, such that for any $x \in D$ there is a neutrosophic duplet $<y, {}_{e}neut(y)>$

included in *D*, such that $x = y$ or $x = {}_{e}neut(y)$.

8. Definition of Neutrosophic Duplet Strong Structures (NDSStr).

Neutrosophic Duplet Strong Structures are structures defined on the neutrosophic duplet strong sets.

9. Definition of Neutrosophic Duplet Weak Structures (NDWStr).

Neutrosophic Duplet Weak Structures are structures defined on the neutrosophic duplet weak sets.

10. Definition of Neutrosophic Extended Duplet Strong Structures (NEDSStr).

Neutrosophic Extended Duplet Strong Structures are structures defined on the neutrosophic extended duplet strong sets.

11. Definition of Neutrosophic Extended Duplet Weak Structures (NEDWStr).

Neutrosophic Extended Duplet Weak Structures are structures defined on the neutrosophic extended duplet weak sets.

References

[1] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[2] Florentin Smarandache, *Neutrosophic Duplet Structures*, Joint Fall 2017 Meeting of the Texas Section of the APS, Texas Section of the AAPT, and Zone 13 of the Society of Physics Students, The University of Texas at Dallas, Richardson, TX, USA, October 20-21, 2017,

http://meetings.aps.org/Meeting/TSF17/Session/F1.32

[3] F. Smarandache, Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications. Pons Editions, Bruxelles, 323 p., 2017;

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Summer 2016