

Neutrosophic weakly G*-Closed Sets

A. Atkinswestley¹, S. Chandrasekar²

¹Department of Mathematics, Roever College Engineering and Technology, Elambalur, Perambalur(DT), Tamil Nadu, India

²Department of Mathematics, Arignar Anna Government Arts college, Namakkal(DT), Tamil Nadu, India.

E-mail: ats.wesly@gmail.com, chandrumat@gmail.com.

Abstract— Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic closed set is called Neutrosophic weakly g*-closed sets in Neutrosophic topological spaces and also discussed about properties and characterization

Keywords— Nu.g* open set, Nu.g*closed set, Nu.weakly g* open set, Nu.weakly g*closed set, Neutrosophic topological spaces

I. INTRODUCTION

A.A.Salama introduced Neutrosophic topological spaces by using Smarandache's Neutrosophic sets. Neutrosophic g closed set introduced by R. Dhavasheelan et.al. and Neutrosophic g*-closed sets presented by A. Atkinswesley et.al. Aim of this present paper is, we introduce and investigate about new kind of Neutrosophic closed set is called Neutrosophic weakly g*-closed sets in Neutrosophic topological spaces and also discussed about properties and characterization

II. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Definition 2.1 [7]

Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form

$$A = \{ \langle x, \eta_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$$

Where $\eta_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent Neutrosophic topological spaces the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.2 [7]

A Neutrosophic set $A = \{ \langle x, \eta_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \eta_A, \sigma_A, \gamma_A \rangle$ in $] -0, 1+[$ on X.

Remark 2.3 [7]

We shall use the symbol

$$A = \langle x, \eta_A, \sigma_A, \gamma_A \rangle \text{ for the Neutrosophic set } A = \{ \langle x, \eta_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}.$$

Example 2.4 [7]

Every Neutrosophic set A is a non-empty set in X is obviously on Neutrosophic set having the form $A = \{ \langle x, \eta_A(x), 1 - ((\eta_A(x) + \gamma_A(x))), \gamma_A(x) \rangle : x \in X \}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set 0_N and 1_N in X as follows:

0_N may be defined as:

$$(0_1) 0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2) 0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3) 0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4) 0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

1_N may be defined as :

$$(1_1) 1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2) 1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3) 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4) 1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$$

Definition 2.5 [8]

Let $A = \langle \eta_A, \sigma_A, \gamma_A \rangle$ be a Neutrosophic set on X , then the complement of the set A A^C defined as

$$A^C = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \eta_A(x) \rangle : x \in X \}$$

Definition 2.6 [8]

Let X be a non-empty set, and Neutrosophic sets A and B in the form

$$A = \{ \langle x, \eta_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \} \text{ and}$$

$$B = \{ \langle x, \eta_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}.$$

Then we consider definition for subsets ($A \subseteq B$).

$A \subseteq B$ defined as: $A \subseteq B \Leftrightarrow \eta_A(x) \leq \eta_B(x), \sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$

Proposition 2.7 [8]

For any Neutrosophic set A , then the following condition are holds:

$$(i) 0_N \subseteq A, 0_N \subseteq 0_N$$

$$(ii) A \subseteq 1_N, 1_N \subseteq 1_N$$

Definition 2.8 [8]

Let X be a non-empty set, and $A = \langle x, \eta_B(x), \sigma_A(x), \gamma_A(x) \rangle$, $B = \langle x, \eta_B(x), \sigma_B(x), \gamma_B(x) \rangle$ be two Neutrosophic sets. Then

$$(i) A \cap B \text{ defined as } : A \cap B = \langle x, \eta_A(x) \wedge \eta_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$$

$$(ii) A \cup B \text{ defined as } : A \cup B = \langle x, \eta_A(x) \vee \eta_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$$

Proposition 2.9 [8]

For all A and B are two Neutrosophic sets then the following condition are true:

$$(i) (A \cap B)^C = A^C \cup B^C$$

$$(ii) (A \cup B)^C = A^C \cap B^C.$$

Definition 2.10 [8]

A Neutrosophic topology is a non-empty set X is a family τ_N of Neutrosophic subsets in X satisfying the following axioms:

$$(i) 0_N, 1_N \in \tau_N,$$

$$(ii) G_1 \cap G_2 \in \tau_N \text{ for any } G_1, G_2 \in \tau_N,$$

$$(iii) \cup G_i \in \tau_N \text{ for any family } \{G_i \mid i \in J\} \subseteq \tau_N.$$

the pair (X, τ_N) is called a Neutrosophic topological space.

The element Neutrosophic topological spaces of τ_N are called Neutrosophic open sets.

A Neutrosophic set A is closed if and only if A^C is Neutrosophic open.

Example 2.11[11]

Let $X = \{x\}$ and

$$A_1 = \{ \langle x, 0.5, 0.6, 0.5 \rangle : x \in X \}$$

$$A_2 = \{ \langle x, 0.4, 0.7, 0.8 \rangle : x \in X \}$$

$$A_3 = \{ \langle x, 0.5, 0.7, 0.5 \rangle : x \in X \}$$

$$A_4 = \{ \langle x, 0.4, 0.6, 0.8 \rangle : x \in X \}$$

Then the family $\tau_N = \{0_N, 1_N, A_1, A_2, A_3, A_4\}$ is called a Neutrosophic topological space on X .

Definition 2.12[11]

Let (X, Nu_{τ}) be Neutrosophic topological spaces and $A = \{ \langle x, \eta_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ be a Neutrosophic set in X . Then the Neutrosophic closure and Neutrosophic interior of A are defined by
 $Nu\text{-}Nu\text{-}cl(A) = \bigcap \{ K : K \text{ is a Neutrosophic closed set in } X \text{ and } A \subseteq K \}$
 $Nu\text{-}Nu\text{-}int(A) = \bigcup \{ G : G \text{ is a Neutrosophic open set in } X \text{ and } G \subseteq A \}$.

Definition 2.13

Let (X, Nu_{τ}) be a Neutrosophic topological space. Then A is called

- (i) Neutrosophic regular Closed set [1] (Neu-RCS in short) if $A = Nu\text{-}Cl(Nu\text{-}Int(A))$,
- (ii) Neutrosophic α -Closed set [1] (Neu- α CS in short) if $Nu\text{-}Cl(Nu\text{-}Int(Nu\text{-}Cl(A))) \subseteq A$,
- (iii) Neutrosophic semi Closed set [8] (Neu-SCS in short) if $Nu\text{-}Int(Nu\text{-}Cl(A)) \subseteq A$,
- (iv) Neutrosophic pre Closed set [18] (Neu-PCS in short) if $Nu\text{-}Cl(Nu\text{-}Int(A)) \subseteq A$,

Definition 2.14

Let (X, Nu_{τ}) be a Neutrosophic topological space. Then A is called

- a) Neutrosophic regular open set [1] (Neu-ROS in short) if $A = Nu\text{-}Int(Nu\text{-}Cl(A))$,
- b) Neutrosophic α -open set [1] (Neu- α OS in short) if $A \subseteq Nu\text{-}Int(Nu\text{-}Cl(Nu\text{-}Int(A)))$,
- c) Neutrosophic semi open set [8] (Neu-SOS in short) if $A \subseteq Nu\text{-}Cl(Nu\text{-}Int(A))$,
- d) Neutrosophic pre open set [18] (Neu-POS in short) if $A \subseteq Nu\text{-}Int(Nu\text{-}Cl(A))$,

Definition 2.15:

An Neutrosophic set A of an Neutrosophic topological space (X, \mathfrak{N}) is called:

- (a) Neutrosophic g -closed [4] if $Nu\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic open.
- (b) Neutrosophic sg -closed [17] if $Nu\text{-}scl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic semi open.
- (c) Neutrosophic g^* -closed [2] if $Nu\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic g -open.
- (d) Neutrosophic αg -closed [9] if $Nu\text{-}\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic α -open.
- (e) Neutrosophic $g\alpha$ -closed [5] if $Nu\text{-}\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic α -open.
- (f) Neutrosophic w -closed [16] if $Nu\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic semi open.
- (g) Neutrosophic gp -closed [10] if $Nu\text{-}pcl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic open.
- (h) Neutrosophic gs -closed [17] if $Nu\text{-}scl(A) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic open.

The complements of the above mentioned closed set are their respective open sets.

Definition 2.16[4]

If A is an Neutrosophic set in Neutrosophic topological space (X, \mathfrak{N}) then

- (a) $Nu\text{-}scl(A) = \bigcap \{ F : A \subseteq F, F \text{ is Neutrosophic semi closed} \}$
- (b) $Nu\text{-}pcl(A) = \bigcap \{ F : A \subseteq F, F \text{ is Neutrosophic pre closed} \}$
- (c) $Nu\text{-}\alpha cl(A) = \bigcap \{ F : A \subseteq F, F \text{ is Neutrosophic } \alpha \text{ closed} \}$

Remark 2.17:

- (a) Every Neutrosophic closed set is Neutrosophic g -closed set.
- (b) Every Neutrosophic α -closed set is Neutrosophic αg -closed set.
- (c) Every Neutrosophic g -closed is Neutrosophic $g\alpha$ -closed set.
- (d) Every Neutrosophic αg -closed is Neutrosophic $g\alpha$ -closed set.
- (e) Every Neutrosophic w -closed set is Neutrosophic g -closed
- (f) Every Neutrosophic w -closed set is Neutrosophic sg -closed set.
- (i) Every Neutrosophic sg -closed set is Neutrosophic gs -closed set.

Lemma 2.18[8]: Let A and B be any two Neutrosophic sets of an Neutrosophic topological space (X, \mathfrak{N}) . Then:

- (a) A is an Neutrosophic closed set in $X \Leftrightarrow Nu\text{-}cl(A) = A$

(b) A is an Neutrosophic open set in $X \Leftrightarrow \text{Nu-int}(A) = A$.

(c) $\text{Nu-cl}(A^C) = (\text{Nu-int}(A))^C$.

(d) $\text{Nu-int}(A^C) = (\text{Nu-cl}(A))^C$.

(e) $A \subseteq B \Rightarrow \text{Nu-int}(A) \subseteq \text{Nu-int}(B)$.

(f) $A \subseteq B \Rightarrow \text{Nu-cl}(A) \subseteq \text{Nu-cl}(B)$.

(g) $\text{Nu-cl}(A \cup B) = \text{Nu-cl}(A) \cup \text{Nu-cl}(B)$.

(h) $\text{Nu-int}(A \cap B) = \text{Nu-int}(A) \cap \text{Nu-int}(B)$

I. NEUTROSOPHICWEAKLY g^* -CLOSED SET

Definition 3.1:

An Neutrosophic set A of an Neutrosophic topological space (X, \mathfrak{T}) is called an Neutrosophic weakly g^* -closed if $\text{Nu-cl}(\text{Nu-int}(A)) \subseteq G$ whenever $A \subseteq G$ and G is Neutrosophic g -open in X .

Theorem 3.2:

Every Neutrosophic w -closed set is Neutrosophic weakly g^* -closed.

Proof:

Let A is Neutrosophic w -closed set. Let $A \subseteq U$ and U Neutrosophic semi-open sets in X . Since every Neutrosophic semi open set is Neutrosophic g -open sets U is Neutrosophic g -open sets. Now by definition of Neutrosophic w -closed sets $\text{Nu-cl}(A) \subseteq U$. But $\text{Nu-cl}(\text{Nu-int}(A)) \subseteq \text{Nu-cl}(A) \subseteq U$. We have $\text{Nu-cl}(\text{Nu-int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is Neutrosophic g -open in X . Therefore A is Neutrosophic weakly g^* -closed set.

Remark 3.3:

The converse of above theorem need not be true as from the following example.

Example 3.4:

Let $X = \{a, b\}$ and $\mathfrak{T} = \{0, U, 1\}$ be an Neutrosophic topology on X , where

$U = \langle x, (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$ Then the Neutrosophic set

$A = \langle x, (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$ is Neutrosophic weakly g^* -closed but it is not Neutrosophic w -closed.

Theorem 3.5:

Every Neutrosophic g^* -closed set is Neutrosophic weakly g^* -closed sets..

Proof:

Let A is Neutrosophic g^* -closed set. Let $A \subseteq U$ and U is Neutrosophic g -open sets in X . Now by definition of Neutrosophic g^* -closed sets $\text{Nu-cl}(A) \subseteq U$. But $\text{Nu-cl}(\text{Nu-int}(A)) \subseteq \text{Nu-cl}(A) \subseteq U$. We have $\text{Nu-cl}(\text{Nu-int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is Neutrosophic g -open in X . Therefore A is Neutrosophic weakly g^* -closed set.

Remark 3.6:

The converse of above theorem need not be true as from the following example.

Example 3.7:

Let $X = \{a, b, c, d\}$ and Neutrosophic sets A_1, A_2, A_3, A_4 defined as follows

$A_1 = \langle x, (\frac{9}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$

$A_2 = \langle x, (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{0}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$

$$A_3 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$A_4 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$\mathfrak{T} = \{0, A_1, A_2, A_3, A_4, 1\}$ be an Neutrosophic topology on X. Then the Neutrosophic set

$A = \langle x, \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$ is Neutrosophicweakly g^* -closed but it is not Neutrosophic g^* -closed.

Theorem 3.8:

Every Neutrosophic g -closed set is Neutrosophicweakly g^* -closed sets.

Proof:

Let A is Neutrosophic g -closed set. Let $A \subseteq U$ and U Neutrosophic-open sets in X. Since every Neutrosophic open set is Neutrosophic g -open sets U is Neutrosophic g -open sets. Now by definition of Neutrosophic g -closed sets $Nu-cl(A) \subseteq U$. But $Nu-cl(Nu-int(A)) \subseteq Nu-cl(A) \subseteq U$. We have $Nu-cl(Nu-int(A)) \subseteq U$ whenever $A \subseteq U$ and U is Neutrosophic g -open in X. Therefore A is Neutrosophicweakly g^* -closed set.

Remark 3.9:

The converse of above theorem need not be true as from the following example

Example 3.10:

Let $X = \{a, b, c, d, e\}$ and Neutrosophic sets A_1, A_2, A_3 , defined as follows

$$A_1 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$A_2$$

$$A_3$$

Let $\mathfrak{T} = \{0, A_1, A_2, A_3, 1\}$ be an Neutrosophic topology on X. Then the Neutrosophic set

$A = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$ is Neutrosophicweakly g^* -closed but it is not Neutrosophic g -closed.

Theorem 3.11:

Every Neutrosophic αg -closed set is Neutrosophicweakly g^* -closed sets.

Proof:

Let A is Neutrosophic αg -closed set. Let $A \subseteq U$ and U Neutrosophic -open sets in X. Since every Neutrosophic open set is Neutrosophic g -open sets U is Neutrosophic g -open sets. Now by definition of Neutrosophic αg -closed sets $Nu-\alpha cl(A) \subseteq U$. But $Nu-\alpha cl(A) \subseteq Nu-cl(A)$ therefore $Nu-cl(A) \subseteq A$. Now $Nu-cl(Nu-int(A)) \subseteq Nu-cl(A) \subseteq U$. We have $Nu-cl(Nu-int(A)) \subseteq U$ whenever $A \subseteq U$ and U is Neutrosophic g -open in X. Therefore A is Neutrosophicweakly g^* -closed set.

Remark 3.12:

The converse of above theorem need not be true as from the following example

Example 3.13:

Let $X = \{a, b, c, d\}$ and Neutrosophic sets A_1, A_2 defined as follows

$$A_1 = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$A_2 = \langle x, \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

Let $\mathfrak{T} = \{0, A_1, A_2, 1\}$ be an Neutrosophic topology on X. Then the Neutrosophic set

$$A = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

is Neutrosophicweakly g^* -closed but it is not Neutrosophic αg -closed.

Theorem 3.14:

Every Neutrosophic $g\alpha$ -closed set is Neutrosophicweakly g^* -closed sets.

Proof:

It follows from theorem 3.11 the fact that every Neutrosophic $g\alpha$ -closed set is Neutrosophic αg -closed sets.

Theorem 3.15:

Every Neutrosophic $g p$ -closed set is Neutrosophicweakly g^* -closed sets.

Proof:

Let A is Neutrosophic $g p$ -closed set. Let $A \subseteq U$ and U Neutrosophic-open sets in X . Since every Neutrosophic open set is Neutrosophic g -open sets U is Neutrosophic g -open sets. Now by definition of Neutrosophic $g p$ -closed sets $Nu-pcl(A) \subseteq U$. But $Nu-pcl(A) \subseteq Nu-cl(A)$ therefore $Nu-cl(A) \subseteq A$. Now $Nu-cl(Nu-int(A)) \subseteq Nu-cl(A) \subseteq U$. We have $Nu-cl(Nu-int(A)) \subseteq U$ whenever $A \subseteq U$ and U is Neutrosophic g -open in X . Therefore A is Neutrosophicweakly g^* -closed set.

Remark 3.16:

The converse of above theorem need not be true as from the following example.

Example 3.17:

Let $X = \{a, b\}$ and $\mathfrak{T} = \{0_N, A_1, 1_N\}$ be an Neutrosophic topology on X , where

$$A_1 = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right) \rangle.$$

Then the Neutrosophic set $A_2 = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$ is Neutrosophicweakly g^* -closed but it is not Neutrosophic $g p$ -closed.

Corollary 3.20:

Every Neutrosophic closed set is Neutrosophicweakly g^* -closed set.

Every Neutrosophic α -closed set is Neutrosophicweakly g^* -closed set.

Every Neutrosophic pre-closed set is Neutrosophic weakly g^* -closed set.

Every Neutrosophic regular-closed set is Neutrosophic weakly g^* -closed set.

Proof: Obvious

Remark 3.25: The intersection of two Neutrosophicweakly g^* -closed sets in an Neutrosophic topological space (X, \mathfrak{T}) may not be Neutrosophicweakly g^* -closed. For,

Example 3.26: Let $X = \{a, b, c, d\}$ and Neutrosophic sets A_1, A_2, A_3, A_4 defined as follows

$$A_1 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$A_2 = \langle x, \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$A_3 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$A_4 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$\mathfrak{T} = \{0, A_1, A_2, A_3, A_4, 1, \}$ be an Neutrosophic topology on X . Then the Neutrosophic set

$$A = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

$$B = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right) \rangle$$

are Neutrosophicweakly g^* -closed in (X, \mathfrak{T}) but $A \cap B$ is not Neutrosophicweakly g^* -closed.

Theorem 3.27:

Let A be an Neutrosophicweakly g^* -closed set in an Neutrosophic topological space (X, \mathfrak{T}) and $A \subseteq B \subseteq Nu-cl(Nu-int(A))$. Then B is Neutrosophicweakly g^* -closed in X .

Proof:

Let G be an Neutrosophic g -open set in X such that $B \subseteq G$. Then $A \subseteq G$ and since A is Neutrosophic weakly g^* -closed, $Nu-cl(Nu-int(A)) \subseteq G$. Now $B \subseteq Nu-cl(Nu-int(A)) \Rightarrow Nu-cl(Nu-int(B)) \subseteq Nu-cl(Nu-int(Nu-cl(Int(A)))) = Nu-cl(Nu-int(A))$, $Nu-cl(Nu-int(B)) \subseteq Nu-cl(Nu-int(A)) \subseteq G$. Consequently B is Neutrosophic weakly g^* -closed.

Definition 3.28: An Neutrosophic set A of an Neutrosophic topological space (X, \mathfrak{T}) is called Neutrosophic g^* -open if and only if its complement A^c is Neutrosophic weakly g^* -closed.

Remark 3.29:

Every Neutrosophic w -open set is Neutrosophic weakly g^* -open but its converse may not be true.

Example 3.30:

Let $X = \{a, b\}$ and $\mathfrak{T} = \{0, A_1, 1\}$ be an Neutrosophic topology on X , where $A_1 = \langle x, (\frac{7}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$. Then the Neutrosophic set $A_2 = \langle x, (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{1}{10}, \frac{5}{10}, \frac{8}{10}) \rangle$ is Neutrosophic weakly g^* -open in (X, \mathfrak{T}) but it is not Neutrosophic w -open in (X, \mathfrak{T}) .

Theorem 3.31:

An Neutrosophic set A of an Neutrosophic topological space (X, \mathfrak{T}) is Neutrosophic weakly g^* -open if $F \subseteq Nu-cl(Nu-int(A))$ whenever F is Neutrosophic g -closed and $F \subseteq A$.

Proof: Follows from definition 3.1 and Lemma 2.18

Theorem 3.32:

Let A be an Neutrosophic weakly g^* -open set of an Neutrosophic topological space (X, \mathfrak{T}) and $Nu-cl(Nu-int(A)) \subseteq B \subseteq A$. Then B is Neutrosophic weakly g^* -open.

Proof:

Suppose A is an Neutrosophic weakly g^* -open in X and $Nu-cl(Nu-int(A)) \subseteq B \subseteq A \Rightarrow A^c \subseteq B^c \subseteq (Nu-cl(Nu-int(A)))^c \subseteq A^c \subseteq B^c \subseteq Nu-cl(Nu-int(A^c))$ by Lemma 2.18 and A^c is Neutrosophic weakly g^* -closed it follows from theorem that B^c is Neutrosophic weakly g^* -closed. Hence B is Neutrosophic weakly g^* -open.

IV. CONCLUSION

The theory of g -closed sets plays an important role in general topology. Since its inception many weak and strong forms of g -closed sets have been introduced in general topology as well as fuzzy topology and Neutrosophic topology. The present paper investigated a new weak form of Neutrosophic g -closed sets called Neutrosophic weakly g^* -closed sets which has been compared with the classes of Neutrosophic closed sets, Neutrosophic pre closed sets, Neutrosophic α -closed sets, Neutrosophic w -closed sets, Neutrosophic gp -closed sets, Neutrosophic ag -closed sets, Neutrosophic $g\alpha$ -closed sets, Neutrosophic g^* -closed sets. Several properties and application of Neutrosophic weakly g^* -closed sets are studied. Many examples are given to justify the result.

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