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New form of single valued neutrosophic uncertain linguistic variables aggregation operators for decision-making

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Abstract

A neutrosophic uncertain linguistic variable is composed of an uncertain linguistic variable and a neutrosophic number, which has more advantages than both of uncertain linguistic variable and neutrosophic number in expression. Therefore, the decision-making theory and its related methods based on neutrosophic uncertain linguistic variables have received wide attention of scholars. This paper mainly studies the new expression and operations of single value neutrosophic uncertain linguistic variables and its application in multiple attribute group decision-making (MAGDM). First, a new form of single value neutrosophic uncertain linguistic set (NFSVNULS) and its operational rules are defined. Then, a new form of single value neutrosophic uncertain linguistic variable weighted arithmetic average (NFSVNULVWAA) operator and a new form of single value neutrosophic uncertain linguistic variable weighted geometric average (NFSVNULVWGA) operator are proposed. Furthermore, a MAGDM method based on the proposed aggregation operators is put forward. Finally, an example of investment is used to demonstrate the feasibility and effectiveness of the proposed method.

Keywords: NFSVNULS; NFSVNULVWAA operator; NFSVNULVWGA operator; MAGDM

1. Introduction

While dealing with the decision-making problems, people like to use the precise values to describe the attributes. However, in a real decision environment, there are many complex and changeable factors, and then decision makers have to use linguistic evaluations for their fuzzy expressions. For example, decision makers like to use "excellent", "good", and "poor" to represent the evaluation values. Hence, Zadeh (1975a, 1975b) proposed a linguistic variable set $S = \{S_0, S_1, S_2, S_3, \cdots S_p\}$ (p is an even number) to express the evaluation values. Xu (2004) proposed the concept of uncertain linguistic variables and defined the operational rules. Wang (2007) used the binary semantic and evidenced reasoning method to construct the decision model for multiple attribute group decision-making (MAGDM) problems. Fan and Liu (2010) designed a function of multi-granularity uncertain linguistic mapping to a trapezoidal fuzzy number and constructed the MAGDM method. Zhang and Guo (2012) studied multiple granularity decision-making problems with the incompletely known weights and uncertain linguistic values of attributes.

In the linguistic evaluation environment, the degree of a linguistic variable only indicates the linguistic evaluation value of a decision maker, but cannot depict the uncertain/hesitant degree of decision makers. In order to overcome this drawback, the combinations of linguistic variables and other sets have been put forward. For examples, Ye (2014) proposed an interval neutrosophic linguistic set (INLS) and an interval neutrosophic linguistic number (INLN); Ye (2015) proposed a single valued neutrosophic

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linguistic set (SVNLS) and a single valued neutrosophic linguistic number (SVNLN); Said, Ye, and Smarandache (2015) and Ye (2017a) proposed the interval neutrosophic uncertain linguistic variables; Smarandache (2015) defined the concept of neutrosophic linguistic numbers and Ye (2016) proposed the Neutrosophic linguistic number weighted arithmetic average (NLNWAA) operator and Neutrosophic linguistic number weighted geometric average (NLNWGA) operator; Ye, Mahmood, and Khan (2017) presented the idea of hesitant single valued neutrosophic uncertain linguistic sets and hesitant single valued neutrosophic uncertain linguistic elements (HSVNULEs); Ye (2018a) defined hesitant neutrosophic linguistic numbers; Ye (2017b) proposed the concept of hesitant interval neutrosophic linguistic sets by combining the three concepts of the HFS, interval neutrosophic set, and linguistic set; Ye (2017c) proposed the concept of a linguistic neutrosophic cubic number (LNCN), including an internal LNCN and external LNCN; Liu, Khan, Ye, and Mahmood (2018) proposed the concept of hesitant interval neutrosophic uncertain linguistic sets (HINULSs) and hesitant interval neutrosophic uncertain linguistic elements (HINULEs) and so on (Fan & Ye, 2018; Fan, Ye, Hu, & Fan, 2017; Fang & Ye, 2017; Liu & Shi, 2017; Liu, 2016; Teng, 2016; Ye, 2018b). Neutrosophic set is a very powerful tool to deal with incomplete and indeterminate information proposed by Smarandache (1998) and has attracted the attention of many scholars (Abdel-Basset & Mohamed, 2018; Abdel-Basset, Mohamed, Zhou, & Hezam, 2017; Abdel-Basset, Gunasekaran, Mohamed, & Chilamkurti, 2018; Abdel-Basset, Gunasekaran, Mohamed, & Smarandache, 2018; Abdel-Basset, Mohamed, & Smarandache, 2018a, 2018b; Abdel-Basset, Mohamed, & Chang, 2018; Abdel-Basset, Mohamed, Gamal, & Smarandache, 2018; Abdel-Basset, Mohamed, Smarandache, & Chang, 2018; Abdel-Basset, Zhou, Mohamed, & Chang, 2018; Fan, Fan, & Ye, 2018; Muhiuddin, Bordbar, Smarandache, & Jun, 2018; Pramanik, Dey, & Smarandache, 2018; Pramanik, Roy, Roy, & Smarandache, 2018a, 2018b), which can provide the credibility of the given linguistic evaluation value and linguistic set can provide qualitative evaluation values.

In this paper, a new form of single value neutrosophic uncertain linguistic set (NFSVNULS) and a new form of the single value neutrosophic uncertain linguistic variable (NFSVNULV) are proposed. In NFSVNULS, we adopt two dimensions to express the evaluation of each expert, the uncertain linguistic part is applied to express the linguistic evaluation and weight for different experts, and the single value neutrosophic part is used to depict the degree of linguistic evaluations. Then, we define its operational rules and propose two aggregation operators based on NFSVNULV. Compared to relevant aggregation operators for linguistic variables, on the one hand, we add the weights of linguistic evaluations to depict the important level of different experts in NFSVNULV. Thus, it can not only reflect the linguistic evaluation of each expert, but also can depict the important degree of different experts. On the other hand, we use the expanding function to ensure that the linguistic information aggregation results do not appear "distortion" and "transboundary" phenomenon. The main purposes of this paper are: (i) to propose the concepts of NFSVNULS and NFSVNULV; (ii) to propose the operational rules of NFSVNULS; (iii) to propose a new form of single value neutrosophic uncertain linguistic variable weighted arithmetic average (NFSVNULVWAA) operator and a new form of single value neutrosophic uncertain linguistic variable weighted geometric average (NFSVNULVWGA) operator based on the operational rules of NFSVNULS; (iv) to put forward a MAGDM method based on two aggregation operators, and (v) to apply the proposed method into an example of investment to demonstrate the feasibility and effectiveness of the proposed method.

The organizations of this paper are as following: Section 2 describes some concepts. Section 3 proposes two aggregation operators based on NFSVNULS. Section 4 establishes a MAGDM method based on two aggregation operators. Section 5 provides an illustrative example to demonstrate the application of the proposed method. Section 6 contains conclusions.

2. Some concepts

2.1. Linguistic set

Definition 1 (*Zadeh* (1975*a*, 1975*b*)). Set $S = \{s_j | j \in [0, q]\}$ as a linguistic variable set, in which q is a sufficiently large even number. Then, for any two linguistic variables $s_m, s_n \in S$ and $m, n \in [0, q]$, they satisfy the following properties:

Order: if $m \leq n$, then $s_m \leq s_n$;
The negative operator: $Neg(s_m) = s_n, n = q - m$;
The max operator: if $s_m \leq s_n$, then max $\{s_m, s_n\}$ $= s_n$;
The min operator: if $s_m \leq s_n$, then min $\{s_m, s_n\}$ $= s_m$.

2.2. Uncertain linguistic variable

Definition 2 Ye (2017c). Set $\tilde{S} = [s_g, s_h], s_g, s_h \in S; 0 \leq g \leq h \leq q$, where q is a sufficiently large positive integer, s_g is the lower limit of $\tilde{S}$ and s_h is the upper limit of $\tilde{S}$, then $\tilde{S}$ is an uncertain linguistic variable.

For the sake of convenience, we suppose $\tilde{S}$ is a set of all uncertain linguistic variables.

Set $\tilde{S}_1 = [s_{g1}, s_{h1}]$ and $\tilde{S}_2 = [s_{g2}, s_{h2}]$ as two uncertain linguistic variables, the number $\lambda \geq 0$, they have the following operational rules:

$$\widetilde{S}_1 \oplus \widetilde{S}_2 = \left[s_{g1+g2-\frac{g1*g2}{q}}, s_{h1+h2-\frac{h1*h2}{q}}\right]; \quad (1)$$

$$\widetilde{S}_1 \otimes \widetilde{S}_2 = \left[s_{\frac{g1\times g2}{q}}, s_{\frac{h1\times h2}{q}}\right]; \quad (2)$$

$$\lambda\widetilde{S}_1 = \left[s_{q-q*(1-\frac{g1}{q})^{\lambda}}, s_{q-q*(1-\frac{h1}{q})^{\lambda}}\right]. \quad (3)$$

2.3. The new uncertain linguistic variable set

Definition 3 Liu (2016). Set $\widetilde{S} = [S_{g(l)}, S_{h(l)}]$ as an uncertain linguistic variable, L as an universe of discourse, $\forall l \in L, S_{g(l)}, S_{h(l)} \in S, 0 \le g(l) \le h(l) \le q$, $S_{g(l)}$ is the lower limit of $\widetilde{S}$ and $S_{h(l)}$ is the upper limit of $\widetilde{S}$, then we can get a new uncertain linguistic variable set $\widehat{S}$ as following:

$$\hat{S} = \{\langle w(l), [S_{g(l)}\ , S_{h(l)}]\rangle | l \in L\}$$

In which w (l): L → [0,1] means the important level of l in the universe L and $\sum_{l\in L} w(l) = 1$.

2.4. The fuzzy set (FS)

Definition 4 Zadeh (1965). Let L be a universe of discourse, a fuzzy set in L is defined as $A = \{\langle\ l, u_A(l)\rangle | l \in L\}$, where $u_A(l)$ is the membership function of the fuzzy set A, and $0 \le u_A(l) \le 1$.

2.5. The intuitionistic fuzzy set (IFS)

Definition 5 Atanassov (1986). Let L be a universe of discourse, an IFS in L is defined as: $A = \{\langle\ l, u_A(l), v_A(l)\rangle | l \in L\}$, where $u_A(l)$: L → [0,1] and $v_A(l)$: L → [0,1] are the membership function and non-membership of IFS A, respectively.

2.6. New linguistic intuition fuzzy set (NLIFS)

Definition 6 Liu (2016). Set $S = \{s_j | j \in [0, q]\}$ as an LS; $q+1$ is the cardinality of S. Set L as a universe of discourse, $\forall l \in L, S_{g(l)} \in S$, then a NLIFS is defined as following:

$$R = \{\langle [w(l), S_{g(l)})], u(l), v(l)\rangle | l \in L\}$$

In which w (l): L → [0,1] means the important level of l in the universe L; $u(l) : L \to [0,1]$ and $v(l) : L \to [0,1]$ mean membership and non-membership of a linguistic variable $S_{g(l)}$, respectively. $\sum_{l\in L} w(l) = 1, u(l) + v(l) \in [0,1]$.

For the simplicity of description, we denote the new linguistic intuition fuzzy number (NLIFN) into $r = \langle S_{g(l)}^{w(l)}, u(l), v(l)\rangle$.

Set $r_1 = \langle S_{g(l_1)}^{w(l_1)}, u(l_1), v(l_1)\rangle$, $r_2 = \langle S_{g(l_2)}^{w(l_2)}, u(l_2), v(l_2)\rangle$ *and* $r_3 = \langle S_{g(l_3)}^{w(l_3)}, u(l_3), v(l_3)\rangle$ as three NLIFNs, $f\ (t)$ is expanding function and $\lambda \in [0,1]$, the operational rules are listed as follows:

$$r_1 \oplus r_2 = \langle S^{f^{-1}(f(w(l_1))+f(w(l_2)))}_{\frac{f(w(l_1))*g(l_1)+f(w(l_2))*g(l_2)}{f(w(l_1))+f(w(l_2))}}, 1-(1-u(l_1))(1-u(l_2)), v(l_1)v(l_2)\rangle. \quad (4)$$

$$\lambda r_1 = \langle S_{g(l_1)}^{f^{-1}(\lambda f(w(l_1)))}, 1-(1-u(l_1))^{\lambda}, v(l_1)^{\lambda}\rangle \quad (5)$$

According to the operational rules of NLIFNs, they have the following properties:

(1) (Commutative law) $r_1 \oplus r_2 = r_2 \oplus r_1$;
(2) (Associative law) $(r_1 \oplus r_2) \oplus r_3 = r_1 \oplus (r_2 \oplus r_3)$;
(3) (Distributive law) $\lambda(r_1 \oplus r_2) = \lambda r_1 \oplus \lambda r_2$.

Definition 7 Liu (2016). Set $r = \langle S_{g(l)}^{w(l)}, u(l), v(l)\rangle$ as a NLIFN, then the expectation E (r) and the accuracy H (r) can be defined as following:

$$E(r) = \frac{1}{2}(1 + u(l) - v(l))w(l)\frac{g(l)}{q} \quad (6)$$

$$H(r) = \left(w(l)\frac{g(l)}{q}\right)(u(l) + v(l)) \quad (7)$$

2.7. Expanding function

In order to ensure the results of linguistic information aggregation do not appear "distortion" and "transboundary" phenomenon, we propose the concept of expanding function.

Definition 8. Set function f: [0,1] → [0,∞) and f (t) is continuous function in [0,1), which is called expanding functions if and only if satisfies the following conditions:

(1) f (t) is strict monotonically increasing in [0,1);
(2) f (0) = 0;
(3) f (1) = +∞.

Based on Definition 8, it shows that $f\ (t)$ has the inverse function. Some expanding functions are list in Table 1.

In this paper, we use $f(t) = \frac{1}{1-t} - 1$ as the expanding function and its inverse function is $f^{-1}(t) = 1 - \frac{1}{1+t}$, which is easy for calculation.

2.8. Single-value neutrosophic set

Definition 9 (Smarandache (1998), Smarandache et al. (2009), Kandasamy and Smarandache (2011a), Kandasamy and Smarandache (2011b)). Let X be a given universe, and then R= $\{\langle x, T_R(x), I_R(x), F_R(x)\rangle | x \in X\}$ is a single-value neutrosophic set (SVNS). $T_R(x)$ is a truth-membership function, $F_R(x)$ is a falsity-membership function and $I_R(x)$ is an indeterminacy-membership function. $T_R(x), I_R(x), F_R(x) \in [0,1]$ *and* $3 \geq T_R(x) + I_R(x) + F_R(x) \geq 0$.

The operational rules of two SVNSs M and N can be defined as following:

$M \oplus N = \ T_M(x) + T_N(x) - T_M(x)T_N(x), \quad I_M(x)I_N(x), F_M(x)F_N(x), \forall x \in X;$

$\lambda M = \langle 1-(1-T_M(x))^{\lambda}, (I_M(x))^{\lambda}, (F_M(x))^{\lambda}\rangle, \lambda > 0, \forall x \in X.$

$M^{\lambda} = \langle (T_M(x))^{\lambda}, 1-(1-I_M(x))^{\lambda}, 1-(1-F_M(x))^{\lambda}\rangle, \lambda > 0, \forall x \in X$

2.9. A new form of single value neutrosophic uncertain linguistic set

As an extension of Definition 7, we present a new form of single value neutrosophic uncertain linguistic set (NFSVNULS) as following:

Definition 10. Set S $= \{s_{\alpha} | \alpha \in [0,q]\}$ is a continuous linguistic variable set, set L as a universe of discourse, $\forall l \in L, [S_{g(l)}, S_{h(l)}] \in S$, and then an NFSVNULS is defined as:

$$\tilde{R} = \{\langle\ w(l), ([S_{g(l)}, S_{h(l)}], t_{R^{\sim}}(l), i_{R^{\sim}}(l), f_{R^{\sim}}(l))\rangle | l \in L\} \quad (8)$$

In which w(l): L → [0,1] means the important level of l in universe L, $t_{\tilde{R}}(l)$: L → [0,1]; $f_{\tilde{R}}(l)$: L → [0,1] and $i_{\tilde{R}}(l)$: L → [0,1] mean truth membership function, falsity membership function and indeterminacy membership of linguistic variable $S_{g(l)}, S_{h(l)}$, respectively, $\sum_{l \in L} w(l) = 1$ and $0 \leq t_{\tilde{R}}(l) + i_{\tilde{R}}(l) + f_{\tilde{R}}(l) \leq 3$.

For the simplicity of description, we simplify the new form of the single value neutrosophic uncertain linguistic variable (NFSVNULV) into $\tilde{r} = \langle [S_{g(l)}^{w(l)}, S_{h(l)}^{w(l)}], w(l)(t(l), i(l), f(l))\rangle = \langle [S_{g(l)}^{w(l)}, S_{h(l)}^{w(l)}], T(l), I(l), F(l)\rangle$, where $T(l) = 1-(1-t(l))^{w(l)}, \ I(l) = (i(l))^{w(l)}, \ F(l) = (f(l))^{w(l)}$.

Set $\tilde{r}_1 = \langle [S_{g(l_1)}^{w(l_1)}, S_{h(l_1)}^{w(l_1)}], w(l_1)(t(l_1), i(l_1), f(l_1))\rangle$, $\tilde{r}_2 = \langle [S_{g(l_2)}^{w(l_2)}, S_{h(l_2)}^{w(l_2)}], w(l_2)(t(l_2), i(l_2), f(l_2))\rangle$ and $\tilde{r}_3 = \langle [S_{g(l_3)}^{w(l_3)}, S_{h(l_3)}^{w(l_3)}], w(l_3)(t(l_3), i(l_3), f(l_3))\rangle$ as three NFSVNULVs, $f(t)$ is an expanding function and $\lambda \in [0,1]$, the operation rules of NFSVNULVs are listed as following:

Table 1
Five kinds of expanding fuctions.

$f(t)(0 \leq t \leq 1)$	$f^{-1}(t)$
$tan(\frac{\pi}{2}t)$	$\frac{2}{\pi}arctan(t)$
$\frac{1}{1-t} - 1$	$1 - \frac{1}{1+t}$
$\frac{1}{(1-t)^2} - 1$	$1 - \frac{1}{\sqrt{1+t}}$
$-\ln(1-t)$	$1 - e^{-t}$
$e^{\frac{1}{1-t}} - e$	$1 - \frac{1}{ln(e+t)}$

$$\tilde{r}_1 \oplus \tilde{r}_2 = \langle [S_{\frac{f(w(l_1))*g(l_1)+f(w(l_2))*g(l_2)}{f(w(l_1))+f(w(l_2))}}^{f^{-1}(f(w(l_1))+f(w(l_2)))}, S_{\frac{f(w(l_1))*h(l_1)+f(w(l_2))*h(l_2)}{f(w(l_1))+f(w(l_2))}}^{f^{-1}(f(w(l_1))+f(w(l_2)))}],$$
$$T(l_1) + T(l_2) - T(l_1)T(l_2), I(l_1)I(l_2), F(l_1)F(l_2)\rangle \quad (9)$$

$$\lambda\tilde{r}_1 = \langle [S_{g(l_1)}^{f^{-1}(\lambda f(w(l_1)))}, S_{h(l_1)}^{f^{-1}(\lambda f(w(l_1)))}],$$
$$1-(1-T(l_1))^{\lambda}, (I(l_1))^{\lambda}, (F(l_1))^{\lambda}\rangle \quad (10)$$

$$\tilde{r}_1^{\ \lambda} = \langle [S_{g(l_1)}^{f^{-1}\left(f(w(l_1))^{\lambda}\right)}, S_{h(l_1)}^{f^{-1}\left(f(w(l_1))^{\lambda}\right)}], (T(l_1))^{\lambda}, 1$$
$$-(1-I(l_1))^{\lambda}, 1-(1-F(l_1))^{\lambda}\rangle \quad (11)$$

where $T(l_i) = 1-(1-t(l_i))^{w(l_i)}, I(l_i) = (i(l_i))^{w(l_i)}, F(l_i) = (f(l_i))^{w(l_i)}$ for $i = 1, 2$

These operational rules of NFSVNULVs satisfy the following properties:

$$\tilde{r}_1 \oplus \tilde{r}_2 = \tilde{r}_2 \oplus \tilde{r}_1; \quad (12)$$

$$\lambda\left(\tilde{r}_1 \oplus \tilde{r}_2\right) = \lambda\tilde{r}_1 \oplus \lambda\tilde{r}_2; \quad (13)$$

$$(\tilde{r}_1 \oplus \tilde{r}_2) \oplus \tilde{r}_3 = \tilde{r}_1 \oplus (\tilde{r}_2 \oplus \tilde{r}_3) \quad (14)$$

Proof. (1) According to Formula (9), we can easily obtain it.

(2) $\lambda\left(\tilde{r}_1 \oplus \tilde{r}_2\right) = \lambda\Bigg(\langle [S_{\frac{f(w(l_1))*g(l_1)+f(w(l_2))*g(l_2)}{f(w(l_1))+f(w(l_2))}}^{f^{-1}(f(w(l_1))+f(w(l_2)))} S_{\frac{f(w(l_1))*h(l_1)+f(w(l_2))*h(l_2)}{f(w(l_1))+f(w(l_2))}}^{f^{-1}(f(w(l_1))+f(w(l_2)))}],$
$T(l_1) + T(l_2) - T(l_1)T(l_2), I(l_1)I(l_2), F(l_1)F(l_2)\rangle)$

$= \langle [S_{\frac{f(w(l_1))*g(l_1)+f(w(l_2))*g(l_2)}{f(w(l_1))+f(w(l_2))}}^{f^{-1}(\lambda(f(w(l_1))+f(w(l_2))))}, S_{\frac{f(w(l_1))*h(l_1)+f(w(l_2))*h(l_2)}{f(w(l_1))+f(w(l_2))}}^{f^{-1}(\lambda(f(w(l_1))+f(w(l_2))))}],$

$1-(1-(T(l_1)+T(l_2)-T(l_1)T(l_2)))^{\lambda}, (I(l_1)I(l_2))^{\lambda},$
$(F(l_1)F(l_2))^{\lambda}\rangle$

$\lambda\tilde{r}_1 \oplus \lambda\tilde{r}_2 = \langle [S_{g(l_1)}^{f^{-1}(\lambda f(w(l_1)))}, S_{h(l_1)}^{f^{-1}(\lambda f(w(l_1)))}],$

$1-(1-T(l_1))^{\lambda}, (I(l_1))^{\lambda}, (F(l_1))^{\lambda}\rangle \oplus \langle [S_{g(l_2)}^{f^{-1}(\lambda f(w(l_2)))},$

$S_{h(l_2)}^{f^{-1}(\lambda f(w(l_2)))}], 1-(1-T(l_2))^{\lambda}, (I(l_2))^{\lambda},$

$(F(l_2))^{\lambda}\rangle = \langle [S_{\frac{f(f^{-1}(\lambda f(w(l_1))))g(l_1)+f(f^{-1}(\lambda f(w(l_2))))g(l_2)}{f(f^{-1}(\lambda f(w(l_1))))+f(f^{-1}(\lambda f(w(l_2))))}}^{f^{-1}\left(f\left(f^{-1}(\lambda f(w(l_1)))\right)+f\left(f^{-1}(\lambda f(w(l_2)))\right)\right)},$

$S_{\frac{f(f^{-1}(\lambda f(w(l_1))))h(l_1)+f(f^{-1}(\lambda f(w(l_2))))h(l_2)}{f(f^{-1}(\lambda f(w(l_1))))+f(f^{-1}(\lambda f(w(l_2))))}}^{f^{-1}\left(f\left(f^{-1}(\lambda f(w(l_1)))\right)+f\left(f^{-1}(\lambda f(w(l_2)))\right)\right)}],$

$$\left(1-(1-T(l_1))^{\lambda}\right) + \left(1-(1-T(l_2))^{\lambda}\right)$$
$$+ \left(1-(1-T(l_1))^{\lambda}\right)\left(1-(1-T(l_2))^{\lambda}\right),$$

$(I(l_1))^{\lambda}(I(l_2))^{\lambda}, (F(l_1))^{\lambda}(F(l_2))^{\lambda}\rangle$

Because $f\left(f^{-1}(x)\right)=x$, so

$$\lambda\widetilde{r}_1\oplus\lambda\widetilde{r}_2=\langle[S^{f^{-1}\left(f\left(f^{-1}(\lambda f(w(l_1)))\right)+f\left(f^{-1}(\lambda f(w(l_2)))\right)\right)}_{\frac{f(f^{-1}(\lambda f(w(l_1))))g(l_1)+f(f^{-1}(\lambda f(w(l_2))))g(l_2),}{f(f^{-1}(\lambda f(w(l_1))))+f(f^{-1}(\lambda f(w(l_2))))}}$$
$$\times S^{f^{-1}\left(f\left(f^{-1}(\lambda f(w(l_1)))\right)+f\left(f^{-1}(\lambda f(w(l_2)))\right)\right)}_{\frac{f(f^{-1}(\lambda f(w(l_1))))h(l_1)+f(f^{-1}(\lambda f(w(l_2))))h(l_2)}{f(f^{-1}(\lambda f(w(l_1))))+f(f^{-1}(\lambda f(w(l_2))))}}],\left(1-(1-T(l_1))^{\lambda}\right)$$
$$+\left(1-(1-T(l_2))^{\lambda}\right)+\left(1-(1-T(l_1))^{\lambda}\right)\left(1-(1-T(l_2))^{\lambda}\right),$$
$$(I(l_1))^{\lambda}(I(l_2))^{\lambda},(F(l_1))^{\lambda}(F(l_2))^{\lambda}\rangle$$

$$=\langle[S^{f^{-1}(\lambda f(w(l_1))+\lambda f(w(l_2)))}_{\frac{f(w(l_1))g(l_1)+f(w(l_2))g(l_2)}{f(w(l_1))+f(w(l_2))}},S^{f^{-1}(\lambda f(w(l_1))+\lambda f(w(l_2)))}_{\frac{f(w(l_1))h(l_1)+f(w(l_2))h(l_2)}{f(w(l_1))+f(w(l_2))}}],$$
$$1-(1-(T(l_1)+T(l_2)-T(l_1)T(l_2)))^{\lambda},(I(l_1)I(l_2))^{\lambda},$$
$$\left(F(l_1)F(l_2))^{\lambda}\right)\rangle$$

Then, we can get $\lambda(\widetilde{r}_1\oplus\widetilde{r}_2)=\lambda\widetilde{r}_1\oplus\lambda\widetilde{r}_2$.

$$(\widetilde{r}_1\oplus\widetilde{r}_2)\oplus\widetilde{r}_3=\langle[S^{f^{-1}(f(w(l_1))+f(w(l_2)))}_{\frac{f(w(l_1))g(l_1)+f(w(l_2))g(l_2)}{f(w(l_1))+f(w(l_2))}},S^{f^{-1}(f(w(l_1))+f(w(l_2)))}_{\frac{f(w(l_1))h(l_1)+f(w(l_2))h(l_2)}{f(w(l_1))+f(w(l_2))}}],$$
$$T(l_1)+T(l_2)-T(l_1)T(l_2),I(l_1)I(l_2),$$
$$F(l_1)F(l_2)\rangle\oplus\langle[S^{w(l_3)}_{g(l_3)},S^{w(l_3)}_{h(l_3)}],T(l_3),I(l_3),$$
$$F(l_3)\rangle$$

$$=\langle[S^{f^{-1}\left(f\left(f^{-1}(f(w(l_1))+f(w(l_2)))\right)+f(w(l_3))\right)}_{\frac{f(f^{-1}(f(w(l_1))+f(w(l_2))))*\frac{f(w(l_1))g(l_1)+f(w(l_2))g(l_2)}{f(w(l_1))+f(w(l_2))}+f(w(l_3)*g(l_3)}{f(f^{-1}(f(w(l_1))+f(w(l_2))))+f(w(l_3))}},$$
$$\times S^{f^{-1}\left(f\left(f^{-1}(f(w(l_1))+f(w(l_2)))\right)+f(w(l_3))\right)}_{\frac{f(f^{-1}(f(w(l_1))+f(w(l_2))))*\frac{f(w(l_1))h(l_1)+f(w(l_2))h(l_2)}{f(w(l_1))+f(w(l_2))}+f(w(l_3))*h(l_3)}{f(f^{-1}(f(w(l_1))+f(w(l_2))))+f(w(l_3))}}],$$
$$T(l_1)+T(l_2)-T(l_1)T(l_2)+T(l_3)-(T(l_1)$$
$$+T(l_2)-T(l_1)T(l_2))T(l_3),I(l_1)I(l_2)I(l_3),$$
$$F(l_1)F(l_2)F(l_3)\rangle$$

$$=\langle[S^{f^{-1}(f(w(l_1))+f(w(l_2))+f(w(l_3))}_{\frac{f(w(l_1)*g(l_1)+f(w(l_2))*g(l_2))+f(w(l_3)*g(l_3)}{f(w(l_1))+f(w(l_2))+f(w(l_3)}},S^{f^{-1}(f(w(l_1))+f(w(l_2))+f(w(l_3))}_{\frac{f(w(l_1)*h(l_1)+f(w(l_2))*h(l_2))+f(w(l_3)*h(l_3)}{f(w(l_1))+f(w(l_2))+f(w(l_3)}}],$$
$$T(l_1)+T(l_2)+T(l_3)-T(l_1)T(l_2)-T(l_1)T(l_3)$$
$$-T(l_2)T(l_3)+T(l_1)T(l_2)T(l_3),I(l_1)I(l_2)I(l_3),$$
$$F(l_1)F(l_2)F(l_3)\rangle$$

$$\widetilde{r}_1\oplus\left(\widetilde{r}_2\oplus\widetilde{r}_3\right)=\langle[S^{w(l_1)}_{g(l_1)},S^{w(l_1)}_{h(l_1)}],T(l_1),I(l_1),F(l_1)>\oplus$$
$$<[S^{f^{-1}(f(w(l_2))+f(w(l_3)))}_{\frac{f(w(l_2))*g(l_2)+f(w(l_3))*g(l_3)}{f(w(l_2))+f(w(l_3))}},S^{f^{-1}(f(w(l_2))+f(w(l_3)))}_{\frac{f(w(l_2)*h(l_2)+f(w(l_3))*h(l_3)}{f(w(l_2))+f(w(l_3))}}],$$
$$T(l_2)+T(l_3)-T(l_2)T(l_3),I(l_2)I(l_3),$$
$$F(l_2)F(l_3)\rangle$$

$$=\langle[S^{f^{-1}\left(f(f^{-1}(f(w(l_2))+f(w(l_3))))+f(w(l_1))\right)}_{\frac{f(f^{-1}(f(w(l_2))+f(w(l_3))))*\frac{f(w(l_2))*g(l_2)+f(w(l_3))*g(l_3)}{f(w(l_2))+f(w(l_3))}+f(w(l_1))*g(l_1)}{f(f^{-1}(f(w(l_1))+f(w(l_2))))+f(w(l_3))}},$$
$$\times S^{f^{-1}\left(f(f^{-1}(f(w(l_2))+f(w(l_3))))+f(w(l_1))\right)}_{\frac{f(f^{-1}(f(w(l_2))+f(w(l_3))))*\frac{f(w(l_2))*h(l_2)+f(w(l_3))*h(l_3)}{f(w(l_2))+f(w(l_3))}+f(w(l_1))*h(l_1)}{f(f^{-1}(f(w(l_1))+f(w(l_2))))+f(w(l_3))}}],$$
$$\times T(l_2)+T(l_3)-T(l_2)T(l_3)+T(l_1)-(T(l_2)+T(l_3)$$
$$-T(l_2)T(l_3))T(l_1),I(l_2)I(l_3)I(l_1),F(l_2)F(l_3)F(l_1)\rangle$$

$$=\langle[S^{f^{-1}(f(w(l_1))+f(w(l_2))+f(w(l_3)))}_{\left(\frac{f(w(l_1))*g(l_1)+f(w(l_2))*g(l_2)+f(w(l_3))*g(l_3)}{f(w(l_1))+f(w(l_2))+f(w(l_3))}\right)},$$
$$S^{f^{-1}(f(w(l_1))+f(w(l_2))+f(w(l_3)))}_{\left(\frac{f(w(l_1))*h(l_1)+f(w(l_2))*h(l_2)+f(w(l_3))*h(l_3)}{f(w(l_1))+f(w(l_2))+f(w(l_3))}\right)}],$$
$$T(l_1)+T(l_2)+T(l_3)-T(l_1)T(l_2)-T(l_1)T(l_3)$$
$$-T(l_2)T(l_3)+T(l_1)T(l_2)T(l_3),I(l_1)I(l_2)I(l_3),$$
$$F(l_1)F(l_2)F(l_3)\rangle$$

Then we can get $\left(\widetilde{r}_1\oplus\widetilde{r}_2\right)\oplus\widetilde{r}_3=\widetilde{r}_1\oplus\left(\widetilde{r}_2\oplus\widetilde{r}_3\right)$. □

Definition 11. Set $\widetilde{r}=<[S^{w(l)}_{g(l)},S^{w(l)}_{h(l)}],w(l)(t(l),i(l),f(l))>$ as an NFSVNULV,- $T(l)=1-(1-t(l))^{w(l)},I(l)=(i(l))^{w(l)},F(l)=(f(l))^{w(l)}$, then the expectation$E(\widetilde{r})$ and the accuracy $H(\widetilde{r})$ can be defined as following:

$$E\left(\widetilde{r}\right)=\frac{1}{6}(2+T(l)-I(l)-F(l))(w(l)\frac{g(l)}{q}+w(l)\frac{h(l)}{q}) \tag{15}$$

$$H\left(\widetilde{r}\right)=\frac{1}{4}\left(w(l)\frac{g(l)}{q}+w(l)\frac{h(l)}{q}\right)(T(l)+I(l)) \tag{16}$$

In which $q+1$ is odd cardinality of an LS $\{s_\alpha|\alpha\in[0,q]\}$.

Based on the Definition 11, we propose a sorting method for NFSVNULVs.

Definition 12. Set $\widetilde{r}_1=\langle[S^{w(l_1)}_{g(l_1)},S^{w(l_1)}_{h(l_1)}],w(l_1)(t(l_1),i(l_1),f(l_1))\rangle,\widetilde{r}_2=\langle[S^{w(l_2)}_{g(l_2)},S^{w(l_2)}_{h(l_2)}],w(l_2)(t(l_2),i(l_2),f(l_2))\rangle$ as two NFSVNULVs, then

If $E(\widetilde{r}_1)>E(\widetilde{r}_2)$, then$\widetilde{r}_1\succ\widetilde{r}_2$;
If $E(\widetilde{r}_1)=E(\widetilde{r}_2)$ then
If $H(\widetilde{r}_1)>H(\widetilde{r}_2)$, then$\widetilde{r}_1\succ\widetilde{r}_2$;
If $H(\widetilde{r}_1)=H(\widetilde{r}_2)$, then$\widetilde{r}_1\sim\widetilde{r}_2$;
If $H(\widetilde{r}_1)<H(\widetilde{r}_2)$, then$\widetilde{r}_1\prec\widetilde{r}_2$.

3. Two aggregation operators for NFSVNULVs

Definition 13. Set $\widetilde{r}_k=\langle[S^{w(l_k)}_{g(l_k)},S^{w(l_k)}_{h(l_k)}],w(l_k)(t(l_k),i(l_k),f(l_k))\rangle\in\widetilde{R}(k=1,2,\cdots,n)$ as a NFSVNULV, f (t) is expanding function, then the mapping NFSVNULVAA: $\widetilde{R}^*\rightarrow\widetilde{R}$ as the arithmetic average operator of the NFSVNULV, which can satisfy the following conditions:

$$\mathrm{NFSVNULVAA}\left(\tilde{r}_1,\tilde{r}_2,\cdots,\tilde{r}_n\right)=\tilde{r}_1\oplus\tilde{r}_2\oplus\cdots\oplus\tilde{r}_n$$
$$=\langle[S^{f^{-1}\left(\sum_{i=1}^{n}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{n}f(w(l_i))*g(l_i)}{\sum_{i=1}^{n}f(w(l_i))}},S^{f^{-1}\left(\sum_{i=1}^{n}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{n}f(w(l_i))*h(l_i)}{\sum_{i=1}^{n}f(w(l_i))}}],$$
$$1-\prod_{i=1}^{n}(1-T(l_i)),\ \prod_{i=1}^{n}I(l_i),\ \prod_{i=1}^{n}F(l_i)\rangle, \tag{17}$$
$$T(l_i)=1-(1-t(l_i))^{w(l_i)},\ I(l_i)=(i(l_i))^{w(l_i)},\ F(l_i)=(f(l_i))^{w(l_i)}.$$

Proof. (1) When $n=1$, the result is obvious.

(2) When $n=2$, according to Formula (9), we can get

$$\mathrm{NFSVNULVAA}(\tilde{r}_1,\tilde{r}_2)=\tilde{r}_1\oplus\tilde{r}_2$$
$$=\langle[S^{f^{-1}(f(w(l_1))+f(w(l_2)))}_{\frac{f(w(l_1))*g(l_1)+f(w(l_2))*g(l_2)}{f(w(l_1))+f(w(l_2))}},S^{f^{-1}(f(w(l_1))+f(w(l_2)))}_{\frac{f(w(l_1))*h(l_1)+f(w(l_2))*h(l_2)}{f(w(l_1))+f(w(l_2))}}],$$
$$T(l_1)+T(l_2)-T(l_1)T(l_2),I(l_1)I(l_2),F(l_1)F(l_2)\rangle$$

If $k=m$, Formula (17) is established, then we can get

$$\mathrm{NFSVNULVAA}(\tilde{r}_1,\tilde{r}_2,\cdots,\tilde{r}_m)=\tilde{r}_1\oplus\tilde{r}_2\oplus\cdots\oplus\tilde{r}_m$$
$$=\langle[S^{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{m}f(w(l_i))*g(l_i)}{\sum_{i=1}^{m}f(w(l_i))}},S^{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{m}f(w(l_i))*h(l_i)}{\sum_{i=1}^{m}f(w(l_i))}}],1-\prod_{i=1}^{m}(1-T(l_i)),\prod_{i=1}^{m}I(l_i),\prod_{i=1}^{m}F(l_i)\rangle$$

Then

$$\mathrm{NFSVNULVAA}(\tilde{r}_1,\tilde{r}_2,\cdots,\tilde{r}_m,\tilde{r}_{m+1})$$
$$=\tilde{r}_1\oplus\tilde{r}_2\oplus\cdots\oplus\tilde{r}_m\oplus\tilde{r}_{m+1}$$
$$=\langle[S^{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{m}f(w(l_i))*g(l_i)}{\sum_{i=1}^{m}f(w(l_i))}},S^{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{m}f(w(l_i))*h(l_i)}{\sum_{i=1}^{m}f(w(l_i))}}],$$
$$1-\prod_{i=1}^{m}(1-T(l_i)),\prod_{i=1}^{m}I(l_i),\prod_{i=1}^{m}F(l_i)\rangle\oplus\langle[S^{w(l_{m+1})}_{g(l_{m+1})},S^{w(l_{m+1})}_{h(l_{m+1})}],$$
$$T(l_{m+1}),I(l_{m+1}),F(l_{m+1})\rangle$$

$$=\langle\left[S^{f^{-1}\left(f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)+f(w(l_{m+1}))\right)}_{\frac{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)*\frac{\sum_{i=1}^{m}f(w(l_i))*g(l_i)}{\sum_{i=1}^{m}f(w(l_i))}+w(l_{m+1})*g(l_{m+1})}{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)+f(w(l_{m+1}))}},S^{f^{-1}\left(f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)+f(w(l_{m+1}))\right)}_{\frac{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)*\frac{\sum_{i=1}^{m}f(w(l_i))*h(l_i)}{\sum_{i=1}^{m}f(w(l_i))}+w(l_{m+1})*h(l_{m+1})}{f^{-1}\left(\sum_{i=1}^{m}f(w(l_i))\right)+f(w(l_{m+1}))}}\right],$$
$$1-\prod_{i=1}^{m}(1-T(l_i))+T(l_{m+1})-(1-\prod_{i=1}^{m}(1-T(l_i)))*T(l_{m+1})+\prod_{i=1}^{m}T(l_i),\prod_{i=1}^{m}I(l_i)*I(l_{m+1}),\prod_{i=1}^{m}F(l_i)*F(l_{m+1}))\rangle$$

$$=\langle[S^{f^{-1}\left(\sum_{i=1}^{m+1}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{m+1}f(w(l_i))*g(l_i)}{\sum_{i=1}^{m+1}f(w(l_i))}},S^{f^{-1}\left(\sum_{i=1}^{m+1}f(w(l_i))\right)}_{\frac{\sum_{i=1}^{m+1}f(w(l_i))*h(l_i)}{\sum_{i=1}^{m+1}f(w(l_i))}}],$$
$$1-\prod_{i=1}^{m+1}(1-T(l_i)),\prod_{i=1}^{m+1}I(l_i),\prod_{i=1}^{m+1}F(l_i)\rangle$$

To sum up, when $i=m+1$, Formula (17) is true, and then following with the mathematical induction, the result of aggregation is also true. □

Definition 14. Set $\tilde{r}_k=\langle[S^{w(l_k)}_{g(l_k)},S^{w(l_k)}_{h(l_k)}],w(l_k)(t(l_k),i(l_k),f(l_k))\rangle\in\tilde{R}(k=1,2,\cdots,n)$ as an NFSVNULV, f (t) is expanding function, then the mapping $NFSVNULVWAA{:}\tilde{R}^*\to\tilde{R}$ as the weighted arithmetic average operator of the NFSVNULV, which can satisfy the following conditions:

$$NFSVNULVWAA(\tilde{r}_1,\tilde{r}_2,\cdots,\tilde{r}_n)=\sum_{i=1}^{n}\lambda_i\tilde{r}_i$$
$$=\langle[S^{f^{-1}\left(\sum_{i=1}^{n}\lambda_if(w(l_i))\right)}_{\frac{\sum_{i=1}^{n}f(w(l_i))*g(l_i)}{\sum_{i=1}^{n}f(w(l_i))}},S^{f^{-1}\left(\sum_{i=1}^{n}\lambda_if(w(l_i))\right)}_{\frac{\sum_{i=1}^{n}f(w(l_i))*h(l_i)}{\sum_{i=1}^{n}f(w(l_i))}}],$$
$$1-\prod_{i=1}^{n}(1-T(l_i))^{\lambda_i},\prod_{i=1}^{n}I(l_i)^{\lambda_i},\prod_{i=1}^{n}F(l_i)^{\lambda_i}\rangle \tag{18}$$

where $T(l_i)=1-(1-t(l_i))^{w(l_i)},I(l_i)=(i(l_i))^{w(l_i)},F(l_i)=(f(l_i))^{w(l_i)}$, λ_i is the relative weight of $\tilde{r}_i,\lambda_i\in([0,1])$ and $\sum_{i=1}^{n}\lambda_i=1$.

Definition 15. Set $\tilde{r}_k=\langle[S^{w(l_k)}_{g(l_k)},S^{w(l_k)}_{h(l_k)}],w(l_k)(t(l_k),i(l_k),f(l_k))\rangle\in\tilde{R}(k=1,2,\cdots,n)$ as an NFSVNULV, $f(t)$ is expanding function, then the mapping$NFSVNULVWGA{:}\tilde{R}^*\to\tilde{R}$ as the weighted geometric average operator of the $NFSVNULV$, which can satisfy the following conditions:

$$NFSVNULVWGA(\tilde{r}_1,\tilde{r}_2,\cdots,\tilde{r}_n)=\sum_{i=1}^{n}\tilde{r}^{\lambda_i}$$
$$=\langle[S^{f^{-1}\left(\sum_{i=1}^{n}f(w(l_i)^{\lambda_i})\right)}_{\frac{\sum_{i=1}^{n}f(w(l_i))*g(l_i)}{\sum_{i=1}^{n}f(w(l_i))}},S^{f^{-1}\left(\sum_{i=1}^{n}f(w(l_i)^{\lambda_i})\right)}_{\frac{\sum_{i=1}^{n}f(w(l_i))*h(l_i)}{\sum_{i=1}^{n}f(w(l_i))}}],\prod_{i=1}^{n}T(l_i)^{\lambda_i},$$
$$1-\prod_{i=1}^{n}(1-I(l_i))^{\lambda_i},1-\prod_{i=1}^{n}(1-F(l_i))^{\lambda_i}\rangle, \tag{19}$$

where $T(l_i) = 1 - (1 - t(l_i))^{w(l_i)}, I(l_i) = (i(l_i))^{w(l_i)}, F(l_i) = (f(l_i))^{w(l_i)}, \lambda_i$ is the relative weight of $\tilde{r}_i \lambda_i \in ([0,1])$, and $\sum_{i=1}^{n} \lambda_i = 1$.

Proofs of Definition 14 and 15 are similar to that of Definition 13, so we don't repeat it here.

4. MAGDM of NFSVNULVS

In this section, we consider a MAGDM problem under the NFSVNULS environment. We set $B = \{B_1, B_2, \cdots, B_m\}$as the solution set, $C = \{C_1, C_2, \cdots, C_n\}$ as the attribute set, $L = \{l_1, l_2, \cdots, l_p\}$ as the expert set and $S = \{s_\alpha | \alpha \in [0, q]\}$as the linguistic assessment set.$\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)^T$with$\lambda_i \geq 0and\sum_{i=1}^{n} \lambda_i = 1$, $w = (w_1, w_2, \cdots, w_p)^T$ with $w_j \geq 0$and$\sum_{j=1}^{p} w_j = 1$express the weights of attributes and the weights of experts, respectively. Set $r_{ij}^{(k)}$asthe evaluated value of solution B_i with attribute C_j made by expertl_k, $r_{ij}^{(k)} = \langle \left[S_{g_{ij}(l_k)}, S_{h_{ij}(l_k)} \right], t_{ij}(l_k), i_{ij}(l_k), f_{ij}(l_k) \rangle \in \tilde{R} \, (k = 1, 2, \cdots, p), S_{g_{ij}(l_k)}, S_{h_{ij}(l_k)} \in S$, $t_{ij}(l_k) \in [0,1], i_{ij}(l_k) \in [0,1], f_{ij}(l_k) \in [0,1], t_{ij}(l_k) + i_{ij}(l_k) + f_{ij}(l_k) \in [0,3]$. Then we can get the neutrosophic linguistic decision evaluation matrix, which is shown in Table 2.

Step1: According to the weight vector $w = (w_1, w_2, \cdots, w_p)^T$ of experts, we change $R_k = (r_{ij}^{(k)})_{m*n}$ into new neutrosophic linguistic decision evaluation matrix$\bar{R}_k = (\bar{r}_{ij}^{(k)})_{m*n}$, then we can get$(\bar{r}_{ij}^{(k)})_{m*n} = \langle [S_{g_{ij}(l_k)}^{w(l_k)}, S_{h_{ij}(l_k)}^{w(l_k)}], w(l_k)(t_{ij}(l_k), i_{ij}(l_k), f_{ij}(l_k)) \rangle$, (k = 1, 2, 3,..., p).

Step2: Selecting the expanding function f (t) and using NFSVNULVAA, we aggregate $\bar{R}_k$ of each expert, then we can get the comprehensive neutrosophic linguistic decision evaluation matrix $\bar{R} = (\bar{r}_{ij})_{m*n}$, in which:

$$\begin{aligned} \bar{r}_{ij} &= \text{NFSVNULVAA}\left(\bar{r}_{ij}^{(1)}, \bar{r}_{ij}^{(2)}, \cdots, \bar{r}_{ij}^{(p)}\right) \\ &= \bar{r}_{ij}^{(1)} \oplus \bar{r}_{ij}^{(2)} \oplus \cdots \oplus \bar{r}_{ij}^{(p)} \\ &= \langle [S_{f^{-1}\left(\frac{\sum_{k=1}^{p} f(w(l_k)) * g_{ij}(l_k)}{\sum_{k=1}^{p} f(w(l_k))}\right)}, S_{f^{-1}\left(\frac{\sum_{k=1}^{p} f(w(l_k)) * h_{ij}(l_k)}{\sum_{k=1}^{p} f(w(l_k))}\right)}], \\ &\quad 1 - \prod_{k=1}^{p}(1 - T_{ij}(l_k)), \prod_{k=1}^{p} I_{ij}(l_k), \\ &\quad \prod_{k=1}^{p} F_{ij}(l_k) \rangle (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n), \\ &\quad T_{ij}(l_k) = 1 - \left(1 - t_{ij}(l_k)\right)^{w(l_k)}, I_{ij}(l_k) = \left(i_{ij}(l_k)\right)^{w(l_k)}, \\ &\quad F_{ij}(l_k) = \left(f_{ij}(l_k)\right)^{w(l_k)} \end{aligned}$$

Step3: Using the NFSVNULVWAA operator or the NFSVNULVWGA operator, we assemble the number i row neutrosophic linguistic decision evaluation information of the comprehensive neutrosophic linguistic decision evaluation matrix $\bar{R}$, and then we can get the evaluation value $\bar{r}_i$ of the solution B_i:

$$\begin{aligned} \bar{r}_i &= NFSVNULVWAA(\bar{r}_{i1}, \bar{r}_{i2}, \cdots, \bar{r}_{in}) = \sum_{j=1}^{n} \lambda_j \bar{r}_{ij} \\ &= \langle [S_{f^{-1}\left(\frac{\sum_{j=1}^{n} \lambda_j f(w(l_i)) * g(l_i)}{\sum_{j=1}^{n} f(w(l_i))}\right)}, S_{f^{-1}\left(\frac{\sum_{j=1}^{n} \lambda_j f(w(l_i)) * h(l_i)}{\sum_{j=1}^{n} f(w(l_i))}\right)}], \\ &\quad 1 - \prod_{j=1}^{n}(1 - T_{ij}(l_i))^{\lambda_j}, \prod_{j=1}^{n}(I(l_i))^{\lambda_j}, \\ &\quad \prod_{j=1}^{n}(F(l_i))^{\lambda_j} \rangle (i = 1, 2, \cdots, m \end{aligned}$$

Or

$$\bar{r}_i = NFSVNULVWGA(\bar{r}_{i1}, \bar{r}_{i2}, \cdots, \bar{r}_{in}) = \sum_{j=1}^{n} \bar{r}_{ij}^{\lambda_j} =$$

Table 2
The decision matrix R of expert l.

	C_1	$\cdots$	C_n
B_1	$\langle [S_{g_{11}}, S_{h_{11}}], t_{11}(l), i_{11}(l), f_{11}(l) \rangle$	$\cdots$	$\langle [S_{g_{1n}}, S_{h_{1n}}], t_{1n}(l), i_{1n}(l), f_{1n}(l) \rangle$
B_2	$\langle [S_{g_{21}}, S_{h_{21}}], t_{21}(l), i_{21}(l), f_{21}(l) \rangle$	$\cdots$	$\langle [S_{g_{2n}}, S_{h_{1n}}], t_{2n}(l), i_{2n}(l), f_{2n}(l) \rangle$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
B_m	$\langle [S_{g_{m1}}, S_{h_{m1}}], t_{m1}(l), i_{m1}(l), f_{m1}(l) \rangle$	$\cdots$	$\langle [S_{g_{mn}}, S_{h_{mn}}], t_{mn}(l), i_{mn}(l), f_{mn}(l) \rangle$

Table 3
The decision matrix of expert l_1.

	C_1	C_2	C_3	C_4
B_1	$\langle [S_4, S_5], 0.5, 0.2, 0.3 \rangle$	$\langle [S_5, S_5], 0.6, 0.1, 0.3 \rangle$	$\langle [S_5, S_6], 0.7, 0.1, 0.1 \rangle$	$\langle [S_4, S_4], 0.7, 0.1, 0.1 \rangle$
B_2	$\langle [S_5, S_5], 0.6, 0.1, 0.2 \rangle$	$\langle [S_5, S_6], 0.7, 0.1, 0.1 \rangle$	$\langle [S_4, S_5], 0.6, 0.1, 0.2 \rangle$	$\langle [S_4, S_5], 0.6, 0.2, 0.2 \rangle$
B_3	$\langle [S_4, S_4], 0.7, 0.1, 0.1 \rangle$	$\langle [S_4, S_4], 0.6, 0.1, 0.2 \rangle$	$\langle [S_5, S_5], 0.6, 0.1, 0.2 \rangle$	$\langle [S_5, S_5], 0.7, 0.1, 0.2 \rangle$
B_4	$\langle [S_4, S_4], 0.7, 0.2, 0.1 \rangle$	$\langle [S_3, S_3], 0.5, 0.2, 0.2 \rangle$	$\langle [S_4, S_4], 0.6, 0.1, 0.2 \rangle$	$\langle [S_3, S_4], 0.6, 0.2, 0.2 \rangle$

$$\langle \left[S_{\frac{\sum_{j=1}^{n} f(w(l_i)*g(l_i))}{\sum_{j=1}^{n} f(w(l_i))}}^{f^{-1}\left(\sum_{j=1}^{n} f(w(l_i)^{\lambda_j}\right))}, S_{\frac{\sum_{j=1}^{n} f(w(l_i)*h(l_i))}{\sum_{j=1}^{n} f(w(l_i))}}^{f^{-1}\left(\sum_{j=1}^{n} f(w(l_i)^{\lambda_j}\right))} \right], \prod_{j=1}^{n} (T(l_i))^{\lambda_j},$$

$$1 - \prod_{j=1}^{n} (1 - I(l_i))^{\lambda_j}, 1 - \prod_{j=1}^{n} (1 - F(l_i))^{\lambda_j} \rangle (i = 1, 2, \cdots, m$$

Step 4: According to the Formulas (15) and (16), we calculate the expectation $E(\bar{r}_i)$ and the accuracy $H(\bar{r}_i)$ of $\bar{r}_i (i = 1, 2, \cdots, m)$,then we can determine the rank of solutions based on the value $E(\bar{r}_i)$ and$H(\bar{r}_i)$.

5. An illustrative example and comparative analysis

5.1. An illustrative example

In this section, we discuss a decision-making problem adapted from the literature (Teng, 2016). There are four companies such as car-company(B_1), computer-company(B_2), food-company (B_3) and TV-selling-company(B_4). A science-technology company wants to determine which company can be selected for investment. Then, three experts are invited as a set of the decision makers $L = \{l_1, l_2, l_3\}$ and the importance of three decision makers is given with a weight vector $w = (0.4, 0.32, 0.28)^T$. Then three experts evaluate these four companies based on four main attributes. The first attribute is the venture analysis, the second is the grown analysis, the third is the social and political impact analysis and the last is the environmental impact analysis. The weight vector of them is $\lambda = (0.32, 0.26, 0.18, 0.24)^T$.the linguistic set is $S = \{s_0 = \text{very low}, s_1 = \text{low}, s_2 = \text{medium low}, s_3 = \text{fair}, s_4 = \text{medium good}, s_5 = \text{good}, s_6 = \text{very good}\}$. The evaluation values of each expert are list in Tables 3–5.

Step 1: Changing the neutrosophic linguistic decision evaluation matrix $R_k = (r_{ij}^{(k)})_{4*4}$ into new neutrosophic linguistic decision evaluation matrix $\bar{R}_k = (\bar{r}_{ij}^{(k)})_{4*4}$, *(k = 1, 2, 3)* with $w = (0.4, 0.32, 0.28)^T$, which are shown in Tables 6–8.

Step 2: Selecting the expanding function $f(t) = \frac{1}{1-t} - 1$ and using NFSVNULVAA, we assemble $\bar{R}_k$ of every expert, and then we can get the comprehensive neutrosophic linguistic decision evaluation matrix $\bar{R} = (\bar{r}_{ij})_{4*4}$, which is shown in Table 9.

Step 3: Using NFSVNULVWAA to assemble the number i row of$\bar{R}$, and then we can get the comprehensive evaluation value $\bar{r}_i$ of solutionB_i:

$\bar{r}_1 = \langle [S_{4.61}^{0.6}, S_{5.12}^{0.6}], 0.6816, 0.1186, 0.1530 \rangle$

$\bar{r}_2 = \langle [S_{4.69}^{0.6}, S_{5.30}^{0.6}], 0.6615, 0.1469, 0.1396 \rangle$

$\bar{r}_3 = \langle [S_{4.41}^{0.6}, S_{4.49}^{0.6}], 0.6944, 0.1144, 0.1568 \rangle$

$\bar{r}_4 = \langle [S_{3.76}^{0.6}, S_{4.07}^{0.6}], 0.6615, 0.1645, 0.1373 \rangle$

Step 4: Using Formula (15) to calculate the expectations $E(\bar{r}_i)$ of$\bar{r}_i (i = 1, 2, \cdots, m)$:

$E(\bar{r}_1) = 0.3908$; $E(\bar{r}_2) = 0.3954$; $E(\bar{r}_3)$=0.3594; $E(\bar{r}_4) = 0.3079$;

According to the results, we can rank $E(\bar{r}_2) > E(\bar{r}_1) > E(\bar{r}_3) > E(\bar{r}_4)$, so company B_2 is the best choose to invest.

Now, we use the NFSVNULVWGA aggregation operator.

Step 1′: Just as step 1;

Step 2′: Just as step 2;

Step 3′: Using NFSVNULVWGA to assemble the number i row of $\bar{R}$, and then we can get the comprehensive evaluation value $\bar{r}_i$ of solution B_i:

$\bar{r}_1 = \langle [S_{4.625}^{0.6}, S_{5.132}^{0.6}], 0.6788, 0.1193, 0.1647 \rangle$

$\bar{r}_2 = \langle [S_{4.694}^{0.6}, S_{5.302}^{0.6}], 0.6612, 0.1505, 0.1408 \rangle$

$\bar{r}_3 = \langle [S_{4.421}^{0.6}, S_{5.000}^{0.6}], 0.6898, 0.1165, 0.1587 \rangle$

$\bar{r}_4 = \langle [S_{3.755}^{0.6}, S_{4.063}^{0.6}], 0.6548, 0.1710, 0.1383 \rangle$

Step 4′: Using Formula (15) to calculate the expectations $E(\bar{r}_i)$ of$\bar{r}_i (i = 1, 2, \cdots, m)$:

$E(\bar{r}_1) = 0.3894$; $E(\bar{r}_2) = 0.3948$; $E(\bar{r}_3)$=0.3791; $E(\bar{r}_4) = 0.3056$;

According to the results, we can rank $E(\bar{r}_2) > E(\bar{r}_1) > E(\bar{r}_3) > E(\bar{r}_4)$, so company B_2 is the best choose to invest.

Through this example, we can see that the proposed decision-making method based on NFSVNULVWAA and NFSVNULVWGA can get the same result, so this method is feasible for dealing with the multiple attribute

Table 4
The decision matrix of expert l_2.

	C_1	C_2	C_3	C_4
B_1	$\langle [S_5, S_6], 0.6, 0.1, 0.3 \rangle$	$\langle [S_4, S_5], 0.6, 0.1, 0.2 \rangle$	$\langle [S_4, S_5], 0.8, 0.1, 0.1 \rangle$	$\langle [S_5, S_5], 0.7, 0.1, 0.1 \rangle$
B_2	$\langle [S_5, S_5], 0.7, 0.2, 0.2 \rangle$	$\langle [S_4, S_5], 0.7, 0.1, 0.2 \rangle$	$\langle [S_4, S_5], 0.7, 0.1, 0.1 \rangle$	$\langle [S_6, S_6], 0.7, 0.2, 0.1 \rangle$
B_3	$\langle [S_5, S_5], 0.7, 0.1, 0.2 \rangle$	$\langle [S_4, S_5], 0.8, 0.1, 0.1 \rangle$	$\langle [S_4, S_4], 0.6, 0.1, 0.2 \rangle$	$\langle [S_4, S_4], 0.7, 0.2, 0.2 \rangle$
B_4	$\langle [S_5, S_5], 0.7, 0.1, 0.2 \rangle$	$\langle [S_4, S_4], 0.5, 0.2, 0.2 \rangle$	$\langle [S_3, S_3], 0.8, 0.1, 0.1 \rangle$	$\langle [S_3, S_4], 0.7, 0.2, 0.1 \rangle$

Table 5
The decision matrix of expert l_3.

	$\boldsymbol{C}_1$	$\boldsymbol{C}_2$	$\boldsymbol{C}_3$	$\boldsymbol{C}_4$
B_1	$\langle [S_5, S_5], 0.8, 0.1, 0.1\rangle$	$\langle [S_5, S_5], 0.8, 0.1, 0.1\rangle$	$\langle [S_5, S_5], 0.7, 0.2, 0.1\rangle$	$\langle [S_5, S_6], 0.7, 0.2, 0.1\rangle$
B_2	$\langle [S_5, S_6], 0.7, 0.2, 0.1\rangle$	$\langle [S_5, S_6], 0.6, 0.2, 0.1\rangle$	$\langle [S_5, S_5], 0.6, 0.2, 0.1\rangle$	$\langle [S_5, S_5], 0.7, 0.2, 0.1\rangle$
B_3	$\langle [S_5, S_5], 0.7, 0.1, 0.2\rangle$	$\langle [S_5, S_5], 0.8, 0.1, 0.1\rangle$	$\langle [S_4, S_4], 0.6, 0.2, 0.2\rangle$	$\langle [S_4, S_4], 0.7, 0.2, 0.1\rangle$
B_4	$\langle [S_4, S_5], 0.7, 0.1, 0.1\rangle$	$\langle [S_4, S_4], 0.7, 0.2, 0.1\rangle$	$\langle [S_4, S_5], 0.7, 0.2, 0.1\rangle$	$\langle [S_5, S_5], 0.7, 0.3, 0.1\rangle$

Table 6
The new decision matrix of expert l_1.

	$\boldsymbol{C}_1$	$\boldsymbol{C}_2$	$\boldsymbol{C}_3$	$\boldsymbol{C}_4$
B_1	$\langle [S_4^{0.4}, S_5^{0.4}], 0.4(0.5, 0.2, 0.3)\rangle$	$\langle [S_5^{0.4}, S_5^{0.4}], 0.4(0.6, 0.1, 0.3)\rangle$	$\langle [S_5^{0.4}, S_6^{0.4}], 0.4(0.7, 0.1, 0.1)\rangle$	$\langle [S_4^{0.4}, S_4^{0.4}], 0.4(0.7, 0.1, 0.1)\rangle$
B_2	$\langle [S_5^{0.4}, S_5^{0.4}], 0.4(0.6, 0.1, 0.2)\rangle$	$\langle [S_5^{0.4}, S_6^{0.4}], 0.4(0.7, 0.1, 0.1)\rangle$	$\langle [S_4^{0.4}, S_5^{0.4}], 0.4(0.6, 0.1, 0.2)\rangle$	$\langle [S_4^{0.4}, S_5^{0.4}], 0.4(0.6, 0.2, 0.2)\rangle$
B_3	$\langle [S_4^{0.4}, S_4^{0.4}], 0.4(0.7, 0.1, 0.1)\rangle$	$\langle [S_4^{0.4}, S_4^{0.4}], 0.4(0.6, 0.1, 0.2)\rangle$	$\langle [S_5^{0.4}, S_5^{0.4}], 0.4(0.6, 0.1, 0.2)\rangle$	$\langle [S_5^{0.4}, S_5^{0.4}], 0.4(0.7, 0.1, 0.2)\rangle$
B_4	$\langle [S_4^{0.4}, S_4^{0.4}], 0.4(0.7, 0.2, 0.1)\rangle$	$\langle [S_3^{0.4}, S_3^{0.4}], 0.4(0.5, 0.2, 0.2)\rangle$	$\langle [S_4^{0.4}, S_4^{0.4}]], 0.4(0.6, 0.1, 0.2)\rangle$	$\langle [S_3^{0.4}, S_4^{0.4}], 0.4(0.6, 0.2, 0.2)\rangle$

Table 7
The new decision matrix of expert l_2.

	C_1	C_2	C_3	C_4
B_1	$\langle [S_5^{0.32}, S_6^{0.32}], 0.32(0.6, 0.1, 0.3)\rangle$	$\langle [S_4^{0.32}, S_5^{0.32}], 0.32(0.6, 0.1, 0.2)\rangle$	$\langle [S_4^{0.32}, S_5^{0.32}], 0.32(0.8, 0.1, 0.1)\rangle$	$\langle [S_5^{0.32}, S_5^{0.32}], 0.32(0.7, 0.1, 0.1)\rangle$
B_2	$\langle [S_5^{0.32}, S_5^{0.32}], 0.32(0.7, 0.2, 0.2)\rangle$	$\langle [S_4^{0.32}, S_5^{0.32}], 0.32(0.7, 0.1, 0.2)\rangle$	$\langle [S_4^{0.32}, S_5^{0.32}], 0.32(0.7, 0.1, 0.1)\rangle$	$\langle [S_6^{0.32}, S_6^{0.32}], 0.32(0.7, 0.2, 0.1)\rangle$
B_3	$\langle [S_5^{0.32}, S_5^{0.32}], 0.32(0.7, 0.1, 0.2)\rangle$	$\langle [S_4^{0.32}, S_5^{0.32}], 0.32(0.8, 0.1, 0.1)\rangle$	$\langle [S_4^{0.32}, S_4^{0.32}], 0.32(0.6, 0.1, 0.2)\rangle$	$\langle [S_4^{0.32}, S_4^{0.32}], 0.32(0.7, 0.2, 0.2)\rangle$
B_4	$\langle [S_5^{0.32}, S_5^{0.32}], 0.32(0.7, 0.1, 0.2)\rangle$	$\langle [S_4^{0.32}, S_4^{0.32}], 0.32(0.5, 0.2, 0.2)\rangle$	$\langle [S_3^{0.32}, S_3^{0.32}], 0.32(0.8, 0.1, 0.1)\rangle$	$\langle [S_3^{0.32}, S_4^{0.32}], 0.32(0.7, 0.2, 0.1)\rangle$

Table 8
The new decision matrix of expert l_3.

	$\boldsymbol{C}_1$	$\boldsymbol{C}_2$	$\boldsymbol{C}_3$	$\boldsymbol{C}_4$
B_1	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.8, 0.1, 0.1)\rangle$	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.8, 0.1, 0.1)\rangle$	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$	$\langle [S_5^{0.28}, S_6^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$
B_2	$\langle [S_5^{0.28}, S_6^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$	$\langle [S_5^{0.28}, S_6^{0.28}], 0.28(0.6, 0.2, 0.1)\rangle$	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.6, 0.2, 0.1)\rangle$	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$
B_3	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.7, 0.1, 0.2)\rangle$	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.8, 0.1, 0.1)\rangle$	$\langle [S_4^{0.28}, S_4^{0.28}], 0.28(0.6, 0.2, 0.2)\rangle$	$\langle [S_4^{0.28}, S_4^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$
B_4	$\langle [S_4^{0.28}, S_5^{0.28}], 0.28(0.7, 0.1, 0.1)\rangle$	$\langle [S_4^{0.28}, S_4^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$	$\langle [S_4^{0.28}, S_5^{0.28}], 0.28(0.7, 0.2, 0.1)\rangle$	$\langle [S_5^{0.28}, S_5^{0.28}], 0.28(0.7, 0.3, 0.1)\rangle$

Table 9
The comprehensive decision evaluation matrix $\bar{R}$.

	$\boldsymbol{C}_1$	$\boldsymbol{C}_2$	$\boldsymbol{C}_3$	$\boldsymbol{C}_4$
B_1	$\langle [S_{4.55}^{0.6}, S_{5.29}^{0.6}], 0.6398, 0.1320, 0.2206\rangle$	$\langle [S_{4.67}^{0.6}, S_{4.98}^{0.6}], 0.6706, 0.1000, 0.1937\rangle$	$\langle [S_{4.67}^{0.6}, S_{5.42}^{0.6}], 0.7365, 0.1214, 0.1000\rangle$	$\langle [S_{4.55}^{0.6}, S_{4.8}^{0.6}], 0.7000, 0.1214, 0.1000\rangle$
B_2	$\langle [S_{4.98}^{0.6}, S_{5.24}^{0.6}], 0.6634, 0.1516, 0.1647\rangle$	$\langle [S_{4.67}^{0.6}, S_{5.67}^{0.6}], 0.6748, 0.1214, 0.1248\rangle$	$\langle [S_{4.24}^{0.6}, S_{4.98}^{0.6}], 0.6352, 0.1214, 0.1320\rangle$	$\langle [S_{4.86}^{0.6}, S_{5.29}^{0.6}], 0.6634, 0.2000, 0.1320\rangle$
B_3	$\langle [S_{4.55}^{0.6}, S_{4.55}^{0.6}], 0.7000, 0.1000, 0.1516\rangle$	$\langle [S_{4.24}^{0.6}, S_{4.55}^{0.6}], 0.7361, 0.1000, 0.1320\rangle$	$\langle [S_{4.42}^{0.6}, S_{4.42}^{0.6}], 0.6000, 0.1214, 0.2000\rangle$	$\langle [S_{4.42}^{0.6}, S_{4.42}^{0.6}], 0.7000, 0.1516, 0.1647\rangle$
B_4	$\langle [S_{4.30}^{0.6}, S_{4.55}^{0.6}], 0.7000, 0.1320, 0.1248\rangle$	$\langle [S_{3.56}^{0.6}, S_{3.56}^{0.6}], 0.5666, 0.2000, 0.1647\rangle$	$\langle [S_{3.69}^{0.6}, S_{3.93}^{0.6}], 0.7044, 0.1214, 0.1320\rangle$	$\langle [S_{3.50}^{0.6}, S_{4.24}^{0.6}], 0.6634, 0.2240, 0.1320\rangle$

group decision-making problems under NFSVNULS environment.

5.2. Comparative analysis

Now, we use the method based on Heronian Mean (HM) operator utilized in literature (Liu & Shi, 2017; Teng, 2016), then we can get the following results:

$\bar{r}_1 = \langle [S_{4.604}^{0.6}, S_{5.112}^{0.6}], 0.6814, 0.1189, 0.1585\rangle$

$\bar{r}_2 = \langle [S_{4.725}^{0.6}, S_{5.313}^{0.6}], 0.6610, 0.1489, 0.1398\rangle$

$\bar{r}_3 = \langle [S_{4.412}^{0.6}, S_{4.493}^{0.6}], 0.6909, 0.1162, 0.1585\rangle$

$\bar{r}_4 = \langle [S_{3.787}^{0.6}, S_{4.089}^{0.6}], 0.6575, 0.1689, 0.1381\rangle$

Calculating the expectations $E(\bar{r}_i)$ of $\bar{r}_i (i = 1, 2, \cdots, m)$, we can get:

$E(\bar{r}_1) = 0.3893$; $E(\bar{r}_2) = 0.3968$; $E(\bar{r}_3)$=0.3586; $E(\bar{r}_4) = 0.3086$;

According to the results, we can rank $E(\bar{r}_2) > E(\bar{r}_1) > E(\bar{r}_3) > E(\bar{r}_4)$, and then the ranking order is $B_2 \succ B_1 \succ B_3 \succ B_4$, which is according with the ranking result of this paper.

Compared to the literature (Liu & Shi, 2017; Teng, 2016); on the one hand, we add the weights of linguistic evaluations to depict the important level of different experts in NFSVNULV. Thus, it can not only reflect the linguistic evaluation of each expert, but also can depict the important degree of different experts. When we make aggregation, we consider the weights of experts in uncertain linguistic part and the neutrosophic number part, which provides the more reasonable decision-making information for decision makers. On the other hand, we use the expanding function to ensure that the linguistic information aggregation results do not appear "distortion" and "transboundary" phenomenon. However, in this paper, the proposed method based on the NFSVNULVWAA operator and the NFSVNULVWGA operator provides a new approach for decision makers under NFSVNULV environment.

6. Conclusions

In this paper, we first defined a new form of single value neutrosophic uncertain linguistic set (NFSVNULS) and gave its operational rules. In NFSVNULS, we expressed the weight of each expert in the uncertain linguistic part, which not only reflected the linguistic evaluation of each expert, but also depicted the important degree of different experts. When we made aggregation, we consider the weights of experts in uncertain linguistic part and the neutrosophic number part, which provides the more reasonable decision-making information for decision makers. Then, we put forward NFSVNULVWAA and NFSVNULVWGA operators and discussed their relevant properties. Based on these two operators, we further proposed a MAGDM method in the NFSVNULS environment. Finally, we used an instance to demonstrate the feasibility and effectiveness of the proposed method. In addition, we did some comparative analysis by using HM aggregation operator. According to the results, we can see that the proposed method can resolve the multiple attribute group decision-making problems under NFSVNULS environment.

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Author contributions

Changxing Fan originally proposed the NFSVNULVWAA and NFSVNULVWGA operators and investigated their properties; En Fan and Keli Hu provided the calculation and comparative analysis; and we wrote the paper together.

Conflicts of interest

The author declares no conflict of interest.

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