Neutrosophic Sets: An Overview

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ABSTRACT

In this study, we give some concepts concerning the neutrosophic sets, single valued neutrosophic sets, interval-valued neutrosophic sets, bipolar neutrosophic sets, neutrosophic hesitant fuzzy sets, inter-valued neutrosophic hesitant fuzzy sets, refined neutrosophic sets, bipolar neutrosophic refined sets, multi-valued neutrosophic sets, simplified neutrosophic linguistic sets, neutrosophic over/off/under sets, rough neutrosophic sets, rough bipolar neutrosophic sets, rough neutrosophic hyper-complex set, and their basic operations. Then we introduce triangular neutrosophic numbers, trapezoidal neutrosophic fuzzy number and their basic operations. Also some comparative studies between the existing neutrosophic sets and neutrosophic number are provided.

KEYWORDS: **Neutrosophic sets (NSs), Single valued neutrosophic sets (SVNSs), Interval-valued neutrosophic sets (IVNSs), Bipolar neutrosophic sets (BNSs), Neutrosophic hesitant fuzzy sets (NHFSs), Interval valued neutrosophic hesitant fuzzy sets (IVNHFSs), Refined neutrosophic sets (RNSs), Bipolar neutrosophic refined sets (BNRSs), Multi-valued neutrosophic sets (MVNSs), Simplified neutrosophic linguistic sets, Neutrosophic numbers, Neutrosophic over/off/under sets, Rough neutrosophic sets, Bipolar rough neutrosophic sets, Rough neutrosophic sets, Bipolar rough neutrosophic sets, Rough neutrosophic hyper-complex set**

1. INTRODUCTION

The concept of fuzzy sets was introduced by L. Zadeh (1965). Since then the fuzzy sets and fuzzy logic are used widely in many applications involving uncertainty. But it is observed that there still remain some situations which cannot be covered by fuzzy sets and so the concept of interval valued fuzzy sets (Zadeh, 1975) came into force to capture those situations, Although Fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all sorts of uncertainties prevailing in different real physical problems such as problems involving incomplete information. Further generalization of the fuzzy set was made by Atanassov (1986), which is known as intuitionistic fuzzy sets (IFs). In IFS, instead of one membership grade, there is also a non-membership grade attached with each element. Further there is a restriction that the sum of these two grades is less or equal to unity. The conception of IFS can be viewed as an appropriate/ alternative approach in case where available information is not sufficient to define the impreciseness by the conventional fuzzy sets. Later on intuitionistic fuzzy sets were extended to interval valued intuitionistic fuzzy sets (Atanassov & Gargov, 1989). Neutrosophic sets (NSs) proposed by (Smarandache, 1998, 1999, 2002, 2005, 2006, 2010) which is a generalization of fuzzy sets and intuitionistic fuzzy set, is a powerful tool to deal with incomplete, indeterminate and inconsistent information which exist in the real world. Neutrosophic sets are characterized by truth membership function (T), indeterminacy membership function (I) and falsity membership function (F). This theory is very important in many application areas since indeterminacy is quantified explicitly and the truth membership function, indeterminacy membership function and falsity membership functions are independent. Wang, Smarandache, Zhang, & Sunderraman (2010) introduced the concept of single valued neutrosophic set. The single-valued neutrosophic set can independently express truth-membership degree, indeterminacy-membership degree and falsity-membership degree and deals with incomplete, indeterminate and inconsistent information. All the factors described by the single-valued neutrosophic set are very suitable for human thinking due to the imperfection of knowledge that human receives or observes from the external world.

Single valued neutrosophic set has been developing rapidly due to its wide range of theoretical elegance and application areas; see for examples (Sodenkamp, 2013; Kharal, 2014; Broumi & Smarandache, 2014; Broumi & Smarandache, 2013; Hai-Long, Zhi-Lian, Yanhong, & Xiuwu, 2016; Biswas, Pramanik, & Giri, 2016a, 2016b, 2016c; 2017; Ye, 2014a, 2014b, 2014c, 2015a, 2016).
Wang, Smarandache, Zhang, & Sunderraman (2005) proposed the concept of interval neutrosophic set (INS) which is an extension of neutrosophic set. The interval neutrosophic set (INS) can represent uncertain, imprecise, incomplete and inconsistent information which exists in real world.
Single valued neutrosophic number is an extension of fuzzy numbers and intuitionistic fuzzy numbers. Single valued fuzzy number is a special case of single valued neutrosophic set and is of importance for decision making problems. Ye (2015b) and Biswas, Pramanik, and Giri (2014) studied the concept of trapezoidal neutrosophic fuzzy number as a generalized representation of trapezoidal fuzzy numbers, trapezoidal intuitionistic fuzzy numbers, triangular fuzzy numbers and triangular intuitionistic fuzzy numbers and applied them for dealing with multi-attribute decision making (MADM) problems. Deli & Subas (2017) and Biswas et al. (2016b) studied the ranking of single valued neutrosophic trapezoidal numbers and applied the concept to solve MADM problems. Liang, Wang, & Zhang (2017) presented a multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information.

Ye (2014b) proposed the concept of single valued neutrosophic hesitant fuzzy sets (SVNHFS). As a combination of hesitant fuzzy sets (HFS) and singled valued neutrosophic sets (SVNs), the single valued neutrosophic hesitant fuzzy set (SVNHF) is an important concept to handle uncertainty and vague information existing in real life which consists of three membership functions and encompass the fuzzy set (FS), intuitionistic fuzzy sets (IFS), hesitant fuzzy set (HFs), dual hesitant fuzzy set (DHFs) and single valued neutrosophic set (SVNS). Theoretical development and applications of such concepts can be found in (Wang & Li, 2016; Ye, 2016). Peng, Wang, Wu, Wang, & Chen, 2014; Peng &Wang, 2015) introduced the concept of multi-valued neutrosophic set as a new branch of NSs which is the same concept of neutrosophic hesitant fuzzy set. Multi-valued neutrosophic sets can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications.

Tian, Wang, Zhang, Chen, & Wang (2016) defined the concept of simplified neutrosophic linguistic sets which combine the concept of simplified neutrosophic sets and linguistic term sets. Simplified neutrosophic linguistic sets have enabled great progress in describing linguistic information to some extent. It may be considered to be an innovative construct.

Deli, Ali, and Smarandache (2015a) defined the concept of bipolar neutrosophic set and its score, certainty and accuracy functions. In the same study, Deli et al. (2015a) proposed the A_w and G_w operators to aggregate the bipolar neutrosophic information. Furthermore, based on the A_w and G_w operators and the score, certainty and accuracy functions, Deli et al. (2015a) developed a bipolar neutrosophic multiple criteria decision-making approach, in which the evaluation values of alternatives on the attributes assume the form of bipolar neutrosophic numbers. Some theoretical and applications using bipolar neutrosophic sets are studied by several authors (Uluçay, Deli, & Şahin,2016; Dey, Pramanik, & Giri, 2016a; Pramanik, Dey, Giri, & Smarandache, 2017;).

Maji (2013) defined neutrosophic soft set. The development of decision making algorithms using neutrosophic soft set theory has been reported in the literature (Deli & Broumi, 2015; Dey, Pramanik, & Giri, 2015, 2016b, 2016c; Pramanik & Dalapati (2016), Das, Kumar, Kar, & Pal, 2017).

Broumi, Smarandache, and Dhar (2014a, 2014b) defined rough neutrosophic set and proved its basic properties. Some theoretical advancement and applications have been reported in the literature (Mondal & Pramanik, 2014, 2015a, 2015b, 2015c, 2015d, 2015e, 2015f, 2015g, 2015h); Mondal, Pramanik, and Smarandache (2016a, 2016b, 2016c, 2016d); Pramanik & Mondal (2015a, 2015b, 2015c); Pramanik, Roy, Roy, &Smarandache (2017); Pramanik, Roy, & Roy (2017).

Ali, Deli, and Smarandache (2016) and Jun, Smarandache, and Kim (2017) proposed neutrosophic cubic set by extending the concept of cubic set. Some studies in neutrosophic cubic set environment have been reported in the literature (Banerjee, Giri, Pramanik, & Smarandache (2017); Pramanik, Dey, Giri, & Smarandache (2017b); Pramanik, Dalapati, Alam, & Roy (2017a, 2017b); Pramanik, Dalapati, Alam, Roy & Smarandache (2017); Ye (2017); Lu & Ye (2017).

Another extension of neutrosophic set namely, neutrosophic refined set and its appilication was studied by several researchers (Deli, Broumi, & Smarandache, 2015b; Broumi & Smarandache, 2014b; Broumi,& Deli,2014;. Uluçay, Deli, & Şahin, 2016, Pramanik, S., Banerjee, D., & Giri, 2016a, 2016b; Mondal & Pramanik, 2015h, 2015i.; Ye & Smarandache, 2016., Chen, Ye, & Du, 2017).

Later on, several extensions of neutrosophic set have been proposed in the literature by researchers to deal with different type of problems such as bipolar neutrosophic refined sets (Deli & Şubaş, 2016), tri-complex rough neutrosophic set (Mondal & Pramanik, 2015g), rough neutrosophic hyper-complex set (Mondal, Pramanik & Smarandache, 2016d), rough bipolar neutrosophic set.(Pramanik a& Mondal, 2016) simplified neutrosophic linguistic sets (SNLS) (Tian, Wang, Zhang, Chen, & Wang, 2016), quadripartitioned single valued neutrosophic sets (Chatterjee, Majumdar, Samanta, 2016). Smarandache (2016a. 2016b) proposed new version of neutrosophic sets such as neutrosophic off/under/over sets. To have a glimpse of new trends of neutrosophic theory and applications, readers can see the latest editorial book (Smarandache & Pramanik, 2016). Interested readers can find a variety of applications of single valued neutrosophic sets and their hybrid extensions in the website of the Journal "Neutrosophic Sets and Systems" namely, http://fs.gallup.unm.edu/nss.

BASIC AND FUNDAMENTAL CONCEPTS

2.1. Neutrosophic sets (Smarandache, 1998)

Let ξ be the universe. A neutrosophic set (NS) A in ξ is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A where T_A, I_A and F_A are real standard elements of [0,1]. It can be written as

$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A, I_A, F_A \in \left]^{-}0, 1^{+}\right[\ \}$

There is no restriction on the sum of T_A(x), I_A (x)and F_A(x) and so $0^- \le T_A(x) + I_A(x) + F_A(x)) \le 3^+$

2.2 Single valued neutrosophic sets (Wang et al., 2010)

Let X be a space of points (objects) with generic elements in ξ denoted by x. A single valued neutrosophic set A (SVNS) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in ξ, $T_A(x)$, $I_A(x)$, $F_A(x)$ $\in$ [0, 1]. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in \xi\}$$

2.3 Interval valued neutrosophic sets (Wang et al., 2005)

Let ξ be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (IVNS A) is characterized by an interval truth-membership function $T_A(x) = \left[T_A^L, T_A^U\right]$, an interval indeterminacy-membership function $I_A(x) = \left[I_A^L, \mathrm{I}_A^U\right]$, and an interval falsity-membership function $F_A(x) = \left[F_A^L, \mathrm{F}_A^U\right]$. For each point $x \in X$ $T_A(x)$, $I_A(x)$, $F_A(x) \subset$ [0, 1]. An IVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in \xi\}$$

Numerical Example: Assume that $X = \{x_1, x_2, x_3\}$, x_1 is capability, x_2 trustworthiness, x_3 price. The values of x_1, x_2 and x_3 are in [0,1].They are obtained from questionnaire of some domain experts and the result can be obtained as the degree of good, degree of indeterminacy and the degree of poor. Then an interval neutrosophic set can be obtained as

$$A = \begin{cases} < x_1, [0.5, 0.3], [0.1, 0.6], [0.4, 0.2] >, \\ < x_2, [0.3, 0.2], [0.4, 0.3], [0.4, 0.5] >, \\ < x_3, [0.6, 0.3], [0.4, 0.1], [0.5, 0.4] > \end{cases}$$

2.3 Bipolar neutrosophic sets (Deli et al., 2015)

A bipolar neutrosophic set A in ξ is defined as an object of the form

A={<x, $T^p(x)$, $I^p(x)$, $F^p(x)$, $T^n(x)$, $I^n(x)$, $F^n(x)$ >: x $\in \xi$}, where T^p, I^p, $F^p : \xi \rightarrow$ [1, 0] and T^n, I^n, $F^n : \xi \rightarrow$ [-1, 0] .The positive membership degree $T^p(x)$, $I^p(x)$, $F^p(x)$ denote the truth membership, indeterminate membership and false membership of an element $\in \xi$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^n(x)$, $I^n(x)$, $F^n(x)$ denotes the truth membership, indeterminate membership and false membership of an element $\in \xi$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

An empty bipolar neutrosophic set $\tilde{A}_1 =< T_1^p, I_1^p, F_1^p, T_1^n, I_1^n, F_1^n >$ is defined as $T_1^p = 0, I_1^p = 0, F_1^p = 1$ and $T_1^n = -1, I_1^n = 0, F_1^n = 0$.

Numerical Example: Let $X = \{x_1, x_2, x_3\}$ then

$$A = \begin{cases} < x_1, 0.5, 0.3, 0.1, -0.6, -0.4, -0.01 >, \\ < x_2, 0.3, 0.2, 0.4, -0.03, -0.004, -0.05 >, \\ < x_3, 0.6, 0.5, 0.4, -0.1, -0.5, -0.004 > \end{cases}$$

is a bipolar neutrosophic number.

2.4 Neutrosophich hesitant fuzzy set (Ye, 2014)

Let ξ be a non-empty fixed set, a neutrosophic hesitant fuzzy set (NHFS) on X is expressed by: $N = \{\langle x, \tilde{t}(x), \tilde{\imath}(x), \tilde{f}(x)\rangle | x \in \xi\}$, where $\tilde{t}(x) = \{\tilde{\gamma} | \tilde{\gamma} \in \tilde{t}(x)\}, \tilde{\imath}(x) = \{\tilde{\delta} | \tilde{\delta} \in \tilde{\imath}(x)\}$ and $\tilde{f}(x) = \{\tilde{\vartheta} | \tilde{\vartheta} \in \tilde{f}(x)\}$ are three sets with some values in interval [0,1], which represents the possible truth-membership hesitant degrees,indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of the element $x \in \xi$ to the set N, and satisfies these limits :

$$\tilde{\gamma} \in [0,1], \tilde{\delta} \in [0,1], \tilde{\vartheta} \in [0,1] \text{and } 0 \le sup\tilde{\gamma}^+ + sup\tilde{\delta}^+ + sup\tilde{\vartheta}^+ \le 3$$

where $\tilde{\gamma}^+ = \bigcup_{\tilde{\gamma} \in \tilde{t}(x)} max\{\tilde{\gamma}\}, \tilde{\delta}^+ = \bigcup_{\tilde{\delta} \in \tilde{\imath}(x)} max\{\tilde{\delta}\}$ and $\tilde{\vartheta}^+ = \bigcup_{\tilde{\vartheta} \in \tilde{\imath}(x)} max\{\tilde{\vartheta}\}$ for $x \in X$.

The $\tilde{n} = \{\tilde{t}(x), \tilde{\imath}(x), \tilde{f}(x)\}$ is called a neutrosophic hesitant fuzzy element (NHFE) which is the

basic unit of the NHFS and is denoted by the symbol $\tilde{n} = \{\tilde{t}, \tilde{\imath}, \tilde{f}\}$.

2.5 Interval neutrosophic hesitant fuzzy set (Ye, 2016)

Let ξ be a fixed set, an INHFS on ξ is defined as

$N = \{\langle x, \tilde{t}(x), \tilde{\imath}(x), \tilde{f}(x)\rangle | x \in \xi\}$.

Here $\tilde{t}(x), \tilde{\imath}(x)$ and $\tilde{f}(x)$ are sets of some different interval values in [0, 1], representing the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of the element $x \in \xi$ to the set N, respectively. Then $\tilde{t}(x)$ reads $\tilde{t}(x) = \{\tilde{\gamma} | \tilde{\gamma} \in \tilde{t}(x)\}, where\ \tilde{\gamma} = [\tilde{\gamma}^L, \tilde{\gamma}^U]$ is an interval number, $\tilde{\gamma}^L = inf\tilde{\gamma}$ and $\tilde{\gamma}^U = sup\tilde{\gamma}$ represent the lower and upper limits of $\tilde{\gamma}$, respectively; $\tilde{\imath}(x)$ reads $\tilde{\imath}(x) = \{\tilde{\delta} | \tilde{\delta} \in \tilde{t}(x)\}, where \tilde{\delta} = [\tilde{\delta}^L, \tilde{\delta}^U]$ is an interval number, $\tilde{\delta}^L = inf\tilde{\delta}$ and $\tilde{\delta}^U = sup\tilde{\delta}$ represent the lower and upper limits of $\tilde{\delta}$, respectively; $\tilde{f}(x)$ reads $\tilde{f}(x) = \{\tilde{\vartheta} | \tilde{\vartheta} \in \tilde{t}(x)\}, where \tilde{\vartheta} = [\tilde{\vartheta}^L, \tilde{\vartheta}^U]$ is an interval number, $\tilde{\vartheta}^L = inf\tilde{\vartheta}$ and $\tilde{\vartheta}^U = sup\tilde{\vartheta}$ represent the lower and upper limits of $\tilde{\vartheta}$, respectively. Hence, there is the condition

$$0 \le sup\tilde{\gamma}^+ + sup\tilde{\delta}^+ + sup\tilde{\vartheta}^+ \le 3$$

where $\tilde{\gamma}^+ = \bigcup_{\tilde{\gamma} \in \tilde{t}(x)} max\{\tilde{\gamma}\}, \tilde{\delta}^+ = \bigcup_{\tilde{\delta} \in \tilde{\imath}(x)} max\{\tilde{\delta}\}$ and $\tilde{\vartheta}^+ = \bigcup_{\tilde{\vartheta} \in \tilde{\imath}(x)} max\{\tilde{\vartheta}\}$ for $x \in X$.

For convenience, $\tilde{n} = \{\tilde{t}(x), \tilde{\imath}(x), \tilde{f}(x)\}$ is called an interval neutrosophic hesitant fuzzy element (INHFE), which is denoted by the simplified symbol $\tilde{n} = \{\tilde{t}, \tilde{\imath}, \tilde{f}\}$.

2.6 Multi-valued neutrosophic sets (Wang & Li, 2015; Peng & Wang, 2015)

Let X be a space of points (objects) with generic elements in X denoted by x, then multi-valued neutrosophic sets A in X is characterized by a truth-membership function $\tilde{T}_A(x)$, a indeterminacy-membership function $\tilde{I}_A(x)$, and a falsity-membership function $\tilde{F}_A(x)$. Multi-valued neutrosophic sets can be defined as the following form:

$$A = \{\langle x, \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)\rangle | x \in X\},$$

where $\tilde{T}_A(x) \in [0,1]$, $\tilde{I}_A(x) \in [0,1]$, $\tilde{F}_A(x) \in [0,1]$, are sets of finite discrete values, and satisfies the condition $0 \le \gamma, \eta, \xi \le 1$, $0 \le \gamma^+ + \eta^+ + \xi^+ \le 3$, $\gamma \in \tilde{T}_A(x)$, $\eta \in \tilde{I}_A(x)$, $\xi \in \tilde{F}_A(x)$, $\gamma^+ = \sup \tilde{T}_A(x)$, $\eta^+ = \sup \tilde{I}_A(x)$, $\xi^+ = \sup \tilde{F}_A(x)$. For the sake of simplicity, $A = \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle$ is called as multi-valued neutrosophic number.

If $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, $\tilde{F}_A(x)$ has only one value, the multi-valued neutrosophic sets is single valued neutrosophic sets. If $\tilde{T}_A(x) = \varnothing$, the multi-valued neutrosophic sets is double hesitant fuzzy sets. If $\tilde{T}_A(x) = \tilde{F}_A(x) = \varnothing$, the multi-valued neutrosophic sets is hesitant fuzzy sets.

Numerical example: Investment company have four options (to invest): the car company, the food company, the computer company, and the arms company, and it considers three criteria: the risk control capability, the growth potential, and the environmental impact. Then the decision matrix based on the multi-valued neutrosophic numbers is R.

$$R = \begin{bmatrix} \langle\{0.4, 0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.4\}, \{0.2, 0.3\}, \{0.3\}\rangle & \langle\{0.2\}, \{0.2\}, \{0.5\}\rangle \\ \langle\{0.6\}, \{0.1, 0.2\}, \{0.2\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.1, 0.2\}\rangle \\ \langle\{0.3, 0.4\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2, 0.3\}, \{0.2\}\rangle \\ \langle\{0.7\}, \{0.1, 0.2\}, \{0.1\}\rangle & \langle\{0.6\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.4\}, \{0.3\}, \{0.2\}\rangle \end{bmatrix}.$$

2.7 Neutrosophic overset/ underset/offset (Smarandache, 2016a)

2.7.1. Definition of neutrosophic overset: Let ξ be a universe of discourse and the neutrosophic set $A \subset \xi$. Let $T_A(x)$, $I_A(x)$, $F_A(x)$ be the functions that describe the degree of membership, indterminate membership and non-membership respectively of a generic element $x \in \xi$ with respect to the neutrosophic set A. A neutrosophic overset (NOVs) A on the universe of discourse ξ is defined as:

$$A = \{(x, T_A(x), I_A(x), F_A(x)), x \in \xi \text{ and } T(x), I(x), F(x) \in [0, \Omega\,]\}, \text{ where}$$

$T(x), I(x), F(x)\colon \xi \to [0, \Omega\,]$, $0 < 1 < \Omega$ and Ω is called over limit. Then there exist at least one element in A such that it has at least one neutrosophic component > 1, and no element has neutrosophic component < 0.

2.7.2 Definition of neutrosophic underset: Let ξ be a universe of discourse and the neutrosophic set $A \subset \xi$. Let $T_A(x)$, $I_A(x)$, $F_A(x)$ be the functions that describe the degree of membership, indterminate membership and non-membership respectively of a generic element $x \in \xi$ with respect to the neutrosophic set A. A neutrosophic under set (NUs) A on the universe of discourse ξ is defined as:

$$A=\{(x, T_A(x), I_A(x), F_A(x)), x \in \xi \text{ and } T(x), I(x), F(x) \in [\Psi, 1\,]\}$$

Where

$T(x), I(x), F(x)\colon \xi \to [\Psi, 1\,]$, $\Psi<0<1$ and Ψ is called lowerlimit.Then there exist at least one element in A such that it has at least one neutrosophic component < 0, and no element has neutrosophic component > 1.

2.7.3 Definition of neutrosophic offset: Let ξ be a universe of discourse and the neutrosophic set $A \subset \xi$. Let $T_A(x)$, $I_A(x)$, $F_A(x)$ be the functions that describe the degree of membership, indterminate membership and non-membership respectively of a generic element $x \in \xi$ with respect to the neutrosophic set A. A neutrosophic offset (NOFFs) A on the universe of discourse ξ is defined as:

$$A=\{(x, T_A(x), I_A(x), F_A(x)), x \in \xi \text{ and } T(x), I(x), F(x) \in [\Psi, \Omega\,]\}, \text{ where}$$

$T(x), I(x), F(x)\colon \xi \to [\Psi, 1\,]$, $\Psi <0< 1< \Omega$ and Ψ is called underlimit while Ω is called overlimit. Then there existe some elments in A such that at least one neutrosophic component > 1, and at least another neutrosophic component < 0.

Numerical example: A={(x_1,<1.2, 0.4,0.1>),(x_2,<0.2, 0.3,-0.7>)}, since $T(x_1) = 1.2 > 1$, $F(x_2) = -0.7 < 0$.

2.7.4 Some operations of neutrosophic over/off/under sets

Definition 1: The complement of a neutrosophic overset/ underset/offset A is denoted by C(A) and is defined by

$$C(A) =\{(x, <F_A(x), \Psi + \Omega - I_A(x), T_A(x)), x \in \xi\}.$$

Definition 2: The intersection of two neutrosophic overset/ underset/offset A and B is a neutrosophic overset/ underset/offset denoted C and is denoted by

$C = A \cap B$ and is defined by

$$C= A\cap B =\{(x,<\min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x))), x \in \xi\}.$$

Definition 3: The union of two overset/ underset/offset A and B is a neutrosophic overset/ underset/offset denoted C and is denoted by

$C = A \cup B$ and is defined by

C= A∪ B ={(x,<max($T_A(x), T_B(x)$), min($I_A(x), I_B(x)$), min($F_A(x), F_B(x)$)), x∈ ξ}.

Let ξ be a universe of discourse and A the neutrosophic set A⊂U. Let $T_A(x)$, $I_A(x)$,$F_A(x)$ be the functions that describe the degree of membership, indterminate membership and non-membership respectively of a generic element x ∈ $\xi$with respect to the neutrosophic set A. A neutrosophic overset (NOV) A on the universe of discourse U is defined as:

$$A=\{(x, T_A(x), I_A(x), F_A(x)), x \in \xi \text{ and } T(x), I(x), F(x) \in [0,\Omega\,]\}, \text{ where}$$

$T(x), I(x), F(x)$: ξU→ $[0,\Omega\,]$, $0 < 1 < \Omega$ and Ω is called overlimit.Then there exist at least one element in A such that it has at least one neutrosophic component >1, and no element has neutrosophic component <0.

2. OPERATIONS ON SOME NEUTROSOPHIC NUMBERS AND NEUTROSOPHIC SETS

2.1 Single valued neutrosophic number

Let $\tilde{A}_1 = (T_1, I_1, F_1)$ and $\tilde{A}_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic number. Then, the operations for SVNNs are defined as below;

i. $\tilde{A}_1 \oplus \tilde{A}_2 =< T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2 >$
ii. $\tilde{A}_1 \otimes \tilde{A}_2 =< T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2) >$
iii. $\lambda\tilde{A}_1 =< 1-(1-T_1)^{\lambda}), I_1^{\lambda}, F_1^{\lambda}) >$
iv. $\tilde{A}_1^{\lambda} = (T_1^{\lambda}, 1-(1-I_1)^{\lambda}, 1-(1-F_1)^{\lambda})$ where $\lambda > 0$

It is to be noted here that 0_n may be defined as follow:

$0_n = \{< x, (0,1,1) >: x \in X\}$.

2.2 Neutrosophic hesitant fuzzy set (Ye, 2014)

For two NHFEs $\tilde{n}_1 = \{\tilde{t}_1, \tilde{\imath}_1, \tilde{f}_1\}$, $\tilde{n}_2 = \{\tilde{t}_2, \tilde{\imath}_2, \tilde{f}_2\}$ and a positivescale > 0 , the operations operations can be defined as follows:

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \{\tilde{t}_1 \oplus \tilde{t}_2, \tilde{\imath}_1 \otimes \tilde{\imath}_2, \tilde{f}_1 \otimes \tilde{f}_2\} = \cup_{\tilde{\gamma}_1\in\tilde{t}_1,\tilde{\delta}_1\in\tilde{\imath}_1,\tilde{\vartheta}_1\in\tilde{f}_1,\tilde{\gamma}_2\in\tilde{t}_2,\tilde{\delta}_2\in\tilde{\imath}_2,\tilde{\vartheta}_2\in\tilde{f}_2}\{\tilde{\gamma}_1 + \tilde{\gamma}_2 - \tilde{\gamma}_1.\tilde{\gamma}_2, \tilde{\delta}_1.\tilde{\delta}_2, \tilde{\vartheta}_1.\tilde{\vartheta}_2\}$

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \{\tilde{t}_1 \otimes \tilde{t}_2, \tilde{\imath}_1 \oplus \tilde{\imath}_2, \tilde{f}_1 \oplus \tilde{f}_2\} = \cup_{\tilde{\gamma}_1\in\tilde{t}_1,\tilde{\delta}_1\in\tilde{\imath}_1,\tilde{\vartheta}_1\in\tilde{f}_1,\tilde{\gamma}_2\in\tilde{t}_2,\tilde{\delta}_2\in\tilde{\imath}_2,\tilde{\vartheta}_2\in\tilde{f}_2}\{\tilde{\gamma}_1.\tilde{\gamma}_2-, \tilde{\delta}_1 + \tilde{\delta}_2 - \tilde{\delta}_1.\tilde{\delta}_2, \tilde{\vartheta}_1 + \tilde{\vartheta}_2 - \tilde{\vartheta}_1.\tilde{\vartheta}_2\}$

(3) $k\tilde{n}_1 = \cup_{\tilde{\gamma}_1\in\tilde{t}_1,\tilde{\delta}_1\in\tilde{\imath}_1,\tilde{\vartheta}_1\in\tilde{f}_1}\left\{1-(1-\tilde{\gamma}_1)^k, \tilde{\delta}_1{}^k, \tilde{\vartheta}_1{}^k\right\}$

(4) $\tilde{n}_1{}^k = \cup_{\tilde{\gamma}_1\in\tilde{t}_1,\tilde{\delta}_1\in\tilde{\imath}_1,\tilde{\vartheta}_1\in\tilde{f}_1}\left\{\tilde{\gamma}_1{}^k, 1-\left(1-\tilde{\delta}_1\right)^k, 1-\left(1-\tilde{\vartheta}_1\right)^k\right\}$.

2.3 Interval neutrosophic hesitant fuzzy set [Ye, 2016]

For two INHFEs$\tilde{n}_1 = \{\tilde{t}_1, \tilde{\imath}_1, \tilde{f}_1\}$, $\tilde{n}_2 = \{\tilde{t}_2, \tilde{\imath}_2, \tilde{f}_2\}$ and a positive scale > 0 , the follwing operations can be given as follows:

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \{\tilde{t}_1 \oplus \tilde{t}_2, \tilde{\iota}_1 \otimes \tilde{\iota}_2, \tilde{f}_1 \otimes \tilde{f}_2\} = \cup_{\tilde{\gamma}_1 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{\iota}_1, \tilde{\vartheta}_1 \in \tilde{f}_1, \tilde{\gamma}_2 \in \tilde{t}_2, \tilde{\delta}_2 \in \tilde{\iota}_2, \tilde{\vartheta}_2 \in \tilde{f}_2} \left\{ [\tilde{\gamma}_1{}^L + \tilde{\gamma}_2{}^L - \tilde{\gamma}_1{}^L . \tilde{\gamma}_2{}^L, \tilde{\gamma}_1{}^U + \tilde{\gamma}_2{}^U - \tilde{\gamma}_1{}^U . \tilde{\gamma}_2{}^U], \left[\tilde{\delta}_1{}^L . \tilde{\delta}_2{}^L, \tilde{\delta}_1{}^U . \tilde{\delta}_2{}^U\right], \left[\tilde{\vartheta}_1{}^L . \tilde{\vartheta}_2{}^L, \tilde{\vartheta}_1{}^U . \tilde{\vartheta}_2{}^U\right] \right\}$

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \{\tilde{t}_1 \otimes \tilde{t}_2, \tilde{\iota}_1 \oplus \tilde{\iota}_2, \tilde{f}_1 \oplus \tilde{f}_2\} = \cup_{\tilde{\gamma}_1 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{\iota}_1, \tilde{\vartheta}_1 \in \tilde{f}_1, \tilde{\gamma}_2 \in \tilde{t}_2, \tilde{\delta}_2 \in \tilde{\iota}_2, \tilde{\vartheta}_2 \in \tilde{f}_2} \left\{ [\tilde{\gamma}_1{}^L . \tilde{\gamma}_2{}^L, \tilde{\gamma}_1{}^U . \tilde{\gamma}_2{}^U] -, \left[\tilde{\delta}_1{}^L + \tilde{\delta}_2{}^L - \tilde{\delta}_1{}^L . \tilde{\delta}_2{}^L, \tilde{\delta}_1{}^U + \tilde{\delta}_2{}^U - \tilde{\delta}_1{}^U . \tilde{\delta}_2{}^U\right], \left[\tilde{\vartheta}_1{}^L + \tilde{\vartheta}_2{}^L - \tilde{\vartheta}_1{}^L . \tilde{\vartheta}_2{}^L, \tilde{\vartheta}_1{}^U + \tilde{\vartheta}_2{}^U - \tilde{\vartheta}_1{}^U . \tilde{\vartheta}_2{}^U\right] \right\}$

(3) $k\tilde{n}_1 = \cup_{\tilde{\gamma}_1 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{\iota}_1, \tilde{\vartheta}_1 \in \tilde{f}_1} \left\{ \left[1 - (1 - \tilde{\gamma}_1{}^L)^k, 1 - (1 - \tilde{\gamma}_1{}^U)^k\right], \left[\left(\tilde{\delta}_1{}^L\right)^k, \left(\tilde{\delta}_1{}^U\right)^k\right], \left[\left(\tilde{\vartheta}_1{}^L\right)^k, \left(\tilde{\vartheta}_1{}^U\right)^k\right] \right\}$

(4) $\tilde{n}_1{}^k = \cup_{\tilde{\gamma}_1 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{\iota}_1, \tilde{\vartheta}_1 \in \tilde{f}_1} \left\{ \left[(\tilde{\gamma}_1{}^L)^k, (\tilde{\gamma}_1{}^U)^k\right], \left[1 - \left(1 - \tilde{\delta}_1{}^L\right)^k, 1 - \left(1 - \tilde{\delta}_1{}^U\right)^k\right], \left[1 - \left(1 - \tilde{\vartheta}_1{}^L\right)^k, 1 - \left(1 - \tilde{\vartheta}_1{}^U\right)^k\right] \right\}.$

4. SCORE FUNCTION, ACCURACY FUNCTION AND CERTAINTY FUNCTION OF NEUTROSOPHIC NUMBERS

A convenient method for comparing of single valued neutrosophic number is described as follows:

Let $\tilde{A}_1 = (T_1, I_1, F_1)$ be a single valued neutrosophic number. Then, the score function $s(\tilde{A}_1)$, accuracy function $a(\tilde{A}_1)$ and certainty function $c(\tilde{A}_1)$ of a SVNN are defined as follows:

(i) $s(\tilde{A}_1) = \dfrac{2 + T_1 - I_1 - F_1}{3}$

(ii) $a(\tilde{A}_1) = T_1 - F_1$

(iii) $c(\tilde{A}_1) = T_1$.

5. RANKING OF NEUTROSOPHIC NUMBERS

Suppose that $\tilde{A}_1 = (T_1, I_1, F_1)$ and $\tilde{A}_2 = (T_2, I_2, F_2)$ are two single valued neutrosophic numbers. Then, the ranking method is defiend as follows:

i. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$,and $a(\tilde{A}_1) \succ a(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

iii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) \succ c(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

iv. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) = c(\tilde{A}_2)$ then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$

6. DIFFERENT TYPES OF NEUTROSOPHIC NUMBERS AND RELATED TERMS ASSOCIATED WITH THEM

6.1 Single valued-triangular neutrosophic numbers (Ye 2015b)

A single valued triangular neutrosophic number (SVTrN-number) $\tilde{a} = \langle (a_1, b_1, c_1); T_a, I_a, F_a \rangle$ is a special neutrosophic set on the real number set *R*, whose truth membership, indeterminacy-membership, and a falsity-membership are given as follows:

$$T_a(x) = \begin{cases} (x-a_1)T_a/(b_1-a_1) & (a_1 \le x \le b_1) \\ T_a & (x = b_1) \\ (c_1-x)T_a/(c_1-b_1) & (b_1 \le x \le c_1) \\ 0 & otherwise \end{cases}$$

$$I_a(x) = \begin{cases} (b_1-x+I_a(x-a_1))/(b_1-a_1) & (a_1 \le x \le b_1) \\ I_a & (x = b_1) \\ (x-b_1+I_a(c_1-x))/(c_1-b_1) & (b_1 \le x \le c_1) \\ 1 & otherwise \end{cases}$$

$$F_a(x) = \begin{cases} (b_1-x+F_a(x-a_1))/(b_1-a_1) & (a_1 \le x \le b_1) \\ F_a & (x = b_1) \\ (x-c_1+F_a(c_1-x))/(c_1-b_1) & (b_1 \le x \le c_1) \\ 1 & otherwise \end{cases}$$

where $0 \le T_a \le 1$; $0 \le I_a \le 1$; $0 \le F_a \le 1$ and $0 \le T_a + I_a + F_a \le 3$; $a_1, b_1, c_1 \in R$

Numerical Example:

Let $\bar{a} = \langle (2, 4, 6); 0.3, 0.4, 0.5 \rangle$ be a single valued triangular neutrosophic number, then the truth membership, indeterminacy membership and falsity membership are expressed as follows

$$T_a(x) = \begin{cases} \frac{0.3(x-2)}{2}, 2 \le x < 4 \\ 0.3, x = 4 \\ 0.3(5-x), 4 < x \le 5 \\ 0, otherwise \end{cases}$$

$$I_a(x) = \begin{cases} \frac{4-x+0.3(x-2)}{2}, 2 \le x < 4 \\ 0.4, x = 4 \\ x-4+0.4(5-x), 4 < x \le 5 \\ 1, otherwise \end{cases}$$

$$F_a(x) = \begin{cases} \dfrac{4-x+0.5(x-2)}{2}, 2 \le x < 4 \\ 0.5, x = 4 \\ x-4+0.4(5-x), 4 < x \le 5 \\ 1, otherwise \end{cases}$$

6.1.1 Operations on singled valued triangular neutrosophic numbers

Let $\tilde{A}_1 = <(a_1, a_2, a_3); T_1, I_1, F_1>$ and $\tilde{A}_2 = <(b_1, b_2, b_3); T_2, I_2, F_2>$ be two single valued triangular neutrosophic numbers. Then, the operations for SVTrN-numbers are defined as below;

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = <(a_1 + b_1, a_2 + b_2, a_3 + b_3); \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2)>$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = <(a_1 b_1, a_2 b_2, a_3 b_3); \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2))$

(iii) $\lambda \tilde{A}_1 = <(\lambda a_1, \lambda a_2, \lambda a_3); \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2)>$

6.1.2 Score function and accuracy function of single valued triangular neutrosophic numbers

The convenient method for comparing of two single valued triangular neutrosophic numbers is described as follows:

Let $\tilde{A}_1 = <(a_1, a_2, a_3); T_1, I_1, F_1>$ be a single valued triangular neutrosophic number. Then, the score function $s(\tilde{A}_1)$ and accuracy function $a(\tilde{A}_1)$ of a SVTrN-numbers are defined as follows:

(i) $s(\tilde{A}_1) = \left(\frac{1}{12}\right)\left[a_1 + 2a_2 + a_3\right] \times \left[2 + T_1 - I_1 - F_1\right]$

(ii) $a(\tilde{A}_1) = \left(\frac{1}{12}\right)\left[a_1 + 2a_2 + a_3\right] \times \left[2 + T_1 - I_1 + F_1\right]$

6.1.3 Ranking of single valued triangular neutrosophic numbers

Let $\tilde{A}_1$ and $\tilde{A}_2$ be two SVTrN-numbers. The ranking of $\tilde{A}_1$ and $\tilde{A}_2$ by score function and accuracy function is defined as follows:

(i) If $s(\tilde{A}_1) \prec s(\tilde{A}_2)$, then $\tilde{A}_1 \prec \tilde{A}_2$

(ii) If $s(\tilde{A}_1) = s(\tilde{A}_2)$ and if

(1) $a(\tilde{A}_1) \prec a(\tilde{A}_2)$, then $\tilde{A}_1 \prec \tilde{A}_2$

(2) $a(\tilde{A}_1) \succ a(\tilde{A}_2)$, then $\tilde{A}_1 \succ \tilde{A}_2$

(3) $a(\tilde{A}_1) = a(\tilde{A}_2)$, then $\tilde{A}_1 = \tilde{A}_2$.

6.2 Single valued-trapezoidal neutrosophic numbers (Deli & Subas, 2017)

A single valued trapezoidal neutrosophic number (SVTN-number) $\tilde{a} = < (a_1, b_1, c_1, d_1); T_a, I_a, F_a >$ is a special neutrosophic set on the real number set *R*, whose truth membership, indeterminacy-membership, and a falsity-membership are given as follows

$$T_a(x) = \begin{cases} (x-a_1)T_a \big/ (b_1-a_1) & (a_1 \le x \le b_1) \\ T_a & (b_1 \le x \le c_1) \\ (d_1-x)T_a \big/ (d_1-c_1) & (c_1 \le x \le d_1) \\ 0 \quad otherwise \end{cases}$$

$$I_a(x) = \begin{cases} (b_1 - x + I_a(x-a_1)) \big/ (b_1-a_1) & (a_1 \le x \le b_1) \\ I_a & (b_1 \le x \le c_1) \\ (x - c_1 + I_a(d_1-x)) \big/ (d_1-c_1) & (c_1 \le x \le d_1) \\ 1 \quad otherwise \end{cases}$$

$$F_a(x) = \begin{cases} (b_1 - x + F_a(x-a_1)) \big/ (b_1-a_1) & (a_1 \le x \le b_1) \\ F_a & (b_1 \le x \le c_1) \\ (x - c_1 + F_a(d_1-x)) \big/ (d_1-c_1) & (c_1 \le x \le d_1) \\ 1 \quad otherwise \end{cases}$$

where $0 \le T_a \le 1$; $0 \le I_a \le 1$; $0 \le F_a \le 1$ and $0 \le T_a + I_a + F_a \le 3$; $a_1, b_1, c_1, d_1 \in R$.

Numerical example:

Let $\bar{a} = < (1, 2, 5, 6); 0.8, 0.6, 0.4 >$ be a single valued trapezoidal neutrosophic number. Then the truth membership, indeterminacy membership and falsity membership are expressed as follows:

$$T_a(x) = \begin{cases} 0.8(x-1), 1 \le x < 2 \\ 0.8, 2 \le x \le 5 \\ 0.8(6-x), 5 < x \le 6 \\ 0, otherwise \end{cases} \quad I_a(x) = \begin{cases} 1.4 - 0.4x, 1 \le x < 2 \\ 0.6, 2 \le x \le 5 \\ 0.8x - 1.4, 5 < x \le 6 \\ 1, otherwise \end{cases}$$

$$F_a(x) = \begin{cases} 1.6 - 0.6x, 1 \le x < 2 \\ 0.4, 2 \le x \le 5 \\ 0.6x - 2.6, 5 < x \le 6 \\ 1, otherwise \end{cases}.$$

6.2.1 Operation on single valued trapezoidal neutrosophic numbers.

Let $\tilde{A}_1 = < (a_1, a_2, a_3, a_4); T_1, I_1, F_1 >$ and $\tilde{A}_2 = < (b_1, b_2, b_3, b_4); T_2, I_2, F_2 >$ be two single valued trapezoidal neutrosophic numbers. Then, the operations for SVTN-numbers are defined as below;

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = <(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2)>$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = <(a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2))$

(iii) $\lambda \tilde{A}_1 = <(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \min(T_1, T_2), \max(I_1, I_2), \max(F_1, F_2)>$

6.2.2Score function and accuracy function of single valued trapezoidal neutrosophic numbers

The convenient method for comparing of two single valued t**rapezoidal** neutrosophic numbers is described as follows:

Let $\tilde{A}_1 = <(a_1, a_2, a_3, a_4); T_1, I_1, F_1>$ be a single valued trapezoidal neutrosophic number. Then, the score function $s(\tilde{A}_1)$ and accuracy function $a(\tilde{A}_1)$ of a SVTN-numbers are defined as follows:

(i) $s(\tilde{A}_1) = \left(\frac{1}{12}\right)\left[a_1 + a_2 + a_3 + a_4\right] \times \left[2 + T_1 - I_1 - F_1\right]$

(ii) $a(\tilde{A}_1) = \left(\frac{1}{12}\right)\left[a_1 + a_2 + a_3 + a_4\right] \times \left[2 + T_1 - I_1 + F_1\right]$

6.2.3 Ranking of single valued trapezoidal neutrosophic numbers

Let $\tilde{A}_1$ and $\tilde{A}_2$ be two SVTN-numbers. The ranking of $\tilde{A}_1$ and $\tilde{A}_2$ by score function is defined as follows:

(i) If $s(\tilde{A}_1) \prec s(\tilde{A}_2)$ then $\tilde{A}_1 \prec \tilde{A}_2$

(ii) If $s(\tilde{A}_1) = s(\tilde{A}_2)$ and if

(1) $a(\tilde{A}_1) \prec a(\tilde{A}_2)$ then $\tilde{A}_1 \prec \tilde{A}_2$

(2) $a(\tilde{A}_1) \succ a(\tilde{A}_2)$ then $\tilde{A}_1 \succ \tilde{A}_2$

(3) $a(\tilde{A}_1) = a(\tilde{A}_2)$ then $\tilde{A}_1 = \tilde{A}_2$

Later on, Liang et al. (2017) redefined the score function, accuracy function and certainty function as follows:

Let $\tilde{a} = <[a_1, a_2, a_3, a_4], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})>$ be a SVTNN. Then, the score function, accuracy function, and certainty function of SVTNN ã are defined, respectively, as:

$E(\tilde{a}) = COG(K) \times \frac{(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}})}{3}$

$A(\tilde{a}) = COG(K) \times (T_{\tilde{a}} - F_{\tilde{a}})$

$C(\tilde{a}) = COG(K) \times T_{\tilde{a}}$

where (COG) denotes the center of gravity of K and can be defined as follows:

$$\text{COG(K)}=\begin{cases} a & \text{if} \quad a_1 = a_2 = a_3 = a_4 \\ \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{a_4 a_3 - a_2 a_1}{a_4 + a_3 - a_2 - a_1}\right], & \text{otherwise} \end{cases}$$

6.3 Interval valued neutrosophic number

6.3.1Operations on interval valued neutrosophic number

Let $\tilde{A}_1 = < \left[T_1^L, T_1^U\right], \left[I_1^L, I_1^U\right], \left[F_1^L, F_1^U\right] >$ and $\tilde{A}_2 = < \left[T_2^L, T_2^U\right], \left[I_2^L, I_2^U\right], \left[F_2^L, F_2^U\right] >$ be two interval valued neutrosophic numbers. Then, the operations for IVNNs are defined as below;

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = < \left[T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U\right], \left[I_1^L I_2^L, I_1^U I_2^U\right], \left[F_1^L F_2^L, F_1^U F_2^U\right] >$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = < \left[T_1^L T_2^L, T_1^U T_2^U\right], \left[I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U\right],$
$\left[F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U\right] >$

(iii) $\lambda \tilde{A} = < \left[1-(1-T_1^L)^\lambda, 1-(1-T_1^U)^\lambda\right], \left[(I_1^L)^\lambda, (I_1^U)^\lambda\right], \left[(F_1^L)^\lambda, (F_1^U)^\lambda\right] >$

(iv) $\tilde{A}_1^\lambda = < \left[(T_1^L)^\lambda, (T_1^U)^\lambda\right], \left[1-(1-I_1^L)^\lambda, 1-(1-I_1^U)^\lambda\right], \left[1-(1-F_1^L)^\lambda, 1-(1-F_1^U)^\lambda\right] >$ where $\lambda > 0$

An interval valued neutrosophic number $\tilde{A}_1 = < \left[T_1^L, T_1^U\right], \left[I_1^L, I_1^U\right], \left[F_1^L, F_1^U\right] >$ is said to be empty if and only if

$T_1^L = 0, T_1^U = 0, I_1^L = 1, I_1^U = 1,$ and $F_1^L = F_1^U$ and is denoted by

$0_n = \{< x, < [0,\ 0],\ [1,\ 1],\ [1,\ 1] >: x \in X\}$

6.3.2 Score function and accuracy functions of interval valued neutrosophic number

The convenient method for comparing of interval valued neutrosophic numbers is described as follows:

Let $\tilde{A}_1 = < \left[T_1^L, T_1^U\right], \left[I_1^L, I_1^U\right], \left[F_1^L, F_1^U\right] >$ be a single valued neutrosophic number. Then, the score function $s(\tilde{A}_1)$ and accuracy function $\mathrm{H}(\tilde{A}_1)$ of an IVNN are defined as follows:

(i) $s(\tilde{A}_1) = \left(\frac{1}{4}\right) \times \left[2 + T_1^L + T_1^U - 2I_1^L - 2I_1^U - F_1^L - F_1^U\right]$

(ii) $\mathrm{H}(\tilde{A}_1) = \frac{T_1^L + T_1^U - I_1^U(1 - T_1^U) - I_1^L(1 - T_1^L) - F_1^U(1 - I_1^U) - F_1^L(1 - I_1^L)}{2}$

6.3.3 Ranking of interval valued neutrosophic numbers

Let $\tilde{A}_1 = < \left[T_1^L, T_1^U\right], \left[I_1^L, I_1^U\right], \left[F_1^L, F_1^U\right] >$ and $\tilde{A}_2 = < \left[T_2^L, T_2^U\right], \left[I_2^L, I_2^U\right], \left[F_2^L, F_2^U\right] >$ are two interval valued neutrosophic numbers. Then, the ranking method for comparing two IVNS is defiend as follows:

v. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

vi. If $s(\tilde{A}_1) = s(\tilde{A}_2)$,and $H(\tilde{A}_1) \succ H(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.

6.4 Bipolar neutrosophic Number

6.4.1Operation on bipolar neutrosophic numbers

Let $\tilde{A}_1 =< T_1^p, I_1^p, F_1^p, T_1^n, I_1^n, F_1^n >$ and $\tilde{A}_2 =< T_2^p, I_2^p, F_2^p, T_2^n, I_2^n, F_2^n >$ be two bipolar neutrosophic numbers and $\lambda > 0$. Then, the operations of these numbers defined as below;

(i) $\tilde{A}_1 \oplus \tilde{A}_2 =< T_1^p + T_2^p - T_1^p T_2^p, I_1^p I_2^p, F_1^p F_2^p$
$-T_1^n T_2^n, -(-I_1^p - I_2^p - I_1^p I_2^p), -(-F_1^p - F_2^p - F_1^p F_2^p) >$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 =< T_1^p T_2^p, I_1^p + I_2^p - I_1^p I_2^p, F_1^p + F_2^p - F_1^p F_2^p$
$-(-T_1^n - T_2^n - T_1^n T_2^n), -I_1^n I_2^n, -F_1^n F_2^n >$

(iii) $\lambda \tilde{A}_1 =< 1-(1-T_1^p)^\lambda, (I_1^p)^\lambda, (F_1^p)^\lambda, -(-T_1^n)^\lambda, -(-I_1^n)^\lambda, -(1-(1-(-F_1^n))^\lambda) >$

(iv) $\tilde{A}_1^\lambda =< (T_1^p)^\lambda, 1-(1-I_1^p)^\lambda, 1-(1-F_1^p)^\lambda, -(1-(1-(-T_1^n))^\lambda), -(-I_1^n)^\lambda, -(-F_1^n))^\lambda) >$ where $\lambda > 0$.

6.4.2 Score function, accuracy function and certainty function of bipolar neutrosophic number

In order to make comparison between two BNNs. Deli et al. (2015) introduced a concept of score function. The score function is applied to compare the grades of BNS. This function shows that greater is the value, the greater is the bipolar neutrosophic sets and by using this concept paths can be ranked. Let $\tilde{A} =< T^p, I^p, F^p, T^n, I^n, F^n >$ be a bipolar neutrosophic number. Then, the score function $s(\tilde{A})$, accuracy function $a(\tilde{A})$ and certainty function $c(\tilde{A})$ of an BNN are defined as follows:

(i) $s(\tilde{A}) = \left(\frac{1}{6}\right) \times \left[T^p + 1 - I^p + 1 - F^p + 1 + T^n - I^n - F^n\right]$

(ii) $a(\tilde{A}) = T^p - F^p + T^n - F^n$

(iii) $c(\tilde{A}) = T^p - F^n$

6.4.3 Comparison of bipolar neutrosophic numbers

Let $\tilde{A}_1 =< T_1^p, I_1^p, F_1^p, T_1^n, I_1^n, F_1^n >$ and $\tilde{A}_2 =< T_2^p, I_2^p, F_2^p, T_2^n, I_2^n, F_2^n >$ be two bipolar neutrosophic numbers. then

vii. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

viii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$,and $a(\tilde{A}_1) \succ a(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

ix. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) \succ c(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

x. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, $a(\tilde{A}_1) = a(\tilde{A}_2)$, and $c(\tilde{A}_1) = c(\tilde{A}_2)$ then $\tilde{A}_1$ is equal to $\tilde{A}_2$, that is, $\tilde{A}_1$ is indifferent to $\tilde{A}_2$, denoted by $\tilde{A}_1 = \tilde{A}_2$.

7. TRAPEZOIDAL NEUTROSOPHIC SETS (Ye, 2015b; Biswas et al., 2014)

Assume that X be the finite universe of discourse and F [0, 1] be the set of all trapezoidal fuzzy numbers on [0, 1]. A trapezoidal fuzzy neutrosophic set (TrFNS) $\tilde{A}$ in X is represented as:

$\tilde{A}$ = {< x: $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, $\tilde{F}_A(x)$>, x $\in$X}, where $\tilde{T}_A(x): X \to F[0,1]$, $\tilde{I}_A(x): X \to F[0,1]$ and $\tilde{F}_A(x): X \to F[0,1]$.

The trapezoidal fuzzy numbers $\tilde{T}_A(x) = (T_A^1(x), T_A^2(x), T_A^3(x), T_A^4(x))$, $\tilde{I}_A(x) = (I_A^1(x), I_A^2(x), I_A^3(x), I_A^4(x))$ and $\tilde{F}_A(x) = (F_A^1(x), F_A^2(x), F_A^3(x), F_A^4(x))$, respectively, denote the truth-membership, indeterminacy-membership and a falsity-membership degree of x in $\tilde{A}$ and for every x$\in$X, $0 \le T_A^4(x) + I_A^4(x) + F_A^4(x) \le 3$.

For notational convenience, the trapezoidal fuzzy neutrosophic value (TrFNV) $\tilde{A}$ is denoted by $\tilde{A} = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ where,

$(T_A^1(x), T_A^2(x), T_A^3(x), T_A^4(x)) = (a_1, a_2, a_3, a_4)$,

$(I_A^1(x), I_A^2(x), I_A^3(x), I_A^4(x)) = (b_1, b_2, b_3, b_4)$, and

$(F_A^1(x), F_A^2(x), F_A^3(x), F_A^4(x)) = (c_1, c_2, c_3, c_4)$

The parameters satisfy the following relations $a_1 \le a_2 \le a_3 \le a_4$, $b_1 \le b_2 \le b_3 \le b_4$ and $c_1 \le c_2 \le c_3 \le c_4$.

The truth membership function is defined as follows

$$\tilde{T}_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

The indeterminacy membership function is defined as follows:

$$\tilde{I}_A(x) = \begin{cases} \dfrac{x-b_1}{b_2-b_1}, & b_1 \le x \le b_2 \\ 1, & b_2 \le x \le b_3 \\ \dfrac{b_4-x}{b_4-b_3}, & b_3 \le x \le b_4 \\ 0, & otherwise \end{cases}$$

and the falsity membership function is defined as follows:

$$\tilde{F}_A(x) = \begin{cases} \dfrac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1, & c_2 \le x \le c_3 \\ \dfrac{c_4-x}{c_4-c_3}, & c_3 \le x \le c_4 \\ 0, & otherwise \end{cases}$$

A trapezoidal neutrosophic number $\tilde{A} = \langle (a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4) \rangle$ is said to be zero triangular fuzzy neutrosophic number if and only if

(a_1,a_2,a_3,a_4) =(0, 0, 0, 0), (b_1,b_2,b_3,b_4) =(1, 1, 1, 1) and (c_1,c_2,c_3,c_4) =(1, 1, 1, 1).

Remark: The trapezoidal fuzzy neutrosophic number is a particular case of trapezoidal neutrosophic number when all the three vector are equal: $(a_1,a_2,a_3,a_4) = (b_1,b_2,b_3,b_4) = (c_1,c_2,c_3,c_4)$.

7.1 Operation on trapezoidal fuzzy neutrosophic value

Let $\tilde{A}_1 = \langle (a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4) \rangle$ and $\tilde{A}_2 = \langle (e_1,e_2,e_3,e_4),(f_1,f_2,f_3,f_4),(g_1,g_2,g_3,g_4) \rangle$ be two TrFNVs in the set of real numbers, and $\lambda > 0$. Then, the operational rules are defined as follows;

(i) $$\tilde{A}_1 \oplus \tilde{A}_2 = \left\langle \begin{array}{l} \begin{pmatrix} a_1+e_1-a_1e_1, a_2+e_2-a_2e_2, \\ a_3+e_3-a_3e_3, a_4+e_4-a_4e_4 \end{pmatrix}, \\ (b_1 f_1, b_2 f_2, b_3 f_3, b_4 f_4), \\ (c_1g_1, c_2g_2, c_3g_3, c_4g_4) \end{array} \right\rangle$$

(ii) $$\tilde{A}_1 \otimes \tilde{A}_2 = \left\langle \begin{array}{l} (a_1e_1, a_2e_2, a_3e_3, a_4e_4), \\ \begin{pmatrix} b_1+f_1-b_1f_1, b_2+f_2-b_2f_2, \\ b_3+f_3-b_3f_3, b_4+f_4-b_4f_4 \end{pmatrix}, \\ \begin{pmatrix} c_1+g_1-c_1g_1, c_2+g_2-c_2g_2, \\ c_3+g_3-c_3g_3, c_4+g_4-c_4g_4 \end{pmatrix} \end{array} \right\rangle$$

(iii) $\lambda\tilde{A} = \left\langle \begin{pmatrix} (1-(1-a_1)^{\lambda}, 1-(1-a_2)^{\lambda}, \\ 1-(1-a_3)^{\lambda}), 1-(1-a_4)^{\lambda}) \end{pmatrix} \\ (b_1^{\lambda}, b_2^{\lambda}, b_3^{\lambda}, b_4^{\lambda}), (c_1^{\lambda}, c_2^{\lambda}, c_3^{\lambda}, c_4^{\lambda}) \right\rangle$

(iv) $\tilde{A}_1^{\lambda} = \left\langle \begin{matrix} (a_1^{\lambda}, a_2^{\lambda}, a_3^{\lambda}, a_4^{\lambda}), \\ \left((1-(1-b_1)^{\lambda}, 1-(1-b_2)^{\lambda}, 1-(1-b_3)^{\lambda}), 1-(1-b_4)^{\lambda})\right), \\ \left((1-(1-c_1)^{\lambda}, 1-(1-c_2)^{\lambda}, 1-(1-c_3)^{\lambda}), 1-(1-c_4)^{\lambda})\right) \end{matrix} \right\rangle$ where $\lambda > 0$.

Ye (2015b) presented the following definitions of score function and accuracy function. The score function S and the accuracy function H are applied to compare the grades of TrFNSs. These functions show that greater is the value, the greater is the TrFNS.

7.2 Score function and accuracy function of trapezoidal fuzzy neutrosophic value

Let $\tilde{A}_1 = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ be a TrFNV. Then, the score function $s(\tilde{A}_1)$ and an accuracy function $H(\tilde{A}_1)$ of TrFNV are defined as follows:

(i) $s(\tilde{A}_1) = \frac{1}{12}\left[8 + (a_1 + a_2 + a_3 + a_4) - (b_1 + b_2 + b_3 + b_4) - (c_1 + c_2 + c_3 + c_4)\right]$

(ii) $H(\tilde{A}_1) = \frac{1}{4}\left[(a_1 + a_2 + a_3 + a_4) - (c_1 + c_2 + c_3 + c_4)\right]$.

In order to make a comparison between two TrFNV, Ye (2015b) presented the order relations between two TrFNVs.

7.3 Ranking of trapezoidal fuzzy neutrosophic value

Let $\tilde{A}_1 = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ and $\tilde{A}_2 = \langle (e_1, e_2, e_3, e_4), (f_1, f_2, f_3, f_4), (g_1, g_2, g_3, g_4) \rangle$ be two TrFNVs in the set of real numbers. Then, we define a ranking method as follows:

xi. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$

xii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$, and $H(\tilde{A}_1) \succ H(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.

8. TRIANGULAR FUZZY NEUTROSOPHIC SETS (Biswas et al., 2014)

Assume that X be the finite universe of discourse and F [0, 1] be the set of all triangular fuzzy numbers on [0, 1]. A triangular fuzzy neutrosophic set (TFNS) $\tilde{A}$ in X is represented

$\tilde{A}$ = {< x: $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, $\tilde{F}_A(x)$>, x $\in$X} ,

where $\tilde{T}_A(x): X \to F[0,1]$, $\tilde{I}_A(x): X \to F[0,1]$ and $\tilde{F}_A(x): X \to F[0,1]$. The triangular fuzzy numbers

$\tilde{T}_A(x) = (T_A^1(x), T_A^2(x), T_A^3(x))$, $\tilde{I}_A(x) = (I_A^1(x), I_A^2(x), I_A^3(x))$ and

$\tilde{F}_A(x) = (F_A^1(x), F_A^2(x), F_A^3(x))$, respectively, denote the truth-membership, indeterminacy-membership and a falsity-membership degree of x in $\tilde{A}$ and for every $x \in X$

$$0 \le T_A^3(x) + I_A^3(x) + F_A^3(x) \le 3.$$

For notational convenience, the triangular fuzzy neutrosophic value (TFNV) $\tilde{A}$ is denoted by $\tilde{A} = \langle (a, b, c), (e, f, g), (r, s, t) \rangle$ where, $(T_A^1(x), T_A^2(x), T_A^3(x)) = (a, b, c)$,

$(I_A^1(x), I_A^2(x), I_A^3(x)) = (e, f, g)$, and $(F_A^1(x), F_A^2(x), F_A^3(x)) = (r, s, t)$.

8.1 Zero triangular fuzzy neutrosophic number

A triangular fuzzy neutrosophic number $\tilde{A} = \langle (a, b, c), (e, f, g), (r, s, t) \rangle$ is said to be zero triangular fuzzy neutrosophic number if and only if

(a, b, c) = (0, 0, 0), (e, f, g) = (1, 1, 1) and (r, s, t) = (1, 1, 1)

8.2 Operation on triangular fuzzy neutrosophic value

Let $\tilde{A}_1 = \langle (a_1, b_1, c_1), (e_1, f_1, g_1), (r_1, s_1, t_1) \rangle$ and $\tilde{A}_2 = \langle (a_2, b_2, c_2), (e_2, f_2, g_2), (r_2, s_2, t_2) \rangle$ be two TFNVs in the set of real numbers, and $\lambda > 0$. Then, the operational rules are defined as follows;

(i) $$\tilde{A}_1 \oplus \tilde{A}_2 = \left\langle \begin{matrix} (a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2), \\ (e_1 e_2, f_1 f_2, g_1 g_2), (r_1 r_2, s_1 s_2, t_1 t_2) \end{matrix} \right\rangle$$

(ii) $$\tilde{A}_1 \otimes \tilde{A}_2 = \left\langle \begin{matrix} (a_1 a_2, b_1 b_2, c_1 c_2), \\ (e_1 + e_2 - e_1 e_2, f_1 + f_2 - f_1 f_2, g_1 + g_2 - g_1 g_2), \\ (r_1 + r_2 - r_1 r_2, s_1 + s_2 - s_1 s_2, t_1 + t_2 - t_1 t_2) \end{matrix} \right\rangle$$

(iii) $$\lambda \tilde{A} = \left\langle \begin{matrix} \left((1 - (1 - a_1)^{\lambda}, 1 - (1 - b_1)^{\lambda}, 1 - (1 - c_1)^{\lambda})\right), \\ (e_1^{\lambda}, f_1^{\lambda}, g_1^{\lambda}), (r_1^{\lambda}, s_1^{\lambda}, t_1^{\lambda}) \end{matrix} \right\rangle$$

(iv) $$\tilde{A}_1^{\lambda} = \left\langle \begin{matrix} (a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}), \\ \left((1 - (1 - e_1)^{\lambda}, 1 - (1 - f_1)^{\lambda}, 1 - (1 - g_1)^{\lambda})\right), \\ \left((1 - (1 - r_1)^{\lambda}, 1 - (1 - s_1)^{\lambda}, 1 - (1 - t_1)^{\lambda})\right) \end{matrix} \right\rangle \text{ where } \lambda > 0.$$

Ye (2015b) introduced the concept of score function and accuracy function TFNS. The score function S and the accuracy function H are applied to compare the grades of TFNS. These functions show that greater is the value, the greater is the TFNS.

8.3 Score function and accuracy function of triangular fuzzy neutrosophic value

Let $\tilde{A}_1 = \langle (a_1, b_1, c_1), (e_1, f_1, g_1), (r_1, s_1, t_1) \rangle$ be a TFNV. Then, the score function $S(\tilde{A}_1)$ and an accuracy function $H(\tilde{A}_1)$ of TFNV are defined as follows:

(i) $s(\tilde{A}_1) = \frac{1}{12}\left[8 + (a_1 + 2b_1 + c_1) - (e_1 + 2f_1 + g_1) - (r_1 + 2s_1 + t_1)\right]$

(ii) $H(\tilde{A}_1) = \frac{1}{4}\left[(a_1 + 2b_1 + c_1) - (r_1 + 2s_1 + t_1)\right]$

In order to make a comparison between two TFNVs, Ye (2015b) presented the order relations between two TFNVs.

8.4 Ranking of triangular fuzzy neutrosophic values

Let $\tilde{A}_1 = \langle (a_1, b_1, c_1), (e_1, f_1, g_1), (r_1, s_1, t_1) \rangle$ and $\tilde{A}_2 = \langle (a_2, b_2, c_2), (e_2, f_2, g_2), (r_2, s_2, t_2) \rangle$ be two TFNVs in the set of real numbers. Then, the ranking method is defined as follows:

i. If $s(\tilde{A}_1) \succ s(\tilde{A}_2)$, then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$
ii. If $s(\tilde{A}_1) = s(\tilde{A}_2)$,and $H(\tilde{A}_1) \succ H(\tilde{A}_2)$ then $\tilde{A}_1$ is greater than $\tilde{A}_2$, that is, $\tilde{A}_1$ is superior to $\tilde{A}_2$, denoted by $\tilde{A}_1 \succ \tilde{A}_2$.

9. DIFFERENCE BETWEEN TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER AND TRAPEZOIDAL NEUTROSOPHIC FUZZY NUMBER

9.1 Trapezoidal intuitionistic fuzzy number (Nayagam, Jeevaraj,& Sivaraman, 2016)

Definition 1. A trapezoidal intuitionistic fuzzy number $\tilde{a} =< (\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}} >$ is a convex intuitionistic fuzzy set on the set $\Re$ of real numbers, whose membership and non-membership functions are follows

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - \underline{a})w_{\tilde{a}} / (a_1 - \underline{a}) & (\underline{a} \le x < a_1) \\ w_{\tilde{a}} & (a_1 \le x \le a_2) \\ (\overline{a} - x)w_{\tilde{a}} / (\overline{a} - a_2) & (a_2 < a \le \overline{a}) \\ 0 & (x < \underline{a}, x > \overline{a}), \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} [a_1 - x + u_{\tilde{a}}(x - \underline{a})] / (a_1 - \underline{a}) & (\underline{a} \le x < a_1) \\ u_{\tilde{a}} & (a_1 \le x \le a_2) \\ [x - a_2 + u_{\tilde{a}}(\overline{a} - x)] / (\overline{a} - a_2) & (a_2 < x \le \overline{a}) \\ 1 & (x < \underline{a}, x > \overline{a}). \end{cases}$$

where $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$ and $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$, $w_{\tilde{a}}$ and $u_{\tilde{a}}$ respectively represent the maximum membership degree and the minimum membership degree of $\tilde{a}$, $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$ is called as the intuitionistic fuzzy index of an element x in $\tilde{a}$. a_1 and a_2 respectively represent the minimum and maximum values of the most probable value of the fuzzy number $\tilde{a}$, $\underline{a}$ represents the minimum value of the $\tilde{a}$, and $\overline{a}$ represents the maximum value of the $\tilde{a}$.

9.2 Trapezoidal neutrosophic fuzzy number

Definition 2. Let X be a universe of discourse, then a trapezoidal fuzzy neutrosophic set $\tilde{N}$ in X is defined as the following form:

$$\tilde{N} = \{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\rangle | x \in X\},$$

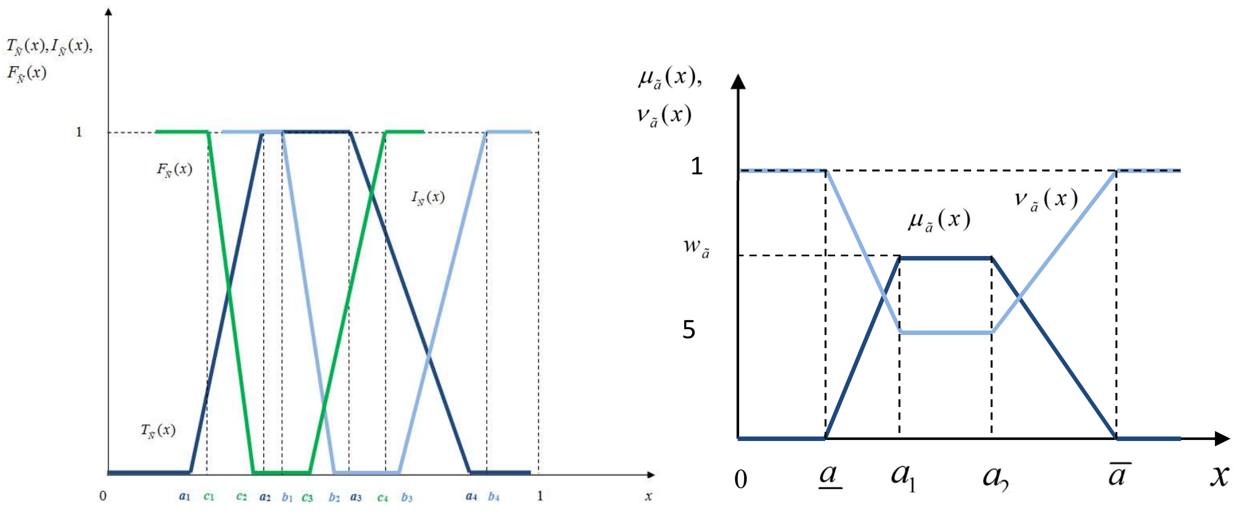
where $T_{\tilde{N}}(x) \subset [0,1]$, $I_{\tilde{N}}(x) \subset [0,1]$ and $F_{\tilde{N}}(x) \subset [0,1]$ are three trapezoidal fuzzy neutrosophic numbers, $T_{\tilde{N}}(x) = \left(t_{\tilde{N}}^1(x), t_{\tilde{N}}^2(x), t_{\tilde{N}}^3(x), t_{\tilde{N}}^4(x)\right) : X \to [0,1]$, $I_{\tilde{N}}(x) = \left(i_{\tilde{N}}^1(x), i_{\tilde{N}}^2(x), i_{\tilde{N}}^3(x), i_{\tilde{N}}^4(x)\right) : X \to [0,1]$,

and $F_{\tilde{N}}(x) = \left(f_{\tilde{N}}^1(x), f_{\tilde{N}}^2(x), f_{\tilde{N}}^3(x), f_{\tilde{N}}^4(x)\right) : X \to [0,1]$ with the condition $0 \le t_{\tilde{N}}^4(x) + i_{\tilde{N}}^4(x) + f_{\tilde{N}}^4(x) \le 3, x \in X$.

9.3 Difference and comparison between trapezoidal intuitionistic fuzzy number and trapezoidal neutrosophic fuzzy number

The difference and comparison between the trapezoidal intuitionistic fuzzy number and trapezoidal neutrosophic fuzzy number are represented in the following way:

First, we give the graphical representation of trapezoidal neutrosophic fuzzy number (TrNFN) and trapezoidal intuitionistic fuzzy number (TrIFN), as shown in Figure 1,



(a) Graphical representation of TrNFN (b) Graphical representation of TrIFN

Fig.1 Graphical representation of trapezoidal neutrosophic fuzzy number and trapezoidal intuitionistic fuzzy number

It can be observed from the Fig. 1, there are some differences between trapezoidal intuitionistic fuzzy number and trapezoidal neutrosophic fuzzy number. On one hand, the membership degree, non-membership degree and hesitancy of trapezoidal intuitionistic fuzzy number are mutually constrained, and the maximum value of the sum of them is not more than 1. However, the truth membership, indeterminacy membership and falsity membership functions of trapezoidal neutrosophic fuzzy number are independent, and their values are between 0 and 3. And the maximum value of their sum is not more than 3. On the other hand, trapezoidal neutrosophic fuzzy number is a generalized representation of trapezoidal fuzzy number and trapezoidal intuitionistic fuzzy number, and trapezoidal intuitionistic fuzzy number is a special case of trapezoidal neutrosophic fuzzy number.

10. DIFFERENCE BETWEEN TRIANGULAR FUZZY NUMBERS, INTUITIONISTIC TRIANGULAR FUZZY NUMBER AND SINGLED VALUED NEUTROSOPHIC SET

Fuzzy sets have been introduced by Zadeh (1965) in order to deal with imprecise numerical quantities in a practical way. A fuzzy number (Kaufmann& Gupta, 1988) is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. A fuzzy number is thus a special case of a convex, normalized fuzzy set of the real line .

10.1 Triangular fuzzy number (Lee, 2005)

A triangular fuzzy number $A=[a_1,a_2,a_3]$ is expressed by the following membership function

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, a_2\le x\le a_3\\ 0, otherwise\end{cases}$$

10.2 Triangular intuitionistic fuzzy number (Li, Nan, & Zhang, 2012)

A TIFN (See Fig. 2) A is a subset of IFS in R with the following membership functions and non-membership function as follows

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, a_2\le x\le a_3\\ 0, otherwise\end{cases}\quad \gamma_A(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'}, a_1'\le x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2}, a_2\le x\le a_3'\\ 1, otherwise\end{cases}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$

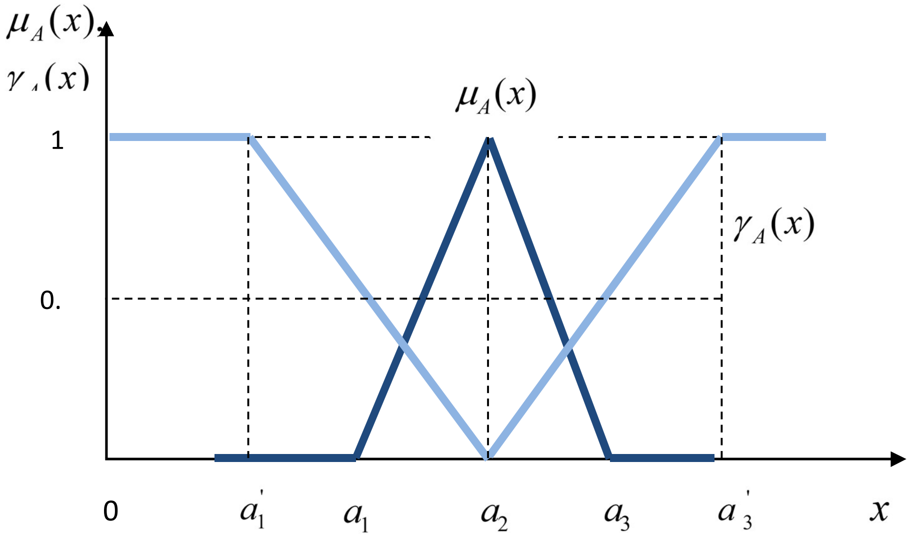


Fig.2. Graphical representation of triangular intuitionistic fuzzy number.

It can be observed from the membership functions that in case of triangular intuitionistic fuzzy number, membership and non-membership degrees are triangular fuzzy numbers. Further it can be noted that the neutrosophic components are best suited in the presentation of indeterminacy and inconsistent information whereas intuitionistic fuzzy sets cannot handle indeterminacy and inconsistent information.
The difference between the fuzzy numbers and singled valued neutrosophic set can be understood clearly with the help of an example. Suppose it is raining continuously for few days in a locality. Then one can guess whether there would be a flood like situation in that area. Observing the rainfall of this year and recalling the incidents of previous years one can only give his judgment on the basis of guess in terms of yes or no but still there remains an indeterminate situation and that indeterminate situation is expressed nicely by the single valued neutrosophic set.

Triangular fuzzy numbers (TFNs) and single valued neutrosophic numbers (SVNNs) are both generalizations of fuzzy numbers that are each characterized by three components. TFNs and SVNNs have been widely used to represent uncertain and vague information in various areas such as engineering, medicine, communication science and decision science. However, SVNNs are far more accurate and convenient to be used to represent the uncertainty and hesitancy that exists in information, as compared to TFNs. SVNNs are characterized by three components, each of which clearly represents the degree of truth membership, indeterminacy membership and falsity membership of a the SVNNs with respect to a an attribute. Therefore, we are able to tell the belongingness of a SVNN to the set of attributes that are being studied, by just looking at the structure of the SVNN. This provides a clear, concise and comprehensive method of representation of the different components of the membership of the number. This is in contrast to the structure of the TFN which only provides us with the maximum, minimum and initial values of the TFN, all of which can only tell us the path of the TFN, but does not tell us anything about the degree of non-belongingness of the TFN with respect to the set of attributes that are being studied. Furthermore, the structure of the TFN is not able to capture the hesitancy that naturally exists within the user in the process of assigning membership values. These reasons clearly show the advantages of SVNNs compared to TFNs.

11. REFINED NEUTROSOPHIC SETS (Smarandache, 2013; Deli et al., 2015b)

Refined neutrosophic sets can be expressed as follows:

Let E be a universe. A neutrosophic refined set (NRS) A on E can be defined as follows

$$A=\left\{\begin{array}{l}<x,(T_A^1(\mathrm{x}),T_A^2(\mathrm{x}),...,T_A^p(\mathrm{x})),(I_A^1(\mathrm{x}),I_A^2(\mathrm{x}),...,I_A^p(\mathrm{x})),\\(F_A^1(\mathrm{x}),F_A^2(\mathrm{x}),...,F_A^p(\mathrm{x}))\end{array}\right\}$$

Where $T_A^1(\mathrm{x}),T_A^2(\mathrm{x}),...,T_A^p(\mathrm{x})$: E → [0, 1], $I_A^1(\mathrm{x}),I_A^2(\mathrm{x}),...,I_A^p(\mathrm{x})$: E → [0, 1] and $F_A^1(\mathrm{x}),F_A^2(\mathrm{x}),...,F_A^p(\mathrm{x})$: E → [0, 1]

12. BIPOLAR NEUTROSOPHIC REFINED SETS

Bipolar neutrosophic refined sets (Deli et al., 2015a) can be described as follows:

Let E be a universe. A bipolar neutrosophic refined set (BNRS) A on E can be defined as follows:

$$A=\left\{\begin{array}{l}<x,(T_A^{1+}(\mathrm{x}),T_A^{2+}(\mathrm{x}),...,T_A^{p+}(\mathrm{x}),T_A^{1-}(\mathrm{x}),T_A^{2-}(\mathrm{x}),...,T_A^{p-}(\mathrm{x})),\\(\mathrm{I}_A^{1+}(\mathrm{x}),\mathrm{I}_A^{2+}(\mathrm{x}),...,\mathrm{I}_A^{p+}(\mathrm{x}),\mathrm{I}_A^{1-}(\mathrm{x}),\mathrm{I}_A^{2-}(\mathrm{x}),...,\mathrm{I}_A^{p-}(\mathrm{x})),\\(\mathrm{F}_A^{1+}(\mathrm{x}),\mathrm{F}_A^{2+}(\mathrm{x}),...,\mathrm{F}_A^{p+}(\mathrm{x}),\mathrm{F}_A^{1-}(\mathrm{x}),\mathrm{F}_A^{2-}(\mathrm{x}),...,\mathrm{F}_A^{p-}(\mathrm{x}))>:x\in X\end{array}\right\}, \text{ where}$$

$(T_A^{1+}(\mathrm{x}),T_A^{2+}(\mathrm{x}),...,T_A^{p+}(\mathrm{x}),T_A^{1-}(\mathrm{x}),T_A^{2-}(\mathrm{x}),...,T_A^{p-}(\mathrm{x}))$: E → [0, 1],

$(\mathrm{I}_A^{1+}(\mathrm{x}),\mathrm{I}_A^{2+}(\mathrm{x}),...,\mathrm{I}_A^{p+}(\mathrm{x}),\mathrm{I}_A^{1-}(\mathrm{x}),\mathrm{I}_A^{2-}(\mathrm{x}),...,\mathrm{I}_A^{p-}(\mathrm{x}))$: E → [0, 1] and

$(\mathrm{F}_A^{1+}(\mathrm{x}),\mathrm{F}_A^{2+}(\mathrm{x}),...,\mathrm{F}_A^{p+}(\mathrm{x}),\mathrm{F}_A^{1-}(\mathrm{x}),\mathrm{F}_A^{2-}(\mathrm{x}),...,\mathrm{F}_A^{p-}(\mathrm{x}))$: E → [0, 1] such that $0\le T_A^i(x)+I_A^i(x)+F_A^i(x)\le 3$ (i =1,2,3,…,p)

$(T_A^{1+}(\mathrm{x}),T_A^{2+}(\mathrm{x}),...,T_A^{p+}(\mathrm{x}),T_A^{1-}(\mathrm{x}),T_A^{2-}(\mathrm{x}),...,T_A^{p-}(\mathrm{x}))$ $(\mathrm{I}_A^{1+}(\mathrm{x}),\mathrm{I}_A^{2+}(\mathrm{x}),...,\mathrm{I}_A^{p+}(\mathrm{x}),\mathrm{I}_A^{1-}(\mathrm{x}),\mathrm{I}_A^{2-}(\mathrm{x}),...,\mathrm{I}_A^{p-}(\mathrm{x}))$

$(\mathrm{F}_A^{1+}(\mathrm{x}),\mathrm{F}_A^{2+}(\mathrm{x}),...,\mathrm{F}_A^{p+}(\mathrm{x}),\mathrm{F}_A^{1-}(\mathrm{x}),\mathrm{F}_A^{2-}(\mathrm{x}),...,\mathrm{F}_A^{p-}(\mathrm{x}))$ is respectively the truth membership sequence, indeterminacy membership sequence and falsity membership sequence of the element x. Also, P is called the dimension of BNRS.

The set of all bipolar neutrosophic refined sets on E is denoted by BNRS(E).

13 MULTI-VALUED NEUTROSOPHIC SETS (Peng & Wang, 2015)

13.1 Operation on multi-valued neutrosophic numbers

Let $A=<\tilde{T}_A(x),\tilde{I}_A(x),\tilde{F}_A(x)>$, $B=<\tilde{T}_B(x),\tilde{I}_B(x),\tilde{F}_B(x)>$ are two multi-valued neutrosophic numbers. If $\forall\tilde{T}_A^a\in\tilde{T}_A$, $\forall\tilde{T}_B^b\in\tilde{T}_B$, $\forall\tilde{I}_A^a\in\tilde{I}_A$, $\forall\tilde{I}_B^b\in\tilde{I}_B$, $\forall\tilde{F}_A^a\in\tilde{F}_A$, $\forall\tilde{F}_B^b\in\tilde{F}_B$, and $\tilde{I}_A^a>\tilde{I}_B^b$, $\tilde{F}_A^a>\tilde{F}_B^b$, $\tilde{T}_A^a>\tilde{T}_B^b$, then *B* is superior to *A*, denoted as $A\prec B$.

Let $A = < \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x) >$, $B = < \tilde{T}_B(x), \tilde{I}_B(x), \tilde{F}_B(x) >$ are any two MVNNs, and $\lambda>0$. The operations for MVNNs are defined as follows.

(1) $\lambda A = \left\langle \cup_{\gamma_A \in \tilde{T}_A} \{1-(1-\gamma_A)^\lambda\}, \cup_{\eta_A \in \tilde{I}_A} \{(\eta_A)^\lambda\}, \cup_{\xi_A \in \tilde{F}_A} \{(\xi_A)^\lambda\} \right\rangle$;

(2) $A^\lambda = \left\langle \cup_{\gamma_A \in \tilde{T}_A} \{(\gamma_A)^\lambda\}, \cup_{\eta_A \in \tilde{I}_A} \{1-(1-\eta_A)^\lambda\}, \cup_{\xi_A \in \tilde{F}_A} \{1-(1-\xi_A)^\lambda\} \right\rangle$;

(3) $A + B = \left\langle \cup_{\gamma_A \in \tilde{T}_A, \gamma_B \in \tilde{T}_B} \{\gamma_A + \gamma_B - \gamma_A \cdot \gamma_B\}, \cup_{\eta_A \in \tilde{I}_A, \eta_B \in \tilde{I}_B} \{\eta_A \cdot \eta_B\}, \cup_{\xi_A \in \tilde{F}_A, \xi_B \in \tilde{F}_B} \{\xi_A \cdot \xi_B\} \right\rangle$;

(4) $A \cdot B = \left\langle \cup_{\gamma_A \in \tilde{T}_A, \gamma_B \in \tilde{T}_B} \{\gamma_A \cdot \gamma_B\}, \cup_{\eta_A \in \tilde{I}_A, \eta_B \in \tilde{I}_B} \{\eta_A + \eta_B - \eta_A \cdot \eta_B\}, \cup_{\xi_A \in \tilde{F}_A, \xi_B \in \tilde{F}_B} \{\xi_A + \xi_B - \xi_A \cdot \xi_B\} \right\rangle$;

13.2 Score function, accuracy function and certainty function of multi-valued neutrosophic number

(1) $s(A) = \dfrac{1}{l_{\tilde{T}_A} \cdot l_{\tilde{I}_A} \cdot l_{\tilde{F}_A}} \times \sum_{\gamma_i \in \tilde{T}_A, \eta_j \in \tilde{I}_A, \xi_k \in \tilde{F}_A} (\gamma_i + 1 - \eta_j + 1 - \xi_k)/3$;

(2) $a(A) = \dfrac{1}{l_{\tilde{T}_A} \cdot l_{\tilde{F}_A}} \times \sum_{\gamma_i \in \tilde{T}_A, \xi_k \in \tilde{F}_A} (\gamma_i - \xi_k)$;

(3) $c(A) = \dfrac{1}{l_{\tilde{T}_A}} \times \sum_{\gamma_i \in \tilde{T}_A} \gamma_i$;

13.3 Comparison of multi-valued neutrosophic numbers

Let $A = < \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x) >$, $B = < \tilde{T}_B(x), \tilde{I}_B(x), \tilde{F}_B(x) >$ are two multi-valued neutrosophic numbers. Then the comparision method can be defined as follows:

i. If $s(A) > s(B)$, then A is greater than B, that is, A is superior to B, denoted by $A \succ B$.
ii. If $s(A) = s(B)$ and $a(A) > a(B)$, then A is greater than B, that is, A is superior to B, denoted by $A \succ B$.
iii. If $s(A) = s(B)$, $a(A) = a(B)$ and $c(A) > c(B)$, then A is greater than B, that is, A is superior to B, denoted by $A \succ B$.
iv. If $s(A) = s(B)$, $a(A) = a(B)$ and $c(A) = c(B)$, then A is equal to B, that is, A is indifferent to B, denoted by $A \sim B$.

14. Simplified neutrosophic linguistic sets (SNLSs) (Tian et al., 2016)

14.1 SNLSs

Definition 1. Let X be a space of points (objects) with a generic element in X, denoted by x and $H = \{h_0, h_1, h_2, \cdots, h_{2t}\}$ be a finite and totally ordered discrete term set, where t is a nonnegative real number. A SNLS A in X is characterized as

$$A = \{< x, h_{\theta(x)}, (t(x), i(x), f(x)) > | x \in X\},$$

where $h_{\theta(x)} \in H$, $t(x) \in [0,1]$, $i(x) \in [0,1]$, $f(x) \in [0,1]$, with the condition $0 \le t(x) + i(x) + f(x) \le 3$ for any $x \in X$. And $t_A(x)$, $i_A(x)$ and $f_A(x)$ represent, respectively, the degree of truth-membership, indeterminacy-membership and falsity-membership of the element x in X to the linguistic term $h_{\theta(x)}$. In addition, if $\|X\|=1$, a SNLS will be degenerated to a SNLN, denoted by $A=<h_\theta,(t,i,f)>$. And A will be degenerated to a linguistic term if $t=1$, $i=0$, and $f=0$.

14.2 Operations of SNLNs

Let $a_i=<h_{\theta_i},(t_i,i_i,f_i)>$ and $a_j=<h_{\theta_j},(t_j,i_j,f_j)>$ be two SNLNs, f^* be a linguistic scale function and $\lambda\geq 0$. Then the following operations of SNLNs can be defined.

(1) $a_i\oplus a_j=\left\langle f^{*-1}(f^*(h_{\theta_i})+f^*(h_{\theta_j})),(\frac{f^*(h_{\theta_i})t_i+f^*(h_{\theta_j})t_j}{f^*(h_{\theta_i})+f^*(h_{\theta_j})},\frac{f^*(h_{\theta_i})i_i+f^*(h_{\theta_j})i_j}{f^*(h_{\theta_i})+f^*(h_{\theta_j})},\frac{f^*(h_{\theta_i})f_i+f^*(h_{\theta_j})f_j}{f^*(h_{\theta_i})+f^*(h_{\theta_j})})\right\rangle$;

(2) $a_i\otimes a_j=\left\langle f^{*-1}(f^*(h_{\theta_i})f^*(h_{\theta_j})),(t_it_j,i_i+i_j-i_ii_j,f_i+f_j-f_if_j)\right\rangle$;

(3) $\lambda a_i=\left\langle f^{*-1}(\lambda f^*(h_{\theta_i})),(t_i,i_i,f_i)\right\rangle$;

(4) $a_i^\lambda=\left\langle f^{*-1}((f^*(h_{\theta_i}))^\lambda),(t_i^\lambda,1-(1-i_i)^\lambda,1-(1-f_i)^\lambda)\right\rangle$;

(5) $neg(a_i)=\left\langle f^{*-1}(f^*(h_{2t})-f^*(h_{\theta_i})),(f_i,1-i_i,t_i)\right\rangle$;

15. COMPARISON ANALYSIS

Refined neutrosophic set is a generalization of fuzzy set, intuitionistic fuzzy set, neutrosophic set, interval-valued neutrosophic set, neutrosophic hesitant fuzzy set and interval-valued neutrosophic hesitant fuzzy set. Also differences and similarities between these sets are given in Table 1.

Table 1. Comparison of fuzzy set anditsextensive set theory

	Fuzzy	intuitionstic fuzzy	Interval-Valued neutrosophic	Interval- Valued neutrosophic HesitantFuzzy Set	Neutro sophic	Neutrosophic HesitantFuzzy Set	Neutrosophicre fined
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Univer se of discour se	Universe of discourse	Universe of discourse
Co-domain	Single-value in [0,1]	Two-value in [0,1]	Unipolar interval in [0,1]	Unipolar interval in [0,1]	$[0,1]^3$	$[0,1]^3$	$[0,1]^3$
Number	Yes	Yes	Yes	Yes	Yes	No	No
Membershipf unction	regular	regular	Regular	irregular	regular	irregular	Regular
Uncertainty	Yes	Yes	Yes	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Falsity	No	Yes	Yes	Yes	Yes	Yes	Yes
Indeterminac y	No	No	Yes	Yes	Yes	Yes	Yes
Negativity	No	No	No	No	Yes in [0,1]	No	No
Membership valued	Membershipv alued	Singlevalue d	İnterv- valued	Singlevalued	Singlev alued	Singlevalued	Multi- valued

Bosc and Pivert (2013) said that “Bipolarity refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired, or considered as being acceptable. On the other hand, negative statements express what is impossible, rejected, or forbidden. Negative preferences correspond to constraints, since they specify which values or objects have to be rejected (i.e., those that do not satisfy the constraints), while positive preferences correspond to wishes, as they specify which objects are more desirable than others (i.e., satisfy user wishes) without rejecting those that do not meet the wishes.” Therefore, Lee (2000, 2009) introduced the concept of bipolar fuzzy sets which is a generalization of the fuzzy sets. Bipolar neutrosophic refined sets which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and bipolar neutrosophic sets. Also differences and similarities between these sets are given in Table 2.

Table 2. Comparison of bipolar fuzzy set and its various extensions

	Bipolar Fuzzy	Bipolar Intuitionistic fuzzy	Bipolar İnterval- Valued neutrosophic	Bipolar Neutrosophic	Bipolar neutrosophic refined
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Co-domain	Single-value in [-1,1]	Two-value in [-1,1]	Unipolar interval in [-1,1]	Bipolar $[-1,1]^3$	Bipolar $[-1,1]^3$
Number	Yes	Yes	Yes	Yes	Yes
Uncertainty	Yes	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes	Yes
Falsity	No	Yes	Yes	Yes	Yes
Membership valued	Singlevalued	Singlevalued	Singlevalued	Singlevalued	Multi valued

Table 3. Comparison of different types of neutrosophic sets

	SVNS	IVNS	BNSs	Multi-valued neutrosophic sets	Trapezoidal Neutrosophic sets	Triangular Fuzzy Neutrosophic sets	**SNLSs**
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of iscourse
Co-domain	$[0,1]^3$	Unipolar İnterval in [0,1]	Bipolar $[-1,1]^3$	$[0,1]^3$	$[0,1]^3$	$[0,1]^3$	[0, 2t] or [-t, t]
Number	Yes	Yes	Yes	Yes	Yes	Yes	No
Uncertainty	Yes	Yes	Yes	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Falsity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Indeterminacy	Yes	Yes	Yes	Yes	Yes	Yes	Yes

CONCLUSIONS

NSs are characterized by truth, indeterminacy, and falsity membership functions which are independent in nature. NSs can handle incomplete, indeterminate, and inconsistent information quite well, whereas IFSs and FSs can only handle incomplete or partial information. However, SVNS, subclass of NSs gain much popularity to apply in concrete areas such as real engineering and scientific problems. Many extensions of NSs have been appeared in the literature. Some of them are discussed in the paper. New hybrid sets derived from neutrosophic sets gain popularity as new research topics. Extensions of neutrosophic sets have been developed by many researchers. This paper presents some of their basic operations. Then, we investigate their properties and the relation between defined numbers and function on neutrosophic sets. We present comparison between bipolar fuzzy sets and its various extensions. We also present the comparison between different types of neutrosophic sets and numbers. The paper can be extended to review different types of neutrosophic hybrid sets and their theoretical development and applications in real world problems.

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