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Neutrosophic generalized b-closed sets in Neutrosophic topological spaces

C.Maheswari[1], M. Sathyabama[2], S.Chandrasekar[3]

[1] Department of Mathematics, Muthayammal College of Arts and Science, Rasipuram,Namakkal(DT),Tamil Nadu, India.

[2] Department of Mathematics, Periyar University Constituent College of Arts and Science,Idappadi,Salem(DT),Tamilnadu, India.

[3] PG & Research Department of Mathematics, Arignar Anna Government Arts College,Namakkal(DT),Tamil Nadu, India

Email:mahi2gobi@ gmail.com [1] ,sathyachezian@gmail.com[2] ,chandrumat@gmail.com[3]

Abstract. Smarandache introduced and developed the new concept of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. Aim of this paper is we introduce and study about Neutrosophic generalized b closed sets in Neutrosophic topological spaces and its properties are discussed details.

1. Introduction

Topology is a classical subject, as a generalization topological spaces many type of topological spaces introduced over the year. C.L. Chang[3] was introduced and developed fuzzy topological space by using L.A. Zadeh's[12] fuzzy sets. Coker[4] introduced the concepts of Intuitionistic fuzzy topological spaces by using Atanassov's[1] Intuitionistic fuzzy set
Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache [6] in 1998. He also defined the Neutrosophic set on three component Neutrosophic topological spaces (T- Truth, F -Falsehood ,I- Indeterminacy). Neutrosophic topological spaces(N-T-S) introduced by Salama [10]et al.In 1996 D. Andrijevic [2] introduced b open sets in topological space, R.Dhavaseelan[5],SaiedJafari are introduced Neutrosophic generalized closed sets.Aim of this paper is we introduced in Neutrosophic b-open sets, Neutrosophic generalized b-open sets in Neutrosophic topological space and also discussed about properties of Neutrosophic gb-interior and Neutrosophic gb-closure in Neutrosophic topological spaces(N-T-S)

2. Preliminaries

In the Second section, we recall needed basic definition and operation of Neutrosophic sets and then fundamental results

Definition 2.1 [10]

Let X be a non-empty fixed set. A Neutrosophic set P is an object having the form

$P = \{\langle x, \mu_P(x),\sigma_P(x),\gamma_P(x)\rangle : x \in X\}$

where $\mu_P(x)$-represents the degree of membership function,

$\sigma_P(x)$- represents degree indeterminacy and then

$\gamma_P(x)$- represents the degree of non-membership function

Remark 2.2 [10]

Neutrosophic set $P=\{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$ can be write to an ordered triple lies in the interval in] -0,1+ [on X.

Remark 2.3[10]

we shall use the symbol

Neutrosophic set $P = \{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle : x \in X\}$ we can be write briefly

Like as $P= \langle x, \mu_P, \sigma_P, \gamma_P \rangle$

Definition 2.4 [10]
In N-T-S, 0_N may be defined like as: $\forall x \in X$
$0_1 = \langle x, 0, 0, 1 \rangle$
$0_2 = \langle x, 0, 1, 1 \rangle$
$0_3 = \langle x, 0, 1, 0 \rangle$
$0_4 = \langle x, 0, 0, 0 \rangle$
1_N may be defined like as: $\forall x \in X$
$1_1 = \langle x, 1, 0, 0 \rangle$
$1_2 = \langle x, 1, 0, 1 \rangle$
$1_3 = \langle x, 1, 1, 0 \rangle$
$1_4 = \langle x, 1, 1, 1 \rangle$
Definition 2.5 [10]
Neutrosophic set $P=\{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle\}$ on X and $\forall x \in X$
then complement of P is
$P^C = \{\langle x, \gamma_P(x), 1 - \sigma_P(x), \mu_P(x)\rangle\}$
Definition 2.6 [10]
Let P and Q are two Neutrosophic sets $\forall x \in X$
$P=\{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle\}$ and
$Q=\{\langle x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x)\rangle\}$.
Then
$P \subseteq Q \Leftrightarrow \mu_P(x) \leq \mu_Q(x), \sigma_P(x) \leq \sigma_Q(x)$ and $\gamma_P(x) \geq \gamma_Q(x)$
Proposition 2.6 [10]
The following results are true for any Neutrosophic set P
(i) $0_N \subseteq P$, $0_N \subseteq 0_N$
(ii) $P \subseteq 1_N$, $1_N \subseteq 1_N$
Definition 2.7 [10]
Let X be a non-empty set, and
Let P and Q be two Neutrosophic sets are
$P=\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x)\rangle$,
$Q=\langle x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x)\rangle$ Then
(i) $P \cap Q=\langle x, \mu_P(x) \wedge \mu_Q(x), \sigma_P(x) \wedge \sigma_Q(x)$ & $\gamma_P(x) \vee \gamma_Q(x)\rangle$
(ii) $P \cup Q=\langle x, \mu_P(x) \vee \mu_Q(x), \sigma_P(x) \vee \sigma_Q(x)$ & $\gamma_P(x) \wedge \gamma_Q(x)\rangle$
Proposition 2.8 [10]
The following conditions are true for all two Neutrosophic sets P and Q are
(i) $(P \cap Q)^C = P^C \cup Q^C$
(ii) $(P \cup Q)^C = P^C \cap Q^C$.
Definition 2.9 [10]
Let X be non-empty set and τ_N be the collection of Neutrosophic subsets of X satisfying the following properties :
(i) $0_N, 1_N \in \tau_N$,
(ii) $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$,
(iii) $\cup T_i \in \tau_N$ for every $\{T_i : i \in J\} \subseteq \tau_N$
Then the space (X, τ_N) is called a Neutrosophic topological space(N-T-S).
The element of τ_N are called Neu-OS (Neutrosophic open set)
and its complement is Neu-CS(Neutrosophic closed set)
Example 2.10 [10]
Let $X=\{x\}$ and $\forall x \in X$
$A_1 = \langle x, 0.6, 0.6, 0.5 \rangle$
$A_2 = \langle x, 0.5, 0.7, 0.9 \rangle$
$A_3 = \langle x, 0.6, 0.7, 0.5 \rangle$

$A_4 = \langle x, 0.5, 0.6, 0.9 \rangle$

Then the collection $\tau_N = \{0_N, A_1, A_2, A_3, A_4, 1_N\}$ is called a N-T-S on X.

Definition 2.11 [10]

(X, τ_N) be N-T-S and ∀x∈X

P ={⟨x, $\mu_P(x)$, $\sigma_P(x)$, $\gamma_P(x)$⟩}be a Neutrosophic set in X. Then the Neutrosophic closure and Then the Neutrosophic closure of P is

Neu-Cl(P)=∩{ H:H is a Neutrosophic closed set in X and P⊆H}

Neutrosophic interior of P is

Neu-Int(P)=∪{M:M is a Neutrosophic open set in X and M⊆P}.

Then

(i) P is Neutrosophic open set iff P=Neu-Int(P) .
(ii) P is Neutrosophic closed set iff P=Neu-Cl(P).

Proposition 2.12 [10]

Let(X, τ_N) be a Neutrosophic topological spaces ,Then for any Neutrosophic set P

(i) Neu-Cl($(P)^C$)= $(\text{Neu-Int}(P))^C$,
(ii) Neu-Int((P^C))= $(\text{Neu-Cl}(P))^C$.

Proposition 2.13 [10]

Let P, Q be two Neutrosophic sets in N-T-S (X, τ_N). Then the following results are true:

(i) Neu-Int(P)⊆P,
(ii) P⊆Neu-Cl(P),
(iii) P⊆Q⇒Neu-Int(P)⊆Neu-Int(Q),
(iv) P⊆Q⇒Neu-Cl(P)⊆Neu-Cl(Q),
(v) Neu-Int(Neu-Int(P))=Neu-Int(P),
(vi) Neu-Cl(Neu-Cl(P))=Neu-Cl(P),
(vii) Neu-Int(P∩Q))=Neu-Int(P)∩Neu-Int(Q),
(viii) Neu-Cl(P∪Q)=Neu-Cl(P)∪Neu-Cl(Q),
(ix) Neu-Int(0_N)=0_N ,
(x) Neu-Int(1_N)=1_N ,
(xi) Neu-Cl(0_N)=0_N ,
(xii) Neu-Cl(1_N)=1_N ,
(xiii) P⊆Q⇒Q^C⊆P^C,
(xiv) Neu-Cl(P∩Q)⊆Neu-Cl(P)∩Neu-Cl(Q),
(xv) Neu-Int(P∪Q)⊇Neu-Int(P)∪Neu-Int(Q).

Definition:2.14[5]

Neutrosophic generalized closed set (Neu-g closed) if Neutrosophic cl(P)⊆G whenever P⊆G and G is Neutrosophic open set in (X,τ_N).

3.Neutrosophic generalized b-open sets

For this third section, we are newly introduce and study the new concept of Neutrosophic generalized b-open sets in N-T-S

Definition:3.1

Let (X, τ_N) be a N-T-S.A Neutrosophic set P is called

Neutrosophic b-open set is

if P⊆Neu-cl [Neu-int(P)]∪Neu-int[Neu-cl(P)]

Neutrosophic b-closed set is

Neu-cl [Neu-int(P)]∩Neu-int[Neu-cl(P)] ⊆P

Definition:3.2

Neutrosophic generalized b-closed Set (Neu-gb-closed set) if Neutrosophic-bcl(P)⊆G whenever P⊆G and G is Neutrosophic open set in (X,τ_N).

Theorem 3.3.

For Every Neutrosophic open sets is Neutrosophic generalized b-open sets.

Proof.
Now Let P is a Neu-OS in N-T-S (X, τ_N) since P⊆Neu-cl(P) and P=Neu-Int(P),Neu-Int(P)⊆Neu-Int(Neu-cl(P)) and then Neu-Int(P)⊆Neu-cl(Neu-Int ((P)) which implies Neu-Int(P)⊆ Neu-cl(Neu-Int(P))∪Neu-Int (Neu-cl(P)).Hence P⊆Neu-Int(P)⊆ Neu-cl(Neu-Int(P))∪Neu-Int(Neu-cl(P)) and P is Neu- gb-open in (X, τ_N) .
But the converse of this theorem is fails
i.e.,For Every Neu-gbOS is not Neutrosophic open sets.
Example 3.4
Here X ={a , b , c} with τ_N={ 0_N, A_1, A_2, 1_N } and $(\tau_N)^C$ ={ 1_N, A_3, A_4, 0_N } where
A_1 = ⟨(0.6, 0.6, 0.4), (0.2, 0.7, 1), (1, 0.6, 0.5) ⟩
A_2 = ⟨(0.1, 0.4, 0.8), (0.2, 0.6, 1), (0.6, 0.5, 0.9)⟩
A_3 = ⟨(0.4, 0.4, 0.6), (1, 0.3, 0.2), (0.5, 0.4, 1)⟩
A_4 = ⟨(0.8, 0.6, 0.1), (1, 0.4, 0.2), (0.9, 0.5, 0.6)⟩.
A_5 = ⟨(0.3, 0.4, 1), (0.1, 0.2, 1), (0.4, 0.2, 1)⟩.
Here the Neu-gbOSs are A_3, A_4 and A_5.
Also A_5 is Neu-gbCS and A_5 is not Neu-CS.
Theorem 3.5
Consider if P and Q are Neu-gbCS, and then P∪Q is Neu-gbCS.
Proof:
If P∪Q⊆K and K is Neutrosophic open set, then P⊆K and Q⊆K.Since P and Q are Neu-gb closed sets, Neu-cl(P)⊆K and Neu-cl(Q)⊆K and hence Neu-cl(P)∪Neu-cl(Q)⊆K.This implies Neu-cl(P ∪ Q)⊆K. Thus P∪Q is Neu-gbCS in X.
Theorem 3.6
Let P is a Neu-gb closed set and then Neu-cl(P)-P ⊈ any nonempty Neu-C-S.
Proof:
Let P is a Neu-gbCS. Let G be a Neu-CS subset of Neu-cl(P)-P.Then P⊆G^C.But P is Neu-gbCS. Therefore Neu-cl(P)⊆G^C.Consequently G⊆$(\text{Neu-cl}(P))^C$.We have G⊆Neu-cl(P). Thus G⊆Neu-cl(P) ∩$(\text{Neu-cl}(P))^C$ =ϕ. Hence G is empty.

4. Neutrosophic generalized b interior in a N-T-S

In this Fourth section, we newly introduce and study about the properties of Neu- gb interior in a N-T-S.
Definition: 4.1
Let (X, τ_N) be a Neutrosophic topological space and P be a Neutrosophic set in X, then the Neu-gb-interior of P is defined as
Neu-gb-int(P) =∪ {M/M is a Neu-gbOS in X and M⊆P}
Theorem:4.2
Neutrosophic subsets P and Q of a N-T-S X we have
(i) Neu-gb-Int(P)⊆P
(ii) P is Neu-gb-open set in X⇔Neu-gb-Int(P)=P
(iii) Neu-gb-Int(Neu-gb-Int(P))=Neu-gb-Int(P)
(iv) If P⊆Q then Neu-gb-Int(P)=Neu-gb-Int(Q)
Proof:
Proof of (i) is directly get the result through the Definition 4.1.
Let P be Neu-gb-open set in X. Then P⊆Neu-gb-Int(P). from 4.2(i) we obtain the result P=Neu-gb-Int(P). Now Conversely we assume that P=Neu-gb-Int(P). From the Definition 4.1,Neutrosophic set P is a Neu-gb-open set in N-T-S X. from this we get the result (ii). From the result (ii), Neu-gb-Int(Neu-gb-Int(P))=Neu-gb-Int(P).we get the result(iii). Since P⊆Q, by using(i), Neu-gb-Int(P)⊆P⊆Q.i.e.,Neu-gb-Int(P)⊆Q. from the result (iii),Neu-gb-Int(Neu-gb-Int(P))⊆Neu-gb-Int(Q).Thus Neu-gb-Int(P)⊆Neu-gb-Int(Q). we get the result (iv).

Theorem 4.3
Let P and Q are two Neutrosophic subsets of N- T-S (X, τ_N) then
(i) Neu-gb-Int(P∩Q)=Neu-gb-Int(P)∩Neu-gb-Int(Q)
(ii) Neu-gb-Int(P∪Q)⊇Neu-gb-Int(P)∪Neu-gb-Int(Q).
Proof :
Since P∩Q⊆P and P∩Q⊆Q, follows from the theorem 4.2(iv), Neu-gb-Int(P∩Q)⊆Neu-gb-Int(P) and Neu-gb-Int(P∩Q)⊆Neu-gb-Int(Q). This implies that Neu-gb-Int(P∩Q)⊆Neu-gb-Int(P) ∩Neu-gb-Int(Q) .....(1).follows from the theorm3.4(i), Neu-gb-Int(P)⊆P and Neu-gb-Int(Q)⊆Q.This implies that Neu-gb-Int(P)∩Neu-gb-Int(Q)⊆P∩Q.Now from theorm4.2(iv),Neu-gb-Int((Neu-gb-Int(P)∩Neu-gb-Int(Q))⊆Neu-gb-Int(P∩Q).By(1), Neu-gb-Int(Neu-gb-Int(P))∩Neu-gb-Int(Neu-gb-Int(Q))⊆Neu-gb-Int(P∩Q). from theorm 4.2(iii), Neu-gb-Int(P)∩Neu-gb-Int(Q)⊆Neu-gb-Int(P∩Q) .....(2).From (1) and (2), Neu-gb-Int(P∩Q)=Neu-gb-Int(P)∩Neu-gb-Int(Q). This implies (i).Since P⊆P∪Q and Q⊆P∪Q,by from theorm 4.2(iv), Neu-gb-Int(P)⊆Neu-gb-Int(P∪Q) and Neu-gb-Int(Q)⊆Neu-gb-Int(P∪Q).This implies that Neu-gb-Int(P)∪Neu-gb-Int(Q)⊆Neu-gb-Int(P∪Q).Hence (ii).
Converse part of Theorem 4.3(ii) is need not be true
Example 4.4
Let X = { *p* , *q* , *r* } and τ_N ={0_N, A_1, A_2, A_3, A_4, 1_N} where τ_N is a Neutrosophic topology in N-T-S
A_1 = ⟨(0.5, 0.7, 0.2), (0.6, 0.6, 0.3), (1, 0.7, 0.4)⟩,
A_2 = ⟨(0.5, 0.6, 0.2), (0.8, 0.7, 0.3), (1, 0.5, 0.2)⟩,
A_3 = ⟨(0.5, 0.7, 0.2), (0.8, 0.7, 0.3), (1, 0.7, 0.2)⟩,
A_4 = ⟨(0.5, 0.6, 0.2), (0.6, 0.6, 0.3), (1, 0.5, 0.4)⟩.
τ_N is a Neutrosophic topology in N-T-S
Consider the Neutrosophic sets
A_5 = ⟨(0.8, 0.6, 0.2), (0.8, 0.6, 0.2), (1, 0.5, 0.1)⟩ and
A_6 = ⟨(0.5, 0.6, 0.2), (0.6, 0.7, 0.3), (1, 0.7, 0.2)⟩.
Then Neu-gbint(A_5)= A_4 and Neu-gbint(A_6)= A_4.
This implies that Neu-gbint(A_5)∪Neu-gbint(A_6)= A_4. Then
A_5∪A_6=⟨(0.8, 0.6, 0.2), (0.8, 0.7, 0.2), (1, 0.7, 0.1)⟩,
it follows that Neu-gbint(A_5∪A_6)= A_2.Then Neu-gbint(A_5∪A_6) ⊈Neu-gbint(A_5)∪Neu-gbint (A_6).

5. Neutrosophic generalized b-closure in N-T-S.
Now In the fifth section, we newly introduce and study the properties and characterization of Neu- gb-closure in N-T-S.
Definition 5.1
Let P is a Neutrosophic subset P of Neutrosophic topological space (X, τ_N)
Neu- gb-closure defined as
Neu-gb-Cl(P) =∩{H:H is a Neu-gb-closed set in X and H⊇P}.
Theorem 5.2
Let P is a Neutrosophic subset of N-T-S (X, τ_N)
(i) $[(\text{Neu-gb-Int}(P)]^C$=Neu-gb-Cl$[(P)]^C$,
(ii) $[\text{Neu-gb-Cl}(P)]^C$=Neu-gb-Int$[(P)]^C$.
Proof :
From the Definition 5.1, Neu-gb-Int(P) =∪{M:M is a Neu-gb-open set in X and M⊆P}. Take complement each both sides, $[(\text{Neu-gb-Int}(P)]^C$ = (∪{ M :M is a Neu-gb open set in X and M⊆P}$)^C$=∩{ M^C:M^C is a Neu-gb-closed set in X and $[(P)]^C \subseteq M^C$ }. Replacing M^C by H, we get $[(\text{Neu-gb-Int}(P)]^C$=∩{ H:H is a Neu-gb-closed set in X and H⊇$[(P)]^C$ }. From the Definition 5.1, $[(\text{Neu-gb-Int}(P)]^C$=Neu-gb-Cl($[(P)]^C$). This proves (i). By using (i), [Neu-gb-Int($(P)^C$)$]^C$=Neu-gb-Cl$[(P)^C]^C$=Neu-gb-Cl(P). Take complement each both sides, Then we obtain Neu-gb-Int((P^C)) = $[\text{Neu-gb-Cl}(P)]^C$ .we obtained result(ii).
Theorem 5.3

If P and Q are Neutrosophic subset of N-T-S (X, τ_N),Then
(i) P⊆Neu-gb-Cl(P)
(ii) P is Neu-gb-CS in X⇔Neu-gb-Cl(P)=P
(iii) Neu-gb-Cl(Neu-gb-Cl(P))=Neu-gb-Cl(P)
(iv)Now, If P⊆Q and then Neu-gb-Cl(P)⊆Neu-gb-Cl(Q)
Proof :
(i) We can easily get result from Definition 5.1.
Let P be Neu-gb-closed set in X. From the theorem 5.3, P^C is Neu-gb-open set in X. From the theorem5.2(ii),Neu-gb-Int($(P)^C$)=$(P)^C$⇔[Neu-gb-Cl(P)]^C=P^C⇔Neu-gb-Cl(P)=P.we obtain the result(ii).By using(ii), Neu-gb-Cl(Neu-gb-Cl(P))=Neu-gb-Cl(P) . we obtain the result (iii).Since P⊆Q,Q^C⊆P^C. From the theorem 4.2(iv),Neu-gb-Int($(Q)^C$)⊆Neu-gb-Int($(P)^C$).apply complement each sides, [Neu-gb-Int((Q^C))]^C⊇[Neu-gb-Int($(P)^C$)]^C. From the theorem 5.2(ii), Neu-gb-Cl(P)⊆Neu-gb-Cl(Q). we obtain the result (iv).
Theorem 5.4
Let *P* be a Neutrosophic set in a N-T-S (X, τ_N).Then Neu-Int(P)⊆Neu-gb-Int(P)⊆P⊆Neu-gb-Cl(P)⊆Neu-Cl(P).
Proof :
We can easily get result from Definition 5.1.
Theroem 5.5
If P and Q are Neutrosophic subset of N-T-S (X, τ_N),Then
(i) Neu-gb-Cl(P∪Q)=Neu-gb-Cl(P)∪Neu-gb-Cl(Q) and
(ii) Neu-gb-Cl(P∩Q)⊆Neu-gb-Cl(P)∩Neu-gb-Cl(Q).
Proof :
Since Neu-gb-Cl(P∪Q)=Neu-gb-Cl($((P∪Q)^C)^C$By From theorem5.2(i),Neu-gb-Cl(P∪Q)=[Neu-gb-Int($(P∪Q)^C$))]^C=[Neu-gb-Int(P^C∩Q^C))]^C. once Again From theorem 3.5(i),Neu-gb-Cl(P∪Q)=[Neu-gb-Int(P^C))∩Neu-gb-Int(Q^C)]^C=[Neu-gb-Int(P^C)]^C∪[Neu-gb-Int(C(Q))]^C. From theorem5.2(i),,Neu-gb-Cl(P∪Q)=Neu-gb-Cl($(P^C)^C$∪Neu-gb-Cl((($(Q^C)^C$))=Neu-gb-Cl(P)∪Neu-gb-Cl(Q).Thus proved(i). Since P∩Q⊆P and P∩Q⊆Q, From theorem5.3(iv), Neu-gb-Cl(P∩Q)⊆Neu-gb-Cl(P) and Neu-gb-Cl(P∩Q)⊆Neu-gb-Cl(Q). This implies that Neu-gb-Cl(P∩Q)⊆Neu-gb-Cl(P)∩Neu-gb-Cl(Q). we obtain the result (ii).
Converse of (ii) is not true ,Neu-gb-Cl(P)∩Neu-gb-Cl(Q) ⊈Neu-gb-Cl(P∩Q)
Example 5.6
Neu-gb-Cl(P)∩Neu-gb-Cl(Q) ⊈Neu-gb-Cl(P∩Q)
Let X = { p , q , r } with τ_N = { 0_N, A_1, A_2, A_3, A_4, 1_N } and $(\tau_N)^C$= {1_N, A_5, A_6, A_7, A_8, 0_N } where
A_1 = ⟨(0.6, 0.6, 0.2), (0.7, 0.7, 0.2), (1, 0.5, 0.3)⟩
A_2 = ⟨(0.5, 0.5, 0.3), (0.9, 0.6, 0.4), (1, 0.7, 0.4)⟩
A_3 = ⟨(0.5, 0.5, 0.3), (0.7, 0.6, 0.4), (1, 0.5, 0.4)⟩
A_4 = ⟨(0.6, 0.6, 0.2), (0.9, 0.7, 0.2), (1, 0.7, 0.3)⟩
A_5 = ⟨(0.2, 0.4, 0.6), (0.2, 0.3, 0.7), (0.3, 0.5, 1)⟩,
A_6 = ⟨(0.3, 0.5, 0.5), (0.4, 0.4, 0.9), (0.4, 0.3, 1)⟩,
A_7 = ⟨(0.3, 0.5, 0.5), (0.4, 0.4, 0.7), (0.4, 0.5, 1)⟩,
A_8 = ⟨(0.2, 0.4, 0.6), (0.2, 0.3, 0.9), (0.3, 0.3, 1)⟩.
Then (X, τ_N) is a N-T-S.
Here we consider the some Neutrosophic sets
A_9 = ⟨(0.2, 0.2, 0.6), (0.3, 0.3, 0.8), (0.4, 0.3, 1)⟩ and
A_{10} = ⟨(0.3, 0.4, 0.9), (0.2, 0.2, 0.9),(0.3, 0.5, 1)⟩.
Then Neu-gbcl(A_9)=A_7 and Neu-gbcl(A_{10})= A_7.
This implies that Neu-gbcl (A_9)∩Neu-gbcl(A_{10}) = A_7.
Now,A_9∩ A_{10} = ⟨(0.2, 0.2, 0.9), (0.2, 0.2, 0.9),(0.3, 0.3,1)⟩, it follows that Neu-gbcl (A_9∩A_{10})= A_8.
Then Neu-gbcl(A_9)∩Neu-gbcl(A_{10}) ⊈ Neu-gbcl(A_9∩A_{10}).
Theorem 5.7

If P and Q are Neutrosophic subset of N-T-S (X, τ_N) then

(i) Neu-gb-Cl(P)⊇P∪Neu-gb-Cl(Neu-gb-Int(P)),

(ii) Neu-gb-Int(P)⊆P∩Neu-gb-Int(Neu-gb-Cl(P)),

(iii) Neu-Int(Neu-gb-Cl(P))⊆Neu-Int(Neu-Cl(P)),

(iv) Neu-Int(Neu-gb-Cl(P))⊇Neu-Int(Neu-gb-Cl(Neu-gb-Int(P))).

Proof:

From theorem 5.3(i),P⊆Neu-gb-Cl(P) .....(1).We use theorem 3.4(i),Neu-gb-Int(P)⊆P. Then Neu-gb-Cl(Neu-gb-Int(P))⊆Neu-gb-Cl(P) .....(2). From (1) &(2) we have, P∪Neu-gb-Cl(Neu-gb-nt(P))⊆Neu-gb-Cl(P). we obtain result (i). From theorem 4.2(i), Neu-gb-Int(P)⊆P.....(3). We get result from theorem 5.3(i), P⊆Neu-gb-Cl(P). Then Neu-gb-Int(P)⊆Neu-gb-Int(Neu-gb-Cl(P)).....(4). From (3) &(4), we have Neu-gb-Int(P)⊆P∩Neu-gb-Int(Neu-gb-Cl(P)). We obtain (ii). From theorem 5.4,Neu-gb-Cl(P)⊆Neu-Cl(P).We obtain Neu-Int(Neu-gb-Cl(P))⊆Neu-Int(Neu-Cl(P)).Hence(iii).By(i),Neu-gb-Cl(P)⊇P∪Neu-gb-Cl(Neu-gb-Int(P)).We have Neu-Int(Neu-gb-Cl(P)⊇Neu-Int(P∪Neu-gb-Cl(Neu -gb-Int(P))).SinceNeu-Int(P∪Q)⊇Neu-Int(P)∪Neu-Int(Q),Neu-Int(Neu-gb-Cl(P)⊇Neu-Int(P)∪Neu-Int(Neu-gb-Cl(Neu-gb-Int(P)))⊇Neu-Int(Neu-gb-Cl(Neu-gb-Int(P))). Hence(iv).

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