Research Article

Neutrosophic *e*-Continuous Maps and Neutrosophic *e*-Irresolute Maps

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Abstract: Aim of this present paper is to introduce and investigate new kind of neutrosophic continuous function called neutrosophic *e*continuous maps in neutrosophic topological spaces and also relate with their near continuous maps. Also, a new irresolute map called neutrosophic *e*-irresolute maps in neutrosophic topological spaces is introduced. Further, discussed about some properties and characterization of neutrosophic *e*-irresolute maps in neutrosophic topological spaces.

Keywords and phrases: Neutrosophic *e*-open sets, neutrosophic *e*-continuous maps, neutrosophic $eU_{\frac{1}{2}}$-space and neutrosophic *e*-irresolute maps.

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1 Introduction

The concept of fuzzy set (briefly, *fs*) was introduced by Lotfi Zadeh in 1965 [17], then Chang depended the fuzzy set to introduce the concept of fuzzy topological space (briefly, *fts*) in 1968 [5]. After that the concept of fuzzy set was developed into the concept of intutionistic fuzzy set (briefly, *Ifs*) by Atanassov in 1983 [2, 3, 4], the intutionistic fuzzy set gives a degree of membership and a degree of non-membership functions. Cokor in 1997 [5] relied on intutionistic fuzzy set to introduced the concept of intutionistic fuzzy topological space (briefly, *Ifts*). In 2005 Smaradache [13] study the concept of neutrosophic set (briefly, *Ns*). After that and as developed the term of neutrosophic set, Salama has studied neutrosophic topological space (briefly, *Nts*) and many of its applications [8, 9, 10, 11]. In 2012 Salama and Alblowi defined neutrosophic topological space [8]. Saha [7] defined δ-open sets in topological spaces. Vadivel et al. [15] introduced δ-open sets in a neutrosophic topological space. In 2008, Ekici [6] introduced the notion of *e*-open sets in a general topology. In 2014, Seenivasan et al. [12] introduced *e*-open sets in a topological space along with *e*-continuity. Vadivel et al. [16] studied fuzzy *e*-open sets in intuitionistic fuzzy topological space. In this paper, we develop the concept of neutrosophic *e* continuity in a topological spaces and also specialized some of their basic properties with examples. Also, we discuss about properties and characterization of neutrosophic *e*-irresolute maps.

2 Preliminaries

The needful basic definitions & properties of neutrosophic topological spaces are discussed in this section.

Definition 2.1 [8] Let Y be a non-empty set. A neutrosophic set (briefly, $N_s s$) L is an object having the form $L = \{hy,\mu_L(y),\sigma_L(y), \nu_L(y)i : y \in Y\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\sigma_L \to [0,1]$ denote the degree of indeterminacy function and $\nu_L \to [0,1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set L and $0 \le \mu_L(y) + \sigma_L(y) + \nu_L(y) \le 3$ for each $y \in Y$.

Remark 2.1 [8] A $N_s s$ $L = \{hy,\mu_L(y),\sigma_L(y),\nu_L(y)i : y \in Y\}$ can be identified to an ordered triple $hy,\mu_L(y),\sigma_L(y),\nu_L(y)i$ in $[0,1]$ on Y .

Definition 2.2 [8] Let Y be a non-empty set & the $N_s s$'s L & M in the form $L = \{hy,\mu_L(y),\sigma_L(y),\nu_L(y)i : y \in Y\}$, $M = \{hy,\mu_M(y),\sigma_M(y),\nu_M(y)i : y \in Y\}$, then

(i) $0_N = hy,0,0,1i$ and $1_N = hy,1,1,0i$,
(ii) $L \subseteq M$ iff $\mu_L(y) \le \mu_M(y)$, $\sigma_L(y) \le \sigma_M(y)$ & $\nu_L(y) \ge \nu_M(y) : y \in Y$,
(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,
(iv) $1_N - L = \{hy,\nu_L(y),1 - \sigma_L(y),\mu_L(y)i : y \in Y\} = L^c$,
(v) $L \cup M = \{hy,\max(\mu_L(y),\mu_M(y)),\max(\sigma_L(y),\sigma_M(y)),\min(\nu_L(y),\nu_M(y))i : y \in Y\}$,
(vi) $L \cap M = \{hy,\min(\mu_L(y),\mu_M(y)),\min(\sigma_L(y),\sigma_M(y)),\max(\nu_L(y),\nu_M(y))i : y \in Y\}$.

Definition 2.3 [8] A neutrosophic topology (briefly, $N_s t$) on a non-empty set Y is a family Ψ_N of neutrosophic subsets of Y satisfying

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(i) $0_N, 1_N \in \Psi_N$.
(ii) $L_1 \cap L_2 \in \Psi_N$ for any $L_1, L_2 \in \Psi_N$.
(iii) $\bigcup L_x \in \Psi_N, \forall L_x : x \in X \subseteq \Psi_N$.

Then (Y,Ψ_N) is called a neutrosophic topological space (briefly, N_sts) in Y. The Ψ_N elements are called neutrosophic open sets (briefly, N_sos) in Y. A N_ss C is called a neutrosophic closed sets (briefly, N_scs) iff its complement C^c is N_sos.

Definition 2.4 [8] Let (Y,Ψ_N) be N_sts on Y and L be an N_ss on Y, then the neutrosophic interior of L (briefly, $N_sint(L)$) and the neutrosophic closure of L (briefly, $N_scl(L)$) are defined as

$$N_sint(L) = \bigcup\{I : I \subseteq L \ \& \ I \text{ is a } N_sos \text{ in } Y\}$$
$$N_scl(L) = \bigcap\{I : L \subseteq I \ \& \ I \text{ is a } N_scs \text{ in } Y\}.$$

Definition 2.5 [1] Let (Y,Ψ_N) be N_sts on Y and L be an N_ss on Y. Then L is said to be a neutrosophic regular open set (briefly, N_sros) if $L = N_sint(N_scl(L))$.

The complement of a N_sros is called a neutrosophic regular closed set (briefly, N_srcs) in Y.

Definition 2.6 [15] A set K is said to be a neutrosophic

(i) δ interior of G (briefly, $N_s\delta int(K)$) is defined by $N_s\delta int(K) = \bigcup\{B : B \subseteq K \ \& \ B \text{ is a } N_sros \text{ in } Y\}$.
(ii) δ closure of K (briefly, $N_s\delta cl(K)$) is defined by $N_s\delta cl(K) = \bigcap\{A : K \subseteq A \ \& \ A \text{ is a } N_srcs \text{ in } Y\}$.

Definition 2.7 [15] A set L is said to be a neutrosophic

(i) δ-open set (briefly, $N_s\delta os$) if $L = N_s\delta int(L)$.
(ii) δ-pre open set (briefly, $N_s\delta Pos$) if $L \subseteq N_sint(N_s\delta cl(L))$.
(iii) δ-semi open set (briefly, $N_s\delta Sos$) if $L \subseteq N_scl(N_s\delta int(L))$.
(iv) e-open set (briefly, N_seos) [14] if $L \subseteq N_scl(N_s\delta int(L)) \cup N_sint(N_s\delta cl(L))$.
(v) e^*-open set (briefly, N_se^*os) if $L \subseteq N_scl(N_sint(N_s\delta cl(L)))$.

The complement of an $N_s\delta os$ (resp. $N_s\delta Pos$, $N_s\delta Sos$, N_seos & N_se^*os) is called a neutrosophic δ (resp. δ-pre, δ-semi, e & e^*) closed set (briefly, $N_s\delta cs$ (resp. $N_s\delta Pcs$, $N_s\delta Scs$, N_secs & N_se^*cs)) in Y.

Definition 2.8 [15] Let (X,Ψ_N) and (Y,Φ_N) be any two Nts's. A map $h : (X,\Psi_N) \to (Y,\Phi_N)$ is said to be neutrosophic (resp. δ, δS, δP & e^*) continuous (briefly, N_sCts [10] (resp. $N_s\delta Cts$, $N_s\delta SCts$, $N_s\delta PCts$ & N_se^*Cts)) if the inverse image of every N_sos in (Y,Φ_N) is a N_sos (resp. $N_s\delta os$, $N_s\delta Sos$, $N_s\delta Pos$ & N_se^*os) in (X,Ψ_N).

3 Neutrosophic *e*-continuous maps in N_sts

Definition 3.1 A map $h : (X,\tau_N) \to (Y,\sigma_N)$ is called neutrosophic e-continuous (N_seCts in short) if $h^{-1}(\lambda)$ is a N_seos in (X,τ_N) for every N_sos λ in (Y,σ_N).

Example 3.1 $X = \{a,b,c\} = Y$ and define N_ss's X_1, X_2 & X_3 in X and Y_1 in Y are

$$X_1 = \langle X, (\tfrac{\mu a}{0.2}, \tfrac{\mu b}{0.3}, \tfrac{\mu c}{0.4}), (\tfrac{\sigma a}{0.5}, \tfrac{\sigma b}{0.5}, \tfrac{\sigma c}{0.5}), (\tfrac{\nu a}{0.8}, \tfrac{\nu b}{0.7}, \tfrac{\nu c}{0.6}) \rangle,$$
$$X_2 = \langle X, (\tfrac{\mu a}{0.1}, \tfrac{\mu b}{0.1}, \tfrac{\mu c}{0.4}), (\tfrac{\sigma a}{0.5}, \tfrac{\sigma b}{0.5}, \tfrac{\sigma c}{0.5}), (\tfrac{\nu a}{0.9}, \tfrac{\nu b}{0.9}, \tfrac{\nu c}{0.6}) \rangle,$$
$$X_3 = \langle X, (\tfrac{\mu a}{0.2}, \tfrac{\mu b}{0.4}, \tfrac{\mu c}{0.4}), (\tfrac{\sigma a}{0.5}, \tfrac{\sigma b}{0.5}, \tfrac{\sigma c}{0.5}), (\tfrac{\nu a}{0.8}, \tfrac{\nu b}{0.6}, \tfrac{\nu c}{0.6}) \rangle,$$
$$Y_1 = \langle Y, (\tfrac{\mu a}{0.2}, \tfrac{\mu b}{0.4}, \tfrac{\mu c}{0.4}), (\tfrac{\sigma a}{0.5}, \tfrac{\sigma b}{0.5}, \tfrac{\sigma c}{0.5}), (\tfrac{\nu a}{0.8}, \tfrac{\nu b}{0.6}, \tfrac{\nu c}{0.6}) \rangle.$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be an identity mapping, then h is N_seCts function.

Proposition 3.1 A map $h : (X,\tau_N) \to (Y,\sigma_N)$, then the statements are hold but the converse does not true.

(i) Every $N_s\delta Cts$ is a N_sCts.
(ii) Every N_sCts is a $N_s\delta SCts$.
(iii) Every N_sCts is a $N_s\delta PCts$.
(iv) Every $N_s\delta SCts$ is a N_seCts.
(v) Every $N_s\delta PCts$ is a N_seCts.
(vi) Every N_seCts is a N_se^*Cts.

Proof. The proof of (i), (ii) & (iii) are studied in [15].

(iv) Let λ be a N_sos in Y. Since h is $N_s\delta SCts$, $h^{-1}(\lambda)$ is a $N_s\delta Sos$ in X. Since every $N_s\delta os$ is a N_seos [14], $h^{-1}(\lambda)$ is a N_seos in X. Hence h is a N_seCts.
(v) Let λ be a N_sos in Y. Since h is $N_s\delta PCts$, $h^{-1}(\lambda)$ is a $N_s\delta Pos$ in X. Since every $N_s\delta Pos$ is a N_seos [14], $h^{-1}(\lambda)$ is a N_seos in X. Hence h is a N_seCts.

(vi) Let λ be a N_sos in h. Since h is N_seCts, $h^{-1}(\lambda)$ is a N_seos in X. Since every N_seos is a N_se^*os [14], $h^{-1}(\lambda)$ is a N_se^*os in X. Hence h is a N_se^*Cts.

(vii) ■

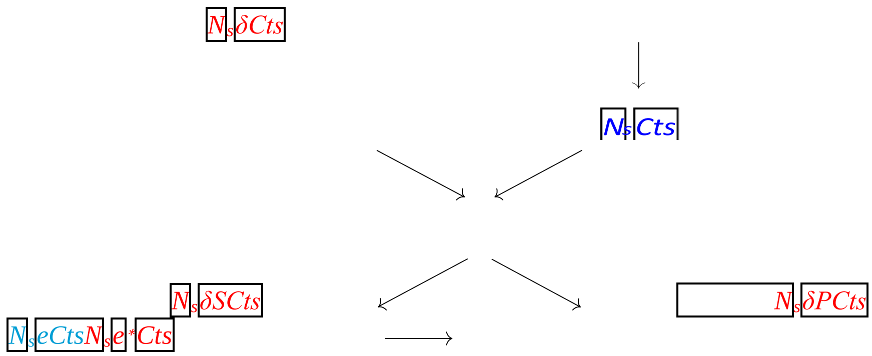

Figure 1: N_seCts maps in N_sts

Example 3.2 In Example 3.1, h is N_seCts but not $N_s\delta PCts$, the set $h^{-1}(Y_1) = X_3$ is a N_seos but not $N_s\delta Pos$.

Example 3.3 $X = \{a,b,c\} = Y$ and define N_ss's X_1, X_2, X_3 & X_4 in X and Y_1 in Y are

$$X_1 = \langle X, (\frac{\mu a}{0.3}, \frac{\mu b}{0.5}, \frac{\mu c}{0.5}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}, \frac{\sigma c}{0.5}), (\frac{\nu a}{0.7}, \frac{\nu b}{0.5}, \frac{\nu c}{0.5})\rangle,$$

$$X_2 = \langle X, (\frac{\mu a}{0.4}, \frac{\mu b}{0.2}, \frac{\mu c}{0.6}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}, \frac{\sigma c}{0.5}), (\frac{\nu a}{0.6}, \frac{\nu b}{0.8}, \frac{\nu c}{0.4})\rangle,$$

$$X_3 = \langle X, (\frac{\mu a}{0.4}, \frac{\mu b}{0.5}, \frac{\mu c}{0.6}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}, \frac{\sigma c}{0.5}), (\frac{\nu a}{0.6}, \frac{\nu b}{0.5}, \frac{\nu c}{0.4})\rangle,$$

$$X_4 = \langle X, (\frac{\mu a}{0.3}, \frac{\mu b}{0.5}, \frac{\mu c}{0.4}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}, \frac{\sigma c}{0.5}), (\frac{\nu a}{0.7}, \frac{\nu b}{0.5}, \frac{\nu c}{0.6})\rangle$$

$$Y_1 = \langle Y, (\frac{\mu^a}{0.3}, \frac{\mu^b}{0.5}, \frac{\mu^c}{0.4}), (\frac{\sigma^a}{0.5}, \frac{\sigma^b}{0.5}, \frac{\sigma^c}{0.5}), (\frac{\nu^a}{0.7}, \frac{\nu^b}{0.5}, \frac{\nu^c}{0.6})\rangle.$$

Then we have $\tau_N = \{0_N, X_1, X_2, X_3, X_1 \cap X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be an identity mapping, then h is N_seCts but not $N_s\delta SCts$, the set $h^{-1}(Y_1) = X_4$ is a N_seos but not $N_s\delta Sos$.

Example 3.4 Let $X = \{a,b\} = Y$ and define N_ss's X_1 & X_2 in X and Y_1 in Y are

$$X1 = \langle X, (\frac{\mu a}{0.3}, \frac{\mu b}{0.2}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}), (\frac{\nu a}{0.5}, \frac{\nu b}{0.5})\rangle,$$

$$X2 = \langle X, (\frac{\mu a}{0.3}, \frac{\mu b}{0.5}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}), (\frac{\nu a}{0.7}, \frac{\nu b}{0.6})\rangle,$$

$$Y1 = \langle Y, (\frac{\mu a}{0.3}, \frac{\mu b}{0.5}), (\frac{\sigma a}{0.5}, \frac{\sigma b}{0.5}), (\frac{\nu a}{0.7}, \frac{\nu b}{0.6})\rangle.$$

Then we have $\tau_N = \{0_N, X_1, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be an identity mapping, then h is N_se^*Cts but not N_seCts, the set $h^{-1}(Y_1) = X_2$ is a N_se^*os but not N_seos.

Theorem 3.1 A map $h : (X,\tau_N) \to (Y,\sigma_N)$ is N_seCts iff the inverse image of each N_scs in Y is N_secs in X.

Proof. Let λ be a N_scs in Y. This implies λ^c is N_sos in Y. Since h is N_seCts, $h^{-1}(\lambda^c)$ is N_seos in X. Since $h^{-1}(\lambda^c) = (h^{-1}(\lambda))^c$, $h^{-1}(\lambda)$ is a N_secs in X.

Conversely, let λ be a N_scs in Y. Then λ^c is a N_sos in Y. By hypothesis $h^{-1}(\lambda^c)$ is N_seos in X. Since $h^{-1}(\lambda^c) = (h^{-1}(\lambda))^c$, $(h^{-1}(\lambda))^c$ is a N_seos in X. Therefore $h^{-1}(\lambda)$ is a N_secs in X. Hence h is N_seCts. ■

Definition 3.2 A N_st (X,τ_N) is said to be an neutrosophic $eU_{\frac{1}{2}}$ (in short $N_seU_{\frac{1}{2}}$)-space, if every N_seos in X is a N_sos in X.

Theorem 3.2 Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be a N_seCts, then h is a N_sCts if X is a $N_seU_{\frac{1}{2}}$-space.

Proof. Let λ be a N_sos in Y. Then $h^{-1}(\lambda)$ is a N_seos in X, by hypothesis. Since X is a $N_seU_{\frac{1}{2}}$-space, $h^{-1}(\lambda)$ is a N_sos in X. Hence h is a N_seCts.■

Theorem 3.3 Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be a N_seCts map and $g : (Y,\sigma_N) \to (Z,\rho_N)$ be an N_seCts, then $g \circ h : (X,\tau_N) \to (Z,\rho_N)$ is a N_seCts.

Proof. Let λ be a N_seos in Z. Then $g^{-1}(\lambda)$ is a N_sos in Y, by hypothesis. Since h is a N_seCts map, $h^{-1}(g^{-1}(\lambda))$ is a N_seos in X. Hence $g \circ h$ is a N_seCts map. ■

Remark 3.1 The composition of two N_seCts maps need not be N_seCts maps shown in following examples.

Example 3.5 Let $X = Y = Z = \{a,b,c\}$ and define N_ss's X_1 & X_2 in X and Y_1, Y_2, Y_3 & Y_4 in Y and Z_1 in Z are

$$X_1 = hx,\left(\frac{\mu a}{0.3},\frac{\mu b}{0.1},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.3},\frac{\nu b}{0.4},\frac{\nu c}{0.4}\right)i,$$

$$X2 = hx,\left(\frac{\mu a}{0.3},\frac{\mu b}{0.2},\frac{\mu c}{0.5}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.2},\frac{\nu b}{0.2},\frac{\nu c}{0.4}\right)i,$$

$$Y1 = hx,\left(\frac{\mu a}{0.4},\frac{\mu b}{0.3},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.4},\frac{\nu b}{0.5},\frac{\nu c}{0.5}\right)i,$$

$$Y_2 = hx,\left(\frac{\mu a}{0.5},\frac{\mu b}{0.5},\frac{\mu c}{0.5}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.5},\frac{\nu b}{0.5},\frac{\nu c}{0.5}\right)i,$$

$$Y3 = hx,\left(\frac{\mu a}{0.5},\frac{\mu b}{0.5},\frac{\mu c}{0.5}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.4},\frac{\nu b}{0.5},\frac{\nu c}{0.5}\right)i,$$

$$Y_4 = hx,\left(\frac{\mu a}{0.4},\frac{\mu b}{0.3},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.5},\frac{\nu b}{0.5},\frac{\nu c}{0.5}\right)i,$$

$$Z_1 = hx,\left(\frac{\mu a}{0.3},\frac{\mu b}{0.4},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.4},\frac{\nu b}{0.4},\frac{\nu c}{0.5}\right)i$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$, $\sigma_N = \{0_N, Y_1, Y_2, Y_3, Y_4, 1_N\}$ and $\rho_N = \{0_N, Z_1, 1_N\}$. Let $h : (X,\tau_N) \to (Y,\sigma_N)$ and $g : (Y,\sigma_N) \to (Z,\rho_N)$ be an identity mapping, then h and g are N_seCts function but $g \circ h$ is not a N_seCts functions.

Theorem 3.4 $h : (X,\tau_N) \to (Y,\sigma_N)$ be a N_seCts map. Then the following conditions are hold.

(i) $h(N_secl(\lambda)) \le N_scl(h(\lambda))$, for all N_scs λ in X.

(ii) $N_secl(h^{-1}(\mu)) \le h^{-1}(N_scl(\mu))$, for all N_scs μ in Y.

Proof. (i) Since $N_secl(h(\lambda))$ is a N_secs in Y and h is N_seCts, then $h^{-1}(N_secl(h(\lambda)))$ is N_sec in Y . Now, since $\lambda \le h^{-1}(N_scl(h(\lambda)))$, $N_secl(\lambda) \le h^{-1}(N_secl(h(\lambda)))$. Therefore, $h(N_secl(\lambda)) \le N_scl(h(\lambda))$.

(ii) By replacing λ with μ in (i), we obtain $h(N_secl(h^{-1}(\mu))) \le N_scl(h(h^{-1}(\mu))) \le N_scl(\mu)$. Hence, $N_secl(h^{-1}(\mu)) \le h^{-1}(N_scl(\mu))$. ■

Remark 3.2 If h is N_seCts, then

(i) $h(N_secl(\lambda))$ is not necessarily equal to $N_scl(h(\lambda))$ where $\lambda \in X$.

(ii) $N_secl(h^{-1}(\mu))$ is not necessarily equal to $h^{-1}(N_scl(\mu))$ where $\mu \in Y$.

Example 3.6 In Example 3.1, h is a N_seCts.

(i) Let λ = 0μ.a2, 0μ.b4, 0μ.c4, 0σ.a5, 0σ.b5, 0σ.c5, 0ν.a8, 0ν.b6, 0ν.c6. Then

$$h(Nsecl(\lambda)) = h\ Nsecl\ \left(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6}\right)$$

$$= h\ \left(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6}\right)$$

$$= \left(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}\right),\left(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}\right),\left(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6}\right).$$

But

$$N_scl(h(\lambda)) = N_scl(h(h(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}),(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}),(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6})i))$$

$$= Nscl(h(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}),(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}),(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6})i)$$

$$= h(\frac{\mu a}{0.8},\frac{\mu b}{0.7},\frac{\mu c}{0.6}),(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}),(\frac{\nu a}{0.2},\frac{\nu b}{0.3},\frac{\nu c}{0.4})i.$$

Thus $h(N_secl(\lambda))$ 6= $N_scl(h(\lambda))$.

(ii) Let η = h(0μ.a2, 0μ.b4, 0μ.c4),(0σ.a5, 0σ.b5, 0σ.c5),(0ν.a8, 0ν.b6, 0ν.c6)i. Then

$$N_secl(h-1(\eta)) \le Nsecl(h-1(h(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}),(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}),(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6})i))$$

$$= Nsecl(h(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}),(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}),(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6})i)$$

$$= h(\frac{\mu a}{0.2},\frac{\mu b}{0.4},\frac{\mu c}{0.4}),(\frac{\sigma a}{0.5},\frac{\sigma b}{0.5},\frac{\sigma c}{0.5}),(\frac{\nu a}{0.8},\frac{\nu b}{0.6},\frac{\nu c}{0.6})i.$$

But

$$h^{-1}(N_s cl(\eta)) = h^{-1}\left(N_s cl\left(\left\langle \left(\frac{\mu^a}{0.2}, \frac{\mu^b}{0.4}, \frac{\mu^c}{0.4}\right), \left(\frac{\sigma^a}{0.5}, \frac{\sigma^b}{0.5}, \frac{\sigma^c}{0.5}\right), \left(\frac{\nu^a}{0.8}, \frac{\nu^b}{0.6}, \frac{\nu^c}{0.6}\right)\right\rangle\right)\right)$$
$$= h^{-1}\left(\left\langle \left(\frac{\mu_a}{0.8}, \frac{\mu_b}{0.7}, \frac{\mu_c}{0.6}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.2}, \frac{\nu_b}{0.3}, \frac{\nu_c}{0.4}\right)\right\rangle\right)$$
$$= \left\langle \left(\frac{\mu_a}{0.8}, \frac{\mu_b}{0.7}, \frac{\mu_c}{0.6}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.2}, \frac{\nu_b}{0.3}, \frac{\nu_c}{0.4}\right)\right\rangle.$$

Thus $N_secl(h^{-1}(\eta)) \neq h^{-1}(N_scl(\eta))$.

Theorem 3.5 If h is N_seCts, then $h^{-1}(N_sint(\mu)) \leq N_seint(h^{-1}(\mu))$, for all N_scs μ in Y.

Proof. If h is N_seCts and $\mu \in \sigma_N$. $N_sint(\mu)$ is N_so in Y and hence, $h^{-1}(N_sint(\mu))$ is N_seo in X. Therefore $N_seint(h^{-1}(N_se\, int(\mu))) = h^{-1}(N_sint(\mu))$. Also, $N_sint(\mu) \leq \mu$, implies that $h^{-1}(N_sint(\mu)) \leq h^{-1}(\mu)$. Therefore $N_seint(h^{-1}(N_sint(\mu))) \leq N_seint(h^{-1}(\mu))$. That is $h^{-1}(N_sint(\mu)) \leq N_seint\,(h^{-1}(\mu))$.

Conversely, let $h^{-1}(N_sint(\mu)) \leq N_seint(h^{-1}(\mu))$ for all subset μ of Y. If μ is N_so in Y, then $N_sint(\mu) = \mu$. By assumption, $h^{-1}(N_sint(\mu)) \leq N_seint(h^{-1}(\mu))$. Thus $h^{-1}(\mu) \leq N_seint(h^{-1}(\mu))$. But $N_seint(h^{-1}(\mu)) \leq h^{-1}(\mu)$. Therefore $N_seint(h^{-1}(\mu)) = h^{-1}(\mu)$. That is, $h^{-1}(\mu)$ is N_seo in X, for all N_sos μ in Y. Therefore h is N_seCts on X. ■

Remark 3.3 If h is N_seCts, then $N_seint(h^{-1}(\mu))$ is not necessarily equal to $h^{-1}(N_sint(\mu))$ where $\mu \in Y$.

Example 3.7 In Example 3.1, h is a $NseCts$. Let $\eta = \left\langle \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\right\rangle$. Then

$$N_seint(h^{-1}(\eta)) \leq N_seint\left(h^{-1}\left(\left\langle \left(\frac{\mu^a}{0.2}, \frac{\mu^b}{0.4}, \frac{\mu^c}{0.4}\right), \left(\frac{\sigma^a}{0.5}, \frac{\sigma^b}{0.5}, \frac{\sigma^c}{0.5}\right), \left(\frac{\nu^a}{0.8}, \frac{\nu^b}{0.6}, \frac{\nu^c}{0.6}\right)\right\rangle\right)\right)$$
$$= N_seint\left(\left\langle \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\right\rangle\right)$$
$$= \left\langle \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\right\rangle.$$

But

$$h^{-1}(N_sint(\eta)) = h^{-1}\left(N_sint\left(\left\langle \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\right\rangle\right)\right)$$
$$= h^{-1}\left(\left\langle \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}\right)\right\rangle\right)$$
$$= \left\langle \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}\right)\right\rangle.$$

Thus $N_seint(h^{-1}(\eta)) \neq h^{-1}(N_sint(\eta))$.

4 Neutrosophic e-irresolute maps in N_sts

In this section we introduce neutrosophic e-irresolute maps and study some of its characterizations.

Definition 4.1 A map $h : (X,\tau_N) \to (Y,\sigma_N)$ is called a neutrosophic e-irresolute (briefly, N_seIrr) map if $h^{-1}(\lambda)$ is a N_seos in (X,τ_N) for every N_seos λ of (Y,σ_N).

Theorem 4.1 Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be a N_seIrr, then h is a N_seCts map. But not conversely.

Proof. Let h be a N_seIrr map. Let λ be any N_sos in Y. Since every N_sos is a N_seos, λ is a N_seos in Y. By hypothesis $h^{-1}(\lambda)$ is a N_seos in Y. Hence h is a N_seCts map. ■

Example 4.1 Let $X = \{a,b,c\} = Y$ and define N_ss's X_1, X_2 & X_3 in X and Y_1 & Y_2 in Y are

$$X_1 = \left\langle X, \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.3}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.7}, \frac{\nu_c}{0.6}\right)\right\rangle,$$
$$X_2 = \left\langle X, \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.1}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.9}, \frac{\nu_c}{0.6}\right)\right\rangle,$$
$$X_3 = \left\langle X, \left(\frac{\mu_a}{0.2}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.8}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.6}\right)\right\rangle,$$
$$Y_1 = \left\langle Y, \left(\frac{\mu^a}{0.1}, \frac{\mu^b}{0.1}, \frac{\mu^c}{0.4}\right), \left(\frac{\sigma^a}{0.5}, \frac{\sigma^b}{0.5}, \frac{\sigma^c}{0.5}\right), \left(\frac{\nu^a}{0.9}, \frac{\nu^b}{0.9}, \frac{\nu^c}{0.6}\right)\right\rangle,$$
$$Y_2 = \left\langle Y, \left(\frac{\mu_a}{0.1}, \frac{\mu_b}{0.4}, \frac{\mu_c}{0.5}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}, \frac{\sigma_c}{0.5}\right), \left(\frac{\nu_a}{0.9}, \frac{\nu_b}{0.6}, \frac{\nu_c}{0.5}\right)\right\rangle.$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be an identity mapping, then h is N_seCts but not N_seIrr, the set Y_2 is a N_seos in Y but $h^{-1}(Y_2)$ is not N_seos in X.

Theorem 4.2 Let $h : (X,\tau_N) \to (Y,\sigma_N)$ be a N_seIrr, then h is a N_sIrr map if X is a $N_seU\frac{1}{2}$-space.

Proof. Let λ be a N_sos in Y . Then λ is a N_seos in Y . Therefore $h^{-1}(\lambda)$ is a N_seos in X, by hypothesis. Since X is a $N_seU\frac{1}{2}$-space, $h^{-1}(\lambda)$ is a N_sos in X. Hence h is a N_sIrr map. ■

Theorem 4.3 Let $h : (X,\tau_N) \rightarrow (Y,\sigma_N)$ and $g : (Y,\sigma_N) \rightarrow (Z,\rho_N)$ be N_seIrr maps, then $g \circ h : (X,\tau_N) \rightarrow (Z,\rho_N)$ is a N_seIrr map.

Proof. Let λ be a N_seos in Z. Then $g^{-1}(\lambda)$ is a N_seos in Y . Since h is a N_seIrr map. $h^{-1}(g^{-1}(\lambda))$ is a N_seos in X. Hence $g \circ h$ is a N_seIrr map. ■

Theorem 4.4 Let $h : (X,\tau_N) \rightarrow (Y,\sigma_N)$ be N_seIrr map and $g : (Y,\sigma_N) \rightarrow (Z,\rho_N)$ be N_seCts map, then $g \circ h : (X,\tau_N) \rightarrow (Z,\rho_N)$ is a N_seCts map.

Proof. Let λ be a N_sos in Z. Then $g^{-1}(\lambda)$ is a N_seos in Y . Since h is a N_seIrr, $h^{-1}(g^{-1}(\lambda))$ is a N_seos in X. Hence $g \circ h$ is a N_seCts map. ■

Theorem 4.5 Let $h : (X,\tau_N) \rightarrow (Y,\sigma_N)$ be a map. Then the following conditions are equivalent if X and Y are $N_seU\frac{1}{2}$-spaces.

(i) h is a N_seIrr map.

(ii) $h^{-1}(\mu)$ is a N_seos in X for each N_seos μ in Y .

(iii) $N_scl(h^{-1}(\mu)) \subseteq h^{-1}(N_scl(\mu))$ for each N_ss μ of Y.

Proof. (i) → (ii): Let μ be any N_seos in Y. Then μ^c is a N_secs in Y. Since h is N_seIrr, $h^{-1}(\mu^c)$ is a N_secs in X. But $h^{-1}(\mu^c) = (h^{-1}(\mu))^c$. Therefore $h^{-1}(\mu)$ is a N_seos in X.

(ii) → (iii): Let μ be a any N_ss in Y and $\mu \leq N_scl(\mu)$. Then $h^{-1}(\mu) \leq h^{-1}(N_scl(\mu))$. Since $N_scl(\mu)$ is a N_scs in Y , $N_scl(\mu)$ is a N_secs in Y. Therefore $(N_scl(\mu))^c$ is a N_seos in Y . By hypothesis, $h^{-1}((N_scl(\mu)))^c$ is a N_seos in X. Since $h^{-1}((N_scl(\mu))^c) = (h^{-1}(N_scl(\mu)))^c$, $h^{-1}(N_scl(\mu))$ is a N_secs in X. Since X is $N_seU\frac{1}{2}$-space, $h^{-1}(N_scl(\mu))$ is a N_scs in X. Hence $N_scl(h^{-1}(\mu)) \subseteq N_scl(h^{-1}(N_scl(\mu))) = h^{-1}(N_scl(\mu))$. That is $N_scl(h^{-1}(\mu)) \subseteq h^{-1}(N_scl(\mu))$.

(iii) → (i): Let μ be any N_secs in Y. Since Y is $N_seU\frac{1}{2}$-space, μ is a N_scs in Y and $N_scl(\mu) = \mu$. Hence $h^{-1}(\mu) = h^{-1}(N_secl(\mu)) \supseteq N_secl(h^{-1}(\mu))$. But clearly $h^{-1}(\mu) \subseteq N_scl(h^{-1}(\mu))$. Therefore $N_scl(h^{-1}(\mu)) = h^{-1}(\mu)$. This implies $h^{-1}(\mu)$ is a N_scs and hence it is a N_secs in X. Thus h is a N_seIrr map. ■

5 Conclusions

In this research paper using N_seos we are defined N_seCts map and analyzed its properties. After that we were compared already existing neutrosophic continuity maps to N_se continuity maps. Furthermore we were extended to this maps to N_se-irresolute maps, Finally this concepts can be extended to future research for some mathematical applications.

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