Neutrosophic Variational Inequalities with Applications in Decision-Making

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Abstract. In this paper, we introduced some new concepts of a neutrosophic set such as neutrosophic convex set, strongly neutrosophic convex set, neutrosophic convex function, strongly neutrosophic convex function, the minimum and maximum of a function f with respect to neutrosophic set, min and max neutrosophic variational inequality, neutrosophic general convex set, neutrosophic general convex function and min, max neutrosophic general variational inequality. We introduced some basic results on these new concepts. Moreover, we discussed the application of neutrosophic set in optimization theory. We developed an algorithm using neutrosophic min and max variational inequality and identified the maximum and minimum profit of the company.

Keyword. Neutrosophic Set, Neutrosophic Convex Set, Neutrosophic Convex Function, max and min Neutrosophic Variational Inequality.

1. Introduction

Zadeh (1965a) suggested the theory of fuzzy sets (FSs) to solve various forms of uncertainties. This theory has now been successfully implemented in different fields (Pedrycz 1990 and Zadeh 1975b). A single value $\mu_A(x) \in [0,1]$ is used by traditional FSs to describe the degree of membership of the fuzzy set A, which is specified on a universal scale, they are unable to manage those instances where it is difficult to describe μ_A by specific value one. Atanassov introduced intuitionist fuzzy sets (IFSs) (2016), which are an extension of Zadeh's FSs, to address the lack of knowledge of non-membership degrees. Moreover, vague sets were described by Gau and Buehrer (9193). Burillo and Bustince (1996) subsequently pointed out that these vague sets and Atanassov's IFSs are mathematically equivalent objects. IFSs have been commonly used to solve multi-criteria decision-making (MCDM) problems (Xu 2012, Zhi 2012, Zeng 2011), medical diagnosis (Shinoj 2012), neural networks (Sotirov 2009), market prediction (Joshi 2012), and color region extraction (Chaira) .

The membership degree, non-membership degree and degree of hesitation are taken into account simultaneously by the IFSs. They are thus, more flexible and realistic than traditional FSs when discussing fuzziness and ambiguity. Moreover, the membership degree, non-membership degree and hesitation degree of an element in IFSs may not be a specific number in some real cases. Thus they were extended to interval-valued intuitive fuzzy sets (2016). Moreover, Torra and Narukawawa introduced hesitant fuzzy sets in order to deal with situations where people are hesitant when expressing their preferences regarding objects in the decision-making process.

Although the theory of FSs has been developed and generalized, in various real-life problems, it does not deal with all uncertainty. For example, it is not possible to deal with certain kinds of uncertainty, such as indeterminate and inconsistent information. For example, when an expert is asked for his or her opinion about a certain statement, he or she may say that the possibility that the statement is true is 0.5, that it is false is 0.6 and the degree that he or she is not sure is 0.2 (Wang 2010a). This problem is beyond the reach of FSs and IFSs, so it needs some new theories.

Samarandache (1999a) suggested neutrosophic (NS) sets and neutrosophic logic. An NS is a set where each element of the universe has the degrees of truth, indeterminacy and falsity and it lies in $]0^-, 1^+[$, the non standard unit interval (2008). This is simply an extension to the standard interval [0, 1] of the IFSs. Moreover, the uncertainty presented here, i.e. the indeterminacy element, is independent of the values of true and falsity, while the incorporated uncertainty depends on the degree of belonging and non-belonging to IFSs (2014). However, in practical situations, NSs are difficult to apply without a particular description. In various areas of knowledge, this theory is being used. See recent examples as Ajay (2020), Crespo Berti (2020) in modeling real life problems; Hatip (2020) and Saqlain et, al., (2020) who developed extensions of it. A new framework for dealing with impreciseness is provided by Neutrosophic Theory. It is well known that statistical concepts and methods can be expanded using a Neutrosophic point of view, see Smarandache (2013b, 2014c), Schweizer (2020), Cacuango et, al., (2020).

Single-value neutrosophic sets (SVNSs), which are a variant of NSs, were proposed in (Majumdar 2014). Moreover, the information energy of SVNSs, their coefficient of correlation and correlation and the process of decision-making used by them were proposed in (Ye 2013a). In addition, Ye (2014b) introduced the Simplified Neutrosophic Sets (SNSs), which can be represented by three real numbers in the real unit interval [0,1], and proposed a method of MCDM using SNS aggregation operators. Moreover, Majumdar and Samant (2014) introduced a measure of SVNS entropy. The definition of Interval Neutrosophic Sets (INSs) was proposed by Wang, Samarandache, Zhang, and Sunderraman (2005b). In addition, Ye (2014c) proposed similarity measures between SVNSs and INSs based on the relationship between measures of similarity and distances.

There are twenty-seven new concepts developed from NS, neutrosophic probability and neutrosophic statistics. Each of these is interactive. The sets derived from NS are intuitionistic set, paraconsistent set, paradoxist set, trivialist set, nihilist set, dialetheist set and faillibilist set. Tautological probability and statistics, intuitionistic probability and statistics, dialetheist probability and statistics, faillibilist probability and statistics, paraconsistent probability and statistics, trivialist probability and statistics and nihilist probability and statistics are derived from neutrosophic probability and statistics. N. A. Nabeeh (2019) proposed an approach that would facilitate a personal selection process by incorporating the process of neutrosophic analytical hierarchy to demonstrate the ideal solution between various options similar to an ideal solution for order preference technique (TOPSIS).

M. A. Baset (2019a) developed a new kind of technique for neutrosophy called neutrosophic numbers of type 2. They suggested a novel T2NN-TOPSIS process, combining type 2 neutrosophic number and TOPSIS, which is very useful in group decision-making. A multi-criteria group decision-making method of the analytical network process method and the Visekriterijusmska Optmzacija I Kommpromisno Resenje method was investigated in a neutrosophical setting dealing with high-order imprecision and incomplete information (Baset 2019b). M. A. Baset introduced a new technique for estimating the GDM selection process for smart medical devices in a vague decision-making environment. Neutrosophic with the TOPSIS strategy is used in decision-making processes to deal with incomplete information, vagueness and ambiguity, taking into account the decision-making criteria in the information obtained by decision-makers in (Baset 2019c). They proposed a robust ranking method with NS to manage the performance of the Supply Chain Management (GSCM) and methods that have been commonly used to promote environmental sustainability and achieve competitive advantages. The principle of the N.S. has been used to handle imprecision, linguistic imprecision, ambiguous details and incomplete information (Baset 2019d). Moreover, M. A. Baset (2018e) et, al., used NS for evaluation techniques and decision-making to identify and analyze factors influencing the selection of suppliers for the supply chain management. T. Bera (2018a) et, al., characterized a neutrosophic norm for a soft linear space known as a neutrosophic soft linear space. They also explore the notion of neutrosophic soft (Ns) prime ideal over a ring. They introduced the idea of N's completely prime ideals, N's fully semi-primary ideals and N's prime K-ideals (Bera 2018b). Moreover, T. Bera (2018c) established the concept of connectedness and compactness in N's topological space along with its various characteristics. R. A. Cruz (2017) et, al., studied the P-OR, P-intersection and P-union and P-AND of neutrosophic cubic sets and their associated properties. N. Shah (2016) et, al., discussed neutrosophic soft graphs. They proposed a connection between the neutrosophic soft sets and the graphs.

In decision making problems, the use of optimization approaches is ubiquitous. The purpose of this article is two-fold. The first half aims to present the theoretical foundations of neutrosophic in optimization such as neutosophic variational inequalities, neutrosophic convex function and the second half aims to present these theoretical foundations and key techniques in convex optimization, decision making, and the principle of the neutrosophic variational inequalities in a coherent manner. The purpose of these innovative concepts is, to provide a new approach with useful mathematical tools to address the fundamental problem of decision-making (e.g maximization and minimization of the problem). The generality of the neutrosophic variational inequalities system is given special importance, illustrating how many interesting optimization decision-making problems can be formulated as a problem of neutrosophic variational inequalities. These applied contexts provide solid evidence of the wide applications of the neutrosophic variational inequality approach to model and research decision-making problems. This article will stimulate the interest in neutrosophic variational inequality and its application in the optimization.

In this paper, we introduce some new concepts of a neutrosophic set such as neutrosophic convex set, strongly neutrosophic convex set, neutrosophic convex function, strongly neutrosophic convex function, the minimum and maximum of a function f with respect to neutrosophic set, min and max neutrosophic variational inequality, neutrosophic general convex set, neutrosophic general convex function and min, max neutrosophic general variational inequality. We study some basic results on these new concepts. Moreover, we discuss the application of neutrosophic set in optimization theory. We propose a method using neutrosophic min and max variational inequality and identify the maximum and minimum profit of the company.

2. Preliminaries

We will define here some new concepts on the neutrosophic set and also discuss particular examples of these new concepts. In this paper, we take the function $f : \tau \to \tau'$, where τ, τ' denotes the collection of neutrosophic sets N.

Definition 1. *(Samarandache 1999a) Let X be a space of points and let $x \in X$. A neutrosophic set N in X is characterized by a truth membership function T_N, an indeterminacy membership function I_N, and a falsity membership function F_N. $T_N(x)$, $I_N(x)$, and $F_N(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$, and F_N. $T_N(x)$,$I_N(x)$,$F_N(x) : X \to]0^-, 1^+[$. The neutrosophic set can be represented as:*

$$N = \{(x, T_N(x), I_N(x), F_N(x)) : x \in X\}.$$

There is no restriction on the sum of $T_N(x)$, $I_N(x)$, and $F_N(x)$, so

$$0^- \leq T_N(x), I_N(x), F_N(x) \leq 3^+.$$

Definition 2. *(Ali 2017) The complement of a neutrosophic set N is denoted by $c(N)$ and is defined by*

$$T_{c(N)}(x) = F_N(x),\ I_{c(N)}(x) = 1 - I_N(x),\ F_{c(N)}(x) = T_N(x) \text{ for all } x \in X.$$

Definition 3. *(Ali 2017) Let N_1 and N_2 be two neutrosophic sets in a universe of discourse X. Then, the union of N_1 and N_2 is denoted by $N_1 \cup N_2$, which is defined by*

$$N_1 \cup N_2 = \{(x, T_{N_1}(x) \vee T_{N_2}(x), I_{N_1}(x) \wedge I_{N_2}(x), F_{N_1}(x) \wedge F_{N_2}(x) : x \in X\}$$

for all $x \in X$, and $\vee$ and $\wedge$ represents the max and min operator, respectively.

Definition 4. *(Ali 2017) Let N_1 and N_2 be two neutrosophic sets in a universe of discourse X. Then, the intersection of N_1 and N_2 is denoted by $N_1 \cap N_2$, which is defined by*

$$N_1 \cap N_2 = \{(x, T_{N_1}(x) \wedge T_{N_2}(x), I_{N_1}(x) \vee I_{N_2}(x), F_{N_1}(x) \vee F_{N_2}(x) : x \in X\}$$

for all $x \in X$, and $\vee$ and $\wedge$ represents the max and min operator, respectively.

Definition 5. *A neutrosophic set N is convex if*

$$N((1-t)x + ty) \geq \min(N(x), N(y).$$

Or

$$\mu_N((1-t)x + ty) \geq \min(\mu_N(x), \mu_N(y))$$

for all $x, y \in \mathbb{R}^n$ and $t \in [0, 1]$.

Definition 6. *A neutrosophic set N is strongly convex if*

$$N((1-t)x + ty) > \min(N(x), N(y).$$

Or

$$\mu_N((1-t)x+ty) > \min(\mu_N(x), \mu_N(y))$$

for all $x \neq y$, $x, y \in \mathbb{R}^n$ and $t \in [0,1]$.

Note: Strongly neutrosophic convex set is neutrosophic convex set but the converse not true.

Definition 7. *Let N be a neutrosophic convex set, then the function f on neutrosophic convex set N is said to be neutrosophic convex function if*

$$f(N((1-t)x+ty)) \geq \min(f(N(x)), f(N(y))$$

for all $x, y \in \mathbb{R}^n$ and $t \in [0,1]$.

Example 1. *The identity function on the neutrosophic convex set N is a neutrosophic convex function.*

Definition 8. *A neutrosophic set $N_i(x) \in \tau$ is called a minimum of f, if $f(N_i(x)) \leq f(N_j(x))$ for all $N_i(x) \in \tau$.*

Definition 9. *Let τ be a collections of neutrosophic convex sets. Then the inequality*

$$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq f(N_i(x)) \circ f(N_j(x)); i \neq j,$$
$$\forall N_1(x), N_2(y) \in \tau$$

is called neutrosophic min variational inequality.

Example 2. *Let $N_1 = [\frac{(0.4,0.7,1)}{x} + \frac{(0.5,0.8,0.2)}{y} + \frac{(0.2,0.5,1)}{z}]$ and $N_2 = [\frac{(0.6,0.9,1)}{x} + \frac{(0.3,0.8,0.6)}{y} + \frac{(0.3,0.7,0.9)}{z}]$ be two neutrosophic sets and f be a function defined by*

$\frac{(0.4,0.7,1)}{x} \xrightarrow{f} \frac{(0.2,0.5,1)}{x}, \frac{(0.5,0.8,0.2)}{y} \xrightarrow{f} \frac{(0.3,0.6,0.2)}{y}, \frac{(0.2,0.5,1)}{z} \xrightarrow{f} \frac{(0,0.3,1)}{z}$
$\frac{(0.6,0.9,1)}{x} \xrightarrow{f} \frac{(0.4,0.7,1)}{x}, \frac{(0.7,0.9,0)}{y} \xrightarrow{f} \frac{(0.5,0.7,0)}{y}, \frac{(0.3,0.7,0.9)}{z} \xrightarrow{f} \frac{(0.1,0.5,0.9)}{z}.$

We have

$f(N_1) = [\frac{(0.2,0.5,1)}{x} + \frac{(0.3,0.6,0.2)}{y} + \frac{(0,0.3,1)}{z}]$ and $f(N_2) = [\frac{(0.4,0.7,1)}{x} + \frac{(0.5,0.7,0)}{y} + \frac{(0.1,0.5,0.9)}{z}]$

then

$$\langle f(N_1), f(N_1) \cap f(N_2)\rangle = \left\langle \begin{array}{c} [\frac{(0.2,0.5,1)}{x} + \frac{(0.3,0.6,0.2)}{y} + \frac{(0,0.3,1)}{z}], \\ [\frac{(0.2,0.5,1)}{x} + \frac{(0.3,0.6,0.2)}{y} + \frac{(0,0.3,1)}{z}] \\ \cap[\frac{(0.4,0.7,1)}{x} + \frac{(0.5,0.7,0)}{y} + \frac{(0.1,0.5,0.9)}{z}] \end{array} \right\rangle$$
$$= \left\langle \begin{array}{c} [\frac{(0.2,0.5,1)}{x} + \frac{(0.3,0.6,0.2)}{y} + \frac{(0,0.3,1)}{z}], \\ [\frac{(0.2,0.7,1)}{x} + \frac{(0.3,0.7,0.2)}{y} + \frac{(0,0.5,1)}{z}] \end{array} \right\rangle$$

$$(1) \quad \langle f(N_1), f(N_1) \cap f(N_2)\rangle = [\frac{(0.04, 0.35, 1)}{x} + \frac{(0.09, 0.42, 0.04)}{y} + \frac{(0, 0.15, 1)}{z}].$$

Now

$$(2) \qquad f(N_1) \circ f(N_2) = [\frac{(0.08, 0.35, 1)}{x} + \frac{(0.15, 0.42, 0)}{y} + \frac{(0, 0.15, 0.9)}{z}]$$

From (1) and (2), we have

$$\langle f(N_1), f(N_1) \cap f(N_2)\rangle \leq f(N_1) \circ f(N_2).$$

Definition 10. *A neutrosophic set $N_i(x) \in \tau$ is called a maximum of f, if $f(N_i(x)) \geq f(N_j(x))$ for all $N_j(x) \in \tau$.*

Definition 11. *Let τ be a collection of neutrosophic convex sets. Then the inequality*

$$\langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x)) \rangle \geq f(N_i(x)) \circ f(N_j(x)); \; i \neq j,$$
$$\forall N(x), N(y) \in \tau$$

is called neutrosophic max variational inequality.

Example 3. *Let $N_1 = [\frac{(0.4,0.7,1)}{x} + \frac{(0.5,0.8,0.2)}{y} + \frac{(0.2,0.5,1)}{z}]$ and $N_2 = [\frac{(0.6,0.9,1)}{x} + \frac{(0.3,0.8,0.6)}{y} + \frac{(0.3,0.7,0.9)}{z}]$ be two neutrosophic sets and f be a function defined by*

$\frac{(0.4,0.7,1)}{x} \xrightarrow{f} \frac{(0.2,0.5,1)}{x}, \frac{(0.5,0.8,0.2)}{y} \xrightarrow{f} \frac{(0.3,0.6,0.2)}{y}, \frac{(0.2,0.5,1)}{z} \xrightarrow{f} \frac{(0,0.3,1)}{z}$
$\frac{(0.6,0.9,1)}{x} \xrightarrow{f} \frac{(0.4,0.7,1)}{x}, \frac{(0.7,0.9,0)}{y} \xrightarrow{f} \frac{(0.5,0.7,0)}{y}, \frac{(0.3,0.7,0.9)}{z} \xrightarrow{f} \frac{(0.1,0.5,0.9)}{z}.$

We have
$f(N_1) = [\frac{(0.7,0.8,0.2)}{x} + \frac{(0.6,0.7,0.4)}{y} + \frac{(1,0.6,0.5)}{z}]$ and $f(N_2) = [\frac{(0.4,0.7,1)}{x} + \frac{(0.5,0.6,0.9)}{y} + \frac{(0.1,0.5,0.6)}{z}]$. Then

$$\langle f(N_1), f(N_1) \cup f(N_2) \rangle = \left\langle \begin{array}{c} [\frac{(0.7,0.8,0.2)}{x} + \frac{(0.6,0.7,0.4)}{y} + \frac{(1,0.6,0.5)}{z}], \\ [\frac{(0.7,0.8,0.2)}{x} + \frac{(0.6,0.7,0.4)}{y} + \frac{(1,0.6,0.5)}{z}] \\ \cup[\frac{(0.4,0.7,1)}{x} + \frac{(0.5,0.6,0.9)}{y} + \frac{(0.1,0.5,0.6)}{z}] \end{array} \right\rangle$$
$$= \left\langle \begin{array}{c} [\frac{(0.7,0.8,0.2)}{x} + \frac{(0.6,0.7,0.4)}{y} + \frac{(1,0.6,0.5)}{z}], \\ [\frac{(0.7,0.7,0.2)}{x} + \frac{(0.6,0.6,0.4)}{y} + \frac{(1,0.5,0.5)}{z}] \end{array} \right\rangle$$

(1)
$$\langle f(N_1), f(N_1) \cup f(N_2) \rangle = [\frac{(0.49, 0.56, 0.04)}{x} + \frac{(0.36, 0.42, 0.16)}{y} + \frac{(1, 0.30, 0.25)}{z}].$$

Now

$$f(N_1) \circ f(N_2) = [\frac{(0.28, 0.56, 0.2)}{x} + \frac{(0.36, 0.42, 0.36)}{y} + \frac{(0.1, 0.30, 0.30)}{z}] \tag{2}$$

From (1) and (2), we have

$$\langle f(N_1), f(N_1) \cup f(N_2) \rangle \geq f(N_1) \circ f(N_2).$$

2.1. **Generalized Convex Set and Convex Function.** In the problems, if the domain set may not be a convex set, in those situations, the non-convex set can be made a convex set with respect to an arbitrary function. These sets are called general convex sets and the function defined on the general convex set is called general convex function.

Definition 12. *A neutrosophic set N is general convex if*

$$N((1-t)g(x) + tg(y)) \geq \min(N(g(x)), N(g(y)).$$

Or

$$\mu_N((1-t)g(x) + tg(y)) \geq \min(\mu_N(g(x)), \mu_N(g(y)))$$

for $g(x), g(y) \in \mathbb{R}^n_g$ and $t \in [0,1]$.

Definition 13. *Let N be a neutrosophic convex set, then the function f on neutrosophic general convex set N is said to be neutrosophic general convex function if*

$$f(N((1-t)g(x)+tg(y))) \geq \min(f(N(g(x)), f(N(g(y)))$$

for all $g(x), g(y) \in \mathbb{R}^n_g$ and $t \in [0,1]$.

Case 1. *If $g = I$, then the neutrosophic general convex function is neutrosophic convex function.*

Definition 14. *Let τ be a collection of neutrosophic general convex sets. Then the inequality*

$$\langle f(N_i(g(x))), f(N_i(g(x))) \cap f(N_j(g(x)))\rangle \leq f(N_i(g(x))) \circ f(N_j(g(x))); i \neq j,$$
$$\forall N(g(x)), N(g(x)) \in \tau$$

is called neutrosophic min general variational inequality.

Definition 15. *Let τ be a collection of neutrosophic general convex set. Then the inequality*

$$\langle f(N_i(g(x))), f(N_i(g(x))) \cup f(N_j(g(x)))\rangle \geq f(N_i(g(x))) \circ f(N_j(g(x)));$$
$$\forall N_i(g(x)), N_j(g(y)) \in \tau$$

is called neutrosophic max general variational inequality.

3. Main Results

Proposition 1. *Let τ be a collection of neutrosophic convex sets and $N_i(x) \in \tau$ be a minimum of the neutrosophic convex function f on τ. Then $N_i(x)$ satisfies the neutrosophic* min *variational inequality.*

Proof. Let $N_i(x) \in \tau$ be the minimum of f. Then

(1) $$f(N_i(x)) \leq f(N_j(x));\ \forall N_j(x) \in \tau.$$

Also from (1), we have

(2) $$(f(N_i(x)) \cap f(N_j(x))) \leq f(N_j(x));\ \forall N_j(x) \in \tau.$$

(2) can be written as

$$f(N_i(x)) \circ (f(N_i(x)) \cap f(N_j(x))) \leq f(N_i(x)) \circ f(N_j(x))$$
$$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq f(N_i(x)) \circ f(N_j(x));$$
$$\forall N_i(x), N_j(y) \in \tau.$$

Thus $N_i(x) \in \tau$ satisfies the neutrosophic min variational inequality. □

Proposition 2. *Let τ be a collection of neutrosophic convex sets and $N_i(x) \in \tau$ be a maximum of the neutrosophic convex function f on τ. Then $N_i(x)$ satisfies the neutrosophic* max *variational inequality.*

Proof. Let $N_i(x) \in \tau$ be the maximum of f. Then

(1) $$f(N_i(x)) \geq f(N_j(x));\ \forall N_j(x) \in \tau.$$

Also from (1), we have

(2) $$(f(N_i(x)) \cup f(N_j(x))) \geq f(N_j(x));\ \forall N_j(x) \in \tau.$$

(2) can be written as

$$f(N_i(x)) \circ (f(N_i(x)) \cup f(N_j(x))) \geq f(N_i(x)) \circ f(N_j(y))$$
$$\langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x)) \rangle \geq f(N_i(x)) \circ f(N_j(x));$$
$$\forall N_i(x), N_j(x) \in \tau.$$

Thus $N_i(x) \in \tau$ satisfies the neutrosophic max variational inequality. □

Proposition 3. *Let τ be a collection of neutrosophic general convex sets and $N_i(g(x)) \in \tau$ be a minimum of the neutrosophic general convex function f on τ. Then $N_i(g(x))$ satisfies the neutrosophic general variational inequality.*

Proof. Let $N_i(g(x)) \in \tau$ be the minimum of f. Then

(1) $$f(N_i(g(x))) \leq f(N_j(g(x))); \ \forall N_j(g(x)) \in \tau.$$

Also from (1), we have

(2) $$(f(N_i(g(x))) \cap f(N_j(g(x))) \leq f(N_j(g(x))); \ \forall N_j(g(x)) \in \tau.$$

(2) can be written as

$$f(N_i(g(x))) \circ (f(N_i(g(x))) \cap f(N_j(g(x))) \leq f(N_i(g(x))) \circ f(N_j(g(x)))$$
$$\langle f(N_i(g(x))), f(N_i(g(x))) \cap f(N_j(g(x))) \rangle \leq f(N_i(g(x))) \circ f(N_j(g(x)));$$
$$\forall N_i(g(x)), N_j(g(x)) \in \tau.$$

Thus $N_i(g(x)) \in \tau$ satisfies the neutrosophic general min variational inequality. □

Proposition 4. *Let τ be a collection of neutrosophic general convex sets and $N_i(g(x)) \in \tau$ be a maximum of the neutrosophic* max *general convex function f on τ. Then $N_i(g(x))$ satisfies the neutrosophic* max *general variational inequality.*

Proof. Let $N_i(g(x)) \in \tau$ be the maximum of f. Then

(1) $$f(N_i(g(x))) \geq f(N_j(g(x))); \ \forall N_j(g(x)) \in \tau.$$

Also from (1), we have

(2) $$(f(N_i(g(x))) \cup f(N_j(g(x))) \geq f(N_j(g(x))); \ \forall N_j(g(x)) \in \tau.$$

(2) can be written as

$$f(N_i(g(x))) \circ (f(N_i(g(x))) \cup f(N_j(g(x))) \geq f(N_i(g(x))) \circ f(N_j(g(x)))$$
$$\langle f(N_i(g(x))), f(N_j(g(x))) \cup f(N_j(g(x))) \rangle \geq f(N_i(g(x))) \circ f(N_j(g(x)));$$
$$\forall N_i(g(x)), N_j(g(x)) \in \tau.$$

Thus $N_i(g(x)) \in \tau$ satisfies the neutrosophic max general variational inequality. □

Proposition 5. *For any two neutrosophic convex sets N_1 and N_2, $N_1 \cup N_2$ is also neutrosophic convex set.*

Proof. Since N_1 is neutrosophic convex set, we have

$$\mu_{N_1}((1-t)x + ty) \geq \min(\mu_{N_1}(x), \mu_{N_1}(y)).$$

Also N_2 is neutrosophic convex set, then

$$\mu_{N_2}((1-t)x + ty) \geq \min(\mu_{N_2}(x), \mu_{N_2}(y)).$$

Now since

$$\mu_{N_1\cup N_2}((1-t)x+ty) \geq \min(\mu_{N_1\cup N_2}(x), \mu_{N_1\cup N_2}(y)).$$

Thus $N_1 \cup N_2$ is neutrosophic convex set. □

Proposition 6. *If $N_i(x) \in \tau$ be a minimum of the neutrosophic convex function f. Then*

$$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq \langle f(N_i(x)), f(N_j(x)) \cup f(N_j(x))\rangle\,; \forall N_j(y) \in \tau.$$

Proof. Assume that $N_i(x) \in \tau$ be a minimum of f. Then

(1) $$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq f(N_i(x)) \circ f(N_j(x)); \forall N_j(x) \in \tau.$$

Since for $N_i(x) \in \tau$, we have

(2) $$f(N_i(x)) \circ f(N_j(x)) \leq \langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x))\rangle\,.$$

From (1) and (2), we have

$$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq \langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x))\rangle\,; \forall N_j(x) \in \tau.$$

□

Proposition 7. *If $N_i(x) \in \tau$ be a maximum of the neutrosophic convex function f. Then*

$$\langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x))\rangle \leq \langle f(N_i(x)), f(N_j(x)) \cap f(N_i(x))\rangle\,; \forall N_j(x) \in \tau.$$

Proof. Assume that $N_i(x) \in \tau$ be a maximum of f. Then

(1) $$\langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x))\rangle \geq f(N_i(x)) \circ f(N_j(x)); \forall N_j(x) \in \tau.$$

Since for $N_i(x) \in \tau$, we have

(2) $$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq f(N_i(x)) \circ f(N_j(x)).$$

From (1) and (2), we have

$$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(x))\rangle \leq \langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x))\rangle\,; \forall N_j(x) \in \tau.$$

□

Theorem 1. *Let $f : \tau \to \tau'$ be a mapping and "$\sim$" be a relation defined in the following way "$f(N_1)$, $f(N_2) \in \tau'$, $f(N_1) \sim f(N_2)$ if the* min *variational inequality is hold. Show that the relation "$\sim$" is an order relation.*

Proof. To prove the relation "$\sim$" is an order relation, we have to show the following.

i). The relation "$\sim$" is reflexive, that is, $f(N_1) \sim f(N_1)$.

(*ii*). The relation "$\sim$" is anti-symmetric, that is, if $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_1)$, then $f(N_1) = f(N_2)$.

(*iii*). The relation "$\sim$" is transitive, that is, if $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_3)$, then $f(N_1) \sim f(N_3)$.

i). **Reflexive**

The relation "$\sim$" is reflexive, since for any $f(N_1) \in \tau'$, we have

$$\langle f(N_1(x)), f(N_1(x)) \cap f(N_1(x))\rangle \leq f(N_1(x)) \circ f(N_1(x)).$$

Hence $f(N_1) \sim f(N_1)$. Thus the relation "$\sim$" is reflexive.

ii). **Anti-symmetric**

Assume that $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_1)$, then

$$\langle f(N_1(x)), f(N_1(x)) \cap f(N_2(x))\rangle \leq f(N_1(x)) \circ f(N_2(x))$$

implies that

(1) $$f(N_1(x) \leq f(N_2(x)$$

and

$$\langle f(N_2(x)), f(N_2(x)) \cap f(N_1(x)) \rangle \leq f(N_2(x)) \circ f(N_1(x))$$

implies that

(2) $$f(N_2(x) \leq f(N_1(x).$$

From (1) and (2), we have $f(N_1) = f(N_2)$. Thus "$\sim$ " is antisymmetric.

iii). **Transitive**

Assume that $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_3)$, then

$$\langle f(N_1(x)), f(N_1(x)) \cap f(N_2(x)) \rangle \leq f(N_1(x)) \circ f(N_2(x))$$

implies that

(3) $$f(N_1(x) \leq f(N_2(x)$$

and

$$\langle f(N_2(x)), f(N_2(x)) \cap f(N_3(x)) \rangle \leq f(N_2(x)) \circ f(N_3(x))$$

implies that

(4) $$f(N_2(x)) \leq f(N_1(x)).$$

From (3) and (4), we have $f(N_1) \leq f(N_3)$. Then

$$\langle f(N_1(x)), f(N_1(x)) \cap f(N_3(x)) \rangle \leq f(N_1(x)) \circ f(N_3(x)).$$

Hence $f(N_1) \sim f(N_3)$. Thus the relation "$\sim$ " is transitive and consequently the relation $\sim$ is order relation. □

Theorem 2. *Let $f : \tau \to \tau'$ be a mapping and "$\sim$ " be a relation defined in the following way "$f(N_1)$, $f(N_2) \in \tau'$, $f(N_1) \sim f(N_2)$ if the* max *variational inequality is hold. Show that the relation $\sim$ is an order relation.*

Proof. To prove the relation "$\sim$ " is an order relation, we have to show the following.

i). The relation "$\sim$ " is reflexive, that is, $f(N_1) \sim f(N_1)$.

(*ii*). The relation "$\sim$ " is anti-symmetric, that is, if $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_1)$, then $f(N_1) = f(N_2)$.

(*iii*). The relation "$\sim$ " is transitive, that is, if $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_3)$, then $f(N_1) \sim f(N_3)$.

i). **Reflexive**

The relation "$\sim$ " is reflexive, since for any $f(N_1) \in \tau'$, we have

$$\langle f(N_1(x)), f(N_1(x)) \cup f(N_1(x)) \rangle \geq f(N_1(x)) \circ f(N_1(x)).$$

Hence $f(N_1) \sim f(N_1)$. Thus the relation "$\sim$ " is reflexive.

ii). **Anti-symmetric**

Assume that $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_1)$, then

$$\langle f(N_1(x)), f(N_1(x)) \cup f(N_2(x)) \rangle \geq f(N_1(x)) \circ f(N_2(x))$$

implies that

(1) $$f(N_1(x) \geq f(N_2(x)$$

and

$$\langle f(N_2(x)), f(N_2(x)) \cup f(N_1(x)) \rangle \geq f(N_2(x)) \circ f(N_1(x))$$

implies that

$$f(N_2(x) \geq f(N_1(x). \tag{2}$$

From (1) and (2), we have $f(N_1) = f(N_2)$. Thus "$\sim$" is antisymmetric.
iii). **Transitive**
Assume that $f(N_1) \sim f(N_2)$ and $f(N_2) \sim f(N_3)$, then

$$\langle f(N_1(x)), f(N_1(x)) \cup f(N_2(x)) \rangle \geq f(N_1(x)) \circ f(N_2(x))$$

implies that

$$f(N_1(x) \geq f(N_2(x) \tag{3}$$

and

$$\langle f(N_2(x)), f(N_2(x)) \cup f(N_3(x)) \rangle \geq f(N_2(x)) \circ f(N_3(x))$$

implies that

$$f(N_2(x)) \geq f(N_1(x)). \tag{4}$$

From (3) and (4), we have $f(N_1) \geq f(N_3)$. Then

$$\langle f(N_1(x)), f(N_1(x)) \cup f(N_3(x)) \rangle \geq f(N_1(x)) \circ f(N_3(x)).$$

Hence $f(N_1) \sim f(N_3)$. Thus the relation "$\sim$" is transitive and consequently the relation "$\sim$" is order relation. □

4. Applications

We are going to discuss a real-life application of newly defined neutrosophic max and neutrosophic min variational Inequalities. In fact, we will discuss that how our novel concepts have real-life applications. Specifically, the neutrosophic max and neutrosophic min variational inequality explain how to get the maximum and minimum profit of the company.

We will discuss the algorithm by using the neutrosophic max variational inequality and neutrosophic min variational inequality. In this algorithm, we will discuss how the trucking company gets maximum profit and minimum profit.

Algorithm

Suppose ABC Trucking is a company that operates 20 trucks for transport and logistics. When they are full and on the track, trucks make the most money for the company. ABC Trucking has the following vector entities or groups:
(*i*). Truck Company(truck type, age, engine size).
(*ii*). Income ($(Euro)_1$, $(Euro)_2$, $(Euro)_3$).
A neutrosophic set N_1, N_2, and N_3 in $X = truck\ type, Y = age, Z = engine\ size$ is characterized by a truth membership function T_{N_1}, T_{N_2}, T_{N_3}, an indeterminacy membership function I_{N_1}, I_{N_2}, I_{N_3}, and a falsity membership function F_{N_1}, F_{N_2}, F_{N_3}. T_{N_1}, T_{N_2}, T_{N_3}, I_{N_1}, I_{N_2}, I_{N_3}, and F_{N_1}, F_{N_2}, F_{N_3} are real standard or non-standard subsets of $]0^-, 1^+[$. A neutrosophic set N'_1, N'_2, and N'_3 in $X' = Euro$, $Y' = Doller$, $Z = Riyal$ is characterized by a truth membership function $T_{N'_1}$, $T_{N'_2}$, $T_{N'_3}$, an indeterminacy membership function $I_{N'_1}$, $I_{N'_2}$, $I_{N'_3}$, and a falsity membership function $F_{N'_1}$, $F_{N'_2}$, $F_{N'_3}$. $T_{N'_1}$, $T_{N'_2}$, $T_{N'_3}$, $I_{N'_1}$, $I_{N'_2}$, $I_{N'_3}$ and $F_{N'_1}$, $F_{N'_2}$, $F_{N'_3}$ are real standard or non-standard subsets of $]0^-, 1^+[$.

The trucking company needs to optimize the use of its trucks and workers for the highest possible profits. To find the probability of the maximum or minimum profit of the trucking company, we define a relation $f : \tau \to \tau'$ by

$$(truck\ type = a, T_{N_1}(a), I_{N_1}(a), F_{N_1}(a)) \xrightarrow{f} ((Euro)_1, T_{N_1'}(x), I_{N_1'}(x), F_{N_1'}(x)),$$

$$(age = b, T_{N_2}(b), I_{N_2}(b), F_{N_2}(b)) \xrightarrow{f} ((Euro)_2, T_{N_2'}(x), I_{N_2'}(x), F_{N_2'}(x)),$$

$$(engine\ size = c, T_{N_3}(c), I_{N_3}(c), F_{N_3}(c)) \xrightarrow{f} ((Euro)_3, T_{N_3'}(x), I_{N_3'}(x), F_{N_3'}(x)).$$

Now if the relation f satisfies the max variational inequality, that is,

(1) $\langle f(N_i(x)), f(N_i(x)) \cup f(N_j(x)) \rangle \geq f(N_i(x)) \circ f(N_j(x));\ \forall N_i(x) \neq N_j(y) \in \tau$

By taking the left hand side of (1), we have

$$\langle f(N_i(x)), f(N_i(x)) \cup f(N_j(y)) \rangle = N' = (z, T_{N'}(z), I_{N'}(z), F_{N'}(z))$$

which gives the maximum profit with a neutrosophic set N' is characterized by a truth membership function $T_{N'}$, an indeterminacy membership function $I_{N'}$, and a falsity membership function $F_{N'}$.

If the relation f satisfies the min variational inequality, that is;

(2) $\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(y)) \rangle \leq f(N_i(x)) \circ f(N_j(y));\ \forall N_i(x) \neq N_j(y) \in N.$

By taking the left hand side of (2), we have

$$\langle f(N_i(x)), f(N_i(x)) \cap f(N_j(y)) \rangle = N'' = (z, T_{N''}(z), I_{N''}(z), F_{N''}(z))$$

which gives the minimum profit is characterized by a neutrosophic set N'' with a truth membership function $T_{N''}$, an indeterminacy membership function $I_{N''}$, and a falsity membership function $F_{N''}$.

Example 4. *Suppose ABC Trucking is a company that operates 20 trucks for transport and logistics. When they are full and on the track, trucks make the most money for the company. ABC Trucking has the following vector entities or groups:*

(i). Truck Company ($X = truck\ type$, $Y = age$, $Z = engine\ size$).
(ii). Income ($X' = (Euro)_1$, $Y' = (Euro)_2$, $Z' = (Euro)_3$).
The neutrosophic sets N_1, N_2, and N_3 in X, Y and Z are

$$N_1 = \frac{(0.4, 0.5, 0.8)}{x},\ N_2 = \frac{(0.2, 0.7, 0.3)}{x},\ N_3 = \frac{(0.5, 0.4, 1)}{x}$$

Let f be a function defined by

$$\frac{(0.4, 0.5, 0.8)}{x} \xrightarrow{f} \frac{(0.8, 0.6, 0.2)}{x}$$
$$\frac{(0.2, 0.7, 0.3)}{x} \xrightarrow{f} \frac{0.7, 0.5, 0.8)}{x}$$
$$\frac{(0.5, 0.4, 1)}{x} \xrightarrow{f} \frac{(0.5, 0.3, 1)}{x}.$$

Now clearly, we have

(1) $\langle f(N_1(x)), f(N_1(x) \cup f(N_j(x)) \rangle \geq f(N_1(x)) \circ f(N_j(x)); j = 2, 3$

Take the left side of the inequality (1), we have

$$\begin{aligned}\langle f(N_1(x)), f(N_1(x) \cup f(N_2(x)\rangle &= \left\langle \frac{(0.8,0.6,0.2)}{x}, \frac{(0.8,0.6,0.2)}{x} \cup \frac{0.7,0.5,0.8)}{x} \right\rangle \\ &= \left\langle \frac{(0.8,0.6,0.2)}{x}, \frac{(0.8,0.5,0.2)}{x} \right\rangle \\ &= \frac{(0.64,0.30,0.04)}{x}.\end{aligned}$$

Also

$$\begin{aligned}\langle f(N_1(x)), f(N_1(x) \cup f(N_3(y)\rangle &= \left\langle \frac{(0.8,0.6,0.2)}{x}, \frac{(0.8,0.6,0.2)}{x} \cup \frac{(0.5,0.3,1)}{x} \right\rangle \\ &= \left\langle \frac{(0.8,0.6,0.2)}{x}, \frac{(0.8,0.3,0.2)}{x} \right\rangle \\ &= \frac{(0.64,0.18,0.04)}{x}.\end{aligned}$$

Now we have two neutrosophic values with respect to $f(N_1(x))$, that is;

$$\frac{(0.64,0.30,0.04)}{x}, \frac{(0.64,0.18,0.04)}{x}.$$

The max value of these two is

$$\max\{\frac{(0.64,0.30,0.04)}{x}, \frac{(0.64,0.18,0.04)}{x}\} = \frac{(0.64,0.30,0.04)}{x}.$$

Thus the maximum profit with a neutrosophic value, $N' = \frac{(0.64,0.30,0.04)}{x}$ is characterized by a truth membership function $T_{N'} = 0.40$, an indeterminacy membership function $I_{N'} = 0.30$, and a falsity membership function $F_{N'} = 0.2$.

Also clearly, we have

$$\langle f(N_3(x)), f(N_3(x) \cap f(N_j(x)\rangle \le f(N_3(x)) \circ f(N_j(x)); j = 1,2. \tag{2}$$

Take the left side of the inequality (1), we have

$$\begin{aligned}\langle f(N_3(x)), f(N_3(x) \cap f(N_1(x)\rangle &= \left\langle \frac{(0.5,0.3,1)}{x}, \frac{(0.5,0.3,1)}{x} \cap \frac{(0.8,0.6,0.2)}{x} \right\rangle \\ &= \left\langle \frac{(0.5,0.3,1)}{x}, \frac{(0.5,0.6,1)}{x} \right\rangle \\ &= \frac{(0.40,0.36,0.2)}{x}.\end{aligned}$$

Also

$$\begin{aligned}\langle f(N_3(x)), f(N_3(x) \cap f(N_2(x)\rangle &= \left\langle \frac{(0.5,0.3,1)}{x}, \frac{(0.5,0.3,1)}{x} \cap \frac{(0.7,0.5,0.8)}{x} \right\rangle \\ &= \left\langle \frac{(0.5,0.3,1)}{x}, \frac{(0.5,0.5,1)}{x} \right\rangle \\ &= \frac{(0.40,0.30,0.2)}{x}.\end{aligned}$$

Now we have two neutrosophic values with respect to $f(N_3(x))$, that is;

$$\frac{(0.40, 0.36, 0.2)}{x}, \frac{(0.40, 0.30, 0.2)}{x}.$$

The min value of these two is

$$\min\{\frac{(0.40, 0.36, 0.2)}{x}, \frac{(0.40, 0.30, 0.2)}{x}\} = \frac{(0.40, 0.30, 0.2)}{x}.$$

Thus the minimum profit with a neutrosophic value, $N'' = (0.40, 0.30, 0.2)$ is characterized by a truth membership function $T_{N''} = 0.40$, an indeterminacy membership function $I_{N''} = 0.30$, and a falsity membership function $F_{N''} = 0.2$.

5. Comparison

The neutrosophic set has many applications in many field of science. Here we discussed the application of neutrosophic set in decision-making problems. In this practical application, one of the main issue is that, how to choose a suitable model. We examined this idea in depth and used the neutrosophic max and min variational inequalities.

In the real world, fuzziness is a common phenomenon and is unavoidable in many realistic fields. In 1965, Zadeh (1965a) suggested the idea of fuzzy sets and developed the theory of fuzzy sets. It is used in many fields, including fuzzy control, fuzzy optimization, fuzzy analysis of data, fuzzy time series, etc. Here are some interesting references: (Bukley 1988a, 1989b) used possibility distribution, (Herrera 1983) used fuzzified constraints and objective functions, transformed a fuzzy linear optimization problem to a classical one by using the structural properties of fuzzy numbers. With the development of computer science and evolutionary computation theory, evolutionary computation methods came into play in fuzzy optimization problems. Razavi et, al., in (2014) discussed the fuzzy linear programing and proposed method is to maximize or minimize the total utility of the objective function, as an aggregated function of its intersection with the minimization and maximization sets. Shirin in (2014) discussed the application of optimization problem which belongs to fuzzy environment. He used the fuzzy linear programming for the setting of company production plan. However, many researchers are using the fuzzy linear programming for the applications of fuzzy optimization. Moreover, Cahkraborty et, al., in (2014) have proposed a new method for solving an intuitionistic fuzzy CCM using chance operators and discussed three different approaches to solve the intuitionistic fuzzy linear programming (IFLPP) using possibility, necessity and credibility measures. The model that presented in this paper for optimization problems (maximization and minimization problems) is unique then the methods previously developed. Here we used a max and min variational inequalities to develop an algorithm for further use in optimization problems. Moreover through this model we determined the maximum value and minimum value separately. However our designed model is not a perfect one, it stuck with a deficiency of theoretical support. The concept of neutrosophic variational inequalities may be useful for applications. Therefore it will be significant for future work.

6. Conclusion

In this paper, we have introduced some new concepts of a neutrosophic set such as neutrosophic convex set, strongly neutrosophic convex set, neutrosophic convex function, strongly neutrosophic convex function, the minimum and maximum of a function f with respect to neutrosophic set, min and max neutrosophic variational inequality, neutrosophic general convex set, neutrosophic general convex function and min, max neutrosophic general variational inequality. We have discussed some basic results on these new concepts. Moreover, we proposed the application of neutrosophic set in optimization theory. This work and further study of neutrosophic max and min variational inequalities will give a new direction of application in the field of optimization.

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