symmetry

MDPI

Article

Neutrosophic Logic Based Quantum Computing

Ahmet Çevik [1], Selçuk Topal [2,*] and Florentin Smarandache [3]

1 Gendarmerie and Coast Guard Academy, Department of Science, Ankara 06805, Turkey, a.cevik@hotmail.com
2 Department of Mathematics, Faculty of Science and Arts, Bitlis Eren University, Bitlis 13000, Turkey
3 Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA; smarand@unm.edu
* Correspondence: s.topal@beu.edu.tr; Tel.: +90-532-709-0239

Received: 18 October 2018; Accepted: 16 November 2018; Published: 20 November 2018

check for updates

Abstract: We introduce refined concepts for neutrosophic quantum computing such as neutrosophic quantum states and transformation gates, neutrosophic Hadamard matrix, coherent and decoherent superposition states, entanglement and measurement notions based on neutrosophic quantum states. We also give some observations using these principles. We present a number of quantum computational matrix transformations based on neutrosophic logic and clarify quantum mechanical notions relying on neutrosophic states. The paper is intended to extend the work of Smarandache by introducing a mathematical framework for neutrosophic quantum computing and presenting some results.

Keywords: neutrosophic computation; neutrosophic logic; quantum computation; computation; logic

---

1. Introduction

1.1. Neutrosophy Theory

Neutrosophic set concept, introduced by Smarandache [1,2], is a more universal structure that extends the concepts of the classic set, fuzzy set [3] and intuitionistic fuzzy set [4]. Unlike intuitionistic fuzzy sets, the indeterminacy is explicitly defined in neutrosophic sets. A neutrosophic set has three basic components defined separately: Truth T, indeterminacy I and falsity F, regarding membership. Neutrosophy was proposed as an ambitious project by Smarandache as a new branch of philosophy as well, concerning "the origin, nature, and scope of neutralities, as well as their mutual effects with different intellectual spectra". The key assumption of neutrosophy is that every idea has not only a certain degree of truth, as is generally taken in many-valued logic contexts, but also degrees of falsity and indeterminacy need to be considered independently from each other. Neutrosophy has settled the baseline for a number of new mathematical theories generalizing both their classical and fuzzy counterparts, such as neutrosophic set theory, geometry, statistics, topology, analysis, probability, and logic. The neutrosophic framework has already been applied to practical applications in many different fields, such as decision-making, semantic web, and data analysis in medicine.

Now, let us look at the concepts of some subfields of neutrosophy. *Neutrosophic set* has a formal definition as follows: Let U be a universe of discourse or space, and M be a set in U. An element x from U is stated related to the set M as $x(T, I, F)$ and belongs to M in the following way: it is t % true in the set, i % indeterminate in the set, and f % false, where t varies in T, i varies in I, f varies in F. Statically T, I, F are subsets, but dynamically T, I, F are functions/operators depending on many known or unknown parameters. *Neutrosophic logic* is a general framework for the unification of many existing logics. The main idea of neutrosophic logic is to characterize each logical statement in a 3-dimensional neutrosophic space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $[0^-, 1^+]$. For instance, a statement can be between $[0.21, 0.55]$ true, 0.23 or between $(0.35, 0.45)$ indeterminate, and either 0.32 or 0.75 false. *Neutrosophic statistics* is the analysis of events characterized by the neutrosophic probability. The function that models the neutrosophic probability of a random variable x is called *neutrosophic distribution*: $NP(x) = (T(x), I(x), F(x))$, where $T(x)$ represents the probability that value x occurs, $F(x)$ represents the probability that value x does not occur, and $I(x)$ represents the indeterminate/unknown probability of value x. *Neutrosophic probability* is an extension of the classical probability and imprecise probability where a case, event or fact A occurs is t % true—where t varies in the subset T, i % indeterminate—where i varies in the subset I, and f % false—where f varies in the subset F. In classical probability $n_s up \leq 1$, while in neutrosophic probability $n_s up \leq 3^+$. In imprecise probability, the probability of an event is a subset T in $[0, 1]$, not a number p in $[0, 1]$, the rest was supposed to be the opposite, subset F (also from the unit interval $[0, 1]$); there is no indeterminate subset I in imprecise probability.

1.2. Quantum Mechanics and Computing

Quantum mechanics was started with Planck [5] and interpreted as real life problem by Einstein [6]. The mechanics was developed by Bohr, Heisenberg, Broglie, Schrödinger, Born, Dirac, Hilbert, Sommerfeld, Dyson, Wien, Pauli, Von Neumann and others [7–12] in the first 30 years of the 20th century. Computers are mechanisms that support transaction information by executing algorithms. An algorithm is a well-defined process to perform an information processing task. The task can always be translated into a realization. When creating complicated algorithms for a variety of tasks, working with some improved computational models is very useful, probably very important. However, when examining the actual limitations of a computation mechanism, it is key to remember the connection between computation and realization. Quantum computation explores how efficiently nature allows us to compute. The standard computational model is based on classical mechanics; the mechanics of the Turing machine relies on classical mechanics. Quantum information processing changes not only the physical paradigm used for computing and communication but also the concepts of knowledge and computation. Quantum computation is not synonymous with quantum effects to make calculations. Actual computing mechanisms of the quantum are based on a larger physical reality than is represented by the idealized computational model. Quantum information processing is the result of the use of the physical reality that quantum theory states to perform tasks that were previously thought to be infeasible or impossible. The mechanisms that perform quantum information processing are known as quantum computers. In the last few decades of the twentieth century, researchers tried to follow two of the most influential and revolutionary theories: information science and quantum mechanics. Their success provided an unfamiliar computation and information range of vision. This new insight has significantly changed how the relationship between quantum information theory, computation, knowledge, and physics is considered and has given rise to new applications and epoch-making algorithms. The theory of information, which contains the foundations of computer science and communication, made possible to address the important issues in computer science and communication. The Turing machine is a classical model that behaves entirely according to classical mechanical principles. Quantum mechanics has become an increasingly significant line in the progress of developing more efficient computing mechanisms. Until recently, the effect of quantum mechanics had been limited to low-level applications and it had no effect on how computation or communication was carried or worked. At the beginning of the 1980s, a number of scientists found that quantum mechanics had eye-opening effects that could be used in information processing. Richard Feynman [13], Yuri Manin [14], and other influential scientists realized that some quantum mechanical phenomena could not be efficiently simulated by a standard Turing machine. This observation has led to speculation that perhaps these quantum phenomena could be used to make computations more efficient in general.

Such programme required re-thinking the underlying theoretical model of informatics and completely removed it from the classical circle. Quantum computing, a field that includes quantum information, quantum algorithms, quantum cryptography, quantum communication, and quantum games, explores the effects of using quantum mechanical phenomena for information modeling and processing instead of using the rules of classical mechanics in computations.

In the following sections, we will introduce a mathematical framework of the unification of neutrosophic theory and quantum theory, in a fully computational approach. In this context, we will reveal how one can have a computational approach to the solution of mathematical and algorithmic problems of a model that can be encountered in both the neutrosophic and quantum universes. In this sense, this paper presents a more computational approach to the neutrosophic quantum concept, i.e., neutrosophic quantum computation, whose groundwork was laid by the work of Smarandache [15].

2. Neutrosophic Quantum Computing

In this part, we define some fundamental notions of neutrosophic quantum computing. Some concepts will involve new interpretations and others will be straightforward generalizations. As also mentioned in Smarandache [15], we should note in the beginning of our paper that the reversibility condition of quantum computing has some challenging issues in the neutrosophic counterpart of this ambitious field. It is mainly due to the fact that neutrosophic states involve indeterminacy, so the inverse function of such states might not always be definable, hence the domain may not be uniquely recovered from the image. We propose an interesting open problem regarding a special case of this issue at the end of the paper.

We assume some basic familiarity with linear algebra and complex numbers including their basic properties like the norm of a complex vector, complex conjugation, complex number multiplication, etc. The reader may refer to Yanofsky and Mannucci's [16] or Nielsen and Chuang's [17] book for a detailed account on quantum computing and quantum information.

Definition 1. *A neutrosophic quantum bit (neutrobit) is a three-dimensional complex vector*

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \alpha|0\rangle + \beta|1\rangle + \gamma|I\rangle$$

such that $\alpha, \beta, \gamma \in \mathbb{C}$ *are called coefficients (or amplitudes) and* $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$, *where we define the basis vectors* $|0\rangle$, $|1\rangle$, $|I\rangle$ *in the canonical basis as*

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad |I\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

In comparison to classical quantum computation, the reader may have noticed a new basis vector $|I\rangle$ introduced above. We call this vector the *indeterminacy basis*.

A *coherent neutrosophic quantum state* $|\psi\rangle$ is a linear combination (superposition) of the basis vectors $|0\rangle$, $|1\rangle$ and $|I\rangle$ which is in the form

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|I\rangle$$

such that $\alpha, \beta, \gamma \in \mathbb{C}$ and that $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$.

Thus, a coherent neutrosophic quantum state is three-dimensional complex vector, which is of unit length.

Quantum systems evolve via special kind of matrix transformations. We define *neutrosophic Pauli gates* as given below:

$$X = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad W = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{bmatrix}.$$

The matrix X is actually the *NOT* gate which the reader might be familiar from classical quantum computation. That is, if $k \in \{0,1\}$, then $X|k\rangle = |1-k\rangle$. Notice that $X|I\rangle = |I\rangle$. Thus, we define the negation of the indeterminacy basis as itself. The next two gates are Y-rotation and Z-rotation (phase change). The new gate here is the W-transformation which can be simply thought of as a rotation around the $|I\rangle$ basis with an equal coefficient distribution of the bases between $|I\rangle$ and the basis on which the rotation is applied. The intuition behind these rotation gates will be understood better once we give the unit ball representation of neutrobits later on.

An important quantum gate in classical quantum computing is the *Hadamard transform*, which is defined as the matrix

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

Standard Hadamard transform is defined on a single qubit since it is a 2×2 matrix. Hadamard matrix used in classical quantum computing is a unitary matrix. Thus, it is reversible, and is actually its own inverse. To introduce the neutrosophic counterpart of this transformation, we first need to define the notion of indeterminate (decoherent) superpositions to make sense of the use of the Hadamard transform in neutrosophic quantum computing. The terms *coherent* and *decoherent* superpositions of neutrobits were first introduced by Smarandache [15] for denoting quantum states with some indeterminacy. We modify these notions to make the Hadamard transform work on neutrobits.

Definition 2. *The reserved three-dimensional vector*

$$|0_I\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_I$$

is called the decoherent state of the $|0\rangle$ basis vector. We define $|1_I\rangle$ similarly. That is,

$$|1_I\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_I$$

is defined to be the decoherent state of $|1\rangle$. Any linear combination that includes either of these vectors is called a decoherent superposition.

The motivation behind this definition is to mix the *coherent* (stable) basis state $|0\rangle$ with the intrinsic property of neutrosophic logic, which is indeterminacy. A quantum system may still have a degree of indeterminancy even if the system appears to be in a pure basis state. A scalar α for any of these decoherent vectors is denoted by α_I. Thus, when we write α_I, for some number α, the reader should understand that we are refering to the coefficient of a decoherent state. For example, the vector

$$|\psi\rangle = \begin{bmatrix} \left(\frac{1}{\sqrt{2}}\right)_I \\ \left(\frac{1}{\sqrt{2}}\right)_I \\ 0 \end{bmatrix}$$

denotes the decoherent superposition state

$$\frac{1}{\sqrt{2}}|0_I\rangle + \frac{1}{\sqrt{2}}|1_I\rangle.$$

We could also define a decoherent state for $|I\rangle$, but, since the state $|I\rangle$ naturally involves an indeterminacy regarding which classical bit the state refers to, there is no need to repeat this decoherence. Thus, we adopt $|0_I\rangle$ and $|1_I\rangle$ as reserved basis vectors that will be used in decoherent superposition states. We should once again emphasize that $|0_I\rangle$ is different than the coherent basis state $|0\rangle$. It is also different than the coherent superposition state $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|I\rangle$. The latter says that the system is in a superposition of basis states $|0\rangle$ and $|I\rangle$, the former says that the system is in a possibly *indetermined* state $|0\rangle$. If $|\psi\rangle = |0\rangle$, this tells us that $|\psi\rangle$ is for certain in the basis state $|0\rangle$. The state $|0_I\rangle + |I\rangle$ says that the system is in a decoherent superposition of $|I\rangle$ and a possibly indetermined state $|0\rangle$. The distinction between coherent and decoherent states should now be clear. However, another way to imagine $|0_I\rangle$ as the state $|0\rangle$ with a bounded error $\epsilon > 0$.

Given the information above, we define the *neutrosophic Hadamard transform* as

$$H_N = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \left(\frac{1}{\sqrt{2}}\right)_I \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \left(\frac{1}{\sqrt{2}}\right)_I \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \end{bmatrix}.$$

Then, it is easy to verify that

$$H_N|0\rangle = \tfrac{1}{\sqrt{3}}|0\rangle + \tfrac{1}{\sqrt{3}}|1\rangle + \tfrac{1}{\sqrt{3}}|I\rangle,$$

$$H_N|1\rangle = \tfrac{1}{\sqrt{3}}|0\rangle - \tfrac{1}{\sqrt{3}}|1\rangle - \tfrac{1}{\sqrt{3}}|I\rangle,$$

$$H_N|I\rangle = \tfrac{1}{\sqrt{2}}|0_I\rangle + \tfrac{1}{\sqrt{2}}|1_I\rangle.$$

3. Observables and Measurement

In classical mechanics, it is intuitively understood what is meant by an observable. An observable in classical mechanics is a quantity like velocity, momentum, position, temperature, etc. It is intuitively clear what these quantities are. In quantum mechanics, one needs to be more specific when talking about observables.

Definition 3. *Let A be an $n \times n$ matrix. We say that A is Hermitian if $A^\dagger A = AA^\dagger$, where $A^\dagger$ is called the Hermitian conjugate of A and is defined as the transpose of the complex conjugate matrix of A. An $n \times n$ matrix A is called unitary if $A^\dagger A = AA^\dagger = Id$, where Id is the identity matrix.*

We note that, in classical quantum computing, state evolution is obtained by applying unitary operators. There are two reasons for this. The first reason is that classical quantum computations are reversible. The second reason is that unitary transformations preserve inner products, hence they preserve the norm of the vectors. As we shall discuss later, this requirement is questionable in neutrosophic quantum computing.

In classical quantum computing, it is assumed that, for every observable, there corresponds a Hermitian operator. We use the same postulate for the neutrosophic case.

Measurement postulate. Observables in neutrosophic quantum computing are Hermitian operators.

Measurements are the outcomes of observables applied on the physical system in consideration. Classical quantum computing usually takes *projective measurements* in the sense that when we measure a state, the new state of the system becomes one of the basis states of the system. Thus, after the measurement, a general superposition state gets *projected* onto one of the basis vectors. We shall not adopt this requirement in neutrosophic quantum computing. The reason is the following. If the outcome were to be projected onto one of the basis states, the logic used here would be no different than the classical interpretation. Even if the state of the quantum system is projected onto a single basis state, we would still require a degree of probability of the same basis state being on other basis states. This is one reason why we should decoherent superposition states into account in neutrosophic quantum computing. It relies on the very nature of neutrosophic logic. For that matter, observables we take into consideration are non-projective.

Measuring an observable on a neutrosophic quantum bit yields not a single classical state, but a probability distribution of the basis states $|0\rangle, |1\rangle, |I\rangle$. This is perhaps one of the most important difference between classical quantum computation and neutrosophic quantum computation. Given a neutrobit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|I\rangle$, making a measurement on state $|\psi\rangle$ yields a triplet

$$\langle p_{|0\rangle}, p_{|1\rangle}, p_{|I\rangle} \rangle,$$

where $p_{|0\rangle}$ denoting the probability of $|\psi\rangle$ being in state $|0\rangle$, $p_{|1\rangle}$ denoting the probability of $|\psi\rangle$ being in state $|1\rangle$, and $p_{|I\rangle}$ denoting the probability of $|\psi\rangle$ being in the indetermined basis state $|I\rangle$. Thus, the outcome of observing a neutrobit gives a probability distribution of basis states. In classical quantum computing, the outcome of measurement on a qubit is a classical bit information.

Let us illustrate this idea. For example, given the neutrobit

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle + \frac{1}{\sqrt{3}}|I\rangle,$$

in a coherent superposition, measuring some observable Ω on the state $|\psi\rangle$ should yield a neutrosophic quantum state $|\psi'\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|I\rangle$, of decoherent superposition.

It should be noted that the neutrosophic quantum state should not be confused with an ordinary superposition state of a classical quantum system. Thus, a pure state in a neutrosophic quantum system always looks like a superposition. A neutrosophic quantum state is in a coherent superposition of three basis states $|0\rangle, |1\rangle, |I\rangle$. However, as soon as we make a measurement on state $|\psi\rangle$, it yields a decoherent superposition, which is merely a triplet containing the information of probability distributions for each basis states. We state this as a theorem.

Theorem 1. *Let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|I\rangle$ be a coherent neutrosophic quantum state. The outcome of a measurement on $|\psi\rangle$ is a three-dimensional real vector, particularly a decoherent neutrosophc quantum superposition.*

Proof. Suppose that we are given a coherent state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|I\rangle$. Without loss of generality, we may assume that the state is in a superposition rather than in a single coherent basis. Assume that we are given a Hermitian operator Ω which is not necessarily unitary and projective. Applying Ω on $|\psi\rangle$, since we assumed that Ω is non-projective, will still yield a linear combination of vectors, particularly a three-dimensional vector. Since the probability of seeing a single coherent basis state is a magnitude square of the coefficient corresponding to that basis vector, the probability of observing

$|0\rangle$ is some $p_{|0\rangle}$. Similarly, the probability of observing $|1\rangle$ is $p_{|1\rangle}$ and the probability of seeing $|I\rangle$ is some $p_{|I\rangle}$. Since Ω is non-projective, we observe a vector containing these probabilites as elements. However, since the outcome is decoherent, it should be that each probability value can be taken to be indetermined. That is, the outcome of the observation will be a vector

$$\begin{bmatrix} (p_{|0\rangle})_I \\ (p_{|1\rangle})_I \\ (p_{|I\rangle})_I \end{bmatrix}.$$

Since $|I\rangle_I = |I\rangle$, we have

$$\begin{bmatrix} (p_{|0\rangle})_I \\ (p_{|1\rangle})_I \\ (p_{|I\rangle}) \end{bmatrix}.$$

The vector above is a decoherent superposition state with numbers $p_{|0\rangle}$, $p_{|1\rangle}$, and $p_{|I\rangle}$. Since each number is the magnitude square of the coefficients of the state vector being measured, they cannot be complex valued. Thus, each of these numbers are real valued. $\square$

4. Tensor Products and Entanglement

The usual tensor product of classical qubits generalizes to the neutrosophic case. Given two neutrobits

$$|\psi\rangle = \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix}, \quad |\phi\rangle = \begin{bmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{bmatrix},$$

the tensor product is defined as

$$|\psi\rangle \otimes |\phi\rangle = \begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} \otimes \begin{bmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \alpha_1\gamma_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \\ \beta_1\gamma_2 \\ \gamma_1\alpha_2 \\ \gamma_1\beta_2 \\ \gamma_1\gamma_2 \end{bmatrix}.$$

The tensor product of measurement outcomes can also be defined. Assume that $|\psi'\rangle = \langle p^1_{|0\rangle}, p^1_{|1\rangle}, p^1_{|I\rangle}\rangle$ and $|\phi'\rangle = \langle p^2_{|0\rangle}, p^2_{|1\rangle}, p^2_{|I\rangle}\rangle$ are two probability distributions of two *decoherent* quantum states. Then, we define

$$p^{1\otimes 2}_{|0\rangle} = p^1_{|0\rangle} \cdot p^2_{|0\rangle},$$

$$p^{1\otimes 2}_{|1\rangle} = p^1_{|1\rangle} \cdot p^2_{|1\rangle},$$

$$p^{1\otimes 2}_{|I\rangle} = p^1_{|I\rangle} \cdot p^2_{|I\rangle}.$$

Then, we write the tensor product as $|\psi'\rangle \otimes |\phi'\rangle = \langle p^{1\otimes 2}_{|0\rangle}, p^{1\otimes 2}_{|1\rangle}, p^{1\otimes 2}_{|I\rangle}\rangle$.

The tensor product of measurement outcomes provides us with the ability to use compound outcome information of multiple neutrobit systems. We shall now look at the neutrosophic entanglement property. In classical quantum computation, a two qubit system is entangled if it is not the tensor product of two single-qubit systems. We adopt the same definition for neutrosophic coherent superposition states. However, entanglement is not defined on decoherent states. Suppose that we are given two neutrobits $|\psi\rangle = \alpha_1|0\rangle + \beta_1|1\rangle + \gamma_1|I\rangle$ and $|\phi\rangle = \alpha_2|0\rangle + \beta_2|1\rangle + \gamma_2|I\rangle$, the tensor product is defined exactly the same as in the classical case. That is,

$$|\psi\rangle \otimes |\phi\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \alpha_1\gamma_2|0I\rangle + \cdots + \gamma_2\gamma_2|II\rangle.$$

This is completely a coherent superposition. If we measure this two-neutrobit system, though, we get a 9-tuple containing probability distributions where each element of the 9-tuple denotes the probability of the compound system $|\psi\rangle \otimes |\phi\rangle$ being in the ith basis state for a two-neutrobit system. The reader should easily be able to verify that, for an n-neutrobit system, there are 3^n basis states.

5. More on Quantum Operators

As noted earlier, most quantum transformations are defined similarly as in the classical case. For a better understanding though, we shall discuss more about the action of the neutrosophic Hadamard transform. The neutrosophic Hadamard transform is defined as

$$H_N = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \left(\frac{1}{\sqrt{2}}\right)_I \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \left(\frac{1}{\sqrt{2}}\right)_I \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \end{bmatrix}.$$

The indeterminate values $\frac{1}{\sqrt{2}}$ in the neutrosophic Hadamard transform denote the indeterminate decoherent counterpart of the basis states $|0\rangle$ and $|1\rangle$. Any state which involves any of these decoherent vectors is also decoherent. Despite that we leave $H_N|0_I\rangle$ and $H_N|1_I\rangle$ undefined, we define the logical NOT operator over the decoherent states as

$$NOT|0_I\rangle = |1_I\rangle,$$

$$NOT|1_I\rangle = |0_I\rangle.$$

We leave the action of H_N on two reserved decoherent vectors $|0_I\rangle$ and $|1_I\rangle$ undefined for the reason that creating a superposition from an already decoherent neutrosophic quantum state might prevent us to obtain the original input decoherence from the output decoherence. Thus, due to this reversibility problem, it is better if we leave the mentioned transformations undefined. Since $|I\rangle$ is a legitimate coherent state in neutrosophic quantum computation, we defined

$$H_N|I\rangle = \frac{1}{\sqrt{2}}|0_I\rangle + \frac{1}{\sqrt{2}}|1_I\rangle.$$

We may imagine a coherent neutrobit as a vector on a three-dimensional unit ball as given in Figure 1.

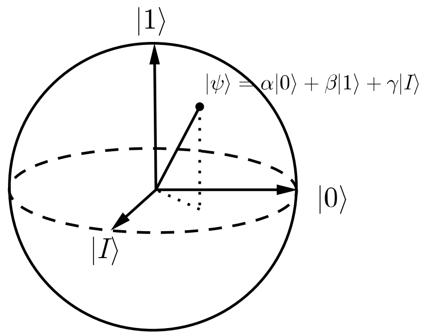

Figure 1. Representation of a neutrobit vector on a unit ball with real coefficients.

Of course, we assume in this image, for simplicity, that the amplitudes are real values. Allowing complex coefficients would require us to represent a neturobit on a four-dimensional geometry since an additional imaginary axis would need to be introduced. The basis vectors here are all mutually orthogonal. That is, the inner product of any of the two basis vectors is 0.

When we make a measurement on the state $|\psi\rangle$, we get a triplet $\langle p_{|0\rangle}, p_{|1\rangle}, p_{|I\rangle}\rangle$ which was defined earlier, where $p_{|0\rangle} = |\alpha|^2$, $p_{|1\rangle} = |\beta|^2$, $p_{|I\rangle} = |\gamma|^2$. The new state of the system in this case is a decoherent superposition of $|0\rangle$, $|1\rangle$ and $|I\rangle$ each with a degree of probability $p_{|0\rangle}$, $p_{|1\rangle}$, $p_{|I\rangle}$, respectively.

6. Results

We introduced a refined mathematical framework for neturosophic quantum computing based on the original work of Smarandache [15] and we gave a few standard transformations and notions that are to be used in neutrosophic quantum computations. Perhaps the most important difference from the classical quantum computation is the involvement of the indeterminacy basis and the separation between coherent and decoherent states. Treating the Hadamard transform as a function creating a superposition from a coherent state, we introduced the reserved decoherent vectors for this purpose. The measurement process is also slightly different in this case. The outcome of any measurement on a neutrobit gives a probability distribution, a decoherent state, of all possible basis states each with a certain degree of probability determined by the corresponding coefficients.

The computational complexity of the neutrosophic quantum gates, when applied to a quantum state, would be the same as their classical counterparts since the size of the transformation matrices in the neutrosophic counterpart does not change asymtotically. That is, for the neutrosophic Hadamard transform for instance, multiplying a 3×3 matrix with a three-dimensional vector does not give any difference in terms of computational complexity compared to its classical counterpart. The same observation can be easily seen with the other gates. The only complexity difference is with the tensor product that, since we are not working on a three-dimensional vector space, the size of the vector space grows by factors of 3 instead of 2 when taking tensor products of n many neutrobits. It should be noted that this is still a constant difference.

A practical application of neutrosophic quantum computing in the future would be used to solve hard problems involving indeterminate cases of multiple states when taken as a whole system. For example, it may not be known which one of the many possible channels that a quantum information is transferred through quantum communication channels. If we were to study the behavior of the transferred superposition quantum state, we would have to use neutrosophic quantum computing notions to describe the state of the transfer process that will involve the probability of the information being transferred on one particular channel, probability of the information not being transferred on the same channel, and a degree of indeterminacy of the information being transferred on that channel. This is required for a single channel. Thus, we would have a superposition of all possible probability distributions if we consider every channel taken together. The entire distribution will naturally define a decoherent quantum superposition state.

As stated in Smarandache [15], satisfying the reversibility condition of quantum computing is more problematic in the neutrosophic case due to the inclusion of indeterminate states. The first attempt to settle this problem is to try to make the neutrosophic Hadamard transform unitary, and hence reversible. We shall give the following open problem, for which we hope to encourage researchers in neutrosophic computation or quantum computing for finding a possible solution.

Open problem. Define a "reasonable" neutrosophic Hadamard transformation matrix, which is unitary.

By "reasonable", we mean preserving the original properties of the standard Hadamard transform such as creating a superposition of basis states, etc.

Another future work is to find a legitimate protocol for the teleportation of the state of a neutrobit from one location to another. This particularly has many applications in networks and communication. A typical quantum teleportation of a standard qubit is performed through classical bit channels. In order to send the state of a qubit, the first party sends two classical bits and the second part recovers the state of a qubit from the received classical bits. What kind of channels do we need to transport the state of a neutrobit? A classical channel may be a solution. A quantum channel, on the other hand, may not be sufficient to teleport a neutrobit due to the fact that the preservation of indeterminate states through the teleportation process becomes questionable. One idea is to separate the indeterminate state from the superposition and treat it as a classical quantum superposition state of all coherent basis states and then use the classical quantum teleportation protocol on this system.

Neutrosophic quantum computing is at its very early stage of development. We believe that this new field will attract many researchers in computer science, physics and mathematics for further advancement along with discovering many useful future applications.

Author Contributions: Conceptualization, A.Ç. and S.T.; Methodology, A.Ç.; Validation, A.Ç., S.T. and F.S.; Investigation, A.Ç. and S.T.; Resources, A.Ç., S.T. and F.S.; Writing—Original Draft Preparation, A.Ç. and S.T.; Writing—Review and Editing, A.Ç., S.T. and F.S.; Supervision, S.T. and F.S.; Funding Acquisition, F.S.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Smarandache, F. *Neutrosophy. Neutrosophic Probability, Set, and Logic*; American Research Press: Rehoboth, DE, USA, 1998; pp. 104–106.
2. Smarandache, F. A generalization of the intuitionistic fuzzy set. *Int. J. Pure Appl. Math.* **2005**, *24*, 287–297.
3. Zadeh, L. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
4. Atanassov, K.T. *Intuitionistic Fuzzy Sets*; Physica: Heidelberg, Germany, 1999; pp. 1–137.
5. Planck, M. *The Theory of Heat Radiation*, 2nd ed.; Blakiston's Son & Co.: Philadelphia, PA, USA, 1989.
6. Albrecht, F.; Einstein, A. *A Biography*, 1st ed.; Viking: New York, NY, USA, 1977.
7. Bohr, N. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **1935**, *48*, 696. [CrossRef]
8. Heisenberg, W. *The Physical Principles of the Quantum Theory*; Courier Corporation: North Chelmsford, MA, USA, 1949.
9. Hanle, P.A. Erwin Schrödinger's Reaction to Louis de broglie's thesis on the quantum theory. *ISIS* **1977**, *68*, 606–609. [CrossRef]
10. Bacciagalup, G.; Valentini, A. *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*; Cambridge University Press: Cambridge, UK, 2009; p. 9184.
11. Jammer, M. *The Conceptual Development of Quantum Mechanics*; MC Grow-Hill: New York, NY, USA, 1966.

12. Jammer, M. *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective*; Wiley: New York, NY, USA, 1974.
13. Feynman, R.P. Quantum mechanical computers. *Opt. News* **1985**, *11*, 11–20. [CrossRef]
14. Manin, Y.I. Classical computing, quantum computing, and Shor's factoring algorithm. In *Seminaire Bourbaki: Volume 1998/99, Exposés 850–864, Asterisque, No. 266 (1998–1999), Talk No. 862, MR 1772680*; Seminaire Bourbaki: Paris, France, 2000; pp. 375–404.
15. Smarandache, F. Neutrosophic Quantum Computer. *Intern. J. Fuzzy Math. Arch.* **2016**, *10*, 139–145.
16. Yanofsky, N.S.; Mannucci, M.A. *Quantum Computing for Computer Scientists*; Cambridge University Press: Cambridge, UK, 2008.
17. Nielsen, M.A.; Chuang, I.L. *Quantum Computation and Quantum Information*; Cambridge University Press: Cambridge, UK, 2000.