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NEUTROSOPHIC SETS IN UP-ALGEBRAS BY MEANS OF INTERVAL-VALUED FUZZY SETS

Metawee Songsaeng and Aiyared Iampan

Abstract. In this paper, we introduce the notion of interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) of UP-algebras, proved some results, and their generalizations. Furthermore, we discuss the relations between interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) and their level subsets.

1. Introduction and Preliminaries

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [**8**], BCI-algebras [**9**], B-algebras [**21**], UP-algebras [**5**] and others. They are strong connected with logic. For example, BCI-algebras introduced by Iséki [**9**] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [**8, 9**] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

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Key words and phrases. UP-algebra, interval-valued neutrosophic UP-subalgebra, interval-valued neutrosophic near UP-filter, interval-valued neutrosophic UP-filter, interval-valued neutrosophic UP-ideal, interval-valued neutrosophic strong UP-ideal.

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The branch of the logical algebra, a UP-algebra was introduced by Iampan [**5**], and it is known that the class of KU-algebras is a proper subclass of the class of UP-algebras. Later Somjanta et al. [**26**] studied fuzzy UP-subalgebras, fuzzy UP-ideals and fuzzy UP-filters of UP-algebras. Guntasow et al. [**4**] studied fuzzy translations of a fuzzy set in UP-algebras. Kesorn et al. [**13**] studied intuitionistic fuzzy sets in UP-algebras. Kaijae et al. [**12**] studied anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras. Tanamoon et al. [**30**] studied Q-fuzzy sets in UP-algebras. Sripaeng et al. [**28**] studied anti Q-fuzzy UP-ideals and anti Q-fuzzy UP-subalgebras of UP-algebras. Dokkhamdang et al. [**3**] studied Generalized fuzzy sets in UP-algebras. Songsaeng and Iampan [**27**] studied $\mathcal{N}$-fuzzy UP-algebras and its level subsets.

A fuzzy set f in a nonempty set S is a function from S to the closed interval $[0, 1]$. The concept of a fuzzy set in a nonempty set was first considered by Zadeh [**32**]. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. Zadeh [**33**] was introduced an interval-value fuzzy sets. An interval-valued fuzzy set is defined by an interval-valued membership function. Wang et al. [**31**] introduced the concept of interval-valued neutrosophic sets. The interval-valued neutrosophic set is an instance of neutrosophic set which can be used in real scientific and engineering applications. Jun et al. [**10**] introduced the notion of interval-valued neutrosophic sets with applications in BCK/BCI-algebra, they also introduced the notion of interval-valued neutrosophic length of an interval-valued neutrosophic set, and investigate their properties and relations. In 2018-2019, Muhiuddin et al. [**15, 16, 17, 18, 19, 20**] applied the notion of neutrosophic sets to semigroups, BCK/BCI-algebras.

In this paper, we apply the concept of interval-valued neutrosophic sets to UP-algebras. We introduce the notion of interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) in UP-algebras, proved some results, and their generalizations. Furthermore, we discuss the relations between interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) and their level subsets.

Before we begin our study, we will give the definition of a UP-algebra.

DEFINITION 1.1. ([**5**]) An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra*, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X (i.e., a nullary operation) if it satisfies the following axioms: for any $x, y, z \in X$,

(UP-1): $(y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0$,
(UP-2): $0 \cdot x = x$,
(UP-3): $x \cdot 0 = 0$, and
(UP-4): $x \cdot y = 0$ and $y \cdot x = 0$ imply $x = y$.

From [**5**], the binary relation $\leqslant$ on a UP-algebra $X = (X, \cdot, 0)$ defined as follows:

$$(\forall x, y \in X)(x \leqslant y \Leftrightarrow x \cdot y = 0).$$

EXAMPLE 1.1. [**23**] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ means the power set of X. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$, where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to* Ω. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to* Ω. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the *power UP-algebra of type 1*, and $(\mathcal{P}(X), *, X)$ is a UP-algebra and we shall call it the *power UP-algebra of type 2*.

EXAMPLE 1.2. ([**3**]) Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases}$$

and

$$x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

For more examples of UP-algebras, see [**1, 2, 6, 22, 23, 24, 25**].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see [**5, 6**]).

(1.1) $(\forall x \in X)(x \cdot x = 0)$,

(1.2) $(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0)$,

(1.3) $(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0)$,

(1.4) $(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0)$,

(1.5) $(\forall x, y \in X)(x \cdot (y \cdot x) = 0)$,

(1.6) $(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x)$,

(1.7) $(\forall x, y \in X)(x \cdot (y \cdot y) = 0)$,

(1.8) $(\forall a, x, y, z \in X)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0)$,

(1.9) $(\forall a, x, y, z \in X)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0)$,

(1.10) $(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0)$,

(1.11) $(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0)$,

(1.12) $(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0)$, and

(1.13) $(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0)$.

DEFINITION 1.2. ([**5, 26, 4, 7**]) A nonempty subset S of a UP-algebra $X = (X, \cdot, 0)$ is called

(1) a *UP-subalgebra* of X if $(\forall x, y \in S)(x \cdot y \in S)$.

(2) a *near UP-filter* of X if it satisfies the following properties:

   (i) the constant 0 of X is in S, and
   (ii) $(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)$.
(3) a *UP-filter* of X if it satisfies the following properties:
   (i) the constant 0 of X is in S, and
   (ii) $(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.
(4) a *UP-ideal* of X if it satisfies the following properties:
   (i) the constant 0 of X is in S, and
   (ii) $(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.
(5) a *strong UP-ideal* (renamed from a strongly UP-ideal) of X if it satisfies the following properties:
   (i) the constant 0 of X is in S, and
   (ii) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [**4**] proved that the notion of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strong UP-ideals. Moreover, they also proved that a UP-algebra X is the only one strong UP-ideal of itself.

In 1965, the concept of a fuzzy set in a nonempty set was first considered by Zadeh [**32**] as the following definition.

Definition 1.3. A *fuzzy set* (briefly, FS) in a nonempty set X (or a fuzzy subset of X) is defined to be a function $\lambda : X \to [0,1]$, where $[0,1]$ is the unit segment of the real line. Denote by $[0,1]^X$ the collection of all fuzzy sets in X. Define a binary relation $\leqslant$ on $[0,1]^X$ as follows:

$$(\forall \lambda, \mu \in [0,1]^X)(\lambda \leqslant \mu \Leftrightarrow (\forall x \in X)(\lambda(x) \leqslant \mu(x))). \tag{1.14}$$

Definition 1.4. ([**26**]) Let λ be a fuzzy set in a nonempty set X. The *complement of* λ, denoted by λ^C, is defined by

$$(\forall x \in X)(\lambda^C(x) = 1 - \lambda(x)). \tag{1.15}$$

Definition 1.5. ([**14**]) Let $\{\lambda_i \mid i \in J\}$ be a family of fuzzy sets in a nonempty set X. We define the *join* and the *meet* of $\{\lambda_i \mid i \in J\}$, denoted by $\vee_{i\in J}\lambda_i$ and $\wedge_{i\in J}\lambda_i$, respectively, as follows:

$$(\forall x \in X)((\vee_{i\in J}\lambda_i)(x) = \sup_{i\in J}\{\lambda_i(x)\}), \tag{1.16}$$

$$(\forall x \in X)((\wedge_{i\in J}\lambda_i)(x) = \inf_{i\in J}\{\lambda_i(x)\}). \tag{1.17}$$

In particular, if λ and μ be fuzzy sets in X, we have the join and meet of λ and μ as follows:

$$(\forall x \in X)((\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}), \tag{1.18}$$

$$(\forall x \in X)((\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}), \tag{1.19}$$

respectively.

An *interval number* we mean a close subinterval $\tilde{a} = [a^-, a^+]$ of $[0, 1]$, where $0 \leqslant a^- \leqslant a^+ \leqslant 1$. The interval number $\tilde{a} = [a^-, a^+]$ with $a^- = a^+$ is denoted by $\mathbf{a}$. Denote by $[[0, 1]]$ the set of all interval numbers.

Definition 1.6. ([**11**]) Let $\{\tilde{a}_i \mid i \in J\}$ be a family of interval numbers. We define the *refined infimum* and the *refined supremum* of $\{\tilde{a}_i \mid i \in J\}$, denoted by $\mathrm{rinf}_{i \in J} \tilde{a}_i$ and $\mathrm{rsup}_{i \in J} \tilde{a}_i$, respectively, as follows:

$$\mathrm{rinf}_{i\in J}\{\tilde{a}_i\} = [\inf_{i\in J}\{a_i^-\}, \inf_{i\in J}\{a_i^+\}], \tag{1.20}$$

$$\mathrm{rsup}_{i\in J}\{\tilde{a}_i\} = [\sup_{i\in J}\{a_i^-\}, \sup_{i\in J}\{a_i^+\}]. \tag{1.21}$$

In particular, if $\tilde{a}_1$ and $\tilde{a}_2$ are interval numbers, we define the *refined minimum* and the *refined maximum* of $\tilde{a}_1$ and $\tilde{a}_2$, denoted by $\mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\}$ and $\mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\}$, respectively, as follows:

$$\mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}], \tag{1.22}$$

$$\mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\} = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}]. \tag{1.23}$$

Definition 1.7. ([**11**]) Let $\tilde{a}_1$ and $\tilde{a}_2$ be interval numbers. We define the symbols "$\succeq$", "$\preceq$", "$=$" in case of $\tilde{a}_1$ and $\tilde{a}_2$ as follows:

$$\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow a_1^- \geqslant a_2^- \text{ and } a_1^+ \geqslant a_2^+, \tag{1.24}$$

and similarly we may have $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. To say $\tilde{a}_1 \succ \tilde{a}_2$ (resp., $\tilde{a}_1 \prec \tilde{a}_2$) we mean $\tilde{a}_1 \succeq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$ (resp., $\tilde{a}_1 \preceq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2$).

Definition 1.8. ([**33**]) Let $\tilde{a}$ be an interval number. The *complement of* $\tilde{a}$, denoted by $\tilde{a}^C$, is defined by the interval number

$$\tilde{a}^C = [1 - a^+, 1 - a^-]. \tag{1.25}$$

In the $[[0, 1]]$, the following assertions are valid (see [**29**]).

(1.26)
$(\forall \tilde{a} \in [[0, 1]])((\tilde{a}^C)^C = \tilde{a})$,
(1.27)
$(\forall \tilde{a} \in [[0, 1]])(\mathrm{rmax}\{\tilde{a}, \tilde{a}\} = \tilde{a} \text{ and } \mathrm{rmin}\{\tilde{a}, \tilde{a}\} = \tilde{a})$,
(1.28)
$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0, 1]])(\mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\} = \mathrm{rmax}\{\tilde{a}_2, \tilde{a}_1\} \text{ and } \mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\} = \mathrm{rmin}\{\tilde{a}_2, \tilde{a}_1\})$,
(1.29)
$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0, 1]])(\mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\} \succeq \tilde{a}_1 \text{ and } \tilde{a}_2 \succeq \mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\})$,
(1.30)
$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0, 1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \tilde{a}_1^C \preceq \tilde{a}_2^C)$,
(1.31)
$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in [[0, 1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_4 \Rightarrow \mathrm{rmin}\{\tilde{a}_1, \tilde{a}_3\} \succeq \mathrm{rmin}\{\tilde{a}_2, \tilde{a}_4\})$,

(1.32)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_2 \Leftrightarrow \mathrm{rmin}\{\tilde{a}_1, \tilde{a}_3\} \succeq \tilde{a}_2),$$
(1.33)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2, \tilde{a}_3 \succeq \tilde{a}_4 \Rightarrow \mathrm{rmax}\{\tilde{a}_1, \tilde{a}_3\} \succeq \mathrm{rmax}\{\tilde{a}_2, \tilde{a}_4\}),$$
(1.34)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_2 \succeq \tilde{a}_1, \tilde{a}_2 \succeq \tilde{a}_3 \Leftrightarrow \tilde{a}_2 \succeq \mathrm{rmax}\{\tilde{a}_1, \tilde{a}_3\}),$$
(1.35)
$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\} = \tilde{a}_2),$$
(1.36)
$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\tilde{a}_1 \succeq \tilde{a}_2 \Leftrightarrow \mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\} = \tilde{a}_1),$$
(1.37)
$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\mathrm{rmin}\{\tilde{a}_1^C, \tilde{a}_2^C\} = \mathrm{rmax}\{\tilde{a}_1, \tilde{a}_2\}^C),$$
(1.38)
$$(\forall \tilde{a}_1, \tilde{a}_2 \in [[0,1]])(\mathrm{rmax}\{\tilde{a}_1^C, \tilde{a}_2^C\} = \mathrm{rmin}\{\tilde{a}_1, \tilde{a}_2\}^C),$$
(1.39)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \preceq \mathrm{rmax}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \succeq \mathrm{rmin}\{\tilde{a}_2^C, \tilde{a}_3^C\}),$$
(1.40)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \mathrm{rmax}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \preceq \mathrm{rmin}\{\tilde{a}_2^C, \tilde{a}_3^C\}),$$
(1.41)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \preceq \mathrm{rmin}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \succeq \mathrm{rmax}\{\tilde{a}_2^C, \tilde{a}_3^C\}), \text{ and}$$
(1.42)
$$(\forall \tilde{a}_1, \tilde{a}_2, \tilde{a}_3 \in [[0,1]])(\tilde{a}_1 \succeq \mathrm{rmin}\{\tilde{a}_2, \tilde{a}_3\} \Leftrightarrow \tilde{a}_1^C \preceq \mathrm{rmax}\{\tilde{a}_2^C, \tilde{a}_3^C\}).$$

In 1975, Zadeh [**33**] introduced interval-valued fuzzy set as the following definition.

Definition 1.9. An *interval-valued fuzzy set* (briefly, an IVFS) in a nonempty set X is an arbitrary function $A : X \to [[0,1]]$. Let $IVFS(X)$ stands for the set of all IVFS in X. For every $A \in IVFS(X)$ and $x \in X, A(x) = [A^-(x), A^+(x)]$ is called the *degree of membership* of an element x to A, where A^-, A^+ are fuzzy sets in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Definition 1.10. ([**11**]) Let A and B be interval-valued fuzzy sets in a nonempty set X. We define the symbols "$\subseteq$", "$\supseteq$", "$=$" in case of A and B as follows:

$$(\forall x \in X)(A \subseteq B \Leftrightarrow A(x) \preceq B(x)), \tag{1.43}$$

and similarly we may have $A \supseteq B$ and $A = B$.

Definition 1.11. ([**33**]) Let A be an interval-valued fuzzy set in a nonempty set X. The *complement of* A, denoted by A^C, is defined as follows: $A^C(x) = A(x)^C$ for all $x \in X$, that is,

$$(\forall x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x)]). \tag{1.44}$$

We note that $A^{C^-}(x) = 1 - A^+(x)$ and $A^{C^+}(x) = 1 - A^-(x)$ for all $x \in X$.

DEFINITION 1.12. ([**33**]) Let $\{A_i \mid i \in J\}$ be a family of interval-valued fuzzy sets in a nonempty set X. We define the *intersection* and the *union* of $\{A_i \mid i \in J\}$, denoted by $\cap_{i\in J} A_i$ and $\cup_{i\in J} A_i$, respectively, as follows:

$$(\forall x \in X)((\cap_{i\in J} A_i)(x) = \mathrm{rinf}_{i\in J}\{A_i(x)\}), \tag{1.45}$$

$$(\forall x \in X)((\cup_{i\in J} A_i)(x) = \mathrm{rsup}_{i\in J}\{A_i(x)\}). \tag{1.46}$$

We note that

$$(\forall x \in X)((\cap_{i\in J} A_i)^-(x) = (\wedge_{i\in J} A_i^-)(x) = \inf_{i\in J}\{A_i^-(x)\})$$

and

$$(\forall x \in X)((\cap_{i\in J} A_i)^+(x) = (\wedge_{i\in J} A_i^+)(x) = \inf_{i\in J}\{A_i^+(x)\}).$$

Similarly,

$$(\forall x \in X)((\cup_{i\in J} A_i)^-(x) = (\vee_{i\in J} A_i^-)(x) = \sup_{i\in J}\{A_i^-(x)\})$$

and

$$(\forall x \in X)((\cup_{i\in J} A_i)^+(x) = (\vee_{i\in J} A_i^+)(x) = \sup_{i\in J}\{A_i^+(x)\}).$$

In particular, if A_1 and A_2 are interval-valued fuzzy sets in X, we have the intersection and the union of A_1 and A_2 as follows:

$$(\forall x \in X)((A_1 \cap A_2)(x) = \mathrm{rmin}\{A_1(x), A_2(x)\}), \tag{1.47}$$

$$(\forall x \in X)((A_1 \cup A_2)(x) = \mathrm{rmax}\{A_1(x), A_2(x)\}). \tag{1.48}$$

2. Interval-Valued Neutrosophic Sets in UP-Algebras

In 2005, the concept of an interval-valued neutrosophic set was first considered by Wang et al. [**31**] as the following definition.

An *interval-valued neutrosophic set* (briefly, IVNS) in a nonempty set X is a structure of the form:

$$\mathbf{A} := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\},$$

where A_T, A_I, and A_F are interval-valued fuzzy sets in X, which are called a *truth membership function*, an *indeterminacy membership function* and a *falsity membership function*, respectively.

For our convenience, we will denote an IVNS as

$$\mathbf{A} = (X, A_T, A_I, A_F) = (X, A_{T,I,F}) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}.$$

Now, we introduce the notions of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

In what follows, let X denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.

Definition 2.1. An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic UP-subalgebra* of X if it holds the following conditions:

$$(\forall x, y \in X)(A_T(x \cdot y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}), \tag{2.1}$$

$$(\forall x, y \in X)(A_I(x \cdot y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\}), \tag{2.2}$$

$$(\forall x, y \in X)(A_F(x \cdot y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}). \tag{2.3}$$

Proposition 2.1. *If $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X, then*

$$(\forall x \in X)(A_T(0) \succeq A_T(x)), \tag{2.4}$$

$$(\forall x \in X)(A_I(0) \preceq A_I(x)), \tag{2.5}$$

$$(\forall x \in X)(A_F(0) \succeq A_F(x)). \tag{2.6}$$

Proof. Let $\mathbf{A}$ be an interval-valued neutrosophic UP-subalgebra of X. By (1.1), we have

$$(\forall x \in X)\left(\begin{array}{c} A_T(0) = A_T(x \cdot x) \succeq \operatorname{rmin}\{A_T(x), A_T(x)\} = A_T(x), \\ A_I(0) = A_I(x \cdot x) \preceq \operatorname{rmin}\{A_I(x), A_I(x)\} = A_I(x), \\ A_F(0) = A_F(x \cdot x) \succeq \operatorname{rmin}\{A_F(x), A_F(x)\} = A_F(x) \end{array}\right).$$

□

Example 2.1. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	1	0	3
3	0	0	0	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.2, 0.5] & [0.3, 0.4] & [0.3, 0.4] \end{pmatrix},$$

$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0, 0.3] & [0.7, 0.8] & [0.2, 0.3] & [0.8, 0.9] \end{pmatrix},$$

$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.7, 1] & [0.1, 0.3] & [0.5, 0.7] & [0.6, 0.7] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X.

Definition 2.2. An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic near UP-filter* of X if it holds the following conditions: (2.4), (2.5), (2.6), and

$$(\forall x, y \in X)(A_T(x \cdot y) \succeq A_T(y)), \tag{2.7}$$

$$(\forall x, y \in X)(A_I(x \cdot y) \preceq A_I(y)), \tag{2.8}$$

$$(\forall x, y \in X)(A_F(x \cdot y) \succeq A_F(y)). \tag{2.9}$$

EXAMPLE 2.2. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	0
2	0	1	0	3
3	0	1	2	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.6, 0.8] & [0.5, 0.6] & [0.4, 0.6] \end{pmatrix},$$
$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0, 0.1] & [0.1, 0.3] & [0.3, 0.4] & [0.5, 0.8] \end{pmatrix},$$
$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.8, 0.9] & [0.6, 0.8] & [0.5, 0.7] & [0.4, 0.6] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X.

DEFINITION 2.3. An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic UP-filter* of X if it holds the following conditions: (2.4), (2.5), (2.6), and

$$(\forall x, y \in X)(A_T(y) \succeq \mathrm{rmin}\{A_T(x \cdot y), A_T(x)\}), \tag{2.10}$$

$$(\forall x, y \in X)(A_I(y) \preceq \mathrm{rmax}\{A_I(x \cdot y), A_I(x)\}), \tag{2.11}$$

$$(\forall x, y \in X)(A_F(y) \succeq \mathrm{rmin}\{A_F(x \cdot y), A_F(x)\}). \tag{2.12}$$

EXAMPLE 2.3. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	3	3
2	0	1	0	0
3	0	1	2	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.5, 0.8] & [0.3, 0.6] & [0.3, 0.6] \end{pmatrix},$$
$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0, 0.1] & [0.2, 0.3] & [0.6, 0.8] & [0.6, 0.8] \end{pmatrix},$$
$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.8, 0.9] & [0.4, 0.5] & [0.3, 0.4] & [0.3, 0.4] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X.

DEFINITION 2.4. An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic UP-ideal* of X if it holds the following conditions: (2.4), (2.5), (2.6), and

$$(\forall x, y, z \in X)(A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(x)\}), \tag{2.13}$$

$$(\forall x, y, z \in X)(A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(x)\}), \tag{2.14}$$

$$(\forall x, y, z \in X)(A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(x)\}). \tag{2.15}$$

EXAMPLE 2.4. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	0
3	0	0	2	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.9, 1] & [0.7, 0.9] & [0.6, 0.8] & [0.6, 0.9] \end{pmatrix},$$

$$A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.1, 0.3] & [0.3, 0.5] & [0.4, 0.7] & [0.3, 0.6] \end{pmatrix},$$

$$A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\ [0.8, 0.9] & [0.5, 0.9] & [0.4, 0.6] & [0.5, 0.8] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X.

DEFINITION 2.5. An IVNS $\mathbf{A}$ in X is called an *interval-valued neutrosophic strong UP-ideal* of X if it holds the following conditions: (2.4), (2.5), (2.6), and

$$(\forall x, y, z \in X)(A_T(x) \succeq \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\}), \tag{2.16}$$

$$(\forall x, y, z \in X)(A_I(x) \preceq \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\}), \tag{2.17}$$

$$(\forall x, y, z \in X)(A_F(x) \succeq \text{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\}). \tag{2.18}$$

EXAMPLE 2.5. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	1	2	0

We define an IVNS $\mathbf{A}$ in X as follows:

$$(\forall x \in X) \begin{pmatrix} A_T(x) = [0.7, 0.9] \\ A_I(x) = [0.3, 0.5] \\ A_F(x) = [0.5, 0.9] \end{pmatrix}.$$

Then $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X.

Definition 2.6. An IVNS $\mathbf{A}$ in a nonempty set X is said to be *constant* if $\mathbf{A}$ is a constant function from X to $[[0,1]]^3$. That is, A_T, A_I, and A_F are constant functions from X to $[[0,1]]$.

Theorem 2.1. *An IVNS $\mathbf{A}$ in X is constant if and only if it is an interval-valued neutrosophic strong UP-ideal of X.*

Proof. Assume that an IVNS $\mathbf{A}$ is constant in X. Then $A_T(x) = A_T(0)$, $A_I(x) = A_I(0)$, and $A_F(x) = A_F(0)$ for all $x \in X$. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$, and for all $x, y, z \in X$,

$$
\begin{aligned}
\operatorname{rmin}\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\} &= \operatorname{rmin}\{A_T(0), A_T(0)\} \\
((1.27)) \qquad &= A_T(0) \\
&= A_T(x), \\
\operatorname{rmax}\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\} &= \operatorname{rmax}\{A_I(0), A_I(0)\} \\
((1.27)) \qquad &= A_I(0) \\
&= A_I(x), \\
\operatorname{rmin}\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\} &= \operatorname{rmin}\{A_F(0), A_F(0)\} \\
((1.27)) \qquad &= A_F(0) \\
&= A_F(x).
\end{aligned}
$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X.

Conversely, assume that $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. Then for all $x \in X$,

$$
\begin{aligned}
A_T(x) &\succeq \operatorname{rmin}\{A_T((x \cdot 0) \cdot (x \cdot x)), A_T(0)\} \\
((\text{UP-3})) \qquad &= \operatorname{rmin}\{A_T(0 \cdot (x \cdot x)), A_T(0)\} \\
((\text{UP-2})) \qquad &= \operatorname{rmin}\{A_T(x \cdot x), A_T(0)\} \\
((1.1)) \qquad &= \operatorname{rmin}\{A_T(0), A_T(0)\} \\
((1.27)) \qquad &= A_T(0) \\
&\succeq A_T(x),
\end{aligned}
$$

$$
\begin{aligned}
A_I(x) &\preceq \operatorname{rmax}\{A_I((x \cdot 0) \cdot (x \cdot x)), A_I(0)\} \\
((\text{UP-3})) \qquad &= \operatorname{rmax}\{A_I(0 \cdot (x \cdot x)), A_I(0)\} \\
((\text{UP-2})) \qquad &= \operatorname{rmax}\{A_I(x \cdot x), A_I(0)\} \\
((1.1)) \qquad &= \operatorname{rmax}\{A_I(0), A_I(0)\} \\
((1.27)) \qquad &= A_I(0) \\
&\preceq A_I(x),
\end{aligned}
$$

$$
\begin{array}{lrl}
 & A_F(x) & \succeq \operatorname{rmin}\{A_F((x\cdot 0)\cdot(x\cdot x)), A_F(0)\} \\
((\text{UP-3})) & & = \operatorname{rmin}\{A_F(0\cdot(x\cdot x)), A_F(0)\} \\
((\text{UP-2})) & & = \operatorname{rmin}\{A_F(x\cdot x), A_F(0)\} \\
((1.1)) & & = \operatorname{rmin}\{A_F(0), A_F(0)\} \\
((1.27)) & & = A_F(0) \\
 & & \succeq A_F(x).
\end{array}
$$

Thus $A_T(0) = A_T(x), A_I(0) = A_I(x)$, and $A_F(0) = A_F(x)$ for all $x \in X$. Hence, $\mathbf{A}$ is constant. □

THEOREM 2.2. *Every interval-valued neutrosophic strong UP-ideal of X is an interval-valued neutrosophic UP-ideal.*

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y, z \in X$. Then

$$
\begin{array}{lrl}
 & A_T(x\cdot z) & \succeq \operatorname{rmin}\{A_T((z\cdot y)\cdot(z\cdot(x\cdot z))), A_T(y)\} \\
((1.5)) & & = \operatorname{rmin}\{A_T((z\cdot y)\cdot 0), A_T(y)\} \\
((\text{UP-3})) & & = \operatorname{rmin}\{A_T(0), A_T(y)\} \\
 & & = A_T(y) \\
 & & \succeq \operatorname{rmin}\{A_T(x\cdot(y\cdot z)), A_T(y)\}, \\
 & A_I(x\cdot z) & \preceq \operatorname{rmax}\{A_I((z\cdot y)\cdot(z\cdot(x\cdot z))), A_I(y)\} \\
((1.5)) & & = \operatorname{rmax}\{A_I((z\cdot y)\cdot 0), A_I(y)\} \\
((\text{UP-3})) & & = \operatorname{rmax}\{A_I(0), A_I(y)\} \\
 & & = A_I(y) \\
 & & \preceq \operatorname{rmax}\{A_T(x\cdot(y\cdot z)), A_T(y)\}, \\
 & A_F(x\cdot z) & \succeq \operatorname{rmin}\{A_F((z\cdot y)\cdot(z\cdot(x\cdot z))), A_F(y)\} \\
((1.5)) & & = \operatorname{rmin}\{A_F((z\cdot y)\cdot 0), A_F(y)\} \\
((\text{UP-3})) & & = \operatorname{rmin}\{A_F(0), A_F(y)\} \\
 & & = A_F(y) \\
 & & \succeq \operatorname{rmin}\{A_F(x\cdot(y\cdot z)), A_F(y)\}.
\end{array}
$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

The following example show that the converse of Theorem 2.2 is not true.

EXAMPLE 2.6. From Example 2.4, we have $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. Since $A_T(1) = [0.7, 0.9] \not\succeq [0.9, 1] = \operatorname{rmin}\{A_T((2\cdot 0)\cdot(2\cdot 1)), A_T(0)\}$, we have $\mathbf{A}$ is not an interval-valued neutrosophic strong UP-ideal of X.

THEOREM 2.3. *Every interval-valued neutrosophic UP-ideal of X is an interval-valued neutrosophic UP-filter.*

PROOF. Assume that **A** is an interval-valued neutrosophic UP-ideal of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$. Then

$$
\begin{aligned}
&((\text{UP-2})) & A_T(y) &= A_T(0 \cdot y) \\
& & &\succeq \text{rmin}\{A_T(0 \cdot (x \cdot y)), A_T(x)\} \\
&((\text{UP-2})) & &= \text{rmin}\{A_T(x \cdot y), A_T(x)\}, \\
&((\text{UP-2})) & A_I(y) &= A_I(0 \cdot y) \\
& & &\preceq \text{rmax}\{A_I(0 \cdot (x \cdot y)), A_I(x)\} \\
&((\text{UP-2})) & &= \text{rmax}\{A_I(x \cdot y), A_I(x)\}, \\
&((\text{UP-2})) & A_F(y) &= A_F(0 \cdot y) \\
& & &\succeq \text{rmin}\{A_F(0 \cdot (x \cdot y)), A_F(x)\} \\
&((\text{UP-2})) & &= \text{rmin}\{A_F(x \cdot y), A_F(x)\}.
\end{aligned}
$$

Hence, **A** is an interval-valued neutrosophic UP-filter of X. $\square$

The following example show that the converse of Theorem 2.3 is not true.

EXAMPLE 2.7. From Example 2.3, we have **A** is an interval-valued neutrosophic UP-filter of X. Since $A_I(3 \cdot 2) = [0.6, 0.8] \npreceq [0.2, 0.3] = \text{rmax}\{A_I(3 \cdot (1 \cdot 2)), A_I(1)\}$, we have **A** is not an interval-valued neutrosophic UP-ideal of X.

THEOREM 2.4. *Every interval-valued neutrosophic UP-filter of X is an interval-valued neutrosophic near UP-filter.*

PROOF. Assume that **A** is an interval-valued neutrosophic UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$. Then

$$
\begin{aligned}
& & A_T(x \cdot y) &\succeq \text{rmin}\{A_T(y \cdot (x \cdot y)), A_T(y)\} \\
&((1.5)) & &= \text{rmin}\{A_T(0), A_T(y)\} \\
& & &= A_T(y), \\
& & A_I(x \cdot y) &\preceq \text{rmax}\{A_I(y \cdot (x \cdot y)), A_I(y)\} \\
&((1.5)) & &= \text{rmax}\{A_I(0), A_I(y)\} \\
& & &= A_I(y), \\
& & A_F(x \cdot y) &\succeq \text{rmin}\{A_F(y \cdot (x \cdot y)), A_F(y)\} \\
&((1.5)) & &= \text{rmin}\{A_F(0), A_F(y)\} \\
& & &= A_F(y).
\end{aligned}
$$

Hence, **A** is an interval-valued neutrosophic near UP-filter of X. $\square$

The following example show that the converse of Theorem 2.4 is not true.

EXAMPLE 2.8. From Example 2.2, we have **A** is an interval-valued neutrosophic near UP-filter of X. Since $A_F(3) = [0.4, 0.6] \nsucceq [0.6, 0.8] = \text{rmin}\{A_F(1 \cdot 3), A_F(1)\}$, we have **A** is not an interval-valued neutrosophic UP-filter of X.

THEOREM 2.5. *Every interval-valued neutrosophic near UP-filter of X is an interval-valued neutrosophic UP-subalgebra.*

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X. Then for all $x \in X, A_T(0) \succeq A_T(x), A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$. By (1.29), we have

$$\begin{aligned} A_T(x \cdot y) &\succeq A_T(y) \succeq \text{rmin}\{A_T(x), A_T(y)\}, \\ A_I(x \cdot y) &\preceq A_I(y) \preceq \text{rmax}\{A_I(x), A_I(y)\}, \\ A_F(x \cdot y) &\succeq A_F(y) \succeq \text{rmin}\{A_F(x), A_F(y)\}. \end{aligned}$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. □

The following example show that the converse of Theorem 2.5 is not true.

EXAMPLE 2.9. From Example 2.1, we have $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. Since $A_F(1 \cdot 3) = [0.5, 0.7] \not\succeq [0.6, 0.8] = A_F(3)$, we have $\mathbf{A}$ is not an interval-valued neutrosophic near UP-filter of X.

By Theorems 2.2, 2.3, 2.4, and 2.5 and Examples 2.6, 2.7, 2.8, and 2.9, we have that the notion of interval-valued neutrosophic UP-subalgebras is a generalization of interval-valued neutrosophic near UP-filters, interval-valued neutrosophic near UP-filters is a generalization of interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-filters is a generalization of interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic UP-ideals is a generalization of interval-valued neutrosophic strong UP-ideals. Moreover, by Theorem 2.1, we obtain that interval-valued neutrosophic strong UP-ideals and constant interval-valued neutrosophic set coincide.

THEOREM 2.6. *If $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X satisfying the following condition:*

$$(\forall x, y \in X)\left(x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \end{cases}\right), \tag{2.19}$$

then $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X.

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X satisfying the condition (2.19). By Theorem 2.1, we have $\mathbf{A}$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$A_T(x \cdot y) = A_T(0) \succeq A_T(y), \tag{(2.4)}$$

$$A_I(x \cdot y) = A_I(0) \preceq A_I(y), \tag{(2.5)}$$

$$A_F(x \cdot y) = A_F(0) \succeq A_F(y). \tag{(2.6)}$$

Case 2: $x \cdot y \neq 0$. By (2.19), it follows that

$$
\begin{aligned}
&((2.1)) && A_T(x \cdot y) \succeq \text{rmin}\{A_T(x), A_T(y)\} \\
&((1.35)) && = A_T(y), \\
&((2.2)) && A_I(x \cdot y) \preceq \text{rmax}\{A_I(x), A_I(y)\} \\
&((1.36)) && = A_I(y), \\
&((2.3)) && A_F(x \cdot y) \succeq \text{rmin}\{A_F(x), A_F(y)\} \\
&((1.35)) && = A_F(y).
\end{aligned}
$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X. $\square$

THEOREM 2.7. *If* $\mathbf{A}$ *is an interval-valued neutrosophic near UP-filter of* X *satisfying the following condition:*

$$A_T = A_I = A_F, \tag{2.20}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-filter of* X.

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X satisfying the condition (2.20). Then $\mathbf{A}$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y \in X$. Then

$$
\begin{aligned}
&((2.20)) && \text{rmin}\{A_T(x \cdot y), A_T(x)\} = \text{rmin}\{A_I(x \cdot y), A_T(x)\} \\
&((2.8)) && \preceq \text{rmin}\{A_I(y), A_T(x)\} \\
&((2.20)) && = \text{rmin}\{A_T(y), A_T(x)\} \\
& && \preceq A_T(y), \\
&((2.20)) && \text{rmax}\{A_I(x \cdot y), A_I(x)\} = \text{rmax}\{A_T(x \cdot y), A_I(x)\} \\
&((2.7)) && \succeq \text{rmax}\{A_T(y), A_I(x)\} \\
&((2.20)) && = \text{rmax}\{A_I(y), A_I(x)\} \\
& && \succeq A_I(y), \\
&((2.20)) && \text{rmin}\{A_F(x \cdot y), A_F(x)\} = \text{rmin}\{A_I(x \cdot y), A_F(x)\} \\
&((2.8)) && \preceq \text{rmin}\{A_I(y), A_F(x)\} \\
&((2.20)) && = \text{rmin}\{A_F(y), A_F(x)\} \\
& && \preceq A_F(y).
\end{aligned}
$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X. $\square$

THEOREM 2.8. *If* $\mathbf{A}$ *is an interval-valued neutrosophic UP-filter of* X *satisfying the following condition:*

$$(\forall x, y, z \in X)\begin{pmatrix} A_T(y \cdot (x \cdot z)) = A_T(x \cdot (y \cdot z)) \\ A_I(y \cdot (x \cdot z)) = A_I(x \cdot (y \cdot z)) \\ A_F(y \cdot (x \cdot z)) = A_F(x \cdot (y \cdot z)) \end{pmatrix}, \tag{2.21}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-ideal of* X.

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X satisfying the condition (2.21). Then $\mathbf{A}$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y, z \in X$. Then

$$\begin{aligned}
&((2.10)) && A_T(x \cdot z) \succeq \mathrm{rmin}\{A_T(y \cdot (x \cdot z)), A_T(y)\} \\
&((2.21) \text{ for } A_T) && \qquad = \mathrm{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \\
&((2.11)) && A_I(x \cdot z) \preceq \mathrm{rmax}\{A_I(y \cdot (x \cdot z)), A_I(y)\} \\
&((2.21) \text{ for } A_I) && \qquad = \mathrm{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}, \\
&((2.12)) && A_F(x \cdot z) \succeq \mathrm{rmin}\{A_F(y \cdot (x \cdot z)), A_F(y)\} \\
&((2.21) \text{ for } A_F) && \qquad = \mathrm{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}.
\end{aligned}$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

Theorem 2.9. *If $\mathbf{A}$ is an IVNS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leqslant x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq \mathrm{rmin}\{A_T(x), A_T(y)\} \\ A_I(z) \preceq \mathrm{rmax}\{A_I(x), A_I(y)\} \\ A_F(z) \succeq \mathrm{rmin}\{A_F(x), A_F(y)\} \end{cases}\right), \tag{2.22}$$

then $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X.

Proof. Assume that $\mathbf{A}$ is an IVNS in X satisfying the condition (2.22). Let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leqslant x \cdot y$. It follows from (2.22) that

$$\begin{aligned}
A_T(x \cdot y) &\succeq \mathrm{rmin}\{A_T(x), A_T(y)\}, \\
A_I(x \cdot y) &\preceq \mathrm{rmax}\{A_I(x), A_I(y)\}, \\
A_F(x \cdot y) &\succeq \mathrm{rmin}\{A_F(x), A_F(y)\}.
\end{aligned}$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. □

Theorem 2.10. *If A is an IVNS in X satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leqslant x \cdot y \Rightarrow \begin{cases} A_T(z) \succeq A_T(y) \\ A_I(z) \preceq A_I(y) \\ A_F(z) \succeq A_F(y) \end{cases}\right), \tag{2.23}$$

then $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X.

Proof. Assume that $\mathbf{A}$ is an IVNS in X satisfying the condition (2.23). Let $x \in X$. By (UP-2) and (1.1), we have $0 \cdot (x \cdot x) = 0$, that is, $0 \leqslant x \cdot x$. It follows from (2.23) that $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Next, let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leqslant x \cdot y$. It follows from (2.23) that $A_T(x \cdot y) \succeq A_T(y), A_I(x \cdot y) \preceq A_I(y)$, and $A_F(x \cdot y) \succeq A_F(y)$. Hence, $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X. □

THEOREM 2.11. *If* $\mathbf{A}$ *is an IVNS in* X *satisfying the following condition:*

$$(\forall x, y, z \in X)\left(z \leqslant x \cdot y \Rightarrow \begin{cases} A_T(y) \succeq \operatorname{rmin}\{A_T(z), A_T(x)\} \\ A_I(y) \preceq \operatorname{rmax}\{A_I(z), A_I(x)\} \\ A_F(y) \succeq \operatorname{rmin}\{A_F(z), A_F(x)\} \end{cases}\right), \tag{2.24}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-filter of* X.

PROOF. Assume that $\mathbf{A}$ is an IVNS in X satisfying the condition (2.24). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leqslant x \cdot 0$. It follows from (2.24) and (1.27) that

$$\begin{aligned} A_T(0) &\succeq \operatorname{rmin}\{A_T(x), A_T(x)\} = A_T(x), \\ A_I(0) &\preceq \operatorname{rmax}\{A_I(x), A_I(x)\} = A_I(x), \\ A_F(0) &\succeq \operatorname{rmin}\{A_F(x), A_F(x)\} = A_F(x). \end{aligned}$$

Next, let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leqslant x \cdot y$. It follows from (2.24) that

$$\begin{aligned} A_T(y) &\succeq \operatorname{rmin}\{A_T(x \cdot y), A_T(x)\}, \\ A_I(y) &\preceq \operatorname{rmax}\{A_I(x \cdot y), A_I(x)\}, \\ A_F(y) &\succeq \operatorname{rmin}\{A_F(x \cdot y), A_F(x)\}. \end{aligned}$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X. □

THEOREM 2.12. *If* $\mathbf{A}$ *is an IVNS in* X *satisfying the following condition:*

$$(\forall a, x, y, z \in X)\left(a \leqslant x \cdot (y \cdot z) \Rightarrow \begin{cases} A_T(x \cdot z) \succeq \operatorname{rmin}\{A_T(a), A_T(y)\} \\ A_I(x \cdot z) \preceq \operatorname{rmax}\{A_I(a), A_I(y)\} \\ A_F(x \cdot z) \succeq \operatorname{rmin}\{A_F(a), A_F(y)\} \end{cases}\right), \tag{2.25}$$

then $\mathbf{A}$ *is an interval-valued neutrosophic UP-ideal of* X.

PROOF. Assume that $\mathbf{A}$ is an IVNS in X satisfying the condition (2.25). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leqslant 0 \cdot (x \cdot 0)$. It follows from (2.25) and (1.27) that

$$A_T(0) = A_T(0 \cdot 0) \succeq \operatorname{rmin}\{A_T(x), A_T(x)\} = A_T(x), \tag{(UP-2)}$$

$$A_I(0) = A_I(0 \cdot 0) \preceq \operatorname{rmax}\{A_I(x), A_I(x)\} = A_I(x), \tag{(UP-2)}$$

$$A_F(0) = A_F(0 \cdot 0) \succeq \operatorname{rmin}\{A_F(x), A_F(x)\} = A_F(x). \tag{(UP-2)}$$

Next, let $x, y, z \in X$. By (1.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leqslant x \cdot (y \cdot z)$. It follows from (2.25) that

$$\begin{aligned} A_T(x \cdot z) &\succeq \operatorname{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \\ A_I(x \cdot z) &\preceq \operatorname{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}, \\ A_F(x \cdot z) &\succeq \operatorname{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}. \end{aligned}$$

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. □

For any fixed interval numbers $\tilde{a}^+, \tilde{a}^-, \tilde{b}^+, \tilde{b}^-, \tilde{c}^+, \tilde{c}^- \in [[0,1]]$ such that $\tilde{a}^+ \succ \tilde{a}^-, \tilde{b}^+ \succ \tilde{b}^-, \tilde{c}^+ \succ \tilde{c}^-$ and a nonempty subset G of X, the IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}] = (X, A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}], A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}], A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}])$ in X, where $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}], A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}]$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}]$ are IVFSs in X which are given as follows:

$$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x) = \begin{cases} \tilde{a}^+ & \text{if } x \in G, \\ \tilde{a}^- & \text{otherwise,} \end{cases}$$

$$A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x) = \begin{cases} \tilde{b}^- & \text{if } x \in G, \\ \tilde{b}^+ & \text{otherwise,} \end{cases}$$

$$A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x) = \begin{cases} \tilde{c}^+ & \text{if } x \in G, \\ \tilde{c}^- & \text{otherwise.} \end{cases}$$

Lemma 2.1. *If the constant* 0 *of* X *is in a nonempty subset* G *of* X*, then the IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *satisfies the conditions* (2.4)*,* (2.5)*, and* (2.6)*.*

Proof. If $0 \in G$, then $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](0) = \tilde{a}^+, A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](0) = \tilde{b}^-$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](0) = \tilde{c}^+$. Thus

$$(\forall x \in X)\begin{pmatrix} A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](0) = \tilde{a}^+ \succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x) \\ A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](0) = \tilde{b}^- \preceq A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x) \\ A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](0) = \tilde{c}^+ \succeq A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x) \end{pmatrix}.$$

Hence, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (2.4), (2.5), and (2.6). $\square$

Lemma 2.2. *If the IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *satisfies the condition* (2.4) *(resp.,* (2.5)*,* (2.6)*), then the constant* 0 *of* X *is in a nonempty subset* G *of* X*.*

Proof. Assume that the IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ in X satisfies the condition (2.4). Then $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](0) \succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)$ for all $x \in X$. Since G is nonempty, there exists $g \in G$. Thus $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](g) = \tilde{a}^+$ and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](0) \succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](g) = \tilde{a}^+ \succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](0)$, that is, $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](0) = \tilde{a}^+$. Hence, $0 \in G$. $\square$

Theorem 2.13. *The IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *is an interval-valued neutrosophic UP-subalgebra of* X *if and only if a nonempty subset* G *of* X *is a UP-subalgebra of* X*.*

Proof. Assume that $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-subalgebra of X. Let $x, y \in G$. Then $A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x) = \tilde{a}^+ = A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)$. Thus

$$
\begin{aligned}
&((2.1)) & A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) &\succeq \operatorname{rmin}\{A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x), A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)\} \\
& & &= \operatorname{rmin}\{\tilde{a}^+, \tilde{a}^+\} \\
&((1.27)) & &= \tilde{a}^+ \\
& & &\succeq A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y)
\end{aligned}
$$

and so $A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+$. Thus $x \cdot y \in G$. Hence, G is a UP-subalgebra of X.

Conversely, assume that G is a UP-subalgebra of X. Let $x, y \in X$.

Case 1: $x, y \in G$. Then

$$
\begin{aligned}
A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x) &= \tilde{a}^+ = A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y), \\
A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x) &= \tilde{b}^- = A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](y), \\
A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x) &= \tilde{c}^+ = A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](y).
\end{aligned}
$$

Since G is a UP-subalgebra of X, we have $x \cdot y \in G$ and so $A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+, A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) = \tilde{b}^-$, and $A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) = \tilde{c}^+$. By (1.27), it follows that

$$
\begin{aligned}
A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) &= \tilde{a}^+ \succeq \tilde{a}^+ = \operatorname{rmin}\{\tilde{a}^+, \tilde{a}^+\} = \operatorname{rmin}\{A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x), A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)\}, \\
A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) &= \tilde{b}^- \preceq \tilde{b}^- = \operatorname{rmax}\{\tilde{b}^-, \tilde{b}^-\} = \operatorname{rmax}\{A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x), A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](y)\}, \\
A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) &= \tilde{c}^+ \succeq \tilde{c}^+ = \operatorname{rmin}\{\tilde{c}^+, \tilde{c}^+\} = \operatorname{rmin}\{A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x), A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.
\end{aligned}
$$

Case 2: $x \notin G$ or $y \notin G$. Then

$$
\begin{aligned}
A^G_T[{}^{\tilde{a}^-}_{\tilde{a}^-}](x) &= \tilde{a}^- \text{ or } A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^-, \\
A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x) &= \tilde{b}^+ \text{ or } A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^+, \\
A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x) &= \tilde{c}^- \text{ or } A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^-.
\end{aligned}
$$

By (1.27), it follows that

$$
\begin{aligned}
\operatorname{rmin}\{A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x), A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)\} &= \operatorname{rmin}\{\tilde{a}^-, \tilde{a}^-\} = \tilde{a}^-, \\
\operatorname{rmax}\{A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x), A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](y)\} &= \operatorname{rmax}\{\tilde{b}^+, \tilde{b}^+\} = \tilde{b}^+, \\
\operatorname{rmin}\{A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x), A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](y)\} &= \operatorname{rmin}\{\tilde{c}^-, \tilde{c}^-\} = \tilde{c}^-.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) &\succeq \tilde{a}^- = \operatorname{rmin}\{A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](x), A^G_T[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)\}, \\
A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) &\preceq \tilde{b}^+ = \operatorname{rmax}\{A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](x), A^G_I[{}^{\tilde{b}^-}_{\tilde{b}^+}](y)\}, \\
A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) &\succeq \tilde{c}^- = \operatorname{rmin}\{A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](x), A^G_F[{}^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.
\end{aligned}
$$

Hence, $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-subalgebra of X. $\square$

THEOREM 2.14. *The IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *is an interval-valued neutrosophic near UP-filter of* X *if and only if a nonempty subset* G *of* X *is a near UP-filter of* X.

PROOF. Assume that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic near UP-filter of X. Since $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the condition (2.4), it follows from Lemma 2.2 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+$. By (2.7)

$$A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) \succeq A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+ \succeq A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y)$$

and so $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+$. Thus $x \cdot y \in G$. Hence, G is a near UP-filter of X.

Conversely, assume that G is a near UP-filter of X. Since $0 \in G$, it follows from Lemma 2.1 that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^+, A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^-$, and $A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^+$. Since G is a near UP-filter of X, we have $x \cdot y \in G$ and so $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) = \tilde{a}^+, A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) = \tilde{b}^-$, and $A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) = \tilde{c}^+$. Thus

$$\begin{aligned}
A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) &= \tilde{a}^+ \succeq \tilde{a}^+ = A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y),\\
A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) &= \tilde{b}^- \preceq \tilde{b}^- = A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y),\\
A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) &= \tilde{c}^+ \succeq \tilde{c}^+ = A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y).
\end{aligned}$$

Case 2: $y \notin G$. Then $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^-, A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^+$, and $A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^-$. Thus

$$\begin{aligned}
A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot y) &\succeq \tilde{a}^- = A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y),\\
A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot y) &\preceq \tilde{b}^+ = A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y),\\
A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot y) &\succeq \tilde{c}^- = A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y).
\end{aligned}$$

Hence, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic near UP-filter of X. □

THEOREM 2.15. *The IVNS* $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ *in* X *is an interval-valued neutrosophic UP-filter of* X *if and only if a nonempty subset* G *of* X *is a UP-filter of* X.

PROOF. Assume that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-filter of X. Since $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the condition (2.4), it follows from Lemma 2.2 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then

$A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y)=\tilde{a}^+=A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)$. Thus

$$\begin{aligned}
((2.10)) \qquad A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y) &\succeq \operatorname{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y), A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)\}\\
&= \operatorname{rmin}\{\tilde{a}^+,\tilde{a}^+\}\\
((1.27)) \qquad &= \tilde{a}^+\\
&\succeq A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)
\end{aligned}$$

and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)=\tilde{a}^+$. Thus $y\in G$. Hence, G is a UP-filter of X.

Conversely, assume that G is a UP-filter of X. Since $0\in G$, it follows from Lemma 2.1 that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x,y\in X$.

Case 1: $x\cdot y\in G$ and $x\in G$. Then

$$\begin{aligned}
A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y)&=\tilde{a}^+=A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x),\\
A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y)&=\tilde{b}^-=A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x),\\
A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y)&=\tilde{c}^+=A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x).
\end{aligned}$$

Since G is a UP-filter of X, we have $y\in G$ and so $A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)=\tilde{a}^+$, $A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)=\tilde{b}^-$, and $A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)=\tilde{c}^+$. By (1.27), it follows that

$$\begin{aligned}
A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)=\tilde{a}^+\succeq\tilde{a}^+=\operatorname{rmin}\{\tilde{a}^+,\tilde{a}^+\}=\operatorname{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y),A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)\},\\
A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)=\tilde{b}^-\preceq\tilde{b}^-=\operatorname{rmax}\{\tilde{b}^-,\tilde{b}^-\}=\operatorname{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y),A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x)\},\\
A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)=\tilde{c}^+\succeq\tilde{c}^+=\operatorname{rmin}\{\tilde{c}^+,\tilde{c}^+\}=\operatorname{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y),A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x)\}.
\end{aligned}$$

Case 2: $x\cdot y\notin G$ or $x\notin G$. Then

$$\begin{aligned}
A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y)=\tilde{a}^- \text{ or } A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)=\tilde{a}^-,\\
A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y)=\tilde{b}^+ \text{ or } A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x)=\tilde{b}^+,\\
A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y)=\tilde{c}^- \text{ or } A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x)=\tilde{c}^-.
\end{aligned}$$

By (1.27), it follows that

$$\begin{aligned}
\operatorname{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y),A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)\}=\operatorname{rmin}\{\tilde{a}^-,\tilde{a}^-\}=\tilde{a}^-,\\
\operatorname{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y),A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x)\}=\operatorname{rmax}\{\tilde{b}^+,\tilde{b}^+\}=\tilde{b}^+,\\
\operatorname{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y),A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x)\}=\operatorname{rmin}\{\tilde{c}^-,\tilde{c}^-\}=\tilde{c}^-.
\end{aligned}$$

Therefore,

$$\begin{aligned}
A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](y)\succeq\tilde{a}^-=\operatorname{rmin}\{A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot y),A_T^G[^{\tilde{a}^+}_{\tilde{a}^-}](x)\},\\
A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](y)\preceq\tilde{b}^+=\operatorname{rmax}\{A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot y),A_I^G[^{\tilde{b}^-}_{\tilde{b}^+}](x)\},\\
A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](y)\succeq\tilde{c}^-=\operatorname{rmin}\{A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot y),A_F^G[^{\tilde{c}^+}_{\tilde{c}^-}](x)\}.
\end{aligned}$$

Hence, $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-filter of X. □

THEOREM 2.16. *The IVNS $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ in X is an interval-valued neutrosophic UP-ideal of X if and only if a nonempty subset G of X is a UP-ideal of X.*

PROOF. Assume that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-ideal of X. Since $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the condition (2.4), it follows from Lemma 2.2 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in G$ and $y \in G$. Then $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)) = \tilde{a}^+ = A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y)$. Thus

$$
\begin{aligned}
&((2.13)) && A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) \succeq \mathrm{rmin}\{A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)), A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y)\} \\
& && = \mathrm{rmin}\{\tilde{a}^+, \tilde{a}^+\} \\
&((1.27)) && = \tilde{a}^+ \\
& && \succeq A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z)
\end{aligned}
$$

and so $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) = \tilde{a}^+$. Thus $x \cdot z \in G$. Hence, G is a UP-ideal of X.

Conversely, assume that G is a UP-ideal of X. Since $0 \in G$, it follows from Lemma 2.1 that $\mathbf{A}^G[^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y, z \in X$.

Case 1: $x \cdot (y \cdot z) \in G$ and $y \in G$. Then

$$
\begin{aligned}
A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)) &= \tilde{a}^+ = A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y), \\
A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)) &= \tilde{b}^- = A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y), \\
A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)) &= \tilde{c}^+ = A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y).
\end{aligned}
$$

Since G is a UP-ideal of X, we have $x \cdot z \in G$ and so $A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) = \tilde{a}^+$, $A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot z) = \tilde{b}^-$, and $A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot z) = \tilde{c}^+$. By (1.27), it follows that

$$
\begin{aligned}
A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot z) = \tilde{a}^+ \succeq \tilde{a}^+ = \mathrm{rmin}\{\tilde{a}^+, \tilde{a}^+\} = \mathrm{rmin}\{A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)), A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y)\}, \\
A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot z) = \tilde{b}^- \preceq \tilde{b}^- = \mathrm{rmax}\{\tilde{b}^-, \tilde{b}^-\} = \mathrm{rmax}\{A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)), A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y)\}, \\
A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot z) = \tilde{c}^+ \succeq \tilde{c}^+ = \mathrm{rmin}\{\tilde{c}^+, \tilde{c}^+\} = \mathrm{rmin}\{A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)), A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y)\}.
\end{aligned}
$$

Case 2: $x \cdot (y \cdot z) \notin G$ or $y \notin G$. Then

$$
\begin{aligned}
A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](x \cdot (y \cdot z)) = \tilde{a}^- \text{ or } A^G_T[^{\tilde{a}^+}_{\tilde{a}^-}](y) = \tilde{a}^-, \\
A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](x \cdot (y \cdot z)) = \tilde{b}^+ \text{ or } A^G_I[^{\tilde{b}^-}_{\tilde{b}^+}](y) = \tilde{b}^+, \\
A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](x \cdot (y \cdot z)) = \tilde{c}^- \text{ or } A^G_F[^{\tilde{c}^+}_{\tilde{c}^-}](y) = \tilde{c}^-.
\end{aligned}
$$

By (1.27), it follows that

$$\begin{aligned} \operatorname{rmin}\{A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot(y\cdot z)), A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)\} &= \operatorname{rmin}\{\tilde{a}^-, \tilde{a}^-\} = \tilde{a}^-,\\ \operatorname{rmax}\{A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot(y\cdot z)), A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](y)\} &= \operatorname{rmax}\{\tilde{b}^+, \tilde{b}^+\} = \tilde{b}^+,\\ \operatorname{rmin}\{A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot(y\cdot z)), A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](y)\} &= \operatorname{rmin}\{\tilde{c}^-, \tilde{c}^-\} = \tilde{c}^-. \end{aligned}$$

Therefore,

$$\begin{aligned} A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot z) \succeq \tilde{a}^- &= \operatorname{rmin}\{A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x\cdot(y\cdot z)), A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](y)\},\\ A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot z) \preceq \tilde{b}^+ &= \operatorname{rmax}\{A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](x\cdot(y\cdot z)), A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](y)\},\\ A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot z) \succeq \tilde{c}^- &= \operatorname{rmin}\{A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](x\cdot(y\cdot z)), A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](y)\}. \end{aligned}$$

Hence, $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic UP-ideal of X. □

THEOREM 2.17. *The IVNS $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ in X is an interval-valued neutrosophic strong UP-ideal of X if and only if a nonempty subset G of X is a strong UP-ideal of X.*

PROOF. Assume that $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic strong UP-ideal of X. By Theorem 2.1, we have $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is constant, that is, $A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}]$ is constant. Since G is nonempty, we have $A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x) = \tilde{a}^+$ for all $x \in X$. Thus $G = X$. Hence, G is a strong UP-ideal of X.

Conversely, assume that G is a strong UP-ideal of X. Then $G = X$, so

$$(\forall x \in X)\begin{pmatrix} A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}](x) = \tilde{a}^+\\ A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}](x) = \tilde{b}^-\\ A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}](x) = \tilde{c}^+ \end{pmatrix}.$$

Thus $A_T^G[{}^{\tilde{a}^+}_{\tilde{a}^-}], A_I^G[{}^{\tilde{b}^-}_{\tilde{b}^+}]$, and $A_F^G[{}^{\tilde{c}^+}_{\tilde{c}^-}]$ are constant, that is, $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is constant. By Theorem 2.1, we have $\mathbf{A}^G[{}^{\tilde{a}^+,\tilde{b}^-,\tilde{c}^+}_{\tilde{a}^-,\tilde{b}^+,\tilde{c}^-}]$ is an interval-valued neutrosophic strong UP-ideal of X. □

3. Level Subsets of Interval-Valued Neutrosophic Sets

In this section, we discuss the relationships among interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, interval-valued neutrosophic strong UP-ideals) of UP-algebras and their level subsets.

Definition 3.1. Let A be an IVFS in a nonempty set X. For any $\tilde{a} \in [[0,1]]$, the sets

$$U(A;\tilde{a}) = \{x \in X \mid A(x) \succeq \tilde{a}\}, \tag{3.1}$$

$$L(A;\tilde{a}) = \{x \in X \mid A(x) \preceq \tilde{a}\}, \tag{3.2}$$

$$E(A;\tilde{a}) = \{x \in X \mid A(x) = \tilde{a}\} \tag{3.3}$$

are called an upper $\tilde{a}$-level subset, a lower $\tilde{a}$-level subset, and an equal $\tilde{a}$-level subset of A, respectively.

Theorem 3.1. *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic UP-subalgebra of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$*, the sets* $U(A_T;\tilde{a}), L(A_I;\tilde{b})$*, and* $U(A_F;\tilde{c})$ *are either empty or UP-subalgebras of* X*.*

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are nonempty.

Let $x, y \in U(A_T;\tilde{a})$. Then $A_T(x) \succeq \tilde{a}$ and $A_T(y) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X and by (1.32), we have

$$A_T(x \cdot y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\} \succeq \tilde{a}.$$

Thus $x \cdot y \in U(A_T;\tilde{a})$.

Let $x, y \in L(A_I;\tilde{b})$. Then $A_I(x) \preceq \tilde{b}$ and $A_I(y) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X and by (1.34), we have

$$A_I(x \cdot y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\} \preceq \tilde{b}.$$

Thus $x \cdot y \in L(A_I;\tilde{b})$.

Let $x, y \in U(A_F;\tilde{c})$. Then $A_F(x) \succeq \tilde{c}$ and $A_F(y) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X and by (1.32), we have

$$A_F(x \cdot y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\} \succeq \tilde{c}.$$

Thus $x \cdot y \in U(A_F;\tilde{c})$.

Hence, $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are UP-subalgebras of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are either empty or UP-subalgebras of X.

Let $x, y \in X$. By (1.29), we have $A_T(x) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}$ and $A_T(y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}$. Thus $x, y \in U(A_T;\operatorname{rmin}\{A_T(x), A_T(y)\})$. By assumption, we have $U(A_T;\operatorname{rmin}\{A_T(x), A_T(y)\})$ is a UP-subalgebra of X. Then $x \cdot y \in U(A_T;\operatorname{rmin}\{A_T(x), A_T(y)\})$. Thus $A_T(x \cdot y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\}$.

Let $x, y \in X$. By (1.29), we have $A_I(x) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\}$ and $A_I(y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\}$. Thus $x, y \in L(A_I;\operatorname{rmax}\{A_I(x), A_I(y)\})$. By assumption, we have $L(A_I;\operatorname{rmax}\{A_I(x), A_I(y)\})$ is a UP-subalgebra of X. Then $x \cdot y \in L(A_I;\operatorname{rmax}\{A_I(x), A_I(y)\})$. Thus $A_I(x \cdot y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\}$.

Let $x, y \in X$. By (1.29), we have $A_F(x) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}$ and $A_F(y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}$. Thus $x, y \in U(A_F;\operatorname{rmin}\{A_F(x), A_F(y)\})$. By assumption, we have $U(A_F;\operatorname{rmin}\{A_F(x), A_F(y)\})$ is a UP-subalgebra of X. Then $x \cdot y \in U(A_F;\operatorname{rmin}\{A_F(x), A_F(y)\})$. Thus $A_F(x \cdot y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\}$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-subalgebra of X. $\square$

THEOREM 3.2. *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic near UP-filter of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$*, the sets* $U(A_T;\tilde{a}), L(A_I;\tilde{b})$*, and* $U(A_F;\tilde{c})$ *are either empty or near UP-filters of* X*.*

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are nonempty.

Let $x \in U(A_T;\tilde{a}), y \in L(A_I;\tilde{b}), z \in U(A_F;\tilde{c})$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a},\ A_I(0) \preceq A_I(y) \preceq \tilde{b},\ A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$

Thus $0 \in U(A_T;\tilde{a}), 0 \in L(A_I;\tilde{b})$, and $0 \in U(A_T;\tilde{a})$.

Let $x \in X$ and $y \in U(A_T;\tilde{a})$. Then $A_T(y) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_T(x \cdot y) \succeq A_T(y) \succeq \tilde{a}.$$

Thus $x \cdot y \in U(A_T;\tilde{a})$.

Let $x \in X$ and $y \in L(A_I;\tilde{b})$. Then $A_I(y) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_I(x \cdot y) \preceq A_I(y) \preceq \tilde{b}.$$

Thus $x \cdot y \in L(A_I;\tilde{b})$.

Let $x \in X$ and $y \in U(A_F;\tilde{c})$. Then $A_F(y) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X, we have

$$A_F(x \cdot y) \succeq A_F(y) \succeq \tilde{c}.$$

Thus $x \cdot y \in U(A_F;\tilde{c})$.

Hence, $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are near UP-filters of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are either empty or near UP-filters of X.

Let $x \in X$. Then $x \in U(A_T;A_T(x)) \neq \emptyset, x \in L(A_I;A_I(x)) \neq \emptyset$, and $x \in U(A_T;A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T;A_T(x)), L(A_I;A_I(x))$, and $U(A_F;A_F(x))$ are near UP-filters of X. Then $0 \in U(A_T;A_T(x)), 0 \in L(A_I;A_I(x))$, and $0 \in U(A_F;A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$.

Let $x, y \in X$. Then $y \in U(A_T;A_T(y)) \neq \emptyset$. By assumption, we have $U(A_T;A_T(y))$ is a near UP-filter of X. Then $x \cdot y \in U(A_T;A_T(y))$. Thus $A_T(x \cdot y) \succeq A_T(y)$.

Let $x, y \in X$. Then $y \in L(A_I;A_I(y)) \neq \emptyset$. By assumption, we have $L(A_I;A_I(y))$ is a near UP-filter of X. Then $x \cdot y \in L(A_I;A_I(y))$. Thus $A_I(x \cdot y) \preceq A_I(y)$.

Let $x, y \in X$. Then $y \in U(A_F;A_F(y)) \neq \emptyset$. By assumption, we have $U(A_F;A_F(y))$ is a near UP-filter of X. Then $x \cdot y \in U(A_F;A_F(y))$. Thus $A_F(x \cdot y) \succeq A_F(y)$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic near UP-filter of X. $\square$

THEOREM 3.3. *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic UP-filter of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$*, the sets* $U(A_T;\tilde{a}), L(A_I;\tilde{b})$*, and* $U(A_F;\tilde{c})$ *are either empty or UP-filters of* X*.*

Proof. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are nonempty.

Let $x \in U(A_T;\tilde{a}), y \in L(A_I;\tilde{b}), z \in U(A_F;\tilde{c})$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a},\ A_I(0) \preceq A_I(y) \preceq \tilde{b},\ A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$

Thus $0 \in U(A_T;\tilde{a}), 0 \in L(A_I;\tilde{b})$, and $0 \in U(A_T;\tilde{a})$.

Let $x, y \in X$ be such that $x \cdot y, x \in U(A_T;\tilde{a})$. Then $A_T(x \cdot y) \succeq \tilde{a}$ and $A_T(x) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_T(y) \succeq \mathrm{rmin}\{A_T(x \cdot y), A_T(x)\} \succeq \tilde{a}.$$

Thus $y \in U(A_T;\tilde{a})$.

Let $x, y \in X$ be such that $x \cdot y, x \in L(A_I;\tilde{b})$. Then $A_I(x \cdot y) \preceq \tilde{b}$ and $A_I(x) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_I(y) \preceq \mathrm{rmax}\{A_I(x \cdot y), A_I(x)\} \preceq \tilde{b}.$$

Thus $y \in L(A_I;\tilde{b})$.

Let $x, y \in X$ be such that $x \cdot y, x \in U(A_F;\tilde{c})$. Then $A_F(x \cdot y) \succeq \tilde{c}$ and $A_F(x) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X, we have

$$A_F(y) \succeq \mathrm{rmin}\{A_F(x \cdot y), A_F(x)\} \succeq \tilde{c}.$$

Thus $y \in U(A_F;\tilde{c})$.

Hence, $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are UP-filters of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T;\tilde{a}), L(A_I;\tilde{b})$, and $U(A_F;\tilde{c})$ are either empty or UP-filters of X.

Let $x \in X$. Then $x \in U(A_T;A_T(x)) \neq \emptyset, x \in L(A_I;A_I(x)) \neq \emptyset$, and $x \in U(A_T;A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T;A_T(x)), L(A_I;A_I(x))$, and $U(A_F;A_F(x))$ are UP-filters of X. Then $0 \in U(A_T;A_T(x)), 0 \in L(A_I;A_I(x))$, and $0 \in U(A_F;A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot y) \succeq \mathrm{rmin}\{A_T(x \cdot y), A_T(x)\}$ and $A_T(x) \succeq \mathrm{rmin}\{A_T(x \cdot y), A_T(x)\}$. Thus $x \cdot y, x \in U(A_T;\mathrm{rmin}\{A_T(x \cdot y), A_T(x)\})$. By assumption, we have $U(A_T;\mathrm{rmin}\{A_T(x \cdot y), A_T(x)\})$ is a UP-filter of X. Then $y \in U(A_T;\mathrm{rmin}\{A_T(x \cdot y), A_T(x)\})$. Thus $A_T(y) \succeq \mathrm{rmin}\{A_T(x \cdot y), A_T(x)\}$.

Let $x, y \in X$. By (1.29), we have $A_I(x \cdot y) \preceq \mathrm{rmax}\{A_I(x \cdot y), A_I(x)\}$ and $A_I(x) \preceq \mathrm{rmax}\{A_I(x \cdot y), A_I(x)\}$. Thus $x \cdot y, x \in L(A_I;\mathrm{rmax}\{A_I(x \cdot y), A_I(x)\})$. By assumption, we have $L(A_I;\mathrm{rmax}\{A_I(x \cdot y), A_I(x)\})$ is a UP-filter of X. Then $y \in L(A_I;\mathrm{rmax}\{A_I(x \cdot y), A_I(x)\})$. Thus $A_I(y) \preceq \mathrm{rmax}\{A_I(x \cdot y), A_I(x)\}$.

Let $x, y \in X$. By (1.29), we have $A_F(x \cdot y) \succeq \mathrm{rmin}\{A_F(x \cdot y), A_F(x)\}$ and $A_F(x) \succeq \mathrm{rmin}\{A_F(x \cdot y), A_F(x)\}$. Thus $x \cdot y, x \in U(A_F;\mathrm{rmin}\{A_F(x \cdot y), A_F(x)\})$. By assumption, we have $U(A_F;\mathrm{rmin}\{A_F(x \cdot y), A_F(x)\})$ is a UP-filter of X. Then $y \in U(A_F;\mathrm{rmin}\{A_F(x \cdot y), A_F(x)\})$. Thus $A_F(y) \succeq \mathrm{rmin}\{A_F(x \cdot y), A_F(x)\}$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-filter of X. $\square$

THEOREM 3.4. *An IVNS* $\mathbf{A}$ *in* X *is an interval-valued neutrosophic UP-ideal of* X *if and only if for all* $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, *the sets* $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, *and* $U(A_F; \tilde{c})$ *are either empty or UP-ideals of* X.

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x \in U(A_T; \tilde{a}), y \in L(A_I; \tilde{b}), z \in U(A_F; \tilde{c})$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a}, \ A_I(0) \preceq A_I(y) \preceq \tilde{b}, \ A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$

Thus $0 \in U(A_T; \tilde{a}), 0 \in L(A_I; \tilde{b})$, and $0 \in U(A_T; \tilde{a})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_T; \tilde{a})$. Then $A_T(x \cdot (y \cdot z)) \succeq \tilde{a}$ and $A_T(y) \succeq \tilde{a}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X, we have

$$A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\} \succeq \tilde{a}.$$

Thus $x \cdot z \in U(A_T; \tilde{a})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in L(A_I; \tilde{b})$. Then $A_I(x \cdot (y \cdot z)) \preceq \tilde{b}$ and $A_I(y) \preceq \tilde{b}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X, we have

$$A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\} \preceq \tilde{b}.$$

Thus $x \cdot z \in L(A_I; \tilde{b})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_F; \tilde{c})$. Then $A_F(x \cdot (y \cdot z)) \succeq \tilde{c}$ and $A_F(y) \succeq \tilde{c}$. Since $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X, we have

$$A_F(x \cdot z) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\} \succeq \tilde{c}.$$

Thus $x \cdot z \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are UP-ideals of X.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-ideals of X.

Let $x \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_I; A_I(x)) \neq \emptyset$, and $x \in U(A_T; A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_I; A_I(x))$, and $U(A_F; A_F(x))$ are UP-ideals of X. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_I; A_I(x))$, and $0 \in U(A_F; A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot (y \cdot z)) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}$ and $A_T(y) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}$. Thus $x \cdot (y \cdot z), y \in U(A_T; \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\})$. By assumption, we have $U(A_T; \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\})$ is a UP-ideal of X. Then $x \cdot z \in U(A_T; \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\})$. Thus $A_T(x \cdot z) \succeq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}$.

Let $x, y \in X$. By (1.29), we have $A_I(x \cdot (y \cdot z)) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}$ and $A_I(y) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}$. Thus $x \cdot (y \cdot z), y \in L(A_I; \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\})$. By assumption, we have $L(A_I; \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(x)\})$ is a UP-ideal of X. Then $x \cdot z \in L(A_I; \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\})$. Thus $A_I(x \cdot z) \preceq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}$.

Let $x, y \in X$. By (1.29), we have $A_F(x \cdot (y \cdot z)) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}$ and $A_F(y) \succeq \text{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}$. Thus $x \cdot (y \cdot z), y \in U(A_F; \text{rmin}\{A_F(x \cdot$

$(y \cdot z)), A_F(y)\})$. By assumption, we have $U(A_F; \mathrm{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\})$ is a UP-ideal of X. Then $x \cdot z \in U(A_F; \mathrm{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\})$. Thus $A_F(x \cdot z) \succeq \mathrm{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}$.

Hence, $\mathbf{A}$ is an interval-valued neutrosophic UP-ideal of X. $\square$

THEOREM 3.5. *An IVNS $\mathbf{A}$ in X is an interval-valued neutrosophic strong UP-ideal if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X.*

PROOF. Assume that $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. By Theorem 2.1, we have $\mathbf{A}$ is constant, that is, A_T, A_I, A_F are constant. Thus

$$(\forall x \in X)\begin{pmatrix} A_T(x) = A_T(0) \\ A_I(x) = A_I(0) \\ A_F(x) = A_F(0) \end{pmatrix}.$$

Hence, $E(A_T; A_T(0)) = X, E(A_I; A_I(0)) = X$, and $E(A_F; A_F(0)) = X$ and so $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X.

Conversely, assume that $E(A_T; A_T(0)), E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of X. Then $E(A_T; A_T(0)) = X$, $E(A_I; A_I(0)) = X$, and $E(A_F; A_F(0)) = X$ and so

$$(\forall x \in X)\begin{pmatrix} A_T(x) = A_T(0) \\ A_I(x) = A_I(0) \\ A_F(x) = A_F(0) \end{pmatrix}.$$

Thus A_T, A_I, A_F are constant, that is, $\mathbf{A}$ is constant. By Theorem 2.1, we have $\mathbf{A}$ is an interval-valued neutrosophic strong UP-ideal of X. $\square$

4. Conclusions and Future Work

In this paper, we have introduced the notions of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals of UP-algebras and investigated some of their important properties. Then, we get the diagram of generalization of IVNSs in UP-algebras as shown in Figure 1.

In our future study, we will apply this notions/results to other type of IVNSs in UP-algebras. Also, we will study the soft set theory of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals.

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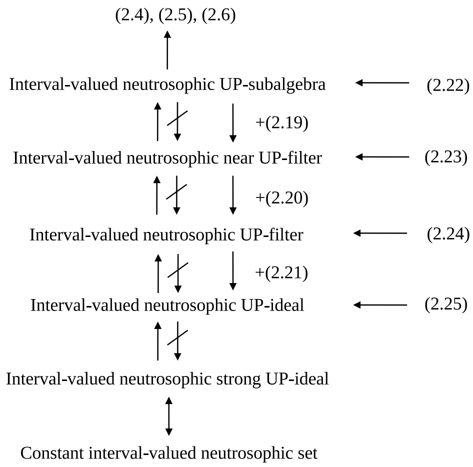

FIGURE 1. IVNSs in UP-algebras

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Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand

E-mail address: metawee.faith@gmail.com

Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand

E-mail address: aiyared.ia@up.ac.th