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Neutrosophic ωα - Closed Sets in Neutrosophic Topological Spaces

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ABSTRACT

Aim of this present paper is, we introduce and investigate a new class of closed sets is called Neutrosophic ωα -closed sets in Neutrosophic topological spaces and its properties and characterization are discussed details.

Mathematics Subject Classification (2010): 03E72.

Keywords: Neutrosophic ω-closed sets, Neutrosophic α -closed sets, Neutrosophic ωα -closed sets, Neutrosophic ωα - open sets, Neutrosophic topological spaces.

1. INTRODUCTION

C.L. Chang[3] was introduced and developed fuzzy topological space by using L.A. Zadeh's[14] fuzzy sets. Coker[4] introduced the notion of Intuitionistic fuzzy topological spaces by using Atanassov's[1] Intuitionistic fuzzy set. Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache[7] in 1998. He also defined the Neutrosophic set on three component Neutrosophic topological spaces (t, f,i) =(Truth, Falsehood, Indeterminacy),The Neutrosophic crisp set concept was converted to Neutrosophic topological spaces by A. A. Salama[10]. I. Arokiarani.[2] *et al*., introduced Neutrosophic α -closed sets. Santhi. R[11] *et al*., introduced and studied about Neutrosophic ω-closed sets in Neutrosophic topological spaces . Aim of this present paper is, we introduce and investigate a new class of closed sets is called Neutrosophic ωα-closed sets in Neutrosophictopological spaces and its properties and characterization are discussed details

2. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Definition 2.1 [7]
Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form
A = {<x, $\mu_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$> : x ∈ X}
Where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent Neutrosophic topological spaces the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element x ∈ X to the set A.

Remark 2.2 [7]
A Neutrosophic set A = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ >: x ∈ X} can be identified to an ordered triple <μ_A, σ_A, γ_A> in α-0,1+ α on X.

Remark 2.3[7]
we shall use the symbol
A =<x, μ_A, σ_A, γ_A> for the Neutrosophic set A = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ >:x∈X}.

Example 2.4 [7]
Every NeutrosophicsetA is a non-empty set in X is obviously on Neutrosophic set having the form A = { <x, $\mu_A(x)$, 1-(($\mu_A(x) + \gamma_A(x)$), $\gamma_A(x)$ > : x ∈ X }**.** Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set0_N and 1_N in X as follows:
0_N may be defined as:
(0_1) 0_N= {<x, 0, 0, 1>: x ∈X}
(0_2) 0_N= {<x, 0, 1, 1>: x ∈X}
(0_3) 0_N = {<x, 0, 1, 0 >:x∈X}
(0_4) 0_N= {<x, 0, 0, 0>: x ∈X}
1_N may be defined as :
(1_1) 1_N = {<x, 1, 0, 0>: x ∈ X}
(1_2) 1_N = {<x, 1, 0, 1 >: x ∈ X}
(1_3) 1_N = {<x, 1, 1, 0 >: x ∈ X}
(1_4) 1_N = {<x, 1, 1, 1 >: x ∈ X}

Definition 2.5 [7]
Let A = <μ_A, σ_A,γ_A> be a Neutrosophic set on X, then the complement of the set A
A^Cdefined as
A^C={<x , $\gamma_A(x)$, 1 - $\sigma_A(x)$, $\mu_A(x)$>: x ∈X}

Definition 2.6 [7]
Let Xbe a non-empty set, and Neutrosophic sets A and B in the form
A = {<x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$>: x∈X} and
B = {<x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$>: x∈X}.
Then we consider definition for subsets (A⊆B).
A⊆B defined as: A⊆B ⟺$\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all x∈X

Proposition 2.7 [7]
For any Neutrosophic set A, then the following condition are holds:
(i) $0_N \subseteq A$, $0_N \subseteq 0_N$
(ii) $A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition 2.8 [7]
Let X be a non-empty set, and $A=<x, \mu_A(x), \sigma_A(x), \gamma_A(x)>$, $B =<x, \mu_B(x), \sigma_B(x), \gamma_B(x)>$ be two Neutrosophic sets. Then
(i) $A \cap B$ defined as :$A \cap B =<x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)>$
(ii) $A \cup B$ defined as :$A \cup B =<x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)>$

Definition 2.9 [7]
We can easily generalize the operation of intersection and union in Definition 2.8 to arbitrary family of Neutrosophic sets as follows:
Let $\{ A_j : j \in J \}$ be an arbitrary family of Neutrosophic sets in X, then
(i) $\cap Aj$ definedas :
$\cap A_j =<x, \wedge_{j \in J} \mu_{Aj}(x), \wedge_{j \in J} \sigma_{Aj}(x), \vee_{j \in J} \gamma_{Aj}(x) >$
(ii) $\cup Aj$ definedas :
$\cup Aj =<x, \vee_{j \in J} \mu_{Aj}(x), \vee_{j \in J} \sigma_{Aj}(x), \wedge_{j \in J} \gamma_{Aj}(x) >$

Proposition 2.10 [9]
For all A and B are two Neutrosophic sets then the following condition are true:
(1) $(A \cap B)^C = A^C \cup B^C$
(2) $(A \cup B)^C = A^C \cap B^C$.

Definition 2.11 [10]
A Neutrosophic topology is a non -empty set X is a family τ_N of Neutrosophic subsets in X satisfying the following axioms:
(i) $0_N, 1_N \in \tau_N$,
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
(iii) $\cup G_i \in \tau_N$ for every $G_i \in \tau_N i \in J$.
the pair(X, τ_N) is called a Neutrosophic topological space.
The element Neutrosophic topological spaces of τ_N are called Neutrosophic open sets.
A Neutrosophic set A is closed if and only if A^C is Neutrosophic open.

Example 2.12[10]
Let $X=\{x\}$ and
$A_1= \{<x, 0.6, 0.6, 0.5>:x \in X\}$
$A_2= \{<x, 0.5, 0.7, 0.9>:x \in X\}$
$A_3= \{<x, 0.6, 0.7, 0.5>:x \in X\}$
$A_4= \{<x, 0.5, 0.6, 0.9>:x \in X\}$
Then the family $\tau_N=\{0_N, 1_N, A_1, A_2, A_3, A_4\}$is called a Neutrosophic topological space on X.

Definition 2.13[10]
Let (X, τ_N) be Neutrosophic topological spaces andA = {<x, μ_A(x), σ_A(x), γ_A(x)>:x∈X} be a Neutrosophic set in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by
Neu-cl(A) =∩{ K :K is a Neutrosophic closed set in X and A⊆K}
Neu-int(A) = ∪{G :G is a Neutrosophic open set in X and G⊆A}.

Definition 2.14[10]
(i) A is Neutrosophic open set if and only if A=Neu-int(A) .
(ii) A is Neutrosophic closed set if and only if A=Neu-cl(A).

Proposition 2.15[10]
For any Neutrosophic set A in(X, τ_N) we have
(i) Neu-cl((A^C))=(Neu-int(A))^C,
(ii) Neu-int((A^C))=(Neu-cl(A))^C.

Proposition 2.16[10]
Let(X, τ_N) be a Neutrosophic topological spaces and A, B be two Neutrosophic sets in X. Then the following properties are holds:
(i) Neu-int(A)⊆A,
(ii) A ⊆Neu-cl(A),
(iii) A⊆B ⇒Neu-int(A)⊆Neu-int(B),
(iv) A⊆B⇒Neu-cl(A)⊆Neu-cl(B),
(v) Neu-int(Neu-int(A))=Neu-int(A),
(vi) Neu-cl(Neu-cl(A))=Neu-cl(A),
(vii) Neu-int(A∩B))=Neu-int(A)∩Neu-int(B),
(viii) Neu-cl(A∪B)=Neu-cl(A)∪Neu-cl(B),
(ix) Neu-int(0_N)=0_N ,
(x) Neu-int(1_N)=1_N ,
(xi) Neu-cl(0_N)=0_N ,
(xii) Neu-cl(1_N)=1_N ,
(xiii) A⊆B⇒B^C⊆A^C,
(xiv) Neu-cl(A∩B)⊆Neu-cl(A)∩Neu-cl(B),
(xv) Neu-int(A∪B)⊇Neu-int(A)∪Neu-int(B).

Definition: 2.17[8]
Let (X, τ_N) be a Neutrosophic topological space. Then A is called
Neutrosophic semi-open if A⊆Neu-cl(Neu-int(A)).
The complement of Neutrosophic semi-open set is called Neutrosophic semi-closed.

Definition: 2.18[10]
Let (X, τ_N) be a Neutrosophic topological space Then A is called
Neutrosophic α-open set if A⊆Neu-int(Neu-cl(Neu-int(A))).
The complement of Neutrosophic α-open set is called Neutrosophic α-closed.

Definition 2.19 [5]
Let (X, τ_N) be a Neutrosophic topological space .Then A is called generalized Neutrosophic closed (Neu g-closed) if Neu-cl(A)⊆U, whenever A⊆U and U is Neu-open set.
The complement of a Neu g-closed set is called the Neu g-open sets.

Definition 2.20 [11]
Let (X, τ_N) be a Neutrosophic topological space. Then A is called Neutrosophic ω-closed set(Neu ω-closed set for short) if Neu-cl(A)⊆U whenever A ⊆ U and U is Neu semi-open set.

Remark:2.21[11]
Let A be an Neutrosophic topological space (X, τN) .Then
(i) Neuα-cl(A) = A ∪ Neu-cl(Neu-int(Neu-cl(A))).
(ii) Neuα-int(A) = A⋂Neu-int(Neu-cl(Neu-int(A))).

3. NEUTROSOPHIC ωα -CLOSED SETS

In this section, we introduce the concept of Neutrosophic ωα-closed sets (Shortly Neu ωα- closed set) and some of their properties are discussed details. Throughout this paper (X, τ_N) represent a Neutrosophic topological spaces.

Definition 3.1
A subset A of a Neutrosophic topological space (X, τ_N) is said to be Neutrosophic ωα -closed set if Neu α-cl(A) ⊆ G whenever A⊆G and G is Neuω-open in(X, τ_N) .
The complement of Neutrosophic ωα-closed sets is called Neutrosophic ωα-open sets

Example 3.2:
Let X={x}. τ_N={0_N,1_N,A_1} be a Neutrosophic topological space
whereA$_1$= {<x, 0.3, 0.6, 0.5>:x∈X}.
S= {<x, t_1, t_2,t_3>:x∈X, t_1∈[0.3,1) , t_2∈[0.6,1) and t_3 ∈(0,0.5]}.
Neutrosophic semi open set is {0_N, 1_N, A_1, S}
Neutrosophicα-open set is {0_N, 1_N, A_1, S}
Neutrosophic ω-open setis {0_N, 1_N, A_1}.
Here A_2= {<x, 0.4, 0.3, 0.5>:x∈X} is Neutrosophic ωα-closed set in(X, τ_N) .

Theorem 3.3:
Every Neutrosophic closed set is Neutrosophic ωα-closed set.

Proof:
Let A be Neutrosophic closed set in aNeutrosophic topological space (X, τ_N). Let G be any Neutrosophic ω - open set in (X, τ_N) such that A⊆G. Since A is closed we have, Neu-cl(A) = A. But Neu α-cl(A) ⊆Neu -cl(A) is always true, so Neu α-cl(A) ⊆Neu-cl(A) ⊆ G. Therefore Neu α-cl(A) ⊆ G. Hence A is Neutrosophic ωα-closed set in (X, τ_N).
The converse of the above theorem need not be true as seen from the following example.

Example: 3.4
Fromthe Example.3.2, Here we consider A= {< x, 0.4, 0.8, 0.5>: x ∈X} which is Neutrosophic ωα-closed set but not Neutrosophic closed set.

Theorem 3.5:
Every Neutrosophic α-closed set is Neutrosophic ωα-closed set.

Proof:
Let A be Neutrosophic α-closed set in a Neutrosophic topological space (X, τ_N). Let G be any Neutrosophic ω-open set in (X, τ_N) such that A⊆G. Since A is Neutrosophic α-closed set, we have, Neu α-cl(A) = A. Therefore Neu α-cl(A) ⊆ G. Hence A is Neutrosophic ωα -closed set in (X, τ_N) .
The converse of the above theorem need not be true as seen from the following example.

Example 3.6
From the Example3.2, Here we considerB= {<x, 0.4, 0.8, 0.5>: x ∈X} which is Neutrosophic ωα-closed set but not Neutrosophic α-closed set.

Remark:3.7
Neutrosophic g- closed, Neutrosophic ω- closed and Neutrosophic-semi closed are independent of Neutrosophic ωα-closed set.

Example 3.8
Let X={x}. τ_N={0_N, 1_N,A_1, A_2}be a Neutrosophic topological space
WhereA_1= {<x, 0.4, 0.3, 0.5 >: x∈X} A_2= {<x, 0.5, 0.7, 0.4>: x ∈X} .
Here weconsiderC = {< x, 0.3, 0.6, 0.5 >: x ∈X} which is Neutrosophic g-closed but not Neutrosophic ωα-closed set.

Example 3.9
Let X={x}. τ_N={0_N, 1_N,A_1, A_2} be a Neutrosophic topological space
WhereA_1= {<x, 0.3, 0.6, 0.5 >: x ∈X},A_2= {<x, 0.5, 0.7, 0.4>: x ∈X}
Here we considerrD= {< x, 0.5, 0.2, 0.4>: x ∈X} Which is Neutrosophic ωα-closed but not Neutrosophic g- closed.

Example 3.10
Let X={x}. τ_N={0_N, 1_N,A_1, A_2} be a Neutrosophic topological space.Where
A_1= {<x, 0.4, 0.3, 0.5 >: x∈X} A_2= {<x, 0.5, 0.7, 0.4>: x ∈X}
Here we considerG = {<x, 0.3, 0.6, 0.5>: x ∈X} which is Neutrosophic ω-closed but not Neutrosophic ωα-closed set.

Example 3.10
Let X={x}. τ_N={0_N, 1_N,A_1, A_2}be a Neutrosophic topological space
whereA_1= {<x, 0.3, 0.6, 0.5 >: x ∈X},A_2= {<x, 0.5, 0.7, 0.4>: x ∈X},
Here we considerH= {< x, 0.4, 0.8, 0.5>,: x ∈X} which is Neutrosophic ωα-closed but not Neutrosophic ω-closed.

Example 3.11

Let X={x}. $\tau_N = \{0_N, 1_N, A_1, A_2, A_3, A_4\}$be aNeutrosophic topological space

$A_1 = \{\langle x, 0.3, 0.6, 0.5\rangle : x \in X\}$

$A_2 = \{\langle x, 0.5, 0.2, 0.4\rangle : x \in X\}$

$A_3 = \{\langle x, 0.5, 0.6, 0.4\rangle : x \in X\}$

$A_4 = \{\langle x, 0.3, 0.2, 0.5\rangle : x \in X\}$

Here we considerI=$\{\langle x, 0.5, 0.2, 0.4\rangle : x \in X\}$ which is Neutrosophic semi-closed but not Neutrosophic ωα-closed set.

Example 3.12

Let X={x}. $\tau_N = \{0_N, 1_N, A_1, A_2\}$ be a Neutrosophic topological space

Where$A_1 = \{\langle x, 0.3, 0.6, 0.5\rangle : x \in X\}$,

$A_2 = \{\langle x, 0.5, 0.7, 0.4\rangle : x \in X\}$

Here we consider F= $\{\langle x, 0.4, 0.8, 0.5\rangle, 0.4 \rangle : x \in X\}$ which is Neutrosophic ωα-closed but not Neutrosophic semi closed.

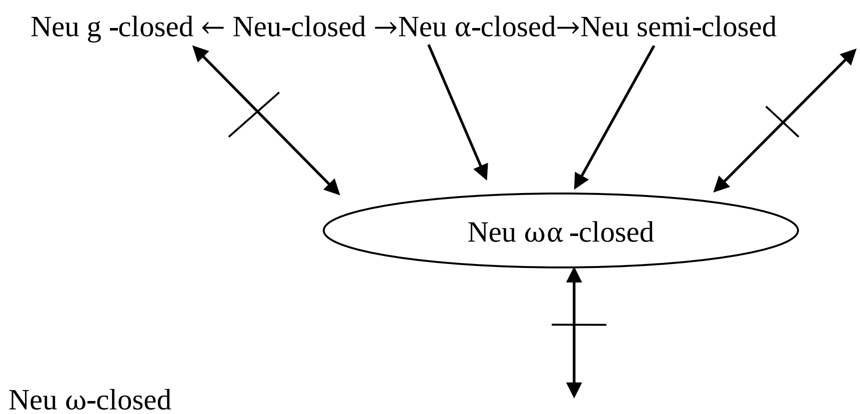

Diagram-I

Where A ⟶ B represents A implies B but B does not implies A

Where A ↮ B represents A independent B each other.

4. PROPERTIES AND CHARACTERIZATION OF NEU ωα-CLOSED SETS

Theorem 4.1

The finite union of Neu ωα-closed sets is Neu ωα-closed.

Proof:

Let A and B be two Neu ωα-closed subsets of a space (X, τ_N).

Then for each Neu ω-open Set U and V in (X,τ_N) containing A and Bsuch that Neu α-cl(A)⊆U and Neu α-cl(B)⊆V respectively .Hence A∪B⊆U∪V. Since arbitrary union of Neu ω-open sets in (X,τ_N) is also Neu ω-open Set in (X,τ_N), U∪V is Neu ω-open in (X, τ_N). Also Neu α-cl(A∪B)=Neu α-cl(A) ∪Neu α-cl(B)⊆U∪V. Therefore A∪B is Neu ωα-closed in (X, τ_N).

Remark 4.2:
Intersection of any two Neu ωα-closed sets in (X, τ_N) need not be Neu ωα-closed.
Let X={x}andτ_N={0_N,1_N,A_1} be a Neutrosophic topological space,from the Example 3.2. Here we consider two subsets A_6= {<x, 0.4, 0.8, 0.5>:x∈X}and A_5= {<x, 0.5, 0.7, 0.4>: x ∈X} are Neu ωα-closed sets but their intersection of two sets A_1= {<x, 0.4, 0.7, 0.5>:x∈X},is not Neu ωα-closed.

Theorem 4.3
Prove that any non-empty Neu ω-closed set does not contain both Neu α-cl(A) and A^C. Where A is Neu ωα-closed set of (X,τ_N).

Proof:
Assume that A is Neu ωα-closed. Assume that contrary , Let G be non-empty Neu ω-closed set contained in Neu α-cl(A) and A^C . Now G^C is Neu ω- open set of (X,τ_N) such that $A \subseteq G^C$. Here A is Neu ωα - closed Set of (X,τ_N). Thus Neu α-cl(A) ⊆ G^C. Therefore G⊆Neu α-cl(A) ⊆G^C.
we get $\mu_G(x) \le \mu_{Neu\ \alpha\text{-}cl(A)}(x)$ and $\gamma_G(x) \ge \gamma_{Neu\ \alpha\text{-}cl(A)}(x)$ for all x∈X. and $\mu_{Neu\ \alpha\text{-}cl(A)}(x) \le \mu_G{}^C(x)$, and $\gamma_{Neu\ \alpha\text{-}cl(A)}(x) \ge \gamma_G{}^C{}_{(x),}$ for all x∈X. which is contradiction because $\gamma_G(x) = \mu_G{}^C(x)$,
Hence non empty Neu ω-closed set does not contain both Neu α-cl(A) and A^C.

Theorem 4.4
If A is Neu ω-open and Neu ωα-closed subset of (X,τ_N) then A is Neu α-closed subset of (X,τ_N).

Proof:
We know that A⊆Neu α-cl(A).Here A is Neu ω-open and Neu ωα-closed,
its implies that Neu α-cl(A) ⊆ A. Hence A is Neu α-closed.

Theorem 4.5
The intersection of a Neu ωα-closed set and a Neu ω-closed set is Neu ωα-closed.

Proof:
Let A be Neu ωα-closed and Let G be Neu ω-closed.Take U is anNeu ω-open set with A∩G⊆U. This implies $A \subseteq U \cup G^C$. Here $U \cup G^C$ is Neu ω-open set. Therefore Neu α-cl(A)⊆$U \cup G^C$ .Now Neu α-cl(A∩G) ⊆Neu α-cl(A)∩Neu α-cl(G)=Neu α-cl(A)∩G⊆ $(U \cup G^C) \cap G = U$. Here A∩G is Neu ωα-closed.

Theorem: 4.6
If A is a Neu ωα-closed set is space (X, τ_N) and A⊆B⊆Neu α-cl(A), then B is also Neu ωα-closed set.

Proof:
Let U be a Neu ω-open set of (X,τ_N) such that B⊆U. Then A⊆U.Since A is Neu ωα-closed set, Neu α-cl(A)⊆U. Also Since B⊆Neu α-cl(A), Neu α-cl(B)⊆Neu α-cl(Neu α-cl(A))=Neu α- cl(A).Hence Neu-αcl(B) ⊆U.Therefore B is also a Neu ωα-closed set.

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