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Non-Dual Multi-Granulation Neutrosophic Rough Set with Applications

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Abstract: Multi-attribute decision-making (MADM) is a part of management decision-making and an important branch of the modern decision theory and method. MADM focuses on the decision problem of discrete and finite decision schemes. Uncertain MADM is an extension and development of classical multi-attribute decision making theory. When the attribute value of MADM is shown by neutrosophic number, that is, the attribute value is complex data and needs three values to express, it is called the MADM problem in which the attribute values are neutrosophic numbers. However, in practical MADM problems, to minimize errors in individual decision making, we need to consider the ideas of many people and synthesize their opinions. Therefore, it is of great significance to study the method of attribute information aggregation. In this paper, we proposed a new theory—non-dual multi-granulation neutrosophic rough set (MS)—to aggregate multiple attribute information and solve a multi-attribute group decision-making (MGDM) problem where the attribute values are neutrosophic numbers. First, we defined two kinds of non-dual MS models, intersection-type MS and union-type MS. Additionally, their properties are studied. Then the relationships between MS, non-dual MS, neutrosophic rough set (NRS) based on neutrosophic intersection (union) relationship, and NRS based on neutrosophic transitive closure relation of union relationship are outlined, and a figure is given to show them directly. Finally, the definition of non-dual MS on two universes is given and we use it to solve a MGDM problem with a neutrosophic number as the attribute value.

Keywords: multi-granulation neutrosophic rough set; non-dual; two universes; multi-attribute group decision making

1. Introduction

Fuzzy sets and rough sets are widely used to solve uncertain problems [1–4]. However, all these theories have their own deficiency, such as in a voting, you may support, not support, be neutral, or abstain from voting, so Smarandache present the definition of the neutrosophic set (NS) [5]. NS is an extensional model of the fuzzy set and intuitionistic fuzzy set. But the original definition of NS is not convenient to solve real-world problems, thus Wang et al. proposed a single-valued neutrosophic set (SVNS) [6]. After that, SVNS was extended and used in many fields. Peng et al. [7] defined simplified NS and obtained some properties. Peng et al. [8] proposed the definition of probability multi-valued NS and studied its properties. Deli et al. [9] defined bipolar NS and studied its properties. Zhang et al. [10] analyzed new inclusion relationships of SVNS and discussed its lattice structure. As an extension of fuzzy sets and rough sets, many scholars combined them and got some results [11–13]. Yang et al. [14] combined SVNS and rough set, then produced a single-valued neutrosophic rough set and discussed its properties. Now NSs and NRSs have been used widely in decision-making problems [15–19].

From the perspective of particle computing, the above rough set theories are essentially defined in a single particle space, and the lower and upper approximations (ULA) of the target concept is shown by the information particles in the particle space induced by a single binary relationship. However, Qian et al. think that, in decision analysis problems, the relationship between the multiple decision makers may be independent of each other, so multiple binary relations are needed to approximate the target. Therefore, they put forward the concept of a multi-granularity rough set (MRS) model [20], and define the optimistic MRS model and pessimistic MRS model, respectively. The biggest difference between MRS and classical rough sets is that MRS can use the knowledge in a multi-granular space to approximate the target. Additionally, because it analyzes the problem from multiple angles and levels, it can obtain a more reasonable and satisfactory solution for the problem, so it has a better application prospect in many practical decision-making problems. Yao et al. [21] studied the rough set models under the multi-granulation approximation space. Now the MRS model has been used widely and has produced some interesting results [22–28].

The ULA operator of most MRS models is dual, there are few articles studying the non-dual MRS model or hybrid MRS model [29,30]. Zhang et al. [31] put forward non-dual MRS (union-type MRS and intersection-type MRS) models and outline the relationships between MRSs. In this paper, we put forward non-dual MS (intersection-type MS and union-type MS) models and study their properties. Then we show the relationships between MS, non-dual MS, NRS-based neutrosophic union relation, and NRS-based neutrosophic intersection relation. Finally, we propose non-dual MS models on two universes and use it to solve MGDM problems with neutrosophic numbers as the attribute values.

The structure of this article is as follows. In Section 2, some basic notions and operations of NRS and MS are introduced. In Section 3, the concepts of non-dual MS are put forward and their qualities are investigated. In Section 4, the relationships between MS, non-dual MS, neutrosophic rough set (NRS) based on neutrosophic intersection (union) relationship, and NRS based on neutrosophic transitive closure relation of union relationship are discussed. In Section 5, non-dual MS models on two universes are proposed and an application to solve the MGDM problem where the attribute values are neutrosophic numbers is outlined. Finally, Section 6 provides our conclusions and outlook.

2. Preliminary

In this section, we look at several basic concepts of NRS and MS.

Definition 1 ([6]). *A SVNS A is denoted by*

$$A = \{(x, T_A(x), I_A(x), F_A(x))|x \in X\} \tag{1}$$

where $T_A(x)$ represents truth-membership function, $I_A(x)$ represents indeterminacy-membership function, $F_A(x)$ represenst falsity-membership function and $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. Additionally, they satisfy the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

In this paper, "SVNS" is abbreviated to "NS" and we use the symbol NS(U) to denote the set of all NSs in U.

Definition 2 ([6]). *For any two NSs A and B, the inclusion relation, union, intersection, and complement operations are defined:*

(1) $A \subseteq B$ *iff* $\forall x \in U$, $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$;
(2) $A \cup B = \{(x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)) \mid x \in U\}$;
(3) $A \cap B = \{(x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)) \mid x \in U\}$;
(4) $A^c = \{(x, F_A(x), 1\text{-}I_A(x), T_A(x)) \mid x \in U\}$.

Definition 3 [14]). *Suppose (U, R) is a neutrosophic approximation space (NAS). ∀ A ∈ NS(U), the LUA of A, denoted by* $\underline{R}(A)$ *and* $\overline{R}(A)$*, are defined as: ∀ x ∈ U,*

$$\underline{R}(A) = \underset{y \in U}{\cap} (R^c(x, y) \cup A(y)), \overline{R}(A) = \underset{y \in U}{\cup} (R(x, y) \cap A(y))$$

The pair $\left(\underline{R}(A), \overline{R}(A)\right)$ *is called the SVNRS of A.*

Definition 4 ([28]). *Suppose (U, R_i) is a multi-granulation neutrosophic approximation space (MAS). A ∈ NS(U), the optimistic ULA of A, represented by* $\underline{MS}^o(A)$ *and* $\overline{MS}^o(A)$*, are defined:*

$$\underline{MS}^o(A)(x) = \overset{m}{\underset{i=1}{\cup}} \left(\underset{y \in U}{\cap} ({R_i}^c(x, y) \cup A(y)) \right) \tag{2}$$

$$\overline{MS}^o(A)(x) = \overset{m}{\underset{i=1}{\cap}} \left(\underset{y \in U}{\cup} (R_i(x, y) \cap A(y)) \right) \tag{3}$$

Then the pair $\left(\underline{MS}^o(A), \overline{MS}^o(A)\right)$ *is called an optimistic* MS *when* $\underline{MS}^o(A) \neq \overline{MS}^o(A)$.

Definition 5 ([28]). *Suppose (U, R_i) is a MAS. ∀A ∈ NS(U), the pessimistic ULA of A, represented by* $\underline{MS}^p(A)$ *and* $\overline{MS}^p(A)$*, are defined:*

$$\underline{MS}^p(A)(x) = \overset{m}{\underset{i=1}{\cap}} \left(\underset{y \in U}{\cap} ({R_i}^c(x, y) \cup A(y)) \right) \tag{4}$$

$$\overline{MS}^p(A)(x) = \overset{m}{\underset{i=1}{\cup}} \left(\underset{y \in U}{\cup} (R_i(x, y) \cap A(y)) \right) \tag{5}$$

Then the pair $\left(\underline{MS}^p(A), \overline{MS}^p(A)\right)$ *is called a pessimistic* MS *when* $\underline{MS}^o(A) \neq \overline{MS}^o(A)$.

Proposition 1 ([28]). *Suppose (U, R_i) be a MAS. ∀ A, B ∈ NS(U), then*

(1) $\underline{MS}^o(A) = \sim \overline{MS}^o(\sim A), \underline{MS}^p(A) = \sim \overline{MS}^p(\sim A)$.
(2) $\overline{MS}^o(A) = \sim \underline{MS}^o(\sim A), \overline{MS}^p(A) = \sim \underline{MS}^p(\sim A)$.
(3) $\underline{MS}^o(A \cap B) = \underline{MS}^o(A) \cap \underline{MS}^o(B), \underline{MS}^p(A \cap B) = \underline{MS}^p(A) \cap \underline{MS}^p(B)$.
(4) $\overline{MS}^o(A \cup B) = \overline{MS}^o(A) \cup \overline{MS}^o(B), \overline{MS}^p(A \cup B) = \overline{MS}^p(A) \cup \overline{MS}^p(B)$.
(5) $A \subseteq B \Rightarrow \underline{MS}^o(A) \subseteq \underline{MS}^o(B), \underline{MS}^p(A) \subseteq \underline{MS}^p(B)$.
(6) $A \subseteq B \Rightarrow \overline{MS}^o(A) \subseteq \overline{MS}^o(B), \overline{MS}^p(A) \subseteq \overline{MS}^p(B)$.
(7) $\underline{MS}^o(A) \cup \underline{MS}^o(B) \subseteq \underline{MS}^o(A \cup B), \underline{MS}^p(A) \cup \underline{MS}^p(B) \subseteq \underline{MS}^p(A \cup B)$.
(8) $\overline{MS}^o(A \cap B) \subseteq \overline{MS}^o(A) \cap \overline{MS}^o(B), \overline{MS}^p(A \cap B) \subseteq \overline{MS}^p(A) \cap \overline{MS}^p(B)$.

Definition 6 ([14]). *If A and B are two neutrosophic numbers in U, the operation of A and B is defined as follows:*

$$(1)\ \lambda A = \left(1 - (1 - T_A)^\lambda, (I_A)^\lambda, (F_A)^\lambda\right) \tag{6}$$

$$(2)\ A \oplus B = (T_A + T_B - T_A \cdot T_B, I_A \cdot I_B, F_A \cdot F_B) \tag{7}$$

Definition 7 ([10]). *Let (t, i, f) be a neutrosophic number, the type-3 score function and type-3 accuracy function are defined:*

$$(1)\ s : D^* \to [0, 1], s(t, i, f) = \frac{t + (1 - f)}{2} \tag{8}$$

$$(2)\ h : D^* \to [0, 1], h(t, i, f) = \frac{t}{t + (1 - f)} \tag{9}$$

Definition 8 ([10]). *Let (t_1, i_1, f_1) and (t_2, i_2, f_2) be two neutrosophic numbers. Then*

(1) *If $s(t_1, i_1, f_1) < s(t_2, i_2, f_2)$, then $(t_1, i_1, f_1) \prec (t_2, i_2, f_2)$.*
(2) *If $s(t_1, i_1, f_1) = s(t_2, i_2, f_2), h(t_1, i_1, f_1) < h(t_2, i_2, f_2)$, then $(t_1, i_1, f_1) \prec (t_2, i_2, f_2)$.*
(3) *If $s(t_1, i_1, f_1) = s(t_2, i_2, f_2)$, $h(t_1, i_1, f_1) = h(t_2, i_2, f_2)$, $i_1 < i_2$, then $(t_1, i_1, f_1) \prec (t_2, i_2, f_2)$; if $i_1 = i_2$, then $(t_1, i_1, f_1) = (t_2, i_2, f_2)$.*

3. Non-Dual Multi-Granulation Neutrosophic Rough Set

In this section, we introduce non-dual MS (intersection-type MS and union-type MS) models and study their properties.

Definition 9. *Let tuple ordered set (U, R_i) $(1 \le i \le m)$ be a MAS. For any $A \in NS(U)$, the intersection-type ULA $\underline{MS}^{(\cap)}(A)$ and $\overline{MS}^{(\cap)}(A)$ in (U, R_i) are defined:*

$$\underline{MS}^{(\cap)}(A)(x) = \bigcap_{i=1}^{m}\left(\bigcap_{y\in U}(R_i{}^c(x,y) \cup A(y))\right) \tag{10}$$

$$\overline{MS}^{(\cap)}(A)(x) = \bigcap_{i=1}^{m}\left(\bigcup_{y\in U}(R_i(x,y) \cap A(y))\right) \tag{11}$$

Obviously, $\underline{MS}^{(\cap)}(A)$ and $\overline{MS}^{(\cap)}(A)$ are two NSs of U. Furthermore, A is called a definable NS on (U, R_i) when $\underline{MS}^{(\cap)}(A) = \overline{MS}^{(\cap)}(A)$. Otherwise, the pair $\left(\underline{MS}^{(\cap)}(A), \overline{MS}^{(\cap)}(A)\right)$ is called intersection-type MS.

Definition 10. *Let tuple ordered set (U, R_i) $(1 \le i \le m)$ be a MAS. For any $A \in NS(U)$, the union-type ULA $\underline{MS}^{(\cup)}(A)$ and $\overline{MS}^{(\cup)}(A)$ in (U, R_i) are defined:*

$$\underline{MS}^{(\cup)}(A)(x) = \bigcup_{i=1}^{m}\left(\bigcap_{y\in U}(R_i{}^c(x,y) \cup A(y))\right), \tag{12}$$

$$\overline{MS}^{(\cup)}(A)(x) = \bigcup_{i=1}^{m}\left(\bigcup_{y\in U}(R_i(x,y) \cap A(y))\right). \tag{13}$$

Obviously, $\underline{MS}^{(\cup)}(A)$ and $\overline{MS}^{(\cup)}(A)$ are two NSs of U. Furthermore, A is called a definable NS on (U, R_i) when $\underline{MS}^{(\cup)}(A) = \overline{MS}^{(\cup)}(A)$. Otherwise, the pair $\left(\underline{MS}^{(\cup)}(A), \overline{MS}^{(\cup)}(A)\right)$ is called union-type MS.

Proposition 2. *Let (U, R_i) be a MAS, R_i $(1 \le i \le m)$ be the neutrosophic relations on U. For any $A, B \in NS(U)$, we have*

(1) $\underline{MS}^{(\cap)}(A \cap B) = \underline{MS}^{(\cap)}(A) \cap \underline{MS}^{(\cap)}(B), \overline{MS}^{(\cap)}(A \cup B) = \overline{MS}^{(\cap)}(A) \cup \overline{MS}^{(\cap)}(B)$;
(2) $\underline{MS}^{(\cup)}(A \cap B) = \underline{MS}^{(\cup)}(A) \cap \underline{MS}^{(\cup)}(B), \overline{MS}^{(\cup)}(A \cup B) = \overline{MS}^{(\cup)}(A) \cup \overline{MS}^{(\cup)}(B)$;
(3) $A \subseteq B \Rightarrow \underline{MS}^{(\cap)}(A) \subseteq \underline{MS}^{(\cap)}(B)$, $A \subseteq B \Rightarrow \overline{MS}^{(\cap)}(A) \subseteq \overline{MS}^{(\cap)}(B)$;
(4) $A \subseteq B \Rightarrow \underline{MS}^{(\cup)}(A) \subseteq \underline{MS}^{(\cup)}(B)$, $A \subseteq B \Rightarrow \overline{MS}^{(\cup)}(A) \subseteq \overline{MS}^{(\cup)}(B)$;
(5) $\underline{MS}^{(\cap)}(A \cup B) \supseteq \underline{MS}^{(\cap)}(A) \cup \underline{MS}^{(\cap)}(B), \overline{MS}^{(\cap)}(A \cap B) \subseteq \overline{MS}^{(\cap)}(A) \cap \overline{MS}^{(\cap)}(B)$;
(6) $\underline{MS}^{(\cup)}(A \cup B) \supseteq \underline{MS}^{(\cup)}(A) \cup \underline{MS}^{(\cup)}(B), \overline{MS}^{(\cup)}(A \cap B) \subseteq \overline{MS}^{(\cup)}(A) \cap \overline{MS}^{(\cup)}(B)$.

Proof. (1) By Definition 9, we have

$$\begin{aligned}
\underline{MS}^{(\cap)}(A\cap B) &= \bigcap_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y)\cup(A\cap B)(y))\right)\\
&= \bigcap_{i=1}^{m}\left(\bigcap_{y\in U}(({R_i}^c(x,y)\cup A(y))\cap({R_i}^c(x,y)\cup B(y)))\right)\\
&= \left(\bigcap_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y)\cup A(y))\right)\right)\cap\left(\bigcap_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y)\cup B(y))\right)\right)\\
&= \underline{MS}^{(\cap)}(A)\cap\underline{MS}^{(\cap)}(B).
\end{aligned}$$

Similarly, by Definition 9, we can get

$$\overline{MS}^{(\cap)}(A\cup B)=\overline{MS}^{(\cap)}(A)\cup\overline{MS}^{(\cap)}(B).$$

(2) By Definition 10, we have

$$\begin{aligned}
\underline{MS}^{(\cup)}(A\cap B) &= \bigcup_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y)\cup(A\cap B)(y))\right)\\
&= \bigcup_{i=1}^{m}\left(\bigcap_{y\in U}(({R_i}^c(x,y)\cup A(y))\cap({R_i}^c(x,y)\cup B(y)))\right)\\
&= \left(\bigcup_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y)\cup A(y))\right)\right)\cap\left(\bigcup_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y)\cup B(y))\right)\right)\\
&= \underline{MS}^{(\cup)}(A)\cap\underline{MS}^{(\cup)}(B).
\end{aligned}$$

Similarly, by Definition 10, we can get

$$\overline{MS}^{(\cup)}(A\cup B)=\overline{MS}^{(\cup)}(A)\cup\overline{MS}^{(\cup)}(B).$$

(3) Suppose $A\subseteq B$, then $T_A(x)\le T_B(x)$, $I_A(x)\ge I_B(x)$, $F_A(x)\ge F_B(x)$,

$$T_{\underline{MS}^{(\cap)}(A)}(x)=\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee T_A(y)\right)\le\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee T_B(y)\right)=T_{\underline{MS}^{(\cap)}(B)}(x).$$

$$I_{\underline{MS}^{(\cap)}(A)}(x)=\bigvee_{i=1}^{m}\bigvee_{y\in U}\left[\left(1-I_{R_i}(x,y)\right)\wedge I_A(y)\right]\ge\bigvee_{i=1}^{m}\bigvee_{y\in U}\left[\left(1-I_{R_i}(x,y)\right)\wedge I_B(y)\right]=I_{\underline{MS}^{(\cap)}(B)}(x).$$

$$F_{\underline{MS}^{(\cap)}(A\cup B)}(x)=\bigvee_{i=1}^{m}\bigvee_{y\in U}\left[T_{R_i}(x,y)\wedge F_A(y)\right]\ge\bigvee_{i=1}^{m}\bigvee_{y\in U}\left[T_{R_i}(x,y)\wedge F_B(y)\right]=F_{\underline{MS}^{(\cap)}(B)}(x).$$

Hence, $\underline{MS}^{(\cap)}(A)\subseteq\underline{MS}^{(\cap)}(B)$.
Similarly, we can get $\overline{MS}^{(\cap)}(A)\subseteq\overline{MS}^{(\cap)}(B)$.
(4) The proof is similar with (3).
(5) According to Definition 9, we have

$$\begin{aligned}
T_{\underline{MS}^{(\cap)}(A\cup B)}(x) &= \bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left[F_{R_i}(x,y)\vee(T_A(y)\vee T_B(y))\right]=\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left[\left(F_{R_i}(x,y)\vee T_A(y)\right)\vee\left(F_{R_i}(x,y)\vee T_B(y)\right)\right]\\
&\ge\left[\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee T_A(y)\right)\right]\vee\left[\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee T_B(y)\right)\right]=T_{\underline{MS}^{(\cap)}(A)}(x)\vee T_{\underline{MS}^{(\cap)}(B)}(x).
\end{aligned}$$

$$\begin{aligned}
I_{\underline{MS}^{(\cap)}(A\cup B)}(x) &= \bigvee_{i=1}^{m}\bigvee_{y\in U}\left[\left(1-I_{R_i}(x,y)\right)\wedge(I_A(y)\wedge I_B(y))\right]\\
&= \bigvee_{i=1}^{m}\bigvee_{y\in U}\left[\left(\left(1-I_{R_i}(x,y)\right)\wedge I_A(y)\right)\wedge\left(\left(1-I_{R_i}(x,y)\right)\wedge I_B(y)\right)\right]\\
&\le\left[\bigvee_{i=1}^{m}\bigvee_{y\in U}\left(\left(1-I_{R_i}(x,y)\right)\wedge I_A(y)\right)\right]\wedge\left[\bigvee_{i=1}^{m}\bigvee_{y\in U}\left(\left(1-I_{R_i}(x,y)\right)\wedge I_B(y)\right)\right]\\
&= I_{\underline{MS}^{(\cap)}(A)}(x)\wedge I_{\underline{MS}^{(\cap)}(B)}(x).
\end{aligned}$$

$$\begin{aligned}F_{\underline{MS}^{(\cap)}(A\cup B)}(x) &= \bigvee_{i=1}^{m}\bigvee_{y\in U}\left[T_{R_i}(x,y)\wedge(F_A(y)\wedge F_B(y))\right] = \bigvee_{i=1}^{m}\bigvee_{y\in U}\left[\left(T_{R_i}(x,y)\wedge F_A(y)\right)\wedge\left(T_{R_i}(x,y)\wedge F_B(y)\right)\right]\\ &\leq \left[\bigvee_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge F_A(y)\right)\right]\wedge\left[\bigvee_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge F_B(y)\right)\right] = F_{\underline{MS}^{(\cap)}(A)}(x)\wedge F_{\underline{MS}^{(\cap)}(B)}(x).\end{aligned}$$

Hence, $\underline{MS}^{(\cap)}(A\cup B)\supseteq\underline{MS}^{(\cap)}(A)\cup\underline{MS}^{(\cap)}(B)$.
Additionally, we have

$$\begin{aligned}T_{\overline{MS}^{(\cap)}(A\cap B)}(x) &= \bigwedge_{i=1}^{m}\bigvee_{y\in U}\left[T_{R_i}(x,y)\wedge(T_A(y)\wedge T_B(y))\right] = \bigwedge_{i=1}^{m}\bigvee_{y\in U}\left[\left(T_{R_i}(x,y)\wedge T_A(y)\right)\wedge\left(T_{R_i}(x,y)\wedge T_B(y)\right)\right]\\ &\leq \left[\bigwedge_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge T_A(y)\right)\right]\wedge\left[\bigwedge_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge T_B(y)\right)\right] = T_{\overline{MS}^{(\cap)}(A)}(x)\wedge T_{\overline{MS}^{(\cap)}(B)}(x).\end{aligned}$$

$$\begin{aligned}I_{\overline{MS}^{(\cap)}(A\cap B)}(x) &= \bigvee_{i=1}^{m}\bigwedge_{y\in U}\left[I_{R_i}(x,y)\vee(I_A(y)\vee I_B(y))\right] = \bigvee_{i=1}^{m}\bigwedge_{y\in U}\left[\left(I_{R_i}(x,y)\vee I_A(y)\right)\vee\left(I_{R_i}(x,y)\vee I_A(y)\right)\right]\\ &\leq \left[\bigvee_{i=1}^{m}\bigwedge_{y\in U}\left(I_{R_i}(x,y)\vee I_A(y)\right)\right]\wedge\left[\bigvee_{i=1}^{m}\bigwedge_{y\in U}\left(I_{R_i}(x,y)\vee I_B(y)\right)\right] = I_{\overline{MS}^{(\cap)}(A)}(x)\wedge I_{\overline{MS}^{(\cap)}(B)}(x).\end{aligned}$$

$$\begin{aligned}F_{\overline{MS}^{(\cap)}(A\cap B)}(x) &= \bigvee_{i=1}^{m}\bigwedge_{y\in U}\left[F_{R_i}(x,y)\vee(F_A(y)\vee F_B(y))\right] = \bigvee_{i=1}^{m}\bigwedge_{y\in U}\left[\left(F_{R_i}(x,y)\vee F_A(y)\right)\vee\left(F_{R_i}(x,y)\vee F_A(y)\right)\right]\\ &\geq \left[\bigvee_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee F_A(y)\right)\right]\vee\left[\bigvee_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee F_B(y)\right)\right] = F_{\overline{MS}^{(\cap)}(A)}(x)\vee F_{\overline{MS}^{(\cap)}(B)}(x).\end{aligned}$$

Hence, $\overline{MS}^{(\cap)}(A\cap B)\subseteq\overline{MS}^{(\cap)}(A)\cap\overline{MS}^{(\cap)}(B)$.
(6) According to Definition 10, we have

$$\begin{aligned}T_{\underline{MS}^{(\cup)}(A\cup B)}(x) &= \bigvee_{i=1}^{m}\bigwedge_{y\in U}\left[F_{R_i}(x,y)\vee(T_A(y)\vee T_B(y))\right] = \bigvee_{i=1}^{m}\bigwedge_{y\in U}\left[\left(F_{R_i}(x,y)\vee T_A(y)\right)\vee\left(F_{R_i}(x,y)\vee T_B(y)\right)\right]\\ &\geq \left[\bigvee_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee T_A(y)\right)\right]\vee\left[\bigvee_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee T_B(y)\right)\right] = T_{\underline{MS}^{(\cup)}(A)}(x)\vee T_{\underline{MS}^{(\cup)}(B)}(x).\end{aligned}$$

$$\begin{aligned}I_{\underline{MS}^{(\cup)}(A\cup B)}(x) &= \bigwedge_{i=1}^{m}\bigvee_{y\in U}\left[\left(1-I_{R_i}(x,y)\right)\wedge(I_A(y)\wedge I_B(y))\right]\\ &= \bigwedge_{i=1}^{m}\bigvee_{y\in U}\left[\left(\left(1-I_{R_i}(x,y)\right)\wedge I_A(y)\right)\wedge\left(\left(1-I_{R_i}(x,y)\right)\wedge I_B(y)\right)\right]\\ &\leq \left[\bigwedge_{i=1}^{m}\bigvee_{y\in U}\left(\left(1-I_{R_i}(x,y)\right)\wedge I_A(y)\right)\right]\wedge\left[\bigwedge_{i=1}^{m}\bigvee_{y\in U}\left(\left(1-I_{R_i}(x,y)\right)\wedge I_B(y)\right)\right]\\ &= I_{\underline{MS}^{(\cup)}(A)}(x)\wedge I_{\underline{MS}^{(\cup)}(B)}(x).\end{aligned}$$

$$\begin{aligned}F_{\underline{MS}^{(\cup)}(A\cup B)}(x) &= \bigwedge_{i=1}^{m}\bigvee_{y\in U}\left[T_{R_i}(x,y)\wedge(F_A(y)\wedge F_B(y))\right] = \bigwedge_{i=1}^{m}\bigvee_{y\in U}\left[\left(T_{R_i}(x,y)\wedge F_A(y)\right)\wedge\left(T_{R_i}(x,y)\wedge F_B(y)\right)\right]\\ &\leq \left[\bigwedge_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge F_A(y)\right)\right]\wedge\left[\bigwedge_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge F_B(y)\right)\right] = F_{\underline{MS}^{(\cup)}(A)}(x)\wedge F_{\underline{MS}^{(\cup)}(B)}(x).\end{aligned}$$

Hence, $\underline{MS}^{(\cup)}(A\cup B)\supseteq\underline{MS}^{(\cup)}(A)\cup\underline{MS}^{(\cup)}(B)$.
Additionally, we have

$$\begin{aligned}T_{\overline{MS}^{(\cup)}(A\cap B)}(x) &= \bigvee_{i=1}^{m}\bigvee_{y\in U}\left[T_{R_i}(x,y)\wedge(T_A(y)\wedge T_B(y))\right] = \bigvee_{i=1}^{m}\bigvee_{y\in U}\left[\left(T_{R_i}(x,y)\wedge T_A(y)\right)\wedge\left(T_{R_i}(x,y)\wedge T_B(y)\right)\right]\\ &\leq \left[\bigvee_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge T_A(y)\right)\right]\wedge\left[\bigvee_{i=1}^{m}\bigvee_{y\in U}\left(T_{R_i}(x,y)\wedge T_B(y)\right)\right] = T_{\overline{MS}^{(\cup)}(A)}(x)\wedge T_{\overline{MS}^{(\cup)}(B)}(x).\end{aligned}$$

$$\begin{aligned}I_{\overline{MS}^{(\cup)}(A\cap B)}(x) &= \bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left[I_{R_i}(x,y)\vee(I_A(y)\vee I_B(y))\right] = \bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left[\left(I_{R_i}(x,y)\vee I_A(y)\right)\vee\left(I_{R_i}(x,y)\vee I_A(y)\right)\right]\\ &\leq \left[\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(I_{R_i}(x,y)\vee I_A(y)\right)\right]\wedge\left[\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(I_{R_i}(x,y)\vee I_B(y)\right)\right] = I_{\overline{MS}^{(\cup)}(A)}(x)\wedge I_{\overline{MS}^{(\cup)}(B)}(x).\end{aligned}$$

$$\begin{aligned}F_{\overline{MS}^{(\cup)}(A\cap B)}(x) &= \bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left[F_{R_i}(x,y)\vee(F_A(y)\vee F_B(y))\right] = \bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left[\left(F_{R_i}(x,y)\vee F_A(y)\right)\vee\left(F_{R_i}(x,y)\vee F_A(y)\right)\right]\\ &\geq \left[\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee F_A(y)\right)\right]\vee\left[\bigwedge_{i=1}^{m}\bigwedge_{y\in U}\left(F_{R_i}(x,y)\vee F_B(y)\right)\right] = F_{\overline{MS}^{(\cup)}(A)}(x)\vee F_{\overline{MS}^{(\cup)}(B)}(x).\end{aligned}$$

Hence, $\overline{MS}^{(\cup)}(A \cap B) \subseteq \overline{MS}^{(\cup)}(A) \cap \overline{MS}^{(\cup)}(B)$. □

4. The Relationships between Multi-Granulation Neutrosophic Rough Set Models

In this section, we discuss the relationships between MS, non-dual MS, neutrosophic rough set (NRS) based on neutrosophic intersection (union) relationship, and NRS based on neutrosophic transitive closure relation of union relationship and show it by a relational graph.

Definition 11. *Suppose U is a non-empty finite universe, and R_i ($1 \le i \le m$) is the binary NR on U. The ULA based on neutrosophic union relationship, represented by $\underline{\overset{m}{\underset{i=1}{\cup}} R_i}(A)$ and $\overline{\overset{m}{\underset{i=1}{\cup}} R_i}(A)$, are defined:*

$$\underline{\overset{m}{\underset{i=1}{\cup}} R_i}(A)(x) = \underset{y \in U}{\cap}\left(\left(\overset{m}{\underset{i=1}{\cup}} R_i(x,y)\right)^c \cup A(y)\right), \tag{14}$$

$$\overline{\overset{m}{\underset{i=1}{\cup}} R_i}(A)(x) = \underset{y \in U}{\cup}\left(\left(\overset{m}{\underset{i=1}{\cup}} R_i(x,y)\right) \cap A(y)\right). \tag{15}$$

Definition 12. *Suppose U is a non-empty finite universe, and R_i ($1 \le i \le m$) is the binary NR on U. The ULA based on neutrosophic intersection relationship, represented by $\underline{\overset{m}{\underset{i=1}{\cap}} R_i}(A)$ and $\overline{\overset{m}{\underset{i=1}{\cap}} R_i}(A)$, are defined:*

$$\underline{\overset{m}{\underset{i=1}{\cap}} R_i}(A)(x) = \underset{y \in U}{\cap}\left(\left(\overset{m}{\underset{i=1}{\cap}} R_i(x,y)\right)^c \cup A(y)\right), \tag{16}$$

$$\overline{\overset{m}{\underset{i=1}{\cap}} R_i}(A)(x) = \underset{y \in U}{\cup}\left(\left(\overset{m}{\underset{i=1}{\cap}} R_i(x,y)\right) \cap A(y)\right). \tag{17}$$

Definition 13 ([32]). *Suppose R is a neutrosophic relation in U. The minimal transitive neutrosophic relation containing R is called transitive closure of R, denoted by t(R).*

Proposition 3. *Suppose R is a neutrosophic relation in U. Then $t(R) = \overset{\infty}{\underset{k=1}{\cup}} R^k$. Where $R^k = R \bullet R \bullet R \bullet \cdots$, $(R \bullet S)(x,z) = \underset{y \in Y}{\cup}(R(x,y) \cap S(y,z))$.*

Definition 14. *Suppose (U, R) is neutrosophic approximation space. Suppose U is a non-empty finite universe, R_i ($1 \le i \le m$) is neutrosophic relations on U, and t(R) denotes the transitive closure of the union of neutrosophic relations R_i on U.$\forall A \in NS(U)$, the ULA of A, denoted by $\underline{t(R)}(A)$ and $\overline{t(R)}(A)$, are defined as: $\forall\, x \in U$,*

$$\underline{t(R)}(A)(x) = \underset{y \in U}{\cap}\left[t(R)^c(x,y) \cup A(y)\right], \overline{t(R)}(A)(x) = \underset{y \in U}{\cup}(t(R)(x,y) \cap A(y)).$$

Proposition 4. *Let (U, R_i) be a MAS, R_i ($1 \le i \le m$) be neutrosophic relations on U. For any $A \in NS$ (U), we have*

(1) $\underline{t(R)}(A) \subseteq \underline{MS}^p(A) = \underline{MS}^{(\cap)}(A) = \underline{\overset{m}{\underset{i=1}{\cup}} R_i}(A) \subseteq \underline{R_i}(A) \subseteq \underline{MS}^o(A) = \underline{MS}^{(\cup)}(A) \subseteq \underline{\overset{m}{\underset{i=1}{\cap}} R_i}(A) \subseteq X$;

(2) $X \subseteq \overline{\overset{m}{\underset{i=1}{\cap}} R_i}(A) \subseteq \overline{MS}^o(A) = \overline{MS}^{(\cap)}(A) \subseteq \overline{R_i}(A) \subseteq \overline{MS}^p(A) = \overline{MS}^{(\cup)}(A) = \overline{\overset{m}{\underset{i=1}{\cup}} R_i}(A) \subseteq \overline{t(R)}(A)$.

Proof. (1) According to Definition 4, Definition 5, Definition 9, and Definition 10, we can get $\overline{MS}^{o}(A) = \overline{MS}^{(\cap)}(A)$, $\overline{MS}^{p}(A) = \overline{MS}^{(\cup)}(A)$. Let $R = R_1 \cup R_2 \cup \cdots \cup R_m$, $t(R) = R \cup R_2 \cup \cdots$, then $(t(R))^c = R^c \cap (R_2)^c \cap \cdots$, so $(t(R))^c \subseteq R^c$, thus

$$\begin{aligned}
\underline{t(R)}(A)(x) &= \bigcap_{y\in U}\Big[t(R)^c(x,y) \cup A(y)\Big] \\
&\subseteq \bigcap_{y\in U}[R^c(x,y) \cup A(y)] = \underline{\bigcup_{i=1}^{m} R_i}(A)(x) \\
&= \bigcap_{y\in U}[({R_1}^c \cap {R_2}^c \cap \cdots \cap {R_m}^c)(x,y) \cup A(y)] \\
&= \bigcap_{y\in U}[({R_1}^c(x,y) \cup A(y)) \cap ({R_2}^c(x,y) \cup A(y)) \cap \cdots \cap ({R_m}^c(x,y) \cup A(y))] \\
&= \left[\bigcap_{y\in U}({R_1}^c(x,y) \cup A(y))\right] \cap \left[\bigcap_{y\in U}({R_2}^c(x,y) \cup A(y))\right] \cap \cdots \cap \left[\bigcap_{y\in U}({R_m}^c(x,y) \cup A(y))\right] \\
&= \bigcap_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y) \cup A(y))\right) = \underline{MS}^{p}(A)(x) = \underline{MS}^{(\cap)}(A) \\
&\subseteq \bigcap_{y\in U}({R_i}^c(x,y) \cup A(y)) = \underline{R_i}(A)(x).
\end{aligned}$$

Additionally, we have

$$\begin{aligned}
\underline{R_i}(A)(x) &= \bigcap_{y\in U}({R_i}^c(x,y) \cup A(y)) \\
&\subseteq \bigcup_{i=1}^{m}\left(\bigcap_{y\in U}({R_i}^c(x,y) \cup A(y))\right) = \underline{MS}^{o}(A)(x) = \underline{MS}^{(\cup)}(A)(x) \\
&\subseteq \bigcap_{y\in U}({R_1}^c(x,y) \cup {R_2}^c(x,y) \cup \cdots \cup {R_m}^c(x,y) \cup A(y)) \\
&= \bigcap_{y\in U}(({R_1}^c \cup {R_2}^c \cup \cdots \cup {R_m}^c)(x,y) \cup A(y)) \\
&= \bigcap_{y\in U}\left(\left(\bigcap_{i=1}^{m} R_i(x,y)\right)^c(x,y) \cup A(y)\right) = \underline{\bigcap_{i=1}^{m} R_i}(A) \subseteq X.
\end{aligned}$$

Then we get the proof.

(2) According to Definition 4, Definition 5, Definition 9, and Definition 10, we can get $\underline{MS}^{p}(A) = \underline{MS}^{(\cap)}(A)$, $\underline{MS}^{o}(A) = \underline{MS}^{(\cup)}(A)$. Let $R = R_1 \cup R_2 \cup \cdots \cup R_m$, $t(R) = R \cup R_2 \cup \cdots$, then $R \subseteq t(R)$, thus

$$\begin{aligned}
X &\subseteq \overline{\bigcap_{i=1}^{m} R_i}(A)(x) = \bigcup_{y\in U}\left(\left(\bigcap_{i=1}^{m} R_i(x,y)\right) \cap A(y)\right) \\
&\subseteq \bigcup_{y\in U}(R_i(x,y) \cap A(y)) = \overline{R_i}(A)(x) \\
&\subseteq \bigcup_{i=1}^{m}\left(\bigcup_{y\in U}(R_i(x,y) \cap A(y))\right) = \overline{MS}^{p}(A)(x) = \overline{MS}^{(\cup)}(A)(x) \\
&= \bigcup_{y\in U}\left(\left(\bigcup_{i=1}^{m} R_i(x,y)\right) \cap A(y)\right) = \overline{\bigcup_{i=1}^{m} R_i}(A)(x).
\end{aligned}$$

Additionally, we have

$$\begin{aligned}
\overline{\bigcup_{i=1}^{m} R_i}(A)(x) &= \bigcup_{y\in U}\left(\left(\bigcup_{i=1}^{m} R_i(x,y)\right) \cap A(y)\right) \\
&= \bigcup_{y\in U}(R(x,y) \cap A(y)) \\
&\subseteq \bigcup_{y\in U}\Big(\big(R \cup R^2 \cup \cdots\big)(x,y) \cap A(y)\Big) \\
&= \overline{t(R)}(A)(x).
\end{aligned}$$

Then we get the proof. □

The above results show that the four kinds of lower and upper approximations equipped with the inclusion relation ⊆ can construct a lattice. This fact can be described by Figure 1, where $i \neq j$, each node denotes an approximation or a concept, and each diagonal line connects two approximations, the lower node is a subset of the upper node.

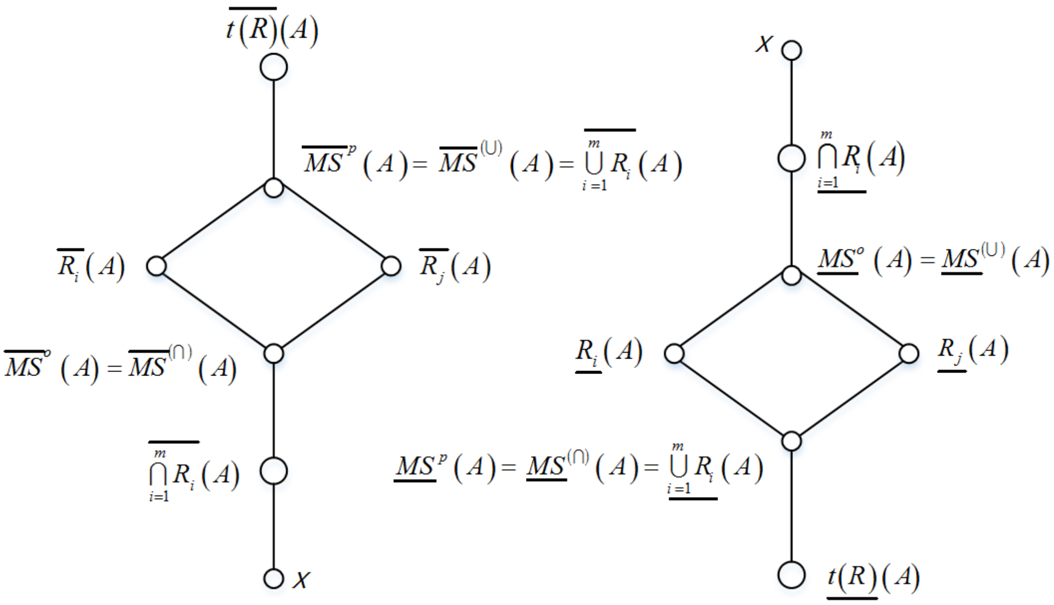

Figure 1. The relationships between neutrosophic rough lower and upper approximations.

5. The Application of Non-Dual Multi-Granulation Neutrosophic Rough Set on Two Universes in MGDM

In this section, we propose the concept of non-dual MS on two universes and we talk about the relationship between non-dual MS on two universes and non-dual MS on a single universe. Additionally, we used non-dual MS on two universes to deal with a MGDM problem where the attribute values are neutrosophic numbers.

Definition 14 ([28]). *Suppose U, V are two non-empty finite universes, and $R_i \in NS\ (U \times V)$ $(1 \leq i \leq m)$ is binary NR. We call (U, V, R_i) the MAS on two universes.*

Definition 15. *Let tuple ordered set (U, V, R_i) $(1 \leq i \leq m)$ be a MAS on two universes. For any $A \in NS(U)$, the intersection-type ULA $\underline{MNRS}^{(\cap)}(A)$ and $\overline{MNRS}^{(\cap)}(A)$ in (U, V, R_i) are defined:*

$$\underline{MNRS}^{(\cap)}(A)(x) = \bigcap_{i=1}^{m}\left(\bigcap_{y\in V}(R_i{}^c(x,y) \cup A(y))\right), \tag{18}$$

$$\overline{MNRS}^{(\cap)}(A)(x) = \bigcap_{i=1}^{m}\left(\bigcup_{y\in V}(R_i(x,y) \cap A(y))\right). \tag{19}$$

Obviously, $\underline{MNRS}^{(\cap)}(A)$ and $\overline{MNRS}^{(\cap)}(A)$ are two NSs. Furthermore, A is called a definable NS on (U, V, R_i) when $\underline{MNRS}^{(\cap)}(A) = \overline{MNRS}^{(\cap)}(A)$. Otherwise, the pair $\left(\underline{MNRS}^{(\cap)}(A), \overline{MNRS}^{(\cap)}(A)\right)$ are called intersection-type MS *on two universes.*

Definition 16. *Let tuple ordered set (U, V, R_i) $(1 \leq i \leq m)$ be a MAS on two universes. For any $A \in NS(U)$, the union-type ULA $\underline{MNRS}^{(\cup)}(A)$ and $\overline{MNRS}^{(\cup)}(A)$ in (U, V, R_i) are defined:*

$$\underline{MNRS}^{(\cup)}(A)(x) = \bigcup_{i=1}^{m}\left(\bigcap_{y\in V}(R_i{}^c(x,y) \cup A(y))\right), \tag{20}$$

$$\overline{MNRS}^{(\cup)}(A)(x) = \bigcup_{i=1}^{m}\left(\bigcup_{y\in V}(R_i(x,y)\cap A(y))\right). \tag{21}$$

Obviously, $\underline{MNRS}^{(\cup)}(A)$ and $\overline{MNRS}^{(\cup)}(A)$ are two NSs. Furthermore, A is called a definable NS on (U, V, R_i) when $\underline{MNRS}^{(\cup)}(A) = \overline{MNRS}^{(\cup)}(A)$. Otherwise, the pair $\left(\underline{MNRS}^{(\cup)}(A), \overline{MNRS}^{(\cup)}(A)\right)$ are called union-type MS *on two universes.*

Proposition 5. *Let (U, V, R_i) be a MAS on two universes. For any A, B ∈ NS(U), we have*

(1) $\underline{MNRS}^{(\cap)}(A\cap B) = \underline{MNRS}^{(\cap)}(A) \cap \underline{MNRS}^{(\cap)}(B)$, $\overline{MNRS}^{(\cap)}(A\cup B) = \overline{MNRS}^{(\cap)}(A) \cup \overline{MNRS}^{(\cap)}(B)$;

(2) $\underline{MNRS}^{(\cup)}(A\cap B) = \underline{MNRS}^{(\cup)}(A) \cap \underline{MNRS}^{(\cup)}(B)$, $\overline{MNRS}^{(\cup)}(A\cup B) = \overline{MNRS}^{(\cup)}(A) \cup \overline{MNRS}^{(\cup)}(B)$;

(3) $A \subseteq B \Rightarrow \underline{MNRS}^{(\cap)}(A) \subseteq \underline{MNRS}^{(\cap)}(B)$, $A \subseteq B \Rightarrow \overline{MNRS}^{(\cap)}(A) \subseteq \overline{MNRS}^{(\cap)}(B)$;

(4) $A \subseteq B \Rightarrow \underline{MNRS}^{(\cup)}(A) \subseteq \underline{MNRS}^{(\cup)}(B)$, $A \subseteq B \Rightarrow \overline{MNRS}^{(\cup)}(A) \subseteq \overline{MNRS}^{(\cup)}(B)$;

(5) $\underline{MNRS}^{(\cap)}(A\cup B) \supseteq \underline{MNRS}^{(\cap)}(A) \cup \underline{MNRS}^{(\cap)}(B)$, $\overline{MNRS}^{(\cap)}(A\cap B) \subseteq \overline{MNRS}^{(\cap)}(A) \cap \overline{MNRS}^{(\cap)}(B)$;

(6) $\underline{MNRS}^{(\cup)}(A\cup B) \supseteq \underline{MNRS}^{(\cup)}(A) \cup \underline{MNRS}^{(\cup)}(B)$, $\overline{MNRS}^{(\cup)}(A\cap B) \subseteq \overline{MNRS}^{(\cup)}(A) \cap \overline{MNRS}^{(\cup)}(B)$.

Proof. The proof is similar with Proposition 2. □

Remark 1. *Note that if the two universes are the same, then the intersection-type (union-type)* MS *on two universes degenerates into the intersection-type (union-type)* MS *on a single universe in Section 3.*

Next, we will use the non-dual MSs to solve the MGDM problems where the attribute values are neutrosophic numbers. For a multiple attribute group decision making problem, let $U = \{x_1, x_2, \ldots, x_n\}$ be the decision set and $V = \{y_1, y_2, \ldots, y_m\}$ be the criteria set, R_l represent l evaluation experts. Here, $R_l \in NR\ (U \times V)$ is NRs from U to V, where $\forall(x_i, y_j) \in U \times V$, $R_l(x_i, y_j)$ denotes the degree of membership about criteria set y_j ($y_j \in V$) with respect to x_i ($x_i \in U$). In the following, we show the process about the non-dual MSs on two universes to solve MGDM problems with neutrosophic numbers as attribute values.

Step 1 Calculate non-dual multi-granulation neutrosophic rough ULA $\underline{MNRS}^{(\cap)}(A)$, $\overline{MNRS}^{(\cap)}(A)$, $\underline{MNRS}^{(\cup)}(A)$, and $\overline{MNRS}^{(\cup)}(A)$.

Step 2 Calculate the sum of non-dual multi-granulation neutrosophic rough ULA $MNRS^{(\cap)} = \lambda\underline{MNRS}^{(\cap)}(A) \oplus (1-\lambda)\overline{MNRS}^{(\cap)}(A)$, $MNRS^{(\cup)} = \lambda\underline{MNRS}^{(\cup)}(A) \oplus (1-\lambda)\overline{MNRS}^{(\cup)}(A)$, $\lambda \in [0, 1]$ according to Definition 6.

Step 3 Make a descending order according to Definitions 7 and 8 for the multi-granulation neutrosophic rough sets in step 2 and use the Borda number scoring method in reference [33] to make a total rank.

In practice, the parameter λ represents a decision maker's preference for risk. In general, the higher the parameter λ is, the more likely the decision maker is to be risk-prone. The smaller the parameter λ is, the less risk the decision maker prefers. Therefore, the value of the parameter λ is determined by the decision maker's preference or by an advance empirical study.

Next, we show the algorithm to calculate the ULA of a union-type multi-granulation neutrosophic rough set.

Algorithm 1 The lower approximation of a union-type multi-granulation neutrosophic rough set

Define the method to acquire a complement for a matrix A:
each neutrosophic number in matrix A do complement the operator according to the following Formula:
$a^c = (F_a, 1 - I_a, T_a)$.
Return matrix C.
Define the method for two matrixes to do union operator:
the union of B and C is the neutrosophic number of each row in C to do union operator with the corresponding neutrosophic number in B according to the Formula (22)

$$a \vee_3 b = (\max(T_a, T_b), \min(I_a, I_b), \min(F_a, F_b)). \tag{22}$$

Return matrix D.
Define the method for one matrix to do intersection operator:
the neutrosophic numbers of each row in D do intersection operator according to the Formula (23)

$$a \wedge_3 b = (\min(T_a, T_b), \max(I_a, I_b), \max(F_a, F_b)). \tag{23}$$

Return matrix E.
Define the method for one matrix to do union operator:
the neutrosophic numbers of each row in E do union operator according to the Formula (22).
Return matrix F.
For the number of iterations is h,
Transfer the method of acquire complement, assign X.
Get Y.
Transfer the method for two matrixes to do union operator, assign Y, Z.
Get M.
Transfer the method to do intersection operator, assign M.
Get N.
End for.
Combine h matrixes N.
Get P.
Transfer the method for one matrix to do union operator, assign P.
Get Q.

X, Y, M are matrixes which line numbers are m, column number is n, and every membership is a neutrosophic number. Z is a matrix which line number is 1, column number is n, and every membership is a neutrosophic number. N and Q are matrixes which line numbers are m, column number is 1, and every membership is a neutrosophic number. P is a matrix which line number is m, column number is h, and every membership is a neutrosophic number.

The lower approximation of a union-type multi-granulation neutrosophic rough set is the transpose of matrix Q.

Algorithm 2 The upper approximation of a union-type multi-granulation neutrosophic rough set

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Define the method for two matrixes to do intersection operator:
the intersection of B and C is the neutrosophic number of each row in C to do intersection operator with the corresponding neutrosophic number in B according to the Formula (23).
Return matrix D.
Define the method for one matrix to do union operator:
the neutrosophic numbers of each row in D do union operator according to the Formula (22).
Return matrix E.
For the number of iterations is h,
Transfer the method for two matrixes to do intersection operator, assign Y, Z.
Get M.
Transfer the method for one matrix to do union operator, assign M.
Get N.
End for.
Combine h matrixes N.
Get P.
Transfer the method for one matrix to do intersection operator, assign P.
Get Q.

Y, M are matrixes which line numbers are m, column number is n, and every membership is a neutrosophic number. Z is a matrix which line number is 1, column number is n, and every membership is a neutrosophic number. N and Q are matrixes which line numbers are m, column number is 1, and every membership is a neutrosophic number. P is a matrix which line number is m, column number is h, and every membership is a neutrosophic number.

The upper approximation of a union-type multi-granulation neutrosophic rough set is the transpose of matrix Q.

With the same method we can get the ULA of an intersection-type multi-granulation neutrosophic rough set. Then, to decide the value of λ, we calculate the sum of ULA of the union-type MS and intersection-type MS according to Formula (6) and (7), and rank them according to Definition 7.

Next, we show an example.

Example 1. *We consider the decision making problem adapted from reference [34]. Suppose* $U = \{x_1, x_2, x_3\}$ *is a criterion set, where* x_1 *represents the ability of salesman,* x_2 *represents the overall condition of the stable supplier, and* x_3 *represents the position of high flow. Let* $V = \{y_1, y_2, y_3, y_4, y_5\}$ *be the decision set, where* y_1 *represents shop 1,* y_2 *represents shop 2,* y_3 *represents shop 3,* y_4 *represents shop 4, and* y_5 *represents shop 5.*

Assume there are three experts. They provide their evaluations shown in Tables 1–3 based on their knowledge and experience. The data of the three tables were adapted from Table 2 in reference [34]. We take the first positive membership and negative membership of the intuitionistic fuzzy set of interval values y_1–y_5 in Table 2 as the true membership and false membership of the neutrosophic set, respectively, and the second negative membership as the uncertain membership of the neutrosophic set. Let $A = \{(0.9, 0.1, 0.2), (0.7, 0.7, 0.3), (0.5, 0.8, 0.6)\}$.

Table 1. Neutrosophic relation R_1.

R_1	x_1	x_2	x_3
y_1	(0.75, 0.14, 0.09)	(0.86, 0.04, 0.01)	(0.66, 0.30, 0.29)
y_2	(0.44, 0.33, 0.29)	(0.51, 0.09, 0.04)	(0.54, 0.29, 0.27)
y_3	(0.54, 0.09, 0.08)	(0.66, 0.14, 0.06)	(0.54, 0.36, 0.34)
y_4	(0.56, 0.19, 0.14)	(0.50, 0.20, 0.12)	(0.44, 0.26, 0.23)
y_5	(0.33, 0.31, 0.30)	(0.43, 0.16, 0.02)	(0.21, 0.61, 0.60)

Table 2. Neutrosophic relation R_2.

R_2	x_1	x_2	x_3
y_1	(0.71, 0.10, 0.08)	(0.57, 0.01, 0.00)	(0.56, 0.09, 0.09)
y_2	(0.39, 0.54, 0.43)	(0.59, 0.11, 0.01)	(0.44, 0.19, 0.18)
y_3	(0.52, 0.17, 0.07)	(0.63, 0.04, 0.02)	(0.37, 0.54, 0.51)
y_4	(0.31, 0.09, 0.08)	(0.52, 0.31, 0.09)	(0.41, 0.29, 0.27)
y_5	(0.10, 0.61, 0.59)	(0.33, 0.33, 0.13)	(0.19, 0.09, 0.07)

Table 3. Neutrosophic relation R_3.

R_3	x_1	x_2	x_3
y_1	(0.89, 0.06, 0.05)	(0.86, 0.01, 0.01)	(0.77, 0.21, 0.20)
y_2	(0.61, 0.30, 0.27)	(0.76, 0.09, 0.01)	(0.56, 0.27, 0.25)
y_3	(0.64, 0.20, 0.10)	(0.63, 0.01, 0.01)	(0.59, 0.33, 0.29)
y_4	(0.68, 0.16, 0.04)	(0.59, 0.03, 0.02)	(0.57, 0.36, 0.31)
y_5	(0.39, 0.23, 0.10)	(0.34, 0.30, 0.19)	(0.29, 0.59, 0.49)

By Definitions 15 and 16, we can compute

$$\underline{MNRS}^{(\cup)}(A) = \begin{array}{l}\{(y_1, 0.50, 0.70, 0.56), (y_2, 0.50, 0.71, 0.44), (y_3, 0.51, 0.70, 0.37), (y_4, 0.50, 0.70, 0.41), \\ (y_5, 0.60, 0.70, 0.30)\}\end{array},$$

$$\overline{MNRS}^{(\cup)}(A) = \begin{array}{l}\{(y_1, 0.89, 0.10, 0.20), (y_2, 0.70, 0.30, 0.27), (y_3, 0.66, 0.10, 0.20), (y_4, 0.68, 0.10, 0.20), \\ (y_5, 0.43, 0.23, 0.20)\}\end{array},$$

$$\underline{MNRS}^{(\cap)}(A) = \begin{array}{l}\{(y_1, 0.50, 0.80, 0.60), (y_2, 0.50, 0.80, 0.56), (y_3, 0.50, 0.70, 0.59), (y_4, 0.50, 0.74, 0.57), \\ (y_5, 0.50, 0.80, 0.30)\}\end{array},$$

$$\overline{MNRS}^{(\cap)}(A) = \begin{array}{l}\{(y_1, 0.71, 0.14, 0.20), (y_2, 0.51, 0.54, 0.30), (y_3, 0.63, 0.20, 0.20), (y_4, 0.52, 0.19, 0.20), \\ (y_5, 0.33, 0.61, 0.30)\}\end{array},$$

Let $\lambda = 0.3$, then

$$MNRS^{(\cup)}(A) = \begin{array}{l}\{(y_1, 0.8267, 01793, 0.2723), (y_2, 0.6525, 0.3885, 0.2896), (y_3, 0.6183, 0.1793, 0.2258), \\ (y_4, 0.6341, 0.1793, 0.2480), (y_5, 0.4874, 0.3212, 0.2236)\}\end{array},$$

$$MNRS^{(\cap)}(A) = \begin{array}{l}\{(y_1, 0.6585, 0.2362, 0.2780), (y_2, 0.5070, 0.6076, 0.3617), (y_3, 0.5950, 0.2912, 0.2767), \\ (y_4, 0.5141, 0.2857, 0.2738), (y_5, 0.3863, 0.6617, 0.3000)\}\end{array},$$

Then, according to Definition 6, we can get

$$s^{(\cup)}(y_1) = 0.5259,\ s^{(\cup)}(y_2) = 0.6814,\ s^{(\cup)}(y_3) = 0.6963,\ s^{(\cup)}(y_4) = 0.6930,\ s^{(\cup)}(y_5) = 0.6319.$$

So, the ranking result for union-type MS is: $y_1 \prec y_5 \prec y_2 \prec y_4 \prec y_3$.

$$s^{(\cap)}(y_1) = 0.6903,\ s^{(\cap)}(y_2) = 0.5726,\ s^{(\cap)}(y_3) = 0.6592,\ s^{(\cap)}(y_4) = 0.6201,\ s^{(\cap)}(y_5) = 0.5432.$$

So, the ranking result for intersection-type MS is: $y_5 \prec y_2 \prec y_4 \prec y_3 \prec y_1$.

Using the Borda counting method, score 4, 3, 2, 1, and 0 for the first, second, third, fourth, and fifth place, respectively, then we can get

$$B(x_1) = 4,\ B(x_2) = 3,\ B(x_3) = 7,\ B(x_4) = 5,\ B(x_5) = 1.$$

So, when $\lambda = 0.3$, the best choice, shop 3, is chosen.

6. Conclusions

The multi-granulation neutrosophic rough set is a useful tool for MGDM problems. In this paper, we proposed non-dual MSs and study their operators and properties. Then we discussed the relationship between NRS, optimistic (pessimistic) MS, non-dual MS, NRS based on intersection (union) NRs, and NRS based on transitive closure relationship of union NRs, we used Figure 1 to show the relationship. Furthermore, we proposed a non-dual MS on two universes and talk about the relationship between non-dual MS on two universes and non-dual MS on a single universe, and we used non-dual MS on two universes to solve a MGDM problem where the attribute values were neutrosophic numbers.

For future orientation, we will research other types of fusions of MRSs and NSs. Additionally, we will study the applications of the concepts in this paper to totally-dependent neutrosophic sets and some algebraic systems and discuss in relation to other algorithms [35–44].

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