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Novel Neutrosophic Cubic Graphs Structures with Application in Decision Making Problems

Muhammad Gulistan[1], Mumtaz Ali[2], Muhammad Azhar[1], Seungmin Raho[3], Seifedine Kadry[4]

[1]Department of Mathematics and Statistics, Hazara University, Mansehra, Pakistan
[2] University of Southern Queensland, Toowoomba, QLD 4300, Australia
[3]Department of software, Sejong University, Seoul, Korea
[4]Department of Mathematics & Computer Science, Beirut Arab University, Lebanon

Corresponding author: Muhammad Gulistan, Seungmin Raho (gulistanmath@hu.edu.pk, smrhosejong.edu)

ABSTRACT: Graphs allows us to study the different patterns of inside the data by making a mental image. The aim of this paper is to develop neutrosophic cubic graph structure which is the extension of neutrosophic cubic graphs. As neutrosophic cubic graphs are defined for one set of edges between vertices while neutrosophic cubic graphs structures are defined for more than one set of edges. Further, we defined some basic operations such as Cartesian product, composition, union, join, cross product, strong product and lexicographic product of two neutrosophic cubic graph structures. Several types of other interesting properties of neutrosophic cubic graph structures are discussed in this paper. Finally, a decision-making algorithm based on the idea of neutrosophic cubic graph structures is constructed. The proposed decision-making algorithm is applied in a decision-making problem to check the validity.

INDEX TERMS: Neutrosophic Cubic Set, Neutrosophic Cubic graphs structures, application.

I. INTRODUCTION

Fuzzy sets: The extension of classical set theory in the form of fuzzy sets was given by Zadeh in 1965 in his seminal paper [1]. Further he introduced the interval-valued fuzzy sets in 1975 [2]. Atanassov use the notion of membership and non-membership of an element in a set X and gave the idea of intuitionistic fuzzy sets. Use of intuitionistic fuzzy sets is helpful in the introduction of additional degrees of freedom (non-membership and hesitation margins) into set description and is extensively use as a tool of intensive research by scholars and scientists from over the so many years. Various theories like theory of probability, fuzzy set theory, intutionistic fuzzy sets, rough set theory etc., are consistently being used as powerful constructive tools to deal with multiform uncertainties and imprecision enclosed in complex systems. But all these above theories do not model undetermined information adequately. Therefore, due to the existence of indeterminacy in various world problems, neutrosophy founds its way into the modern research. Neutrosophy is a generalization of fuzzy set, where the models represented by three types concepts that is truthfulness, falsehood and neutrality. Neutrosophy is a Latin world "neuter" - neutral, Greek "sophia" - skill/wisdom). Neutrosophy is a branch of philosophy, introduced by FlorentinSmarandache which studies the origin, nature, and scope of neutralities, as well as theirinteractions with different ideational spectra. Neutrosophy considers a proposition, theory, event, concept, or entity, "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A". Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.

Inspiring from the realities of real life phenomenons like sport games (winning/ tie/ defeating), votes (yes/ NA/ no) and decision making (making a decision/ hesitating/ not making), Smarandache [3, 4] introduced a new concept of a neutrosophic set and neutrosophic logic (NS in short) in 1999, which is the generalization of a fuzzy sets and intutionistic fuzzy set. NS is described by membership degree, indeterminate degree and non-membership degree. The idea of NS generates the theory of neutrosophic sets by giving representation to indeterminates. This theory is considered as complete representation of almost every model of all real-world problems. Therefore, if uncertainty is involved in a problem we use fuzzy theory while dealing indeterminacy, we need neutrosophic theory. In fact, this theory has several applications in many different fields like control theory, databases, medical diagnosis problem and decision-making problems. These sets models have been studied by many authors. Using Neutrosophic theory, many mathematicians introduced the concept of neutrosophic algebraic structures such as neutrosophic algebraic structures, neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophicbigroups, neutrosophic N-groups, neutrosophicbisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophicbiloops, neutrosophic N-loop, neutrosophic groupoids, neutrosophicbigroupoids and neutrosophic AG-groupoids. In 2012, Jun et al. gave the idea of cubic sets [5]. For more detail of cubic set one can cite [6, 7, 8, 9, 10, 11]. More recently Jun et al. combine neutrosophic set with cubic sets and gave the idea of Neutrosophic cubic set [12] and define different operations [13]. Further interval neutrosophic sets was introduced by Wang et al. [14]. Fuzzy Graphs: In 1975 Rosenfeld [15] extended the idea given by Kauffmann in 1973 [16] and initiate the concept of fuzzy graphs and considered the relations between fuzzy sets. In 1987 Bhattacharya explained some remarks on fuzzy graphs [17]. Mordeson and Nair explained the study of fuzzy graphs and fuzzy hypergraphs in their book in 2001 [18]. Akram et al. gave the idea of interval valued fuzzy graphs [19, 20], intuitionistic fuzzy graphs and bipolar fuzzy graphs [21, 22, 23]. Strong intuitionistic fuzzy graphs were presented by Akram and Davvaz [24]. Intuitionistic fuzzy sets were further generalized by Smarandache [4]. Cayley interval-valued fuzzy threshold graphs were studied by Borzooei and Rashmanlou [25]. Buckley gave the concept of self-centered graphs [26]. Further characterized g-self-centered fuzzy graphs was given by Sunitha et al. [27]. Mishra et al. [28] introduced the idea of coherent category of interval-valued intuitionistic fuzzy graphs. Pal et al. [29] and Pramanik et al. [30, 31] discussed some results to the theory of interval-valued fuzzy graphs. Parvathi et al. [32] defined operations on intuitionistic fuzzy graphs. The idea of product of intuitionistic fuzzy graphs was introduced by Sahoo and Pal [33]. Gulistan et al. [32] presented the idea of neutrophic cubic graphs with real life application in industry. The main role of neutrosophic cubic graph structure theory in computer application is the development of graph algorithms. These algorithms are used to those problems that are modeled in the form of graphs and the corresponding computer science applications problems. Theoretical concept of the neutrosophic cubic graphs structures are highly utilized by computer science application. Especially in research area of computer science such as data mining, image segmentation, clustering, image capturing and networking. The neutrosophic cubic graphs structures are more flexible and compatible then fuzzy graphs due to the fact that they have many applications in networks.

Our approach: In this paper we initiate the idea of neutrosophic cubic graph structures which is extension of neutrosophic cubic graphs. Neutrosophic cubic graphs are defined for one set of edges between vertices while neutrosophic cubic graphs structures are defined for more than one set of edges. We also defined basic operations like Cartesian product, composition, union, join, cross product, strong product and lexicographic product of two neutrosophic cubic graph structures. At the end we discuss the application of neutrosophic cubic graphs in decision making problems.

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II. Preliminaries

We briefly describe few fundamental concepts, ideas and preliminaries of neutrosophic sets, neutrosophic cubic sets and neutrosophic cubic graphs.

Definition 2.1 *[34] Neurosophics set is define as:*

$$A = \{\langle x, F_A(x), T_A(x), I_A(x)\rangle : x \in X\}$$

where X is a universe of discoveries and A is characterized by a truth-membership function $T_A: X \to]0^-, 1^+[$,an indeterminacy-membership function $I_A: X \to]0^-, 1^+[$ and a falsity-membership function $F_A: X \to]0^-, 1^+[$. There is not restriction on the sum of $T_A(x), I_A(x), , F_A(x)$.

Definition 2.2 *[35] A single valued neutrosophics set is define as:*

$$A_{NS} = \{\langle x, F_A(x), T_A(x), I_A(x)\rangle : x \in X\}$$

where X is a universe of discoveries and A_{NS} is characterized by a truth-membership function $T_A: X \to 0,1]$,an indeterminacy-membership function $I_A: X \to 0,1]$ and a falsity-membership function $F_A: X \to 0,1]$. There is not restriction on the sum of $T_A(x), I_A(x), F_A(x)$.

Definition 2.3 *[35]Let us consider two single valued neutrosophic sets*

$$A_{NS} = \{\langle x, F_A(x), T_A(x), I_A(x)\rangle : x \in X\}$$

and

$$B_{NS} = \{\langle x, F_B(x), T_B(x), I_B(x)\rangle : x \in X\}$$

then set theoretical operations for these two single valued nurtrosophic sets are given as;

$(i) A_{NS} \subset B_{NS}$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$.

$(ii) A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$, for any $x \in X$.

(iii) The complement of A_{NS} is denoted by A^c_{NS} and is defined by

$$A^c_{NS} = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle / x \in X\}$$

(iv) The intersection

$A_{NS} B_{NS}$
$= \{\langle x, \min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\}, \max\{F_A(x), F_B(x)\}\rangle : x \in X\}$

(v) The Union

$A_{NS} B_{NS}$
$= \{\langle x, \max\{T_A(x), T_B(x)\}, \min\{I_A(x), I_B(x)\}, \min\{F_A(x), F_B(x)\}\rangle : x \in X\}$.

Definition 2.4 *[2, 36] Let $\tilde{A}_1 = \langle T_1, I_1, F_1\rangle$ and $\tilde{A}_2 = \langle T_2, I_2, F_2\rangle$ be two single valued neutrosophic number. Then, the operations for NNs are defined as below;*

$$\lambda\tilde{A} = \langle 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda\rangle,$$
$$\tilde{A}_1^\lambda = \langle T_1^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda\rangle,$$
$$\tilde{A}_1 + \tilde{A}_2 = \langle T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2\rangle$$
$$\tilde{A}_1\tilde{A}_2 = \langle T_1T_2, I_1 + I_{2-}I_1I_2, F_1 + F_2 - F_1F_2\rangle \text{ where } \lambda > 0.$$

Definition 2.5 *[8]Let X be a non-empty set. A neutrosophic cubic set (NCS) in X is a pair $A = (\boldsymbol{A}, \boldsymbol{\Lambda})$ where $\boldsymbol{A} = \{\langle x, \tilde{A}_T(x), \tilde{A}_I(x), \tilde{A}_F(x)\rangle | x \in X\}$ is an interval neutrosophic set in X and$\boldsymbol{\Lambda} = \{\langle x, \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle | x \in X\}$* is a neutrosophic set in X. Also $[0,0] \preceq \tilde{A}_T + \tilde{A}_I + \tilde{A}_F \preceq [3,3]$ and $0 \le \lambda_T + \lambda_I + \lambda_F \le 3$.

Definition 2.6 *[24] Let $G^* = (V, E)$ be a Graph. By neutrosophic cubic graph of G^*, we mean a pair $G = (M, N)$ where*

$$M = (A, B) = ((\tilde{T}_A, T_B), (\tilde{I}_A, I_B), (\tilde{F}_A, F_B))$$

is the neutrosophic cubic set representation of V and

$$N = (C, D) = ((\tilde{T}_C, T_D), (\tilde{I}_C, I_D), (\tilde{F}_C, F_D))$$

is the neutrosphic cubic set representation of E such that;

(i)$\left(\tilde{T}_C(u_iv_i) \preceq rmin\{\tilde{T}_A(u_i), \tilde{T}_A(v_i)\}, T_D(u_iv_i) \le \max\{T_B(u_i), T_B(v_i)\}\right)$

(ii)$(\tilde{I}_C(u_iv_i) \preceq rmin\{\tilde{I}_A(u_i), \tilde{I}_A(v_i)\}, I_D(u_iv_i) \le \max\{I_B(u_i), I_B(v_i)\})$

(iii)$\left(\tilde{F}_C(u_iv_i) \preceq rmax\{\tilde{F}_A(u_i), \tilde{F}_A(v_i)\}, F_D(u_iv_i) \le \min\{F_B(u_i), F_B(v_i)\}\right)$

Definition 2.7 *[24] Let $G^* = (V, E)$ be a graph and $G = (M, N)$ be a Neutrosophic Cubic Graph on V is said to be truth-internal (T-internal) if the following conditions hold*

$$T_B(x) \in \tilde{T}_A^-(x), \tilde{T}_A^+(x)], \forall x \in V, T_D(e \in \tilde{T}_C^-(e), \tilde{T}_C^+(e)], \forall e \in E$$

indeterminacy-internal (I-internal) if the following conditions hold

$$I_B(x) \in \tilde{I}_A^-(x), \tilde{I}_A^+(x)], \forall x \in V, I_D(e) \in \tilde{I}_C^-(e), \tilde{I}_C^+(e)], \forall e \in E$$

falsity-internal (F-internal) if the following conditions hold

$$F_B(x) \in \tilde{F}_A^-(x), \tilde{F}_A^+(x)], \forall x \in V, F_D(e) \in \tilde{F}_C^-(e), \tilde{F}_C^+(e)], \forall e \in E$$

truth-external (T-external) if the following conditions hold

$$T_B(x) \notin \tilde{T}_A^-(x), \tilde{T}_A^+(x)], \forall x \in V, T_D(e) \notin \tilde{T}_C^-(e), \tilde{T}_C^+(e)], \forall e \in E$$

indeterminacy-external (I-external) if the following conditions hold

$$I_B(x) \notin \tilde{I}_A^-(x), \tilde{I}_A^+(x)], \forall x \in V, I_D(e) \notin \tilde{I}_C^-(e), \tilde{I}_C^+(e)], \forall e \in E$$

falsity-external (F-external) if the following conditions hold

$$F_B(x) \notin \tilde{F}_A^-(x), \tilde{F}_A^+(x)], \forall x \in V, F_D(e) \notin \tilde{F}_C^-(e), \tilde{F}_C^+(e)], \forall e \in E$$

Definition 2.8 *[24] Let $G^* = (V, E)$ be a graph and $G = (M, N)$ be a neutrosophic cubic graph on V is said to be internal if the following conditions hold*

$$\begin{pmatrix} T_B(x) \in \tilde{T}_A^-(x), \tilde{T}_A^+(x)], \\ I_B(x) \in \tilde{I}_A^-(x), \tilde{I}_A^+(x)], \\ F_B(x) \in \tilde{F}_A^-(x), \tilde{F}_A^+(x)] \end{pmatrix} \forall x \in V, \begin{pmatrix} T_D(e) \in T_C^-(e), T_C^+(e)], \\ I_D(e) \in I_C^-(e), I_C^+(e)], \\ F_D(e) \in F_C^-(e), F_C^+(e)] \end{pmatrix} \forall e \in E$$

A neutrosophic cubic graph is said to be internal neutrosophic cubic graph if it is truth-internal, indeterminacy-internal and falsity-internal.

III. Neutrosophic Cubic Graph Structures

In this section we define the extension of neutrosophic cubic graphs to neutrosophic cubic graph structures

Definition 3.1 *Let $\breve{G}^* = (V, E_1, E_2, \ldots, E_n)$ be a graph structure. Then $\breve{G} = (M, N_1, N_2, \ldots, N_n)$ is said to be neutrosophic cubic graph structure of $\breve{G}^*$,where*

$$M = (A, B) = ((\tilde{T}_A, T_B), (\tilde{I}_A, I_B), (\tilde{F}_A, F_B))$$

is the neutrosophic cubic set representation of V and

$$N_1 = (C_1, D_1) = ((\tilde{T}_{C_1}, T_{D_1}), (\tilde{I}_{C_1}, I_{D_1}), (\tilde{F}_{C_1}, F_{D_1}))$$
$$N_2 = (C_2, D_2) = ((\tilde{T}_{C_2}, T_{D_2}), (\tilde{I}_{C_2}, I_{D_2}), (\tilde{F}_{C_2}, F_{D_2}))$$
$$N_n = (C_n, D_n) = ((\tilde{T}_{C_n}, T_{D_n}), (\tilde{I}_{C_n}, I_{D_n}), (\tilde{F}_{C_n}, F_{D_n}))$$

are the neutrosphic cubic set representations of $E_1, E_2, \ldots, E_n$ respectively, if the following conditions are satisfied:

(i) M is a neutrosophic cubic set on V such that $\forall x \in V$

$$0 \le \tilde{T}_A + \tilde{I}_A + \tilde{F}_A \le 3,3], 0 \le T_B + I_B + F_B \le 3$$

(ii) N_n is a neutrosophic cubic set on E_n such that $\forall xy \in E_n, i \in 1,2, \ldots, n$

$$0 \le \tilde{T}_{C_n} + \tilde{I}_{C_n} + \tilde{F}_{C_n} \le 3,3], 0 \le T_{D_n} + I_{D_n} + F_{D_n} \le 3$$

(iii) Also $\forall xy \in E_n, i \in 1,2, \ldots, n$

$\tilde{T}_{C_n}(xy) \preceq rmin\{\tilde{T}_A(x), \tilde{T}_A(y)\}, T_{D_n}(xy) \le \max\{T_B(x), T_B(y)\}$,
$\tilde{I}_{C_n}(xy) \preceq rmin\{\tilde{I}_A(x), \tilde{I}_A(y)\}, I_{D_n}(xy) \le \max\{I_B(x), I_B(y)\}$,
$\tilde{F}_{C_n}(xy) \preceq rmax\{\tilde{F}_A(x), \tilde{F}_A(y)\}, F_{D_n}(xy) \le \min\{F_B(x), F_B(y)\}$

Example: Let $\breve{G}^* = (V, E_1, E_2)$ be a graph structure where

$$V = \{a, b, c, d\},$$
$$E_1 = \{ab, ac, \},$$
$$E_2 = \{ad, bc, bd\}$$

defined as

$$M = \begin{Bmatrix} \{a, ([0.4,0.5], 0.6), ([0.2,0.3], 0.2), ([0.6,0.7], 0.1)\}, \\ \{b, ([0.5,0.6], 0.4), ([0.4,0.5], 0.5), ([0.1,0.2], 0.7)\}, \\ \{c, ([0.3,0.4], 0.9), ([0.9,1.0], 0.2), ([0.7,0.8], 0.3)\}, \\ \{d, ([0.8,0.9], 0.7), ([0.4,0.5], 0.3), ([0.5,0.6], 0.4)\} \end{Bmatrix}, N_1$$
$$= \begin{Bmatrix} \{ab, ([0.4,0.5], 0.6), ([0.2,0.3], 0.5), ([0.6,0.7], 0.1)\}, \\ \{ac, ([0.3,0.4], 0.9), ([0.2,0.3], 0.2), ([0.7,0.8], 0.1)\} \end{Bmatrix},$$
$$N_2 = \begin{Bmatrix} \{ad, ([0.4,0.5],0.7), ([0.2,0.3],0.3), ([0.6,0.7],0.1)\}, \\ \{bc, ([0.3,0.4],0.9), ([0.4,0.5],0.5), ([0.7,0.8],0.3)\}, \\ \{bd, ([0.5,0.6],0.7), ([0.4,0.5],0.5), ([0.5,0.6],0.4)\} \end{Bmatrix}.$$

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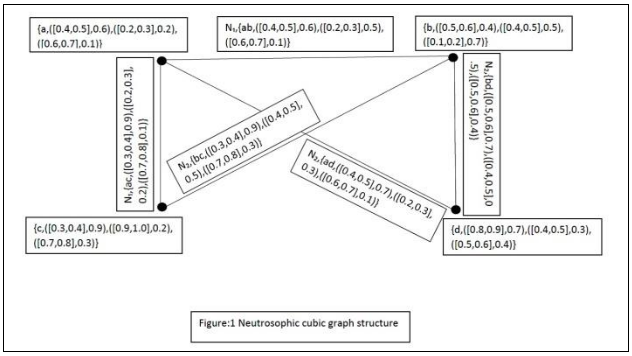

Figure:1 Neutrosophic cubic graph structure

Definition 3.2 *Let $\breve{G}_{S1}=(M_1,N_{11},N_{21},\dots,N_{n1})$ and $\breve{G}_{S2}=(M_2,N_{12},N_{22},\dots,N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^*=(V_1,E_{11},E_{21},\dots,E_{n1})$ and $\breve{G}_2^*=(V_2,E_{12},E_{22},\dots,E_{n2})$ respectively. The cartesian product of $\breve{G}_1^*$ and $\breve{G}_2^*$ is defined as*

$$\begin{aligned}\breve{G}_{S1}\times\breve{G}_{S2}&=(M_1,N_{11},N_{21},\dots,N_{n1})\times(M_2,N_{12},N_{22},\dots,N_{n2})\\&=(M_1\times M_2,N_{11}\times N_{12},N_{21}\times N_{22},\dots,N_{n1}\times N_{n2})\\&=((A_1,B_1)\times(A_2,B_2),(C_{11},D_{11})\times(C_{12},D_{12}),\\&(C_{21},D_{21})\times(C_{22},D_{22}),\dots,(C_{n1},D_{n1})\times(C_{n2},D_{n2}))\\&=((A_1\times A_2,B_1\times B_2,),(C_{11}\times C_{12},D_{11}\times D_{12}),\\&(C_{21}\times C_{22},D_{21}\times D_{22}),\dots,(C_{n1}\times C_{n2},D_{n1}\times D_{n2}))\\&=\\&((\tilde{T}_{A_1\times A_2},T_{B_1\times B_2}),(\tilde{I}_{A_1\times A_2},I_{B_1\times B_2}),(\tilde{F}_{A_1\times A_2},F_{B_1\times B_2})),\\&((\tilde{T}_{C_{11}\times C_{12}},T_{D_{11}\times D_{12}}),(\tilde{I}_{C_{11}\times C_{12}},I_{D_{11}\times D_{12}}),(\tilde{F}_{C_{11}\times C_{12}},F_{D_{11}\times D_{12}})\\&(\tilde{T}_{C_{21}\times C_{22}},T_{D_{21}\times D_{22}}),(\tilde{I}_{C_{21}\times C_{22}},I_{D_{21}\times D_{22}}),(\tilde{F}_{C_{21}\times C_{22}},F_{D_{21}\times D_{22}}),\dots,\\&(\tilde{T}_{C_{n1}\times C_{n2}},T_{D_{n1}\times D_{n2}}),(\tilde{I}_{C_{n1}\times C_{n2}},I_{D_{n1}\times D_{n2}}),(\tilde{F}_{C_{n1}\times C_{n2}},F_{D_{n1}\times D_{n2}}))\end{aligned}$$

and is defined as follow:

(i) $\tilde{T}_{A_1\times A_2}(x,y)=rmin(\tilde{T}_{A_1}(x),\tilde{T}_{A_2}(y)),T_{B_1\times B_2}(x,y)=\max(T_{B_1}(x),T_{B_2}(y))$,

(ii) $\tilde{I}_{A_1\times A_2}(x,y)=rmin(\tilde{I}_{A_1}(x),\tilde{I}_{A_2}(y)),I_{B_1\times B_2}(x,y)=\max(I_{B_1}(x),I_{B_2}(y))$,

(iii) $\tilde{F}_{A_1\times A_2}(x,y)=rmax(\tilde{F}_{A_1}(x),\tilde{F}_{A_2}(y)),F_{B_1\times B_2}(x,y)=\min(F_{B_1}(x),F_{B_2}(y))$

(iv) $\tilde{T}_{C_{n1}\times C_{n2}}((x,y_1)(x,y_2))=rmin(\tilde{T}_{A_1}(x),\tilde{T}_{C_{n2}}(y_1y_2))$
$T_{D_{n1}\times D_{n2}}((x,y_1)(x,y_2))=\max(T_{B_1}(x),T_{D_{n2}}(y_1y_2))$

(v) $\tilde{I}_{C_{n1}\times C_{n2}}((x,y_1)(x,y_2))=rmin(\tilde{I}_{A_1}(x),\tilde{I}_{C_{n2}}(y_1y_2))$
$I_{D_{n1}\times D_{n2}}((x,y_1)(x,y_2))=\max(I_{B_1}(x),I_{D_{n2}}(y_1y_2))$

(vi) $\tilde{F}_{C_{n1}\times C_{n2}}((x,y_1)(x,y_2))=rmax(\tilde{F}_{A_1}(x),\tilde{F}_{C_{n2}}(y_1y_2))$
$F_{D_{n1}\times D_{n2}}((x,y_1)(x,y_2))=\min(F_{B_1}(x),F_{D_{n2}}(y_1y_2))$

(vii) $\tilde{T}_{C_{n1}\times C_{n2}}((x_1,y)(x_2,y))=rmin(\tilde{T}_{C_{n1}}(x_1x_2),\tilde{T}_{A_2}(y))$
$T_{D_{n1}\times D_{n2}}((x_1,y)(x_2,y))=\max(T_{D_{n1}}(x_1x_2),T_{B_2}(y))$

(viii) $\tilde{I}_{C_{n1}\times C_{n2}}((x_1,y)(x_2,y))=rmin(\tilde{I}_{C_{n1}}(x_1x_2),\tilde{I}_{A_2}(y))$
$I_{D_{n1}\times D_{n2}}((x_1,y)(x_2,y))=\max(I_{D_{n1}}(x_1x_2),I_{B_2}(y))$

(ix) $\tilde{F}_{C_{n1}\times C_{n2}}((x_1,y)(x_2,y))=rmax(\tilde{F}_{C_{n1}}(x_1x_2),\tilde{F}_{A_2}(y))$
$F_{D_{n1}\times D_{n2}}((x_1,y)(x_2,y))=\min(F_{D_{n1}}(x_1x_2),F_{B_2}(y))$

$\forall(x,y)\in(V_1,V_2)=V$ for $(i)-(iii),\forall x\in V_1$ and $y_1y_2\in E_{n2};(i\in 1,2,\dots,n)$ for $(iv)-(vi),\forall y\in V_2$ and $x_1x_2\in E_{n1};(i\in 1,2,\dots,n)$ for $(vi)-(ix)$.

Example: Let $\breve{G}_{s1}=(M_1,N_{11},N_{21},N_{31})$ and $\breve{G}_{s2}=(M_2,N_{12},N_{22})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^*$ and $\breve{G}_2^*$ respectively, where

$$M_1=\left\langle\begin{matrix}\{a,([0.3,0.4],0.1),([0.5,0.6],0.2),([0.4,0.5],0.3)\},\\\{b,([0.1,0.2],0.5),([0.7,0.8],0.6),([0.5,0.7],0.4)\},\\\{c,([0.8,0.9],0.3),([0.2,0.3],0.5),([0.3,0.4],0.5)\},\\\{d,([0.2,0.3],0.5),([0.9,1.0],0.4),([0.6,0.7],0.2)\}\end{matrix}\right\rangle$$

$$N_{11}=\left\langle\begin{matrix}\{ab,([0.1,0.2],0.5),([0.5,0.6],0.6),([0.5,0.7],0.3)\},\\\{cd,([0.2,0.3],0.5),([0.2,0.3],0.5),([0.6,0.7],0.2)\}\end{matrix}\right\rangle$$

$$N_{21}=\left\langle\begin{matrix}\{ad,([0.2,0.3],0.5),([0.5,0.6],0.4),([0.6,0.7],0.2)\},\\\{bc,([0.1,0.2],0.5),([0.2,0.3],0.6),([0.5,0.7],0.4)\}\end{matrix}\right\rangle$$

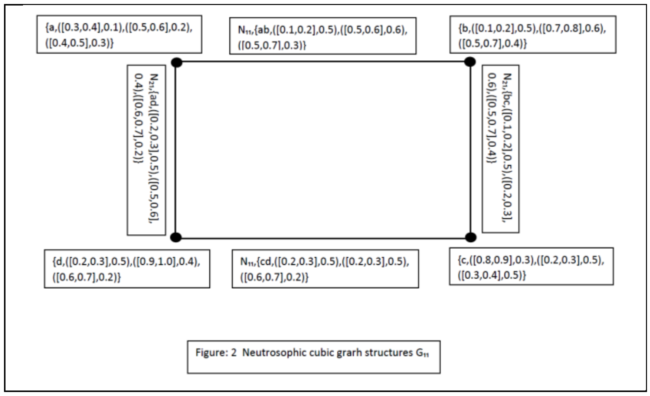

Figure: 2 Neutrosophic cubic grarh structures G_{11}

and

$$M_2=\left\langle\begin{matrix}\{x,([0.4,0.5],0.2),([0.2,0.3],0.7),([0.5,0.6],0.1)\},\\\{y,([0.2,0.3],0.4),([0.7,0.8],0.4),([0.1,0.2],0.5)\},\\\{z,([0.4,0.5],0.8),([0.5,0.6],0.3),([0.8,0.9],0.4)\}\end{matrix}\right\rangle$$

$$N_{12}=\langle\{xy,([0.2,0.3],0.4),([0.2,0.3],0.7),([0.5,0.6],0.1)\}\rangle$$

$$N_{22}=\left\langle\begin{matrix}\{xz,([0.4,0.5],0.8),([0.2,0.3],0.7),([0.8,0.9],0.1)\},\\\{yz,([0.2,0.3],0.8),([0.5,0.6],0.4),([0.8,0.9],0.4)\}\end{matrix}\right\rangle$$

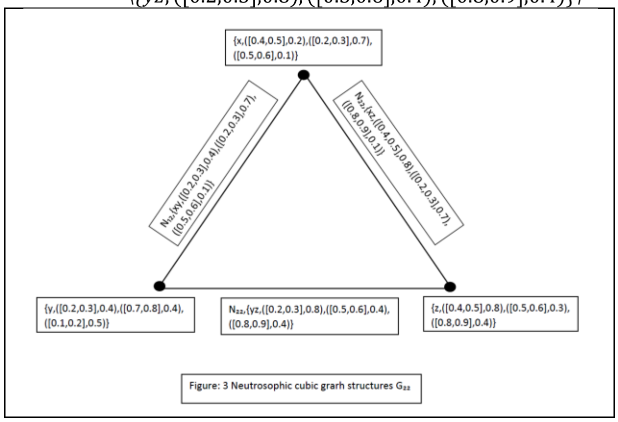

Figure: 3 Neutrosophic cubic grarh structures G_{22}

Then $\breve{G}_{s1}\times\breve{G}_{s2}$ will be

$$M_1\times M_2=\left\langle\begin{matrix}\{(a,x),([0.3,0.4],0.2),([0.2,0.3],0.7),([0.5,0.6],0.7)\},\\\{(a,y),([0.2,0.3],0.4),([0.5,0.6],0.4),([0.4,0.5],0.3)\},\\\{(a,z),([0.3,0.4],0.8),([0.5,0.6],0.3),([0.8,0.9],0.3)\},\\\{(b,x),([0.1,0.2],0.5),([0.2,0.3],0.7),([0.5,0.7],0.1)\},\\\{(b,y),([0.1,0.2],0.5),([0.7,0.8],0.6),([0.5,0.7],0.4)\},\\\{(b,z),([0.1,0.2],0.8),([0.5,0.6],0.6),([0.8,0.9],0.4)\},\\\{(c,x),([0.4,0.5],0.3),([0.2,0.3],0.7),([0.5,0.6],0.1)\},\\\{(c,y),([0.2,0.3],0.4),([0.2,0.3],0.5),([0.3,0.4],0.5)\},\\\{(c,z),([0.4,0.5],0.8),([0.2,0.3],0.5),([0.8,0.9],0.4)\},\\\{(d,x),([0.2,0.3],0.5),([0.2,0.3],0.7),([0.6,0.7],0.1)\},\\\{(d,y),([0.2,0.3],0.5),([0.7,0.8],0.4),([0.6,0.7],0.2)\},\\\{(d,z),([0.2,0.3],0.8),([0.5,0.6],0.4),([0.8,0.9],0.2)\}\end{matrix}\right\rangle$$

$$N_{11}\times N_{12}=\left\langle\begin{matrix}\{((a,x)(a,y)),([0.2,0.3],0.4),([0.2,0.3],0.7),([0.5,0.6],0.1)\},\\\{((b,x)(b,y)),([0.1,0.2],0.5),([0.2,0.3],0.7),([0.5,0.6],0.1)\},\\\{((c,x)(c,y)),([0.2,0.3],0.4),([0.2,0.3],0.7),([0.5,0.6],0.1)\}\end{matrix}\right\rangle$$

$$N_{11}\times N_{22}=\left\langle\begin{matrix}\{((a,z)(b,z)),([0.1,0.2],0.8),([0.5,0.6],0.6),([0.8,0.9],0.3)\},\\\{((c,z)(d,z)),([0.2,0.3],0.8),([0.2,0.3],0.5),([0.8,0.9],0.2)\},\\\{((d,x)(d,y)),([0.2,0.3],0.5),([0.2,0.3],0.7),([0.6,0.7],0.1)\}\end{matrix}\right\rangle$$

$$N_{21}\times N_{12}=\left\langle\begin{matrix}\{((b,y)(c,y)),([0.1,0.2],0.5),([0.2,0.3],0.6),([0.5,0.7],0.4)\},\\\{((a,x)(d,x)),([0.2,0.3],0.5),([0.2,0.3],0.7),([0.6,0.7],0.1)\}\end{matrix}\right\rangle$$

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$$N_{21} \times N_{22} = \left\langle \begin{matrix} \{((a,y)(a,z)),([0.2,0.3],0.8),([0.5,0.6],0.4),([0.8,0.9],0.3)\}, \\ \{((b,x)(b,z)),([0.1,0.2],0.8),([0.2,0.3],0.7),([0.8,0.9],0.1)\}, \\ \{((c,x)(c,z)),([0.4,0.5],0.8),([0.2,0.3],0.7),([0.8,0.9],0.1)\}, \\ \{((d,y)(d,z)),([0.2,0.3],0.8),([0.5,0.6],0.4),([0.8,0.9],0.2)\} \end{matrix} \right\rangle$$

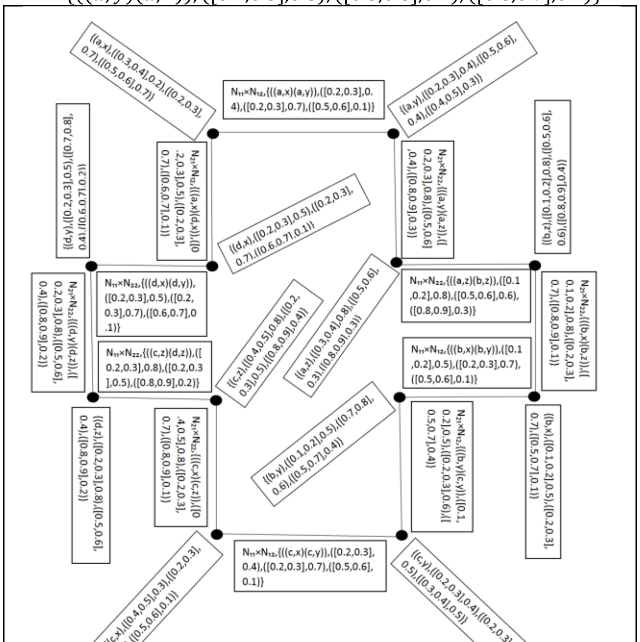

Figure:4 Cartesian product of G_{S1} and G_{S2}

Proposition 3.3 *The cartesian product of two neutrosophic cubic graph structures is also a neutrosophic cubic graph structure.*

Proof. Condition is obvious for $M_1 \times M_2$. Therefore we verify for $N_{n1} \times N_{n2}; n = 1,2,\ldots,n$, where

$$N_{n1} \times N_{n2} = \{((\tilde{T}_{C_{n1}\times C_{n2}}, T_{D_{n1}\times D_{n2}}), (\tilde{I}_{C_{n1}\times C_{n2}}, I_{D_{n1}\times D_{n2}}), (\tilde{F}_{C_{n1}\times C_{n2}}, F_{D_{n1}\times D_{n2}}))\}$$

Let $x \in V_1$ and $x_2y_2 \in E_{n2}$. Then

$$\tilde{T}_{C_{n1}\times C_{n2}}((x,x_2)(x,y_2)) = rmin\{(\tilde{T}_{A_1}(x), \tilde{T}_{C_{n2}}(x_2y_2))\} \le rmin\{(\tilde{T}_{A_1}(x), rmin((\tilde{T}_{A_2}(x_2),(\tilde{T}_{A_2}(y_2))\} = rmin\{rmin((\tilde{T}_{A_1}(x),(\tilde{T}_{A_2}(x_2)), rmin((\tilde{T}_{A_1}(x),(\tilde{T}_{A_2}(y_2))\} = rmin\{(\tilde{T}_{A_1}\times\tilde{T}_{A_2})(x,x_2),((\tilde{T}_{A_1}\times\tilde{T}_{A_2})(x,y_2)\}$$

$$T_{D_{n1}\times D_{n2}}((x,x_2)(x,y_2)) = \max\{(T_{B_1}(x), T_{D_{n2}}(x_2y_2))\} \le \max\{(T_{B_1}(x), \max((T_{B_2}(x_2),(T_{B_2}(y_2))\} = \max\{\max((T_{B_1}(x),(T_{B_2}(x_2)), \max((T_{B_1}(x),(T_{B_2}(y_2))\} = \max\{(T_{B_1}\times T_{B_2})(x,x_2),((T_{B_1}\times T_{B_2})(x,y_2)\}$$

$$\tilde{I}_{C_{n1}\times C_{n2}}((x,x_2)(x,y_2)) = rmin\{(\tilde{I}_{A_1}(x), \tilde{I}_{C_{n2}}(x_2y_2))\} \le rmin\{(\tilde{I}_{A_1}(x), rmin((\tilde{I}_{A_2}(x_2),(\tilde{I}_{A_2}(y_2))\} = rmin\{rmin((\tilde{I}_{A_1}(x),(\tilde{I}_{A_2}(x_2)), rmin((\tilde{I}_{A_1}(x),(\tilde{I}_{A_2}(y_2))\} = rmin\{(\tilde{I}_{A_1}\times\tilde{I}_{A_2})(x,x_2),((\tilde{I}_{A_1}\times\tilde{I}_{A_2})(x,y_2)\}$$

$$I_{D_{n1}\times D_{n2}}((x,x_2)(x,y_2)) = \max\{(I_{B_1}(x), I_{D_{n2}}(x_2y_2))\} \le \max\{(I_{B_1}(x), \max((I_{B_2}(x_2),(I_{B_2}(y_2))\} = \max\{\max((I_{B_1}(x),(I_{B_2}(x_2)), \max((I_{B_1}(x),(I_{B_2}(y_2))\} = \max\{(I_{B_1}\times I_{B_2})(x,x_2),((I_{B_1}\times I_{B_2})(x,y_2)\}$$

$$\tilde{F}_{C_{n1}\times C_{n2}}((x,x_2)(x,y_2)) = rmax\{(\tilde{F}_{A_1}(x), \tilde{F}_{C_{n2}}(x_2y_2))\} \le rmax\{(\tilde{F}_{A_1}(x), rmax((\tilde{F}_{A_2}(x_2),(\tilde{F}_{A_2}(y_2))\} = rmax\{rmax((\tilde{F}_{A_1}(x),(\tilde{F}_{A_2}(x_2)), rmax((\tilde{F}_{A_1}(x),(\tilde{F}_{A_2}(y_2))\} = rmax\{(\tilde{F}_{A_1}\times\tilde{F}_{A_2})(x,x_2),((\tilde{F}_{A_1}\times\tilde{F}_{A_2})(x,y_2)\}$$

$$F_{D_{n1}\times D_{n2}}((x,x_2)(x,y_2)) = \min\{(F_{B_1}(x), F_{D_{n2}}(x_2y_2))\} \le \min\{(F_{B_1}(x), \min((F_{B_2}(x_2),(F_{B_2}(y_2))\} = \min\{\min((F_{B_1}(x),(F_{B_2}(x_2)), \min((F_{B_1}(x),(F_{B_2}(y_2))\} = \min\{(F_{B_1}\times F_{B_2})(x,x_2),(F_{B_1}\times F_{B_2})(x,y_2)\},$$

similarly we can prove it for $z \in V_2$ and $x_1y_1 \in E_{n1}$. □

Definition 3.4 *Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \ldots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \ldots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \ldots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \ldots, E_{2n})$ respectively. The composition of $\breve{G}_1^*$ and $\breve{G}_2^*$ is denoted by $\breve{G}_1[\breve{G}_2]$ and is defined as*

$$\breve{G}_1[\breve{G}_2] = (M_1, N_{11}, N_{21}, \ldots, N_{n1})[(M_2, N_{12}, N_{22}, \ldots, N_{n2})] = \{M_1[M_2], N_{11}[N_{12}], N_{21}[N_{22}], \ldots, N_{n1}[N_{n2}]\} = \left\{ \begin{matrix} (A_1,B_1)[(A_2,B_2)], (C_{11},D_{11})[(C_{12},D_{12})], \\ (C_{21},D_{21})[(C_{22},D_{22})], \ldots, (C_{n1},D_{n1})[(C_{n2},D_{n2})] \end{matrix} \right\} = \left\{ \begin{matrix} (A_1[A_2], B_1[B_2]), (C_{11}[C_{12}], D_{11}[D_{12}]), \\ (C_{21}[C_{22}], D_{21}[D_{22}]), \ldots, (C_{n1}[C_{n2}], D_{n1}[D_{n2}]) \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \left\langle \begin{matrix} ((\tilde{T}_{A_1}\circ\tilde{T}_{A_2}),(T_{B_1}\circ T_{B_2})),((\tilde{I}_{A_1}\circ\tilde{I}_{A_2}),(I_{B_1}\circ I_{B_2})), \\ ((\tilde{F}_{A_1}\circ\tilde{F}_{A_2}),(F_{B_1}\circ F_{B_2})) \end{matrix} \right\rangle, \\ \left\{ \begin{matrix} ((\tilde{T}_{C_{11}}\circ\tilde{T}_{C_{12}}),(T_{D_{11}}\circ T_{D_{12}})),((\tilde{I}_{C_{11}}\circ\tilde{I}_{C_{12}}),(I_{D_{11}}\circ I_{D_{12}})), \\ ((\tilde{F}_{C_{11}}\circ\tilde{F}_{C_{12}}),(F_{D_{11}}\circ F_{D_{12}})) \end{matrix} \right\}, \\ \left\langle \left\{ \begin{matrix} ((\tilde{T}_{C_{21}}\circ\tilde{T}_{C_{22}}),(T_{D_{21}}\circ T_{D_{22}})),((\tilde{I}_{C_{21}}\circ\tilde{I}_{C_{22}}),(I_{D_{21}}\circ I_{D_{22}})), \\ ((\tilde{F}_{C_{21}}\circ\tilde{F}_{C_{22}}),(F_{D_{21}}\circ F_{D_{22}})) \end{matrix} \right\}, \ldots, \right\rangle \\ \left\{ \begin{matrix} ((\tilde{T}_{C_{n1}}\circ\tilde{T}_{C_{n2}}),(T_{D_{n1}}\circ T_{D_{n2}})),((\tilde{I}_{C_{n1}}\circ\tilde{I}_{C_{n2}}),(I_{D_{n1}}\circ I_{D_{n2}})), \\ ((\tilde{F}_{C_{n1}}\circ\tilde{F}_{C_{n2}}),(F_{D_{n1}}\circ F_{D_{n2}})) \end{matrix} \right\} \end{matrix} \right\}$$

where (i) $\forall (x,y) \in (V_1,V_2) = V$

$$(\tilde{T}_{A_1}\circ\tilde{T}_{A_2})(x,y) = rmin(\tilde{T}_{A_1}(x),\tilde{T}_{A_2}(y)), (T_{B_1}\circ T_{B_2})(x,y) = \max(T_{B_1}(x),T_{B_2}(y))$$

$$(\tilde{I}_{A_1}\circ\tilde{I}_{A_2})(x,y) = rmin(\tilde{I}_{A_1}(x),\tilde{I}_{A_2}(y)), (I_{B_1}\circ I_{B_2})(x,y) = \max(I_{B_1}(x),I_{B_2}(y))$$

$$(\tilde{F}_{A_1}\circ\tilde{F}_{A_2})(x,y) = rmax(\tilde{F}_{A_1}(x),\tilde{F}_{A_2}(y)), (F_{B_1}\circ F_{B_2})(x,y) = \min(F_{B_1}(x),F_{B_{F_2}}(y))$$

(ii) $\forall x \in V_1$ and $y_1y_2 \in E_{n2}$

$$(\tilde{T}_{C_{n1}}\circ\tilde{T}_{C_{n2}})((x,y_1)(x,y_2)) = rmin(\tilde{T}_{A_1}(x),\tilde{T}_{C_{n2}}(y_1y_2))$$
$$(T_{D_{n1}}\circ T_{D_{n2}})((x,y_1)(x,y_2)) = \max(T_{B_1}(x),T_{D_{n2}}(y_1y_2))$$
$$(\tilde{I}_{C_{n1}}\circ\tilde{I}_{C_{n2}})((x,y_1)(x,y_2)) = rmin(\tilde{I}_{A_1}(x),\tilde{I}_{C_{n2}}(y_1y_2))$$
$$(I_{D_{n1}}\circ I_{D_{n2}})((x,y_1)(x,y_2)) = \max(I_{B_1}(x),I_{D_{n2}}(y_1y_2))$$
$$(\tilde{F}_{C_{n1}}\circ\tilde{F}_{C_{n2}})((x,y_1)(x,y_2)) = rmax(\tilde{F}_{A_1}(x),\tilde{F}_{C_{n2}}(y_1y_2))$$
$$(F_{D_{n1}}\circ F_{D_{n2}})((x,y_1)(x,y_2)) = \min(F_{B_1}(x),F_{D_{n2}}(y_1y_2))$$

(iii) $\forall y \in V_2$ and $x_1x_2 \in E_{n1}$

$$(\tilde{T}_{C_{n1}}\circ\tilde{T}_{C_{n2}})((x_1,y)(x_2,y)) = rmin(\tilde{T}_{C_{n1}}(x_1x_2),\tilde{T}_{A_2}(y))$$
$$(T_{D_{n1}}\circ T_{D_{n2}})((x_1,y)(x_2,y)) = \max(T_{D_{n1}}(x_1x_2),T_{B_2}(y))$$

$$(\tilde{I}_{C_{n1}}\circ\tilde{I}_{C_{n2}})((x_1,y)(x_2,y)) = rmin(\tilde{I}_{C_{n1}}(x_1x_2),\tilde{I}_{A_2}(y))$$
$$(I_{D_{n1}}\circ I_{D_{n2}})((x_1,y)(x_2,y)) = \max(I_{D_{n1}}(x_1x_2),I_{B_2}(y))$$

$$(\tilde{F}_{C_{n1}}\circ\tilde{F}_{C_{n2}})((x_1,y)(x_2,y)) = rmax(\tilde{F}_{C_{n1}}(x_1x_2),\tilde{F}_{A_2}(y))$$
$$(F_{D_{n1}}\circ F_{D_{n2}})((x_1,y)(x_2,y)) = \min(F_{D_{n1}}(x_1x_2),F_{B_2}(y))$$

(iv) $\forall (x_1,y_1)(x_2,y_2) \in E^0 - E$

$$(\tilde{T}_{C_{n1}}\circ\tilde{T}_{C_{n2}})((x_1,y_1)(x_2,y_2)) = rmin(\tilde{T}_{A_2}(y_1),\tilde{T}_{A_2}(y_2),\tilde{T}_{C_{n1}}(x_1x_2))$$
$$(T_{D_{n1}}\circ T_{D_{n2}})((x_1,y_1)(x_2,y_2)) = \max(T_{B_2}(y_1),T_{B_2}(y_2),T_{D_{n1}}(x_1x_2))$$
$$(\tilde{I}_{C_{n1}}\circ\tilde{I}_{C_{n2}})((x_1,y_1)(x_2,y_2)) = rmin(\tilde{I}_{A_2}(y_1),\tilde{I}_{A_2}(y_2),\tilde{I}_{C_{n1}}(x_1x_2))$$
$$(I_{D_{n1}}\circ I_{D_{n2}})((x_1,y_1)(x_2,y_2)) = \max(I_{B_2}(y_1),I_{B_2}(y_2),I_{D_{n1}}(x_1x_2))$$
$$(\tilde{F}_{C_{n1}}\circ\tilde{F}_{C_{n2}})((x_1,y_1)(x_2,y_2)) = rmax(\tilde{F}_{A_2}(y_1),\tilde{F}_{A_2}(y_2),\tilde{F}_{C_{n1}}(x_1x_2))$$
$$(F_{D_{n1}}\circ F_{D_{n2}})((x_1,y_1)(x_2,y_2)) = \min(F_{B_2}(y_1),F_{B_2}(y_2),F_{D_{n1}}(x_1x_2))$$

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Example: Let $\breve{G}_{s1} = (M_1, N_{11})$ and $\breve{G}_{s2} = (M_2, N_{12}, N_{22})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^*$ and $\breve{G}_2^*$ respectively, where

$$lM_1 = \left\langle \begin{matrix} \{a, ([0.3,0.4],0.7), ([0.5,0.6],0.2), ([0.4,0.5],0.4)\}, \\ \{b, ([0.5,0.6],0.4), ([0.6,0.7],0.3), ([0.1,0.2],0.5)\} \end{matrix} \right\rangle N_{11}$$
$$= \langle \{ab, ([0.3,0.4],0.7), ([0.5,0.6],0.3), ([0.4,0.5],0.4)\} \rangle$$

and

$$lM_2 = \left\langle \begin{matrix} \{x, ([0.4,0.5],0.2), ([0.6,0.7],0.4), ([0.2,0.3],0.6)\}, \\ \{y, ([0.1,0.2],0.7), ([0.8,0.9],0.5), ([0.3,0.4],0.3)\}, \\ \{z, ([0.3,0.4].0.6), ([0.5,0.6],0.2), ([0.7,0.8],0.4)\} \end{matrix} \right\rangle N_{12}$$
$$= \left\langle \begin{matrix} \{xy, ([0.1,0.2],0.7), ([0.6,0.7],0.5), ([0.3,0.4],0.3)\}, \\ \{xz, ([0.3,0.4].0.6), ([0.5,0.6],0.4), ([0.7,0.8],0.4)\} \end{matrix} \right\rangle N_{22}$$
$$= \langle \{yz, ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.7,0.8],0.3)\} \rangle$$

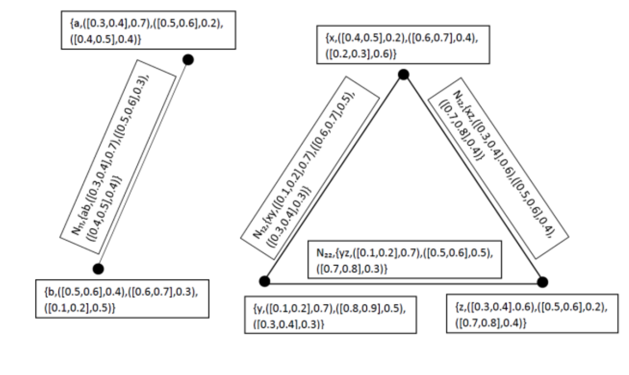

Figure: 5 Nuetrosophic cubic graph structures Gs₁ and Gs₂

Then $\breve{G}_{s1}[\breve{G}_{s2}]$ will be

$$lM_1[M_2]$$
$$= \left\langle \begin{matrix} \{(a,x), ([0.3,0.4],0.7), ([0.5,0.6],0.4), ([0.4,0.5],0.4)\}, \\ \{(a,y), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.4,0.5],0.3)\}, \\ \{(a,z), ([0.3,0.4],0.7), ([0.5,0.6],0.2), ([0.7,0.8],0.4)\}, \\ \{(b,z), ([0.3,0.4],0.6), ([0.5,0.6],0.3), ([0.7,0.8],0.4)\}, \\ \{(b,y), ([0.1,0.2],0.7), ([0.6,0.7],0.5), ([0.3,0.4],0.3)\}, \\ \{(b,x), ([0.4,0.5],0.4), ([0.6,0.7],0.4), ([0.2,0.3],0.5)\} \end{matrix} \right\rangle N_{11}[N_{12}]$$
$$= \left\langle \begin{matrix} \{(a,x)(a,y), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.4,0.5],0.3)\}, \\ \{(a,x)(a,z), ([0.3,0.4],0.7), ([0.5,0.6],0.4), ([0.7,0.8],0.4)\}, \\ \{(a,x)(b,z), ([0.3,0.4],0.7), ([0.5,0.6],0.4), ([0.7,0.8],0.4)\}, \\ \{(a,x)(b,y), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.4,0.5],0.3)\}, \\ \{(a,x)(b,x), ([0.3,0.4],0.7), ([0.5,0.6],0.4), ([0.7,0.8],0.4)\}, \\ \{(a,y)(b,y), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.4,0.5],0.3)\}, \\ \{(a,y)(b,x), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.4,0.5],0.3)\}, \\ \{(a,z)(b,x), ([0.3,0.4],0.7), ([0.5,0.6],0.4), ([0.7,0.8],0.4)\}, \\ \{(b,z)(b,x), ([0.3,0.4],0.6), ([0.5,0.6],0.4), ([0.7,0.8],0.4) \\ \{(b,x)(b,y), ([0.1,0.2],0.7), ([0.6,0.7],0.5), ([0.3,0.4],0.3)\} \end{matrix} \right\rangle N_{11}[N_{22}]$$
$$= \left\langle \begin{matrix} \{(a,y)(a,z), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.4,0.5],0.3)\}, \\ \{(a,y)(b,z), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.7,0.8],0.3)\}, \\ \{(a,z)(b,z), ([0.3,0.4],0.7), ([0.5,0.6],0.3), ([0.7,0.8],0.4)\}, \\ \{(a,z)(b,y), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.7,0.8],0.3)\}, \\ \{(b,y)(b,z), ([0.1,0.2],0.7), ([0.5,0.6],0.5), ([0.7,0.8],0.3)\} \end{matrix} \right\rangle$$

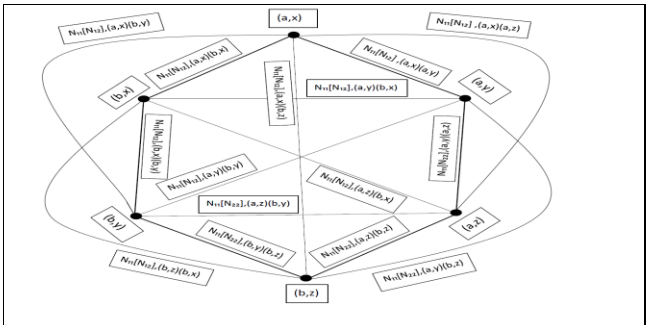

Figure: 6 Composition of two neutrosophic cubic graph structures

Proposition 3.5 *The composition of two neutrosophic cubic graph structures is again a neutrosophic cubic graph structure.*

Proof. Condition is obvious for $M_1 \circ M_2$. we will prove it for $N_{n1} \circ N_{n2}; n = 1,2,\ldots,n$, where
$N_{n1} \circ N_{n2} = \{((\tilde{T}_{C_{n1}\circ C_{n2}}, T_{D_{n1}\circ D_{n2}}), (\tilde{I}_{C_{n1}\circ C_{n2}}, I_{D_{n1}\circ D_{n2}}), (\tilde{F}_{C_{n1}\circ C_{n2}}, F_{D_{n1}\circ D_{n2}}))\}$
(i) Let $x \in V_1$ and $x_2y_2 \in E_{n2}$. Then

$$\tilde{T}_{C_{i1}\circ C_{i2}}((x,x_2)(x,y_2)) = rmin\{(\tilde{T}_{A_1}(x), \tilde{T}_{C_{i2}}(x_2y_2))\}$$
$$\leq rmin\{(\tilde{T}_{A_1}(x), rmin((\tilde{T}_{A_2}(x_2), (\tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{rmin((\tilde{T}_{A_1}(x), (\tilde{T}_{A_2}(x_2)), rmin((\tilde{T}_{A_1}(x), (\tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{T}_{A_1} \circ \tilde{T}_{A_2})(x,x_2), (\tilde{T}_{A_1} \circ \tilde{T}_{A_2})(x,y_2)\}$$
$$T_{D_{i1}\circ D_{i2}}((x,x_2)(x,y_2)) = \max\{(T_{B_1}(x), T_{D_{i2}}(x_2y_2))\}$$
$$\leq \max\{(T_{B_1}(x), \max(T_{B_2}(x_2), (T_{B_2}(y_2))\}$$
$$= \max\{\max((T_{B_1}(x), (T_{B_2}(x_2)), \max((T_{B_1}(x), (T_{B_2}(y_2))\}$$
$$= \max\{(T_{B_1} \circ T_{B_2})(x,x_2), (T_{B_1} \circ T_{B_2})(x,y_2)\}$$
$$\tilde{I}_{C_{i1}\circ C_{i2}}((x,x_2)(x,y_2)) = rmin\{(\tilde{I}_{A_1}(x), \tilde{I}_{C_{i2}}(x_2y_2))\}$$
$$\leq rmin\{(\tilde{I}_{A_1}(x), rmin((\tilde{I}_{A_2}(x_2), (\tilde{I}_{A_2}(y_2))\}$$
$$= rmin\{rmin((\tilde{I}_{A_1}(x), (\tilde{I}_{A_2}(x_2)), rmin((\tilde{I}_{A_1}(x), (\tilde{I}_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{I}_{A_1} \circ \tilde{I}_{A_2})(x,x_2), ((\tilde{I}_{A_1} \circ \tilde{I}_{A_2})(x,y_2)\}$$
$$I_{D_{i1}\circ D_{i2}}((x,x_2)(x,y_2)) = \max\{(I_{B_1}(x), I_{D_{i2}}(x_2y_2))\}$$
$$\leq \max\{(I_{B_1}(x), \max((I_{B_2}(x_2), (I_{B_2}(y_2))\}$$
$$= \max\{\max((I_{B_1}(x), (I_{B_2}(x_2)), \max((I_{B_1}(x), (I_{B_2}(y_2))\}$$
$$= \max\{(I_{B_1} \circ I_{B_2})(x,x_2), ((I_{B_1} \circ I_{B_2})(x,y_2)\}$$
$$\tilde{F}_{C_{i1}\circ C_{i2}}((x,x_2)(x,y_2)) = rmax\{(\tilde{F}_{A_1}(x), \tilde{F}_{C_{i2}}(x_2y_2))\}$$
$$\leq rmax\{(\tilde{F}_{A_1}(x), rmax((\tilde{F}_{A_2}(x_2), (\tilde{F}_{A_2}(y_2))\}$$
$$= rmax\{rmax((\tilde{F}_{A_1}(x), (\tilde{F}_{A_2}(x_2)), rmax((\tilde{F}_{A_1}(x), (\tilde{F}_{A_2}(y_2))\}$$
$$= rmax\{(\tilde{F}_{A_1} \circ \tilde{F}_{A_2})(x,x_2), ((\tilde{F}_{A_1} \circ \tilde{F}_{A_2})(x,y_2)\}$$
$$F_{D_{i1}\circ D_{i2}}((x,x_2)(x,y_2)) = \min\{(F_{B_1}(x), F_{D_{i2}}(x_2y_2))\}$$
$$\leq \min\{(F_{B_1}(x), \min((F_{B_2}(x_2), (F_{B_2}(y_2))\}$$
$$= \min\{\min((F_{B_1}(x), (F_{B_2}(x_2)), \min((F_{B_1}(x), (F_{B_2}(y_2))\}$$
$$= \min\{(F_{B_1} \circ F_{B_2})(x,x_2), (F_{B_1} \circ F_{B_2})(x,y_2)\}$$

for $(x_1,x_2), (x,y_2) \in V_1 \circ V_2$.
(ii) Let $y \in V_2$ and $x_1y_1 \in E_{i1}$

$$(\tilde{T}_{C_{i1}} \circ \tilde{T}_{C_{i2}})((x_1,y)(y_1,y)) = rmin(\tilde{T}_{C_{i1}}(x_1y_1), \tilde{T}_{A_2}(y))$$
$$\leq rmin(rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_1}(y_1)), \tilde{T}_{A_2}(y))$$
$$= rmin\{rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_2}(y)), rmin(\tilde{T}_{A_1}(y_1), \tilde{T}_{A_2}(y))\}$$
$$= rmin\{(\tilde{T}_{A_1} \circ \tilde{T}_{A_2})(x_1,y), (\tilde{T}_{A_1} \circ \tilde{T}_{A_2})(y_1,y)\}$$
$$(T_{D_{i1}} \circ T_{D_{i2}})((x_1,y)(y_1,y)) = \max(T_{D_{i1}}(x_1y_1), T_{B_2}(y))$$
$$\leq \max(\max(T_{B_1}(x_1), T_{B_1}(y_1)), T_{B_2}(y))$$
$$= \max\{\max(T_{B_1}(x_1), T_{B_2}(y)), \max(T_{B_1}(y_1), T_{B_2}(y)\}$$
$$= \max\{(T_{B_1} \circ T_{B_2})(x_1,y), (T_{B_1} \circ T_{B_2})(y_1,y)$$

$$(\tilde{I}_{C_{i1}} \circ \tilde{I}_{C_{i2}})((x_1,y)(y_1,y)) = rmin(\tilde{I}_{C_{i1}}(x_1y_1), \tilde{I}_{A_2}(y))$$

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$$\leq rmin(rmin(\tilde{I}_{A_1}(x_1), \tilde{I}_{A_1}(y_1)), \tilde{I}_{A_2}(y))$$
$$= rmin\{rmin(\tilde{I}_{A_1}(x_1), \tilde{I}_{A_2}(y)), rmin(\tilde{I}_{A_1}(y_1), \tilde{I}_{A_2}(y))\}$$
$$= rmin\{(\tilde{I}_{A_1} \circ \tilde{I}_{A_2})(x_1, y), (\tilde{I}_{A_1} \circ \tilde{I}_{A_2})(y_1, y)\}$$
$$(I_{D_{i1}} \circ I_{D_{i2}})((x_1, y)(y_1, y)) = \max(I_{D_{i1}}(x_1 y_1), I_{B_2}(y))$$
$$\leq \max(\max(I_{B_1}(x_1), I_{B_1}(y_1)), I_{B_2}(y))$$
$$= \max\{\max(I_{B_1}(x_1), I_{B_2}(y)), \max(I_{B_1}(y_1), I_{B_2}(y)\}$$
$$= \max\{(I_{B_1} \circ I_{B_2})(x_1, y), (I_{B_1} \circ I_{B_2})(y_1, y)$$

$$(\tilde{F}_{C_{i1}} \circ \tilde{F}_{C_{i2}})((x_1, y)(y_1, y)) = rmax(\tilde{F}_{C_{i1}}(x_1 y_1), \tilde{F}_{A_2}(y))$$
$$\leq rmax(rmax(\tilde{F}_{A_1}(x_1), \tilde{F}_{A_1}(y_1)), \tilde{F}_{A_2}(y))$$
$$= rmax\{rmax(\tilde{F}_{A_1}(x_1), \tilde{F}_{A_2}(y)), rmax(\tilde{F}_{A_1}(y_1), \tilde{F}_{A_2}(y))\}$$
$$= rmax\{(\tilde{F}_{A_1} \circ \tilde{F}_{A_2})(x_1, y), (\tilde{F}_{A_1} \circ \tilde{F}_{A_2})(y_1, y)\}$$
$$(F_{D_{i1}} \circ F_{D_{i2}})((x_1, y)(y_1, y)) = \min(F_{D_{i1}}(x_1 y_1), F_{B_2}(y))$$
$$\leq \min(\min(F_{B_1}(x_1), F_{B_1}(y_1)), F_{B_2}(y))$$
$$= \min\{\min(F_{B_1}(x_1), F_{B_2}(y)), \min(F_{B_1}(y_1), F_{B_2}(y)\}$$
$$= \min\{(F_{B_1} \circ F_{B_2})(x_1, y), (F_{B_1} \circ F_{B_2})(y_1, y)\}$$

for (x_1, y), $(y_1, y) \in V_1 \circ V_2$.

(iii) Let $(x_1, y_1)(x_2, y_2) \in E^0 - E$

$$(\tilde{T}_{C_{i1}} \circ \tilde{T}_{C_{i2}})((x_1, y_1)(x_2, y_2)) = rmin(\tilde{T}_{A_2}(y_1), \tilde{T}_{A_2}(y_2), \tilde{T}_{C_{i1}}(x_1 x_2))$$
$$\leq rmin\{\tilde{T}_{A_2}(y_1), \tilde{T}_{A_2}(y_2), rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_1}(x_2)\}$$
$$= rmin\{(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_2}(y_1)), (\tilde{T}_{A_1}(x_2), \tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{T}_{A_1} \circ \tilde{T}_{A_2})(x_1, y_1), (\tilde{T}_{A_1} \circ \tilde{T}_{A_2})(x_2, y_2)\}$$
$$(T_{D_{i1}} \circ T_{D_{i2}})((x_1, y_1)(x_2, y_2)) = \max(T_{B_2}(y_1), T_{B_2}(y_2), T_{D_{i1}}(x_1 x_2))$$
$$\leq \max\{T_{B_2}(y_1), T_{B_2}(y_2), \max(T_{B_1}(x_1), T_{B_1}(x_2)\}$$
$$= \max\{(T_{B_1}(x_1), T_{B_2}(y_1)), (T_{B_1}(x_2), T_{B_2}(y_2))\}$$
$$= \max\{(T_{B_1} \circ T_{B_2})(x_1, y_1), (T_{B_1} \circ T_{B_2})(x_2, y_2)\}$$

$$(\tilde{I}_{C_{i1}} \circ \tilde{I}_{C_{i2}})((x_1, y_1)(x_2, y_2)) = rmin(\tilde{I}_{A_2}(y_1), \tilde{I}_{A_2}(y_2), \tilde{I}_{C_{i1}}(x_1 x_2))$$
$$\leq rmin\{\tilde{I}_{A_2}(y_1), \tilde{I}_{A_2}(y_2), rmin(\tilde{I}_{A_1}(x_1), \tilde{I}_{A_1}(x_2)\}$$
$$= rmin\{(\tilde{I}_{A_1}(x_1), \tilde{I}_{A_2}(y_1)), (\tilde{I}_{A_1}(x_2), \tilde{I}_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{I}_{A_1} \circ \tilde{I}_{A_2})(x_1, y_1), (\tilde{I}_{A_1} \circ \tilde{I}_{A_2})(x_2, y_2)\}$$
$$(I_{D_{i1}} \circ I_{D_{i2}})((x_1, y_1)(x_2, y_2)) = \max(I_{B_2}(y_1), I_{B_2}(y_2), I_{D_{i1}}(x_1 x_2))$$
$$\leq \max\{I_{B_2}(y_1), I_{B_2}(y_2), \max(I_{B_1}(x_1), I_{B_1}(x_2)\}$$
$$= \max\{(I_{B_1}(x_1), I_{B_2}(y_1)), (I_{B_1}(x_2), I_{B_2}(y_2))\}$$
$$= \max\{(I_{B_1} \circ I_{B_2})(x_1, y_1), (I_{B_1} \circ I_{B_2})(x_2, y_2)\}$$

$$(\tilde{F}_{C_{i1}} \circ \tilde{F}_{C_{i2}})((x_1, y_1)(x_2, y_2)) = rmax(\tilde{F}_{A_2}(y_1), \tilde{F}_{A_2}(y_2), \tilde{F}_{C_{i1}}(x_1 x_2))$$
$$\leq rmax\{\tilde{F}_{A_2}(y_1), \tilde{F}_{A_2}(y_2), rmax(\tilde{F}_{A_1}(x_1), \tilde{F}_{A_1}(x_2)\}$$
$$= rmax\{(\tilde{F}_{A_1}(x_1), \tilde{F}_{A_2}(y_1)), (\tilde{F}_{A_1}(x_2), \tilde{F}_{A_2}(y_2))\}$$
$$= rmax\{(\tilde{F}_{A_1} \circ \tilde{F}_{A_2})(x_1, y_1), (\tilde{F}_{A_1} \circ \tilde{F}_{A_2})(x_2, y_2)\}$$
$$(F_{D_{i1}} \circ F_{D_{i2}})((x_1, y_1)(x_2, y_2)) = \min(F_{B_2}(y_1), F_{B_2}(y_2), F_{D_{i1}}(x_1 x_2))$$
$$\leq \min\{F_{B_2}(y_1), F_{B_2}(y_2), \min(F_{B_1}(x_1), F_{B_1}(x_2)\}$$
$$= \min\{(F_{B_1}(x_1), F_{B_2}(y_1)), (F_{B_1}(x_2), F_{B_2}(y_2))\}$$
$$= \min\{(F_{B_1} \circ F_{B_2})(x_1, y_1), (F_{B_1} \circ F_{B_2})(x_2, y_2)\}$$

for (x_1, y_1), $(x_1, y_2) \in V_1 \circ V_2$ for $i \in 1,2,\ldots,n$. This proves the result.

Definition 3.6 *Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \ldots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \ldots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \ldots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \ldots, E_{2n})$ respectively. P-union is denoted by $\breve{G}_{S1} \cup_P \breve{G}_{S2}$ and is defined as*

$$\breve{G}_{S1} \cup_P \breve{G}_{S2} = (M_1, N_{11}, N_{21}, \ldots, N_{n1}) \cup_P (M_2, N_{12}, N_{22}, \ldots, N_{n2})$$
$$= (M_1 \cup_P M_2, N_{11} \cup_P N_{12}, N_{21} \cup_P N_{22}, \ldots, N_{n1} \cup_P N_{n2})$$
$$= ((A_1, B_1) \cup_P (A_2, B_2), (C_{11}, D_{11}) \cup_P (C_{12}, D_{12}), (C_{21}, D_{21}) \cup_P (C_{22}, D_{22}), \ldots, (C_{n1}, D_{n1}) \cup_P (C_{n2}, D_{n2}))$$
$$= ((A_1 \cup_P A_2, B_1 \cup_P B_2,), (C_{11} \cup_P C_{12}, D_{11} \cup_P D_{12}), (C_{21} \cup_P C_{22}, D_{21} \cup_P D_{22}), \ldots, (C_{n1} \cup_P C_{n2}, D_{n1} \cup_P D_{n2}))$$
$$= \left\{\begin{array}{l} ((\tilde{T}_{A_1 \cup_P A_2}, T_{B_1 \cup_P B_2}), (\tilde{I}_{A_1 \cup_P A_2}, I_{B_1 \cup_P B_2}), (\tilde{F}_{A_1 \cup_P A_2}, F_{B_1 \cup_P B_2})), \\ ((\tilde{T}_{C_{11} \cup_P C_{12}}, T_{D_{11} \cup_P D_{12}}), (\tilde{I}_{C_{11} \cup_P C_{12}}, I_{D_{11} \cup_P D_{12}}), (\tilde{F}_{C_{11} \cup_P C_{12}}, F_{D_{11} \cup_P D_{12}}), \\ (\tilde{T}_{C_{21} \cup_P C_{22}}, T_{D_{21} \cup_P D_{22}}), (\tilde{I}_{C_{21} \cup_P C_{22}}, I_{D_{21} \cup_P D_{22}}), (\tilde{F}_{C_{21} \cup_P C_{22}}, F_{D_{21} \cup_P D_{22}}), \ldots, \\ (\tilde{T}_{C_{n1} \cup_P C_{n2}}, T_{D_{n1} \cup_P D_{n2}}), (\tilde{I}_{C_{n1} \cup_P C_{n2}}, I_{D_{n1} \cup_P D_{n2}}), (\tilde{F}_{C_{n1} \cup_P C_{n2}}, F_{D_{n1} \cup_P D_{n2}})) \end{array}\right\}$$

where

$$(\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(x) = \begin{cases} \tilde{T}_{A_1}(x) \text{if} x \in V_1 - V_2 \\ \tilde{T}_{A_2}(x) \text{if} x \in V_2 - V_1 \\ rmax\{\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(x)\} \text{if} x \in V_1 \cap V_2 \end{cases}$$
$$(T_{B_1} \cup_P T_{B_2})(x) = \begin{cases} T_{B_1}(x) \text{if} x \in V_1 - V_2 \\ T_{B_2}(x) \text{if} x \in V_2 - V_1 \\ \max\{T_{B_1}(x), T_{B_2}(x)\} \text{if} x \in V_1 \cap V_2 \end{cases}$$

$$(\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(x) = \begin{cases} \tilde{I}_{A_1}(x) \text{if} x \in V_1 - V_2 \\ \tilde{I}_{A_2}(x) \text{if} x \in V_2 - V_1 \\ rmax\{\tilde{I}_{A_1}(x), \tilde{I}_{A_2}(x)\} \text{if} x \in V_1 \cap V_2 \end{cases}$$
$$(I_{B_1} \cup_P I_{B_2})(x) = \begin{cases} I_{B_1}(x) \text{if} x \in V_1 - V_2 \\ I_{B_2}(x) \text{if} x \in V_2 - V_1 \\ \max\{I_{B_1}(x), I_{B_2}(x)\} \text{if} x \in V_1 \cap V_2 \end{cases}$$

$$(\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(x) = \begin{cases} \tilde{F}_{A_1}(x) \text{if} x \in V_1 - V_2 \\ \tilde{F}_{A_2}(x) \text{if} x \in V_2 - V_1 \\ rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{A_2}(x)\} \text{if} x \in V_1 \cap V_2 \end{cases}$$
$$(F_{B_1} \cup_P F_{B_2})(x) = \begin{cases} F_{B_1}(x) \text{if} x \in V_1 - V_2 \\ F_{B_2}(x) \text{if} x \in V_2 - V_1 \\ \max\{F_{B_1}(x), F_{B_2}(x)\} \text{if} x \in V_1 \cap V_2 \end{cases}$$

$$(\tilde{T}_{C_{n1}} \cup_P \tilde{T}_{C_{n2}})(x_2 y_2) = \begin{cases} \tilde{T}_{C_{n1}}(x_2 y_2) \text{if} x_2 y_2 \in V_1 - V_2 \\ \tilde{T}_{C_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in V_2 - V_1 \\ rmax\{\tilde{T}_{C_{n1}}(x_2 y_2), \tilde{T}_{C_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in E_1 \cap E_2 \end{cases}$$
$$(T_{D_{n1}} \cup_P T_{D_{n2}})(x_2 y_2) = \begin{cases} T_{D_{n1}}(x_2 y_2) \text{if} x_2 y_2 \in V_1 - V_2 \\ T_{D_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in V_2 - V_1 \\ \max\{T_{D_{n1}}(x_2 y_2), T_{D_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in E_1 \cap E_2 \end{cases}$$

$$(\tilde{I}_{C_{n1}} \cup_P \tilde{I}_{C_{n2}})(x_2 y_2) = \begin{cases} \tilde{I}_{C_{n1}}(x_2 y_2) \text{if} x_2 y_2 \in V_1 - V_2 \\ \tilde{I}_{C_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in V_2 - V_1 \\ rmax\{\tilde{I}_{C_{n1}}(x_2 y_2), \tilde{I}_{C_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in E_1 \cap E_2 \end{cases}$$
$$(I_{D_{n1}} \cup_P I_{D_{n2}})(x_2 y_2) = \begin{cases} I_{D_{n1}}(x_2 y_2) \text{if} x_2 y_2 \in V_1 - V_2 \\ I_{D_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in V_2 - V_1 \\ \max\{I_{D_{n1}}(x_2 y_2), I_{D_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in E_1 \cap E_2 \end{cases}$$

$$(\tilde{F}_{C_{n1}} \cup_P \tilde{F}_{C_{n2}})(x_2 y_2) = \begin{cases} \tilde{F}_{C_{n1}}(x_2 y_2) \text{if} x_2 y_2 \in V_1 - V_2 \\ \tilde{F}_{C_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in V_2 - V_1 \\ rmax\{\tilde{F}_{C_{n1}}(x_2 y_2), \tilde{F}_{C_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in E_1 \cap E_2 \end{cases}$$
$$(F_{D_{n1}} \cup_P F_{D_{n2}})(x_2 y_2) = \begin{cases} F_{D_{n1}}(x_2 y_2) \text{if} x_2 y_2 \in V_1 - V_2 \\ F_{D_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in V_2 - V_1 \\ \max\{F_{D_{n1}}(x_2 y_2), F_{D_{n2}}(x_2 y_2) \text{if} x_2 y_2 \in E_1 \cap E_2 \end{cases}$$

and R-union is denoted by $\breve{G}_{S1} \cup_R \breve{G}_{S2}$ and is defined as

$$\breve{G}_{S1} \cup_R \breve{G}_{S2} = (M_1, N_{11}, N_{21}, \ldots, N_{n1}) \cup_R (M_2, N_{12}, N_{22}, \ldots, N_{n2})$$
$$= (M_1 \cup_R M_2, N_{11} \cup_R N_{12}, N_{21} \cup_R N_{22}, \ldots, N_{n1} \cup_R N_{n2})$$
$$= \left\{\begin{array}{l} (A_1, B_1) \cup_R (A_2, B_2), (C_{11}, D_{11}) \cup_R (C_{12}, D_{12}), \\ (C_{21}, D_{21}) \cup_R (C_{22}, D_{22}), \ldots, (C_{n1}, D_{n1}) \cup_R (C_{n2}, D_{n2}) \end{array}\right\}$$
$$= \left\{\begin{array}{l} (A_1 \cup_R A_2, B_1 \cup_R B_2,), (C_{11} \cup_R C_{12}, D_{11} \cup_R D_{12}), \\ (C_{21} \cup_R C_{22}, D_{21} \cup_R D_{22}), \ldots, (C_{n1} \cup_R C_{n2}, D_{n1} \cup_R D_{n2}) \end{array}\right\}$$

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$$=\left\{\begin{matrix}((\tilde{T}_{A_1\cup_R A_2},T_{B_1\cup_R B_2}),(\tilde{I}_{A_1\cup_R A_2},I_{B_1\cup_R B_2}),(\tilde{F}_{A_1\cup_R A_2},F_{B_1\cup_R B_2})),\\ ((\tilde{T}_{C_{11}\cup_R C_{12}},T_{D_{11}\cup_R D_{12}}),(\tilde{I}_{C_{11}\cup_R C_{12}},I_{D_{11}\cup_R D_{12}}),(\tilde{F}_{C_{11}\cup_R C_{12}},F_{D_{11}\cup_R D_{12}}),\\ (\tilde{T}_{C_{21}\cup_R C_{22}},T_{D_{21}\cup_R D_{22}}),(\tilde{I}_{C_{21}\cup_R C_{22}},I_{D_{21}\cup_R D_{22}}),(\tilde{F}_{C_{21}\cup_R C_{22}},F_{D_{21}\cup_R D_{22}}),\dots,\\ (\tilde{T}_{C_{n1}\cup_R C_{n2}},T_{D_{n1}\cup_R D_{n2}}),(\tilde{I}_{C_{n1}\cup_R C_{n2}},I_{D_{n1}\cup_R D_{n2}}),(\tilde{F}_{C_{n1}\cup_R C_{n2}},F_{D_{n1}\cup_R D_{n2}}))\end{matrix}\right\}$$

$$(\tilde{T}_{A_1}\cup_R\tilde{T}_{A_2})(x)=\begin{cases}\tilde{T}_{A_1}(x)\text{if}x\in V_1-V_2\\ \tilde{T}_{A_2}(x)\text{if}x\in V_2-V_1\\ rmax\{\tilde{T}_{A_1}(x),\tilde{T}_{A_2}(x)\}\text{if}x\in V_1\cap V_2\end{cases}$$

$$(T_{B_1}\cup_R T_{B_2})(x)=\begin{cases}T_{B_1}(x)\text{if}x\in V_1-V_2\\ T_{B_2}(x)\text{if}x\in V_2-V_1\\ \min\{T_{B_1}(x),T_{B_2}(x)\}\text{if}x\in V_1\cap V_2\end{cases}$$

$$(\tilde{I}_{A_1}\cup_R\tilde{I}_{A_2})(x)=\begin{cases}\tilde{I}_{A_1}(x)\text{if}x\in V_1-V_2\\ \tilde{I}_{A_2}(x)\text{if}x\in V_2-V_1\\ rmax\{\tilde{I}_{A_1}(x),\tilde{I}_{A_2}(x)\}\text{if}x\in V_1\cap V_2\end{cases}$$

$$(I_{B_1}\cup_R I_{B_2})(x)=\begin{cases}I_{B_1}(x)\text{if}x\in V_1-V_2\\ I_{B_2}(x)\text{if}x\in V_2-V_1\\ \min\{I_{B_1}(x),I_{B_2}(x)\}\text{if}x\in V_1\cap V_2\end{cases}$$

$$(\tilde{F}_{A_1}\cup_R M_{T_{F_2}})(x)=\begin{cases}\tilde{F}_{A_1}(x)\text{if}x\in V_1-V_2\\ \tilde{F}_{A_2}(x)\text{if}x\in V_2-V_1\\ rmax\{\tilde{F}_{A_1}(x),\tilde{F}_{A_2}(x)\}\text{if}x\in V_1\cap V_2\end{cases}$$

$$(F_{B_1}\cup_R F_{B_2})(x)=\begin{cases}F_{B_1}(x)\text{if}x\in V_1-V_2\\ F_{B_2}(x)\text{if}x\in V_2-V_1\\ \min\{F_{B_1}(x),F_{B_2}(x)\}\text{if}x\in V_1\cap V_2\end{cases}$$

$$(\tilde{T}_{C_{n1}}\cup_R\tilde{T}_{C_{n2}})(x_2y_2)=\begin{cases}\tilde{T}_{C_{n1}}(x_2y_2)\text{if}x_2y_2\in V_1-V_2\\ \tilde{T}_{C_{n2}}(x_2y_2)\text{if}x_2y_2\in V_2-V_1\\ rmax\{\tilde{T}_{C_{n1}}(x_2y_2),\tilde{T}_{C_{n2}}(x_2y_2)\text{if}x_2y_2\in E_1\cap E_2\end{cases}$$

$$(T_{D_{n1}}\cup_R N_{D_{n2}})(x_2y_2)=\begin{cases}T_{D_{n1}}(x_2y_2)\text{if}x_2y_2\in V_1-V_2\\ T_{D_{n2}}(x_2y_2)\text{if}x_2y_2\in V_2-V_1\\ \min\{T_{D_{n1}}(x_2y_2),T_{D_{n2}}(x_2y_2)\text{if}x_2y_2\in E_1\cap E_2\end{cases}$$

$$(\tilde{I}_{C_{n1}}\cup_R\tilde{I}_{C_{n2}})(x_2y_2)=\begin{cases}\tilde{I}_{C_{n1}}(x_2y_2)\text{if}x_2y_2\in V_1-V_2\\ \tilde{I}_{C_{n2}}(x_2y_2)\text{if}x_2y_2\in V_2-V_1\\ rmax\{\tilde{I}_{C_{n1}}(x_2y_2),\tilde{I}_{C_{n2}}(x_2y_2)\text{if}x_2y_2\in E_1\cap E_2\end{cases}$$

$$(I_{D_{n1}}\cup_R I_{D_{n2}})(x_2y_2)=\begin{cases}I_{D_{n1}}(x_2y_2)\text{if}x_2y_2\in V_1-V_2\\ I_{D_{n2}}(x_2y_2)\text{if}x_2y_2\in V_2-V_1\\ \min\{I_{D_{n1}}(x_2y_2),I_{D_{n2}}(x_2y_2)\text{if}x_2y_2\in E_1\cap E_2\end{cases}$$

$$(\tilde{F}_{C_{n1}}\cup_R\tilde{F}_{C_{n2}})(x_2y_2)=\begin{cases}\tilde{F}_{C_{n1}}(x_2y_2)\text{if}x_2y_2\in V_1-V_2\\ \tilde{F}_{C_{n2}}(x_2y_2)\text{if}x_2y_2\in V_2-V_1\\ rmax\{\tilde{F}_{C_{n1}}(x_2y_2),\tilde{F}_{C_{n2}}(x_2y_2)\text{if}x_2y_2\in E_1\cap E_2\end{cases}$$

$$(F_{D_{n1}}\cup_R F_{D_{n2}})(x_2y_2)=\begin{cases}F_{D_{n1}}(x_2y_2)\text{if}x_2y_2\in V_1-V_2\\ F_{D_{n2}}(x_2y_2)\text{if}x_2y_2\in V_2-V_1\\ \min\{F_{D_{n1}}(x_2y_2),F_{D_{n2}}(x_2y_2)\text{if}x_2y_2\in E_1\cap E_2\end{cases}$$

Example: Let $\breve{G}_{s1}=(M_1,N_{11},N_{21})$ and $\breve{G}_{s2}=(M_2,N_{12},N_{22})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^*$ and $\breve{G}_2^*$ respectively, where

$$lM_1=\left\langle\begin{matrix}\{a,([0.3,0.4],0.8),([0.5,0.6],0.2),([0.4,0.5],0.4)\},\\ \{b,([0.6,0.7],0.2),([0.1,0.2],0.8),([0.5,0.6],0.5)\},\\ \{c,([0.4,0.5],0.3),([0.7,0.8],0.4),([0.2,0.3],0.3)\}\end{matrix}\right\rangle N_{11}=$$

$$\left\langle\begin{matrix}\{ab,([0.3,0.4],0.8),([0.1,0.2],0.8),([0.5,0.6],0.4)\},\\ \{bc,([0.4,0.5],0.3),([0.1,0.2],0.8),([0.5,0.6],0.3)\}\end{matrix}\right\rangle N_{21}=$$

$$\langle\{ac,([0.3,0.4],0.8),([0.5,0.6],0.4),([0.4,0.5],0.3)\}\rangle$$

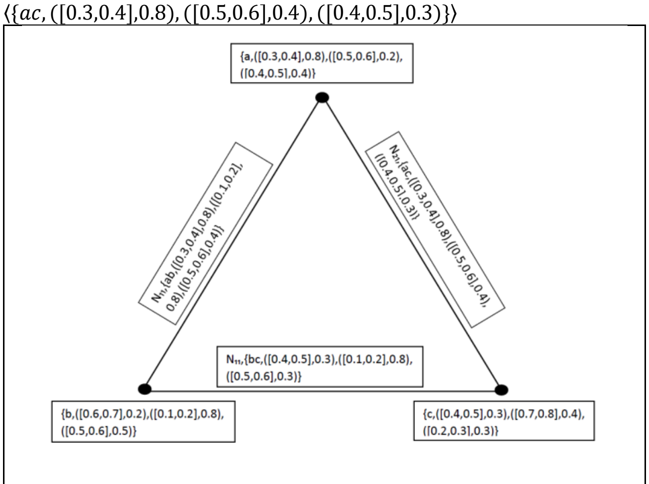

Figure: 7 Neutrosophic cubic graph structure Gs_1

and

$$lM_2=\left\langle\begin{matrix}\{a,([0.2,0.4],0.7),([0.4,0.5],0.3),([0.7,0.8],0.4)\},\\ \{b,([0.5,0.6],0.3),([0.2,0.3],0.6),([0.5,0.6]),0.2)\},\\ \{c,([0.4,0.5],0.4),([0.1,0.2],0.8),([0.6,0.7],0.1)\}\end{matrix}\right\rangle N_{12}=$$

$$\left\langle\begin{matrix}\{ab,([0.2,0.4],0.7),([0.2,0.3],0.6),([0.7,0.8],0.2)\},\\ \{bc,([0.4,0.5],0.4),([0.1,0.2],0.8),([0.6,0.7],0.1)\}\end{matrix}\right\rangle N_{22}=$$

$$\langle\{ac,([0.2,0.4],0.7),([0.1,0.2],0.8),([0.7,0.8],0.1)\}\rangle$$

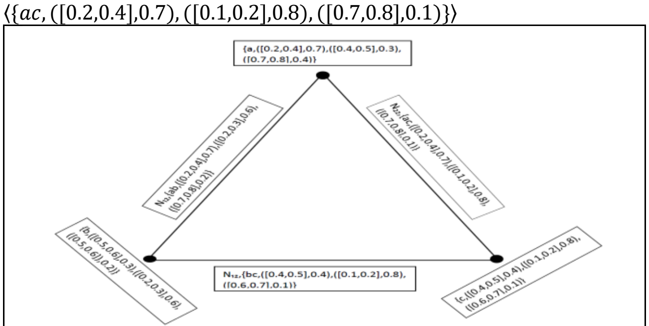

Figure: 8 Neutrosophic cubic graph structure Gs_2

Then $\breve{G}_{s1}\cup_P\breve{G}_{s2}$ will be

$$M_1\cup_P M_2=\left\langle\begin{matrix}\{a,([0.3,0.4],0.8),([0.5,0.6],0.3),([0.7,0.8],0.4)\},\\ \{b,([0.6,0.7],0.3),([0.2,0.3],0.8),([0.5,0.6],0.5)\},\\ \{c,([0.4,0.5],0.4),([0.7,0.8],0.8),([0.6,0.7],0.3)\}\end{matrix}\right\rangle$$

$$N_{11}\cup_P N_{12}=\left\langle\begin{matrix}\{ab,([0.3,0.4],0.8),([0.2,0.3],0.8),([0.7,0.8],0.4)\},\\ \{bc,([0.4,0.5],0.4),([0.1,0.2],0.8),([0.6,0.7],0.3)\}\end{matrix}\right\rangle$$

$$N_{21}\cup_P N_{22}=\langle\{ac,([0.3,0.4],0.8),([0.5,0.6],0.8),([0.7,0.8],0.3)\}\rangle$$

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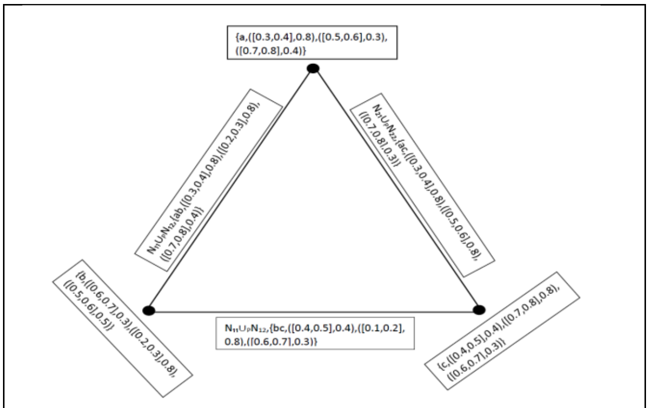

Figure: 9 P-union of two Neutrosophic cubic graph structures

and $\breve{G}_{s1} \cup_R \breve{G}_{s2}$ will be

$$M_1 \cup_R M_2 = \left\langle \begin{matrix} \{a,([0.3,0.4],0.7),([0.5,0.6],0.2),([0.7,0.8],0.4)\}, \\ \{b,([0.6,0.7],0.2),([0.2,0.3],0.6),([0.5,0.6],0.2)\}, \\ \{c,([0.4,0.5],0.3),([0.7,0.8],0.4),([0.6,0.7],0.1)\} \end{matrix} \right\rangle$$

$$N_{11} \cup_R N_{12} = \left\langle \begin{matrix} \{ab,([0.3,0.4],0.7),([0.2,0.3],0.6),([0.7,0.8],0.2)\}, \\ \{bc,([0.4,0.5],0.3),([0.1,0.2],0.8),([0.6,0.7],0.1)\} \end{matrix} \right\rangle$$

$$N_{21} \cup_R N_{22} = \langle\{ac,([0.3,0.4],0.7),([0.5,0.6],0.4),([0.7,0.8],0.1)\}\rangle.$$

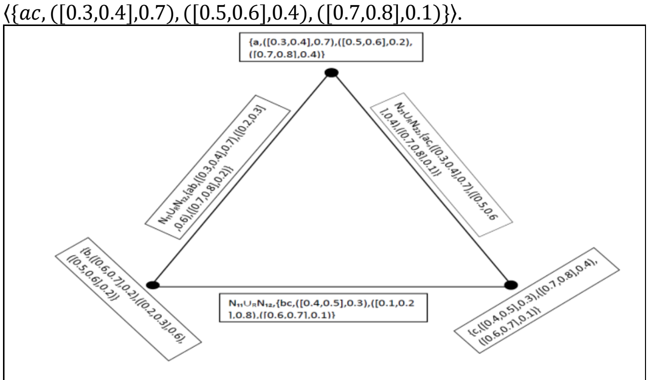

Figure: 10 R-union of two Neutrosophic cubic graph structure

Proposition 3.7 *The P-union of two neutrosophic cubic graph structures is again a neutrosophic cubic graph structure.*

Proof. Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \ldots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \ldots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \ldots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \ldots, E_{2n})$ respectively. Since all the conditions for $M_1 \cup_P M_2$ are satisfied automatically hence, we only verify conditions for $N_{1i} \cup_P N_{2i}$; $i \in 1,2,\ldots,n$. Let $xy \in E_{1i} \cap E_{2i}$ then

$$(\tilde{T}_{C_{i1}} \cup_P \tilde{T}_{C_{i2}})(xy) = rmax\{\tilde{T}_{C_{i1}}(xy), \tilde{T}_{C_{i2}}(xy)\}$$
$$\preceq rmax\{rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{A_1}(y)\}, rmin\{\tilde{T}_{A_2}(x), \tilde{T}_{A_2}(y)\}\}$$
$$= rmin\{rmax\{\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(x)\}, rmax\{\tilde{T}_{A_1}(y), \tilde{T}_{A_2}(y)\}\}$$
$$= rmin\{(\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(x), (\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(y)\}$$
$$(T_{D_{i1}} \cup_P T_{D_{i2}})(xy) = \max\{T_{D_{i1}}(xy), T_{D_{i2}}(xy)\}$$
$$\leq \max\{\max\{T_{B_1}(x), T_{B_1}(y)\}, \max\{T_{B_2}(x), T_{B_2}(y)\}\}$$
$$= \max\{\max\{T_{B_1}(x), T_{B_2}(x)\}, \max\{T_{B_1}(y), T_{B_2}(y)\}\}$$
$$= \max\{(T_{B_1} \cup_P T_{B_2})(x), (T_{B_1} \cup_P T_{B_2})(y)\}$$
$$(\tilde{I}_{C_{i1}} \cup_P \tilde{I}_{C_{i2}})(xy) = rmax\{\tilde{I}_{C_{i1}}(xy), \tilde{I}_{C_{i2}}(xy)\}$$
$$\preceq rmax\{rmin\{\tilde{I}_{A_1}(x), \tilde{I}_{A_1}(y)\}, rmin\{\tilde{I}_{A_2}(x), \tilde{I}_{A_2}(y)\}\}$$
$$= rmin\{rmax\{\tilde{I}_{A_1}(x), \tilde{I}_{A_2}(x)\}, rmax\{\tilde{I}_{A_1}(y), \tilde{I}_{A_2}(y)\}\}$$
$$= rmin\{(\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(x), (\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(y)\}$$
$$(I_{D_{i1}} \cup_P I_{D_{i2}})(xy) = \max\{I_{D_{i1}}(xy), I_{D_{i2}}(xy)\}$$
$$\leq \max\{rmax\{I_{B_1}(x), I_{B_1}(y)\}, rmax\{I_{B_2}(x), I_{B_2}(y)\}\}$$
$$= \max\{rmax\{I_{B_1}(x), I_{B_2}(x)\}, rmax\{I_{B_1}(y), I_{B_2}(y)\}\}$$
$$= \max\{(I_{B_1} \cup_P I_{B_2})(x), (I_{B_1} \cup_P I_{B_2})(y)\}$$
$$(\tilde{F}_{C_{i1}} \cup_P \tilde{F}_{C_{i2}})(xy) = rmax\{\tilde{F}_{C_{i1}}(xy), \tilde{F}_{C_{i2}}(xy)\}$$
$$\preceq rmax\{rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{A_1}(y)\}, rmax\{\tilde{F}_{A_2}(x), \tilde{F}_{A_2}(y)\}\}$$
$$= rmax\{rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{A_2}(x)\}, rmax\{\tilde{F}_{A_1}(y), \tilde{F}_{A_2}(y)\}\}$$
$$= rmax\{(\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(x), (\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(y)\}$$
$$(F_{D_{i1}} \cup_P F_{D_{i2}})(xy) = \max\{F_{D_{i1}}(xy), F_{D_{i2}}(xy)\}$$
$$\leq \max\{\min\{F_{B_1}(x), F_{B_1}(y)\}, \min\{F_{B_2}(x), F_{B_2}(y)\}\}$$
$$= \min\{\max\{F_{B_1}(x), F_{B_2}(x)\}, \max\{F_{B_1}(y), F_{B_2}(y)\}\}$$
$$= \min\{(F_{B_1} \cup_P F_{B_2})(x), (F_{B_1} \cup_P F_{B_2})(y)\}$$

If $xy \in E_{i1}$ and $xy \notin E_{i2}$, then

$$(\tilde{T}_{C_{i1}} \cup_P \tilde{T}_{C_{i2}})(xy) \preceq rmin\{(\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(x), (\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(y)\}$$
$$(T_{D_{i1}} \cup_P T_{D_{i2}})(xy) = \max\{(T_{B_1} \cup_P T_{B_2})(x), (T_{B_1} \cup_P T_{B_2})(y)\}$$
$$(\tilde{I}_{C_{i1}} \cup_P \tilde{I}_{C_{i2}})(xy) \preceq rmin\{(\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(x), (\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(y)\}$$
$$(I_{D_{i1}} \cup_P I_{D_{i2}})(xy) = \max\{(I_{B_1} \cup_P I_{B_2})(x), (I_{B_1} \cup_P I_{B_2})(y)\}$$
$$(\tilde{F}_{C_{i1}} \cup_P \tilde{F}_{C_{i2}})(xy) \preceq rmax\{(\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(x), (\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(y)\}$$
$$(F_{D_{i1}} \cup_P F_{D_{i2}})(xy) = \min\{(F_{B_1} \cup_P F_{B_2})(x), (F_{B_1} \cup_P F_{B_2})(y)\}$$

If $xy \notin E_{i1}$ and $xy \in E_{i2}$, then

$$(\tilde{T}_{C_{i1}} \cup_P \tilde{T}_{C_{i2}})(xy) \preceq rmin\{(\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(x), (\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(y)\}$$
$$(T_{D_{i1}} \cup_P T_{D_{i2}})(xy) = \max\{(T_{B_1} \cup_P T_{B_2})(x), (T_{B_1} \cup_P T_{B_2})(y)\}$$
$$(\tilde{I}_{C_{i1}} \cup_P \tilde{I}_{C_{i2}})(xy) \preceq rmin\{(\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(x), (\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(y)\}$$
$$(I_{D_{i1}} \cup_P I_{D_{i2}})(xy) = \max\{(I_{B_1} \cup_P I_{B_2})(x), (I_{B_1} \cup_P I_{B_2})(y)\}$$
$$(\tilde{F}_{C_{i1}} \cup_P \tilde{F}_{C_{i2}})(xy) \preceq rmax\{(\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(x), (\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(y)\}$$
$$(F_{D_{i1}} \cup_P F_{D_{i2}})(xy) = \min\{(F_{B_1} \cup_P F_{B_2})(x), (F_{B_1} \cup_P F_{B_2})(y)\}$$

Hence the P-union of two nuetrosophic cubic graphs is a neutrosophic cubic graph.

Remark 3.8 *R-union of two neutrosophic cubic graph structures may not be a neutrosophic cubic graph structure as in above example*

$I_{D_{11} \cup_R D_{12}}(ab) = 0.8 \nleq \max\{0.6, 0.4\} = \max\{I_{B_1 \cup_R B_2}(a), I_{B_1 \cup_R B_2}(b)\}$

so it is not a neutrosophic cubic graph structure.

Proposition 3.9: *Let $G^* = (V_1 \cup_P V_2, E_{11} \cup_P E_{12}, E_{21} \cup_P E_{22}, \ldots, E_{n1} \cup_P E_{n2})$ be the P-union of $G_1^* = (V_1, E_{11}, E_{21}, \ldots, E_{n1})$ and $G_2^* = (V_2, E_{12}, E_{22}, \ldots, E_{n2})$. Then every neutrosophic cubic graph structure $\breve{G}_S = (M, N_1, N_2, \ldots, N_n)$ of the G^* is the P-union of a neutrosophic cubic graph structure $\breve{G}_{S1}$ of G_1^* and a neutrosophic cubic graph structure $\breve{G}_{S2}$ of G_2^*.*

Proof. We define M_1, M_2, N_{i1} and N_{i2} for $i = 1,2,\ldots,n$ as;

if $x \in V_1$

$$\tilde{T}_{A_1}(x) = \tilde{T}_A(x)$$
$$T_{B_1}(x) = T_B(x)$$

if $x \in V_2$

$$\tilde{T}_{A_2}(x) = \tilde{T}_A(x)$$
$$T_{B_2}(x) = T_B(x)$$

if $xy \in E_{i1}$

$$\tilde{T}_{C_{i1}}(xy) = \tilde{T}_{C_i}(xy)$$
$$\tilde{T}_{D_{i1}}(xy) = \tilde{T}_{D_i}(xy)$$

if $xy \in E_{i2}$

$$\tilde{T}_{C_{i2}}(xy) = \tilde{T}_{C_i}(xy)$$
$$\tilde{T}_{D_{i2}}(xy) = \tilde{T}_{D_i}(xy)$$

so that M_1, M_2, E_{i1} and E_{i2} are neutosophic cubic sets on V_1, V_2, E_{i1} and E_{i2} also $M = M_1 \cup_P M_2$ and $N_i = N_{i1} \cup_P N_{i2}$ for $i = 1,2,\ldots,n$. Now for $xy \in E_{ij}$; $j = 1,2$ and $i = 1,2,\ldots,n$, we have

$$\tilde{T}_{C_{ij}}(xy) = \tilde{T}_{C_i}(xy)$$
$$\preceq rmin\{\tilde{T}_A(x), \tilde{T}_A(y)\}$$
$$= rmin\{\tilde{T}_{A_j}(x), \tilde{T}_{A_j}(y)\}$$

$$T_{D_{ij}}(xy) = T_{D_i}(xy)$$

$\leq rmin\{T_B(x),T_B(y)\}$
$= rmin\{T_{B_j}(x),T_{B_j}(y)\}$

Similarly we can prove it for $(\breve{I},I)$ and $(\breve{F},F)$. So $\breve{G}_{sj} = (M_j,N_{1j},N_{2j},\dots,N_{nj})$ is a neutrosophic cubic graph structure of $G_j^*; j = 1,2$. Thus a nuetrosophic cubic graph structure of $G^* = G_1^* \cup_P G_2^*$ is the P-union of the neutrosophic cubic graph structures G_1^* and G_2^*. This completes the proof.

Definition 3.10: *Let $\breve{G}_{S1} = (M_1,N_{11},N_{21},\dots,N_{n1})$ and $\breve{G}_{S2} = (M_2,N_{12},N_{22},\dots,N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1,E_{11},E_{12},\dots,E_{1n})$ and $\breve{G}_2^* = (V_2,E_{21},E_{22},\dots,E_{2n})$ respectively. P-join is denoted by $\breve{G}_{S1}+_P\breve{G}_{S2}$ and is defined by*

$$\breve{G}_{S1}+_P\breve{G}_{S2} = (M_1,N_{11},N_{21},\dots,N_{n1})+_P(M_2,N_{12},N_{22},\dots,N_{n2})$$
$$= (M_1+_PM_2,N_{11}+_PN_{12},N_{21}+_PN_{22},\dots,N_{n1}+_PN_{n2})$$
$$= \begin{Bmatrix}(A_1,B_1)+_P(A_2,B_2),(C_{11},D_{11})+_P(C_{12},D_{12}),\\(C_{21},D_{21})+_P(C_{22},D_{22}),\dots,(C_{n1},D_{n1})+_P(C_{n2},D_{n2})\end{Bmatrix}$$
$$= \begin{Bmatrix}(A_1+_PA_2,B_1+_PB_2,),(C_{11}+_PC_{12},D_{11}+_PD_{12}),\\(C_{21}+_PC_{22},D_{21}+_PD_{22}),\dots,(C_{n1}+_PC_{n2},D_{n1}+_PD_{n2})\end{Bmatrix}$$
$$= \begin{Bmatrix}((\tilde{T}_{A_1+_PA_2},T_{B_1+_PB_2}),(\tilde{I}_{A_1+_PA_2},I_{B_1+_PB_2}),(\tilde{F}_{A_1+_PA_2},F_{B_1+_PB_2})),\\((\tilde{T}_{C_{11}+_PC_{12}},T_{D_{11}+_PD_{12}}),(\tilde{I}_{C_{11}+_PC_{12}},I_{D_{11}+_PD_{12}}),(\tilde{F}_{C_{11}+_PC_{12}},F_{D_{11}+_PD_{12}}),\\(\tilde{T}_{C_{21}+_PC_{22}},T_{D_{21}+_PD_{22}}),(\tilde{I}_{C_{21}+_PC_{22}},I_{D_{21}+_PD_{22}}),(\tilde{F}_{C_{21}+_PC_{22}},F_{D_{21}+_PD_{22}}),\dots,\\(\tilde{T}_{C_{n1}+_PC_{n2}},T_{D_{n1}+_PD_{n2}}),(\tilde{I}_{C_{n1}+_PC_{n2}},I_{D_{n1}+_PD_{n2}}),(\tilde{F}_{C_{n1}+_PC_{n2}},F_{D_{n1}+_PD_{n2}}))\end{Bmatrix}$$

where (i) if $x \in V_1 \cup V_2$

$$(\tilde{T}_{A_1}+_P\tilde{T}_{A_2})(x) = (\tilde{T}_{A_1} \cup_P \tilde{T}_{A_2})(x)$$
$$(T_{B_1}+_PT_{B_2})(x) = (T_{B_1} \cup_P T_{B_2})(x)$$
$$(\tilde{I}_{A_1}+_P\tilde{I}_{A_2})(x) = (\tilde{I}_{A_1} \cup_P \tilde{I}_{A_2})(x)$$
$$(I_{B_1}+_PI_{B_2})(x) = (I_{B_1} \cup_P I_{B_2})(x)$$
$$(\tilde{F}_{A_1}+_P\tilde{F}_{A_2})(x) = (\tilde{F}_{A_1} \cup_P \tilde{F}_{A_2})(x)$$
$$(F_{B_1}+_PF_{B_2})(x) = (F_{B_1} \cup_P F_{B_2})(x)$$

(ii) if $xy \in E_{i1} \cup E_{i2}; i = 1,2,\dots,n$

$$(\tilde{T}_{C_{i1}}+_P\tilde{T}_{C_{i2}})(xy) = (\tilde{T}_{C_{i1}} \cup_P \tilde{T}_{C_{i2}})(xy)$$
$$(T_{D_{i1}}+_PT_{D_{i2}})(xy) = (T_{D_{i1}} \cup_P T_{D_{i2}})(xy)$$
$$(\tilde{I}_{C_{i1}}+_P\tilde{I}_{C_{i2}})(xy) = (\tilde{I}_{C_{i1}} \cup_P \tilde{I}_{C_{i2}})(xy)$$
$$(I_{D_{i1}}+_PI_{D_{i2}})(xy) = (I_{D_{i1}} \cup_P I_{D_{i2}})(xy)$$
$$(\tilde{F}_{C_{i1}}+_P\tilde{F}_{C_{i2}})(xy) = (\tilde{F}_{C_{i1}} \cup_P \tilde{F}_{C_{i2}})(xy)$$
$$(F_{D_{i1}}+_PF_{D_{i2}})(xy) = (F_{D_{i1}} \cup_P F_{D_{i2}})(xy)$$

(iii) if $xy \in E_i^*$, where E_i^* is the set of all edges joining the vertices of V_1 and V_2; $i = 1,2,\dots,n$

$$(\tilde{T}_{C_{i1}}+_P\tilde{T}_{C_{i2}})(xy) = rmin\{\tilde{T}_{A_1}(x),\tilde{T}_{A_2}(y)\}$$
$$(T_{D_{i1}}+_PT_{D_{i2}})(xy) = \min\{T_{B_1}(x),T_{B_2}(y)\}$$
$$(\tilde{I}_{C_{i1}}+_P\tilde{I}_{C_{i2}})(xy) = rmin\{\tilde{I}_{A_1}(x),\tilde{I}_{A_2}(y)\}$$
$$(I_{D_{i1}}+_PI_{D_{i2}})(xy) = \min\{I_{B_1}(x),I_{B_2}(y)\}$$
$$(\tilde{F}_{C_{i1}}+_P\tilde{F}_{C_{i2}})(xy) = rmin\{\tilde{F}_{A_1}(x),\tilde{F}_{A_2}(y)\}$$
$$(F_{D_{i1}}+_PF_{D_{i2}})(xy) = \min\{F_{B_1}(x),F_{B_2}(y)\}$$

Definition 3.11 *Let $\breve{G}_{S1} = (M_1,N_{11},N_{21},\dots,N_{n1})$ and $\breve{G}_{S2} = (M_2,N_{12},N_{22},\dots,N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1,E_{11},E_{12},\dots,E_{1n})$ and $\breve{G}_2^* = (V_2,E_{21},E_{22},\dots,E_{2n})$ respectively. R-join is denoted by $\breve{G}_{S1}+_R\breve{G}_{S2}$ and is defined by*

$$\breve{G}_{S1}+_R\breve{G}_{S2} = (M_1,N_{11},N_{21},\dots,N_{n1})+_R(M_2,N_{12},N_{22},\dots,N_{n2})$$
$$= (M_1+_RM_2,N_{11}+_RN_{12},N_{21}+_RN_{22},\dots,N_{n1}+_RN_{n2})$$
$$= \begin{Bmatrix}(A_1,B_1)+_R(A_2,B_2),(C_{11},D_{11})+_R(C_{12},D_{12}),\\(C_{21},D_{21})+_R(C_{22},D_{22}),\dots,(C_{n1},D_{n1})+_R(C_{n2},D_{n2})\end{Bmatrix}$$
$$= \begin{Bmatrix}(A_1+_RA_2,B_1+_RB_2,),(C_{11}+_RC_{12},D_{11}+_RD_{12}),\\(C_{21}+_RC_{22},D_{21}+_RD_{22}),\dots,(C_{n1}+_RC_{n2},D_{n1}+_RD_{n2})\end{Bmatrix}$$
$$= \begin{Bmatrix}((\tilde{T}_{A_1+_RA_2},T_{B_1+_RB_2}),(\tilde{I}_{A_1+_RA_2},I_{B_1+_RB_2}),(\tilde{F}_{A_1+_RA_2},F_{B_1+_RB_2})),\\((\tilde{T}_{C_{11}+_RC_{12}},T_{D_{11}+_RD_{12}}),(\tilde{I}_{C_{11}+_RC_{12}},I_{D_{11}+_RD_{12}}),(\tilde{F}_{C_{11}+_RC_{12}},F_{D_{11}+_RD_{12}}),\\(\tilde{T}_{C_{21}+_RC_{22}},T_{D_{21}+_RD_{22}}),(\tilde{I}_{C_{21}+_RC_{22}},I_{D_{21}+_RD_{22}}),(\tilde{F}_{C_{21}+_RC_{22}},F_{D_{21}+_RD_{22}}),\dots,\\(\tilde{T}_{C_{n1}+_RC_{n2}},T_{D_{n1}+_RD_{n2}}),(\tilde{I}_{C_{n1}+_RC_{n2}},I_{D_{n1}+_RD_{n2}}),(\tilde{F}_{C_{n1}+_RC_{n2}},F_{D_{n1}+_RD_{n2}}))\end{Bmatrix}$$

where (i) if $x \in V_1 \cup V_2$

$$(\tilde{T}_{A_1}+_R\tilde{T}_{A_2})(x) = (\tilde{T}_{A_1} \cup_R \tilde{T}_{A_2})(x)$$
$$(T_{B_1}+_RT_{B_2})(x) = (T_{B_1} \cup_R T_{B_2})(x)$$
$$(\tilde{I}_{A_1}+_R\tilde{I}_{A_2})(x) = (\tilde{I}_{A_1} \cup_R \tilde{I}_{A_2})(x)$$
$$(I_{B_1}+_RI_{B_2})(x) = (I_{B_1} \cup_R I_{B_2})(x)$$
$$(\tilde{F}_{A_1}+_R\tilde{F}_{A_2})(x) = (\tilde{F}_{A_1} \cup_R \tilde{F}_{A_2})(x)$$
$$(F_{B_1}+_RF_{B_2})(x) = (F_{B_1} \cup_R F_{B_2})(x)$$

(ii) if $xy \in E_{i1} \cup E_{i2}; i = 1,2,\dots,n$

$$\begin{cases}(\tilde{T}_{C_{i1}}+_R\tilde{T}_{C_{i2}})(xy) = (\tilde{T}_{C_{i1}} \cup_R \tilde{T}_{C_{i2}})(xy)\\(T_{D_{i1}}+_RT_{D_{i2}})(xy) = (T_{D_{i1}} \cup_R T_{D_{i2}})(xy)\end{cases}$$
$$\begin{cases}(\tilde{I}_{C_{i1}}+_R\tilde{I}_{C_{i2}})(xy) = (\tilde{I}_{C_{i1}} \cup_R \tilde{I}_{C_{i2}})(xy)\\(I_{D_{i1}}+_RI_{D_{i2}})(xy) = (I_{D_{i1}} \cup_R I_{D_{i2}})(xy)\end{cases}$$
$$\begin{cases}(\tilde{F}_{C_{i1}}+_R\tilde{F}_{C_{i2}})(xy) = (\tilde{F}_{C_{i1}} \cup_R \tilde{F}_{C_{i2}})(xy)\\(F_{D_{i1}}+_RF_{D_{i2}})(xy) = (F_{D_{i1}} \cup_R F_{D_{i2}})(xy)\end{cases}$$

(iii) if $xy \in E_i^*$, where E_i^* is the set of all edges joining the vertices of V_1 and V_2; $i = 1,2,\dots,n$

$$\begin{cases}(\tilde{T}_{C_{i1}}+_R\tilde{T}_{C_{i2}})(xy) = rmin\{\tilde{T}_{A_1}(x),\tilde{T}_{A_2}(y)\}\\(T_{D_{i1}}+_RT_{D_{i2}})(xy) = \max\{T_{B_1}(x),T_{B_2}(y)\}\end{cases}$$
$$\begin{cases}(\tilde{I}_{C_{i1}}+_R\tilde{I}_{C_{i2}})(xy) = rmin\{\tilde{I}_{A_1}(x),\tilde{I}_{A_2}(y)\}\\(I_{D_{i1}}+_RI_{D_{i2}})(xy) = \max\{I_{B_1}(x),I_{B_2}(y)\}\end{cases}$$
$$\begin{cases}(\tilde{F}_{C_{i1}}+_R\tilde{F}_{C_{i2}})(xy) = rmin\{\tilde{F}_{A_1}(x),\tilde{F}_{A_2}(y)\}\\(F_{D_{i1}}+_RF_{D_{i2}})(xy) = \max\{F_{B_1}(x),F_{B_2}(y)\}\end{cases}$$

Proposition 3.12 *The P-join of two neutrosophic cubic graph structures is again a neutrosophic cubic graph structure.*

Proof. Straightforward.

Definition 3.13 *Let $\breve{G}_{S1} = (M_1,N_{11},N_{21},\dots,N_{n1})$ and $\breve{G}_{S2} = (M_2,N_{12},N_{22},\dots,N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1,E_{11},E_{12},\dots,E_{1n})$ and $\breve{G}_2^* = (V_2,E_{21},E_{22},\dots,E_{2n})$ respectively. The cross product is denoted by $\breve{G}_{S1} * \breve{G}_{S2}$ and is defined by*

$$\breve{G}_{S1} * \breve{G}_{S2} = (M_1,N_{11},N_{21},\dots,N_{n1}) * (M_2,N_{12},N_{22},\dots,N_{n2})$$
$$= (M_1 * M_2,N_{11} * N_{12},N_{21} * N_{22},\dots,N_{n1} * N_{n2})$$
$$= \begin{Bmatrix}(A_1,B_1) * (A_2,B_2),(C_{11},D_{11}) * (C_{12},D_{12}),\\(C_{21},D_{21}) * (C_{22},D_{22}),\dots,(C_{n1},D_{n1}) * (C_{n2},D_{n2})\end{Bmatrix}$$
$$= \begin{Bmatrix}((A_1 * A_2,B_1 * B_2,),(C_{11} * C_{12},D_{11} * D_{12}),\\(C_{21} * C_{22},D_{21} * D_{22}),\dots,(C_{n1} * C_{n2},D_{n1} * D_{n2})\end{Bmatrix}$$
$$= \begin{Bmatrix}((\tilde{T}_{A_1*A_2},T_{B_1*B_2}),(\tilde{I}_{A_1*A_2},I_{B_1*B_2}),(\tilde{F}_{A_1*A_2},F_{B_1*B_2})),\\((\tilde{T}_{C_{11}*C_{12}},T_{D_{11}*D_{12}}),(\tilde{I}_{C_{11}*C_{12}},I_{D_{11}*D_{12}}),(\tilde{F}_{C_{11}*C_{12}},F_{D_{11}*D_{12}}),\\(\tilde{T}_{C_{21}*C_{22}},T_{D_{21}*D_{22}}),(\tilde{I}_{C_{21}*C_{22}},I_{D_{21}*D_{22}}),(\tilde{F}_{C_{21}*C_{22}},F_{D_{21}*D_{22}}),\dots,\\(\tilde{T}_{C_{n1}*C_{n2}},T_{D_{n1}*D_{n2}}),(\tilde{I}_{C_{n1}*C_{n2}},I_{D_{n1}*D_{n2}}),(\tilde{F}_{C_{n1}*C_{n2}},F_{D_{n1}*D_{n2}}))\end{Bmatrix}$$

where (i) if $xy \in V_1 \times V_2$

$$(\tilde{T}_{A_1} * \tilde{T}_{A_2})(xy) = rmin\{\tilde{T}_{A_1}(x),\tilde{T}_{A_2}(y)\}$$
$$(T_{B_1} * T_{B_2})(xy) = \max\{T_{B_1}(x),T_{B_2}(y)\}$$

$$(\tilde{I}_{A_1} * \tilde{I}_{A_2})(xy) = rmin\{\tilde{I}_{A_1}(x),\tilde{I}_{A_2}(y)\}$$
$$(I_{B_1} * I_{B_2})(xy) = \max\{I_{B_1}(x),I_{B_2}(y)\}$$

$$(\tilde{F}_{A_1} * \tilde{F}_{A_2})(xy) = rmax\{\tilde{F}_{A_1}(x),\tilde{F}_{A_2}(y)\}$$
$$(F_{B_1} * F_{B_2})(xy) = \min\{F_{B_1}(x),F_{B_2}(y)\}$$

(ii) if $x_1x_2 \in E_{i1}$ and $y_1y_2 \in E_{i2}, i = 1,2,\dots,n$

$$(\tilde{T}_{C_{i1}} * \tilde{T}_{C_{i2}})(x_1y_1)(x_2y_2) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2),\tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$(T_{D_{i1}} * T_{D_{i2}})(x_1y_1)(x_2y_2) = \max\{T_{D_{i1}}(x_1x_2),T_{D_{i2}}(y_1y_2)\}$$
$$(\tilde{I}_{C_{i1}} * \tilde{I}_{C_{i2}})(x_1y_1)(x_2y_2) = rmin\{\tilde{I}_{C_{i1}}(x_1x_2),\tilde{I}_{C_{i2}}(y_1y_2)\}$$
$$(I_{D_{i1}} * I_{D_{i2}})(x_1y_1)(x_2y_2) = \max\{I_{D_{i1}}(x_1x_2),I_{D_{i2}}(y_1y_2)\}$$

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$$(\tilde{F}_{C_{i1}} * \tilde{F}_{C_{i2}})(x_1y_1)(x_2y_2) = rmax\{\tilde{F}_{C_{i1}}(x_1x_2), \tilde{F}_{C_{i2}}(y_1y_2)\}$$
$$(F_{D_{i1}} * F_{D_{i2}})(x_1y_1)(x_2y_2) = \min\{F_{D_{i1}}(x_1x_2), F_{D_{i2}}(y_1y_2)\}$$

Example: Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21})$ and $\breve{G}_{S2} = (M_2, N_{12})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^*$ and $\breve{G}_2^*$ respectively, where

$$lM = \left\langle \begin{matrix} \{a, ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.6,0.7],0.5)\}, \\ \{b, ([0.2,0.3],0.6), ([0.4,0.5],0.2), ([0.1,0.2],0.3)\}, \\ \{c, ([0.4,0.6],0.3), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\} \end{matrix} \right\rangle N_{11}$$
$$= \langle \{ab, ([0.2,0.3],0.6), ([0.3,0.4],0.6), ([0.6,0.7],0.3)\} \rangle N_{21}$$
$$= \langle \{bc, ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.7,0.8],0.2)\} \rangle$$

and

$$lM_2 = \left\langle \begin{matrix} \{x, ([0.2,0.3],0.5), ([0.6,0.7],0.1), ([0.5,0.6],0.4)\}, \\ \{y, ([0.5,0.6],0.2), ([0.7,0.8],0.3), ([0.1,0.2],0.5)\} \end{matrix} \right\rangle N_{12}$$
$$= \langle \{xy, ([0.2,0.3],0.5), ([0.6,0.7],0.3), ([0.5,0.6],0.4)\} \rangle$$

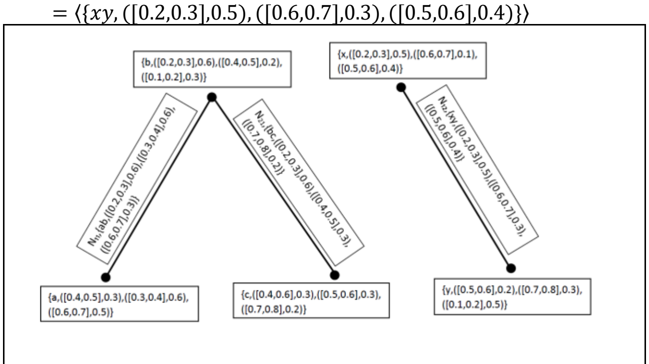

Figure: 11 Two neutrosophic cubic graph structures G_{S1} and G_{S2}

Then $\breve{G}_{S1} * \breve{G}_{S2}$ will be

$$lM_1 * M_2$$
$$= \left\langle \begin{matrix} \{(a,x), ([0.2,0.3],0.5), ([0.3,0.4],0.6), ([0.6,0.7],0.4)\}, \\ \{(a,y), ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.6,0.7],0.5)\}, \\ \{(b,x), ([0.2,0.3],0.6), ([0.4,0.5],0.2), ([0.5,0.6],0.3)\}, \\ \{(b,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.1,0.2],0.3)\}, \\ \{(c,x), ([0.2,0.3],0.5), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\}, \\ \{(c,y), ([0.4,0.6],0.3), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\} \end{matrix} \right\rangle N_{11} * N_{12}$$
$$= \left\langle \begin{matrix} \{(a,x)(b,y), ([0.2,0.3],0.6), ([0.3,0.4],0.6), ([0.6,0.7],0.3)\}, \\ \{(a,y)(b,x), ([0.2,0.3],0.6), ([0.3,0.4],0.6), ([0.6,0.7],0.3)\} \end{matrix} \right\rangle N_{21}$$
$$* N_{12} = \left\langle \begin{matrix} \{(b,x)(c,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.7,0.8],0.2)\}, \\ \{(b,y)(c,x), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.7,0.8],0.2)\} \end{matrix} \right\rangle$$

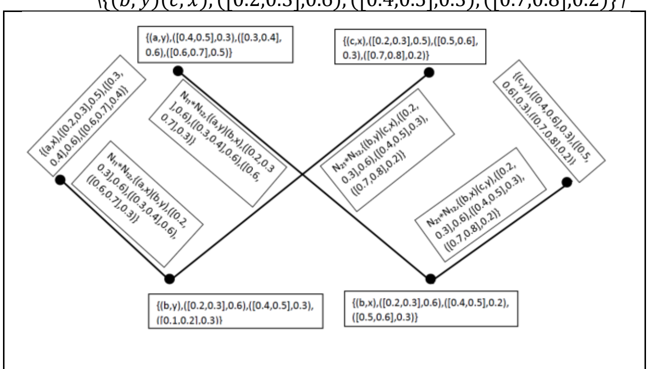

Figure: 12 Cross product of two neutrosophic cubic graph structures

Proposition 3.14 *The cross product of two neutrosophic cubic graph structures is again a neutrosophic cubic graph structure.*

Proof. Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \dots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \dots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \dots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \dots, E_{2n})$ respectively. Condition is obvious for $M_1 * M_2$. Therefore we verify for $N_{n1} * N_{n2}; n = 1,2, \dots, n$, where

$$N_{n1} * N_{n2} = \{((\tilde{T}_{C_{n1}*C_{n2}}, T_{D_{n1}*D_{n2}}), (\tilde{I}_{C_{n1}*C_{n2}}, I_{D_{n1}*D_{n2}}), (\tilde{F}_{C_{n1}*C_{n2}}, F_{D_{n1}*D_{n2}}))\}$$

We consider for $x_1y_1 \in E_{i1}$ and $x_2y_2 \in E_{i2}, i = 1,2, \dots, n$

$$(\tilde{T}_{C_{i1}} * \tilde{T}_{C_{i2}})(x_1x_2)(y_1y_2) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$\preceq rmin\langle rmin\{\tilde{T}_{A_{i1}}(x_1), \tilde{T}_{A_{i1}}(x_2)\}, rmin\{\tilde{T}_{A_{i2}}(y_1), \tilde{T}_{A_{i2}}(y_2)\}\rangle$$
$$= rmin\langle rmin\{\tilde{T}_{A_{i1}}(x_1), \tilde{T}_{A_{i2}}(y_1)\}, rmin\{\tilde{T}_{A_{i1}}(x_2), \tilde{T}_{A_{i2}}(y_2)\}\rangle$$
$$= rmin\{\tilde{T}_{C_{n1}*C_{n2}}(x_1y_1), \tilde{T}_{C_{n1}*C_{n2}}(x_2y_2)\}$$
$$(T_{D_{i1}} * T_{D_{i2}})(x_1x_2)(y_1y_2) = \max\{T_{D_{i1}}(x_1x_2), T_{D_{i2}}(y_1y_2)\}$$
$$= \max\langle \max\{T_{D_{i1}}(x_1), T_{D_{i1}}(x_2)\}, \max\{T_{D_{i2}}(y_1), T_{D_{i2}}(y_2)\}\rangle$$
$$= \max\langle \max\{T_{D_{i1}}(x_1), T_{D_{i2}}(y_1)\}, \max\{T_{D_{i1}}(x_2), T_{D_{i2}}(y_2)\}\rangle$$
$$= \max\{T_{D_{n1}*D_{n2}}(x_1y_1), T_{D_{n1}*D_{n2}}(x_2y_2)\}$$

Similarly we can show it for $(\tilde{I}_{C_{n1}*C_{n2}}, I_{D_{n1}*D_{n2}})$ and $(\tilde{F}_{C_{n1}*C_{n2}}, F_{D_{n1}*D_{n2}})$. This completes the proof. □

Definition 3.15 *Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \dots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \dots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \dots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \dots, E_{2n})$ respectively. The strong product is denoted by $\breve{G}_{S1} \boxtimes \breve{G}_{S2}$ and is defined by*

$$\breve{G}_{S1} \boxtimes \breve{G}_{S2} = (M_1, N_{11}, N_{21}, \dots, N_{n1}) \boxtimes (M_2, N_{12}, N_{22}, \dots, N_{n2})$$
$$= (M_1 \boxtimes M_2, N_{11} \boxtimes N_{12}, N_{21} \boxtimes N_{22}, \dots, N_{n1} \boxtimes N_{n2})$$
$$= \left\langle \begin{matrix} ((A_1, B_1) \boxtimes (A_2, B_2)), ((C_{11}, D_{11}) \boxtimes (C_{12}, D_{12})), \\ ((C_{21}, D_{21}) \boxtimes (C_{22}, D_{22})), \dots, ((C_{n1}, D_{n1}) \boxtimes (C_{n2}, D_{n2})) \end{matrix} \right\rangle$$
$$= \left\langle \begin{matrix} (A_1 \boxtimes A_2, B_1 \boxtimes B_2,), (C_{11} \boxtimes C_{12}, D_{11} \boxtimes D_{12}), \\ (C_{21} \boxtimes C_{22}, D_{21} \boxtimes D_{22}), \dots, (C_{n1} \boxtimes C_{n2}, D_{n1} \boxtimes D_{n2}) \end{matrix} \right\rangle$$
$$= \left\langle \begin{matrix} ((\tilde{T}_{A_1\boxtimes A_2}, T_{B_1\boxtimes B_2}), (\tilde{I}_{A_1\boxtimes A_2}, I_{B_1\boxtimes B_2}), (\tilde{F}_{A_1\boxtimes A_2}, F_{B_1\boxtimes B_2})), \\ \{((\tilde{T}_{C_{11}\boxtimes C_{12}}, T_{D_{11}\boxtimes D_{12}}), (\tilde{I}_{C_{11}\boxtimes C_{12}}, I_{D_{11}\boxtimes D_{12}}), (\tilde{F}_{C_{11}\boxtimes C_{12}}, F_{D_{11}\boxtimes D_{12}})), \\ ((\tilde{T}_{C_{21}\boxtimes C_{22}}, T_{D_{21}\boxtimes D_{22}}), (\tilde{I}_{C_{21}\boxtimes C_{22}}, I_{D_{21}\boxtimes D_{22}}), (\tilde{F}_{C_{21}\boxtimes C_{22}}, F_{D_{21}\boxtimes D_{22}})), \dots, \\ ((\tilde{T}_{C_{n1}\boxtimes C_{n2}}, T_{D_{n1}\boxtimes D_{n2}}), (\tilde{I}_{C_{n1}\boxtimes C_{n2}}, I_{D_{n1}\boxtimes D_{n2}}), (\tilde{F}_{C_{n1}\boxtimes C_{n2}}, F_{D_{n1}\boxtimes D_{n2}}))\} \end{matrix} \right\rangle$$

where (i) if $xy \in V_1 \times V_2$

$$\tilde{T}_{A_1\boxtimes A_2}(xy) = (\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(xy) = rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(y)\}$$
$$T_{B_1\boxtimes B_2}(xy) = (T_{B_1} \boxtimes T_{B_2})(xy) = \max\{T_{B_1}(x), T_{B_2}(y)\}$$

$$\tilde{I}_{A_1\boxtimes A_2}(xy) = (\tilde{I}_{A_1} \boxtimes \tilde{I}_{A_2})(xy) = rmin\{\tilde{I}_{A_1}(x), \tilde{I}_{A_2}(y)\}$$
$$I_{B_1\boxtimes B_2}(xy) = (I_{B_1} \boxtimes I_{B_2})(xy) = \max\{I_{B_1}(x), I_{B_2}(y)\}$$

$$\tilde{F}_{A_1\boxtimes A_2}(xy) = (\tilde{F}_{A_1} \boxtimes \tilde{F}_{A_2})(xy) = rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{A_2}(y)\}$$
$$F_{B_1\boxtimes B_2}(xy) = (F_{B_1} \boxtimes F_{B_2})(xy) = \min\{F_{B_1}(x), F_{B_2}(y)\}$$

(ii) if $x \in V_1$ and $y_1y_2 \in E_{i2}; i = 1,2, \dots, n$

$$\tilde{T}_{C_{i1}\boxtimes C_{i2}}(xy_1)(xy_2) = (\tilde{T}_{C_{i1}} \boxtimes \tilde{T}_{C_{i2}})(xy_1)(xy_2) = rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$T_{D_{i1}\boxtimes D_{i2}}(xy_1)(xy_2) = (T_{D_{i1}} \boxtimes T_{D_{i2}})(xy_1)(xy_2) = \max\{T_{B_1}(x), T_{D_{i2}}(y_1y_2)\}$$

$$\tilde{I}_{C_{i1}\boxtimes C_{i2}}(xy_1)(xy_2) = (\tilde{I}_{C_{i1}} \boxtimes \tilde{I}_{C_{i2}})(xy_1)(xy_2) = rmin\{\tilde{I}_{A_1}(x), \tilde{I}_{C_{i2}}(y_1y_2)\}$$
$$I_{D_{i1}\boxtimes D_{i2}}(xy_1)(xy_2) = (I_{D_{i1}} \boxtimes I_{D_{i2}})(xy_1)(xy_2) = \max\{I_{B_1}(x), I_{D_{i2}}(y_1y_2)\}$$

$$\tilde{F}_{C_{i1}\boxtimes C_{i2}}(xy_1)(xy_2) = (\tilde{F}_{C_{i1}} \boxtimes \tilde{F}_{C_{i2}})(xy_1)(xy_2) = rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{C_{i2}}(y_1y_2)\}$$
$$F_{D_{i1}\boxtimes D_{i2}}(xy_1)(xy_2) = (F_{D_{i1}} \boxtimes F_{D_{i2}})(xy_1)(xy_2) = \min\{F_{B_1}(x), F_{D_{i2}}(y_1y_2)\}$$

(iii) if $x_1x_2 \in E_{i1}$ and $y \in V_2; i = 1,2, \dots, n$

$$\tilde{T}_{C_{i1}\boxtimes C_{i2}}(x_1y)(x_2y) = (\tilde{T}_{C_{i1}} \boxtimes \tilde{T}_{C_{i2}})(x_1y)(x_2y) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{A_2}(y)\}$$

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$$T_{D_{i1}\boxtimes D_{i2}}(x_1y)(x_2y) = (T_{D_{i1}} \boxtimes T_{D_{i2}})(x_1y)(x_2y) = \max\{T_{C_{i1}}(x_1x_2), T_{A_2}(y)\}$$

$$\tilde{I}_{C_{i1}\boxtimes C_{i2}}(x_1y)(x_2y) = (\tilde{I}_{C_{i1}} \boxtimes \tilde{I}_{C_{i2}})(x_1y)(x_2y) = rmin\{\tilde{I}_{C_{i1}}(x_1x_2), \tilde{I}_{A_2}(y)\}$$

$$I_{D_{i1}\boxtimes D_{i2}}(x_1y)(x_2y) = (I_{D_{i1}} \boxtimes I_{D_{i2}})(x_1y)(x_2y) = \max\{I_{C_{i1}}(x_1x_2), I_{A_2}(y)\}$$

$$\tilde{F}_{C_{i1}\boxtimes C_{i2}}(x_1y)(x_2y) = (\tilde{F}_{C_{i1}} \boxtimes \tilde{F}_{C_{i2}})(x_1y)(x_2y) = rmax\{\tilde{F}_{C_{i1}}(x_1x_2), \tilde{F}_{A_2}(y)\}$$

$$F_{D_{i1}\boxtimes D_{i2}}(x_1y)(x_2y) = (F_{D_{i1}} \boxtimes F_{D_{i2}})(x_1y)(x_2y) = \min\{F_{C_{i1}}(x_1x_2), F_{A_2}(y)\}$$

(iv) if $x_1x_2 \in E_{i1}$ and $y_1y_2 \in E_{i2}, i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1}\boxtimes C_{i2}}(x_1y_1)(x_2y_2) = (\tilde{T}_{C_{i1}} \boxtimes \tilde{T}_{C_{i2}})(x_1y_1)(x_2y_2) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{C_{i2}}(y_1y_2)\}$$

$$T_{D_{i1}\boxtimes D_{i2}}(x_1y_1)(x_2y_2) = (T_{D_{i1}} \boxtimes T_{D_{i2}})(x_1y_1)(x_2y_2) = \max\{T_{D_{i1}}(x_1x_2), T_{D_{i2}}(y_1y_2)\}$$

$$\tilde{I}_{C_{i1}\boxtimes C_{i2}}(x_1y_1)(x_2y_2) = (\tilde{I}_{C_{i1}} \boxtimes \tilde{I}_{C_{i2}})(x_1y_1)(x_2y_2) = rmin\{\tilde{I}_{C_{i1}}(x_1x_2), \tilde{I}_{C_{i2}}(y_1y_2)\}$$

$$I_{D_{i1}\boxtimes D_{i2}}(x_1y_1)(x_2y_2) = (I_{D_{i1}} \boxtimes I_{D_{i2}})(x_1y_1)(x_2y_2) = \max\{I_{D_{i1}}(x_1x_2), I_{D_{i2}}(y_1y_2)\}$$

$$\tilde{F}_{C_{i1}\boxtimes C_{i2}}(x_1y_1)(x_2y_2) = (\tilde{F}_{C_{i1}} \boxtimes \tilde{F}_{C_{i2}})(x_1y_1)(x_2y_2) = rmax\{\tilde{F}_{C_{i1}}(x_1x_2), \tilde{F}_{C_{i2}}(y_1y_2)\}$$

$$F_{D_{i1}\boxtimes D_{i2}}(x_1y_1)(x_2y_2) = (F_{D_{i1}} \boxtimes F_{D_{i2}})(x_1y_1)(x_2y_2) = \min\{F_{D_{i1}}(x_1x_2), F_{D_{i2}}(y_1y_2)\}$$

Example: Let $\breve{G}_{s1} = (M_1, N_{11}, N_{21})$ and $\breve{G}_{s2} = (M_2, N_{12})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^*$ and $\breve{G}_2^*$ respectively, where

$$lM = \left\langle \begin{matrix} \{a, ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.6,0.7],0.5)\}, \\ \{b, ([0.2,0.3],0.6), ([0.4,0.5],0.2), ([0.1,0.2],0.3)\}, \\ \{c, ([0.4,0.6],0.3), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\} \end{matrix} \right\rangle N_{11}$$

$$= \langle \{ab, ([0.2,0.3],0.6), ([0.3,0.4],0.6), ([0.6,0.7],0.3)\} \rangle N_{21}$$

$$= \left\langle \begin{matrix} \{bc, ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.7,0.8],0.2)\}, \\ \{ac, ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.7,0.8],0.2)\} \end{matrix} \right\rangle$$

and

$$lM_2 = \left\langle \begin{matrix} \{x, ([0.2,0.3],0.5), ([0.6,0.7],0.1), ([0.5,0.6],0.4)\}, \\ \{y, ([0.5,0.6],0.2), ([0.7,0.8],0.3), ([0.1,0.2],0.5)\} \end{matrix} \right\rangle N_{12}$$

$$= \langle \{xy, ([0.2,0.3],0.5), ([0.6,0.7],0.3), ([0.5,0.6],0.4)\} \rangle$$

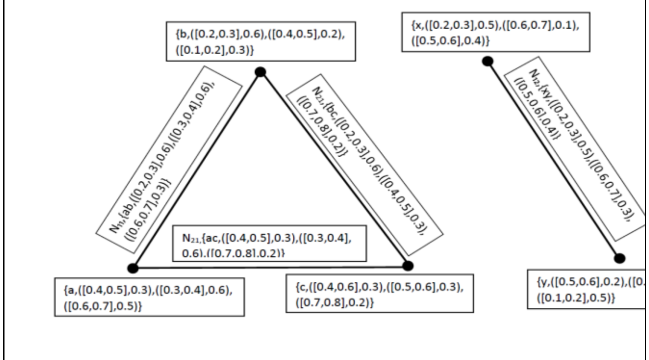

Figure: 13 Two neutrosophic cubic graph structures G_{s1} and G_{s2}

Then $\breve{G}_{s1} \boxtimes \breve{G}_{s2}$ will be

$lM_1 \boxtimes M_2$

$$= \left\langle \begin{matrix} \{(a,x), ([0.2,0.3], 0.5), ([0.3,0.4], 0.6), ([0.6,0.7], 0.4)\}, \\ \{(a,y), ([0.4,0.5], 0.3), ([0.3,0.4], 0.6), ([0.6,0.7], 0.5)\}, \\ \{(b,x), ([0.2,0.3], 0.6), ([0.4,0.5], 0.2), ([0.5,0.6], 0.3)\}, \\ \{(b,y), ([0.2,0.3], 0.6), ([0.4,0.5], 0.3), ([0.1,0.2], 0.3)\}, \\ \{(c,x), ([0.2,0.3], 0.5), ([0.5,0.6], 0.3), ([0.7,0.8], 0.2)\}, \\ \{(c,y), ([0.4,0.6], 0.3), ([0.5,0.6], 0.3), ([0.7,0.8], 0.2)\} \end{matrix} \right\rangle N_{11} \boxtimes N_{12}$$

$$= \left\langle \begin{matrix} \{(a,x)(b,y), ([0.2,0.3], 0.6), ([0.3,0.4], 0.6), ([0.6,0.7], 0.3)\}, \\ \{(a,y)(b,x), ([0.2,0.3], 0.6), ([0.3,0.4], 0.6), ([0.6,0.7], 0.3)\}, \\ \{(a,x)(a,y), ([0.2,0.3], 0.5), ([0.3,0.4], 0.6), ([0.6,0.7], 0.4)\} \end{matrix} \right\rangle$$

$N_{21} \boxtimes N_{12}$

$$= \left\langle \begin{matrix} \{(b,x)(c,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.7,0.8],0.2)\}, \\ \{(b,x)(c,x)\}, ([0.2,0.3],0.6), ([0.4,0.5],0.3), (0.7,0.8],0.2)\}, \\ \{(b,x)(b,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), (0.5,0.6],0.3)\} \end{matrix} \right\rangle$$

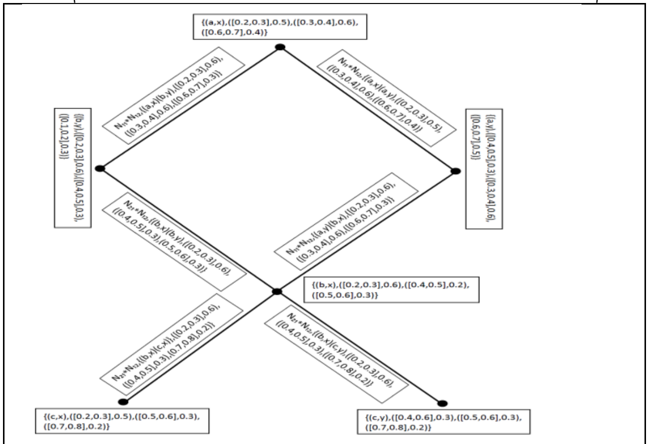

Figure:14 Strong product of two neutrosophic cubic graph structures

Proposition 3.16 *The strong product of two neutrosophic cubic graph structures is again a neutrosophic cubic graph structure.*

Proof. Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \ldots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \ldots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \ldots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \ldots, E_{2n})$ respectively. Condition is obvious for $M_1 \boxtimes M_2$. Therefore we verify for $N_{n1} \boxtimes N_{n2}; n = 1,2,\ldots,n$, where

$$N_{n1} \boxtimes N_{n2} = \{((\tilde{T}_{C_{n1}\boxtimes C_{n2}}, T_{D_{n1}\boxtimes D_{n2}}), (\tilde{I}_{C_{n1}\boxtimes C_{n2}}, I_{D_{n1}\boxtimes D_{n2}}), (\tilde{F}_{C_{n1}\boxtimes C_{n2}}, F_{D_{n1}\boxtimes D_{n2}}))\}$$

(i) Let $x \in V_1$ and $y_1y_2 \in E_{i2}; i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1}\boxtimes C_{i2}}(xy_1)(xy_2) = rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{C_{i2}}(y_1y_2)\} \le rmin\{(\tilde{T}_{A_1}(x), rmin((\tilde{T}_{A_2}(y_1), (\tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{rmin(\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(y_1)), rmin(\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(x, y_1), ((\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(x, y_2)\}$$

$$T_{D_{n1}\boxtimes D_{n2}}((xy_1)(xy_2)) = \max\{(T_{B_1}(x), T_{D_{n2}}(y_1y_2))\} \le \max\{(T_{B_1}(x), \max((T_{B_2}(y_1), (T_{B_2}(y_2))\}$$
$$= \max\{\max((T_{B_1}(x), (T_{B_2}(y_1)), \max((T_{B_1}(x), (T_{B_2}(y_2))\}$$
$$= \max\{(T_{B_1} \boxtimes T_{B_2})(x, y_1), (T_{B_1} \boxtimes T_{B_2})(x, y_2)\}$$

(ii) Let $x_1x_2 \in E_{i1}$ and $y \in V_2; i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1}\boxtimes C_{i2}}(x_1y)(x_2y) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{A_2}(y)\} \le rmin\{rmin((\tilde{T}_{A_1}(x_1), \tilde{T}_{A_1}(x_2)), \tilde{T}_{A_2}(y)\}$$
$$= rmin\{rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_2}(y)), rmin((\tilde{T}_{A_1}(x_2), (\tilde{T}_{A_2}(y))\}$$
$$= rmin\{(\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(x_1y), (\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(x_2y)\}$$

$$T_{D_{n1}\boxtimes D_{n2}}((x_1y)(x_2y)) = \max\{(T_{D_{n1}}(x_1x_2), T_{B_2}(y))\} \le \max\{\max(T_{B_1}(x_1), T_{B_1}(x_2)), T_{B_2}(y)\}$$
$$= \max\{\max(T_{B_1}(x_1), T_{B_2}(y)), \max(T_{B_2}(x_2), T_{B_2}(y))\}$$
$$= \max\{(T_{B_1} \boxtimes T_{B_2})(x_1y), (T_{B_1} \boxtimes T_{B_2})(x_2y)\}$$

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(iii) Let $x_1x_2 \in E_{i1}$ and $y_1y_2 \in E_{i2}; i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1}\boxtimes C_{i2}}(x_1y_1)(x_2y_2) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$\preceq rmin\{rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_1}(x_2)), rmin(\tilde{T}_{A_2}(y_1), \tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_2}(y_1)), rmin(\tilde{T}_{A_1}(x_2), T'_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(x_1y_1), (\tilde{T}_{A_1} \boxtimes \tilde{T}_{A_2})(x_2y_2)$$
$$T_{D_{i1}\boxtimes D_{i2}}(x_1y_1)(x_2y_2) = \max\{T_{D_{i1}}(x_1x_2), T_{D_{i2}}(y_1y_2)\}$$
$$\le \max\{\max(T_{B_1}(x_1), T_{B_1}(x_2)), (T_{B_2}(y_1), T_{B_2}(y_2))\}$$
$$= \max\{\max(T_{B_1}(x_1), T_{B_2}(y_1)), (T_{B_1}(x_2), T_{B_2}(y_2))\}$$
$$= \max\{(T_{B_1} \boxtimes T_{B_2})(x_1y_1), (T_{B_1} \boxtimes T_{B_2})(x_2y_2)\}$$

Similarly we can also show this for $(\tilde{I}_{C_{n1}\boxtimes C_{n2}}, I_{D_{n1}\boxtimes D_{n2}})$ and $(\tilde{F}_{C_{n1}\boxtimes C_{n2}}, F_{D_{n1}\boxtimes D_{n2}})$. This completes the proof.

Definition 3.17 *Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \ldots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \ldots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \ldots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \ldots, E_{2n})$ respectively. The lexicographic product is denoted by $\breve{G}_{S1} \cdot \breve{G}_{S2}$ and is defined by*

$$\breve{G}_{S1} \cdot \breve{G}_{S2} = (M_1, N_{11}, N_{21}, \ldots, N_{n1}) \cdot (M_2, N_{12}, N_{22}, \ldots, N_{n2})$$
$$= (M_1 \cdot M_2, N_{11} \cdot N_{12}, N_{21} \cdot N_{22}, \ldots, N_{n1} \cdot N_{n2})$$
$$= ((A_1, B_1) \cdot (A_2, B_2), (C_{11}, D_{11}) \cdot (C_{12}, D_{12}), (C_{21}, D_{21}) \cdot (C_{22}, D_{22}), \ldots, (C_{n1}, D_{n1}) \cdot (C_{n2}, D_{n2}))$$
$$= ((A_1 \cdot A_2, B_1 \cdot B_2,), (C_{11} \cdot C_{12}, D_{11} \cdot D_{12}), (C_{21} \cdot C_{22}, D_{21} \cdot D_{22}), \ldots, (C_{n1} \cdot C_{n2}, D_{n1} \cdot D_{n2}))$$
$$= \left\{\begin{array}{l} ((\tilde{T}_{A_1 \cdot A_2}, T_{B_1 \cdot B_2}), (\tilde{I}_{A_1 \cdot A_2}, I_{B_1 \cdot B_2}), (\tilde{F}_{A_1 \cdot A_2}, F_{B_1 \cdot B_2})), \\ ((\tilde{T}_{C_{11} \cdot C_{12}}, T_{D_{11} \cdot D_{12}}), (\tilde{I}_{C_{11} \cdot C_{12}}, I_{D_{11} \cdot D_{12}}), (\tilde{F}_{C_{11} \cdot C_{12}}, F_{D_{11} \cdot D_{12}}), \\ (\tilde{T}_{C_{21} \cdot C_{22}}, T_{D_{21} \cdot D_{22}}), (\tilde{I}_{C_{21} \cdot C_{22}}, I_{D_{21} \cdot D_{22}}), (\tilde{F}_{C_{21} \cdot C_{22}}, F_{D_{21} \cdot D_{22}}), \ldots, \\ (\tilde{T}_{C_{n1} \cdot C_{n2}}, T_{D_{n1} \cdot D_{n2}}), (\tilde{I}_{C_{n1} \cdot C_{n2}}, I_{D_{n1} \cdot D_{n2}}), (\tilde{F}_{C_{n1} \cdot C_{n2}}, F_{D_{n1} \cdot D_{n2}})) \end{array}\right\}$$

where (i) if $xy \in V_1 \times V_2$

$$\tilde{T}_{A_1 \cdot A_2}(xy) = (\tilde{T}_{A_1} \cdot \tilde{T}_{A_2})(xy) = rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(y)\}$$
$$T_{B_1 \cdot B_2}(xy) = (T_{B_1} \cdot T_{B_2})(xy) = \max\{T_{B_1}(x), T_{B_2}(y)\}$$

$$\tilde{I}_{A_1 \cdot A_2}(xy) = (\tilde{I}_{A_1} \cdot \tilde{I}_{A_2})(xy) = rmin\{\tilde{I}_{A_1}(x), \tilde{I}_{A_2}(y)\}$$
$$I_{B_1 \cdot B_2}(xy) = (I_{B_1} \cdot I_{B_2})(xy) = \max\{I_{B_1}(x), I_{B_2}(y)\}$$

$$\tilde{F}_{A_1 \cdot A_2}(xy) = (\tilde{F}_{A_1} \cdot \tilde{F}_{A_2})(xy) = rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{A_2}(y)\}$$
$$F_{B_1 \cdot B_2}(xy) = (F_{B_1} \cdot F_{B_2})(xy) = \min\{F_{B_1}(x), F_{B_2}(y)\}$$

(ii) if $x \in V_1$ and $y_1y_2 \in E_{i2}; i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1} \cdot C_{i2}}(xy_1)(xy_2) = (\tilde{T}_{C_{i1}} \cdot \tilde{T}_{C_{i2}})(xy_1)(xy_2) = rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$T_{D_{i1} \cdot D_{i2}}(xy_1)(xy_2) = (T_{D_{i1}} \cdot T_{D_{i2}})(xy_1)(xy_2) = \max\{T_{B_1}(x), T_{D_{i2}}(y_1y_2)\}$$

$$\tilde{I}_{C_{i1} \cdot C_{i2}}(xy_1)(xy_2) = (\tilde{I}_{C_{i1}} \cdot \tilde{I}_{C_{i2}})(xy_1)(xy_2) = rmin\{\tilde{I}_{A_1}(x), \tilde{I}_{C_{i2}}(y_1y_2)\}$$
$$I_{D_{i1} \cdot D_{i2}}(xy_1)(xy_2) = (I_{D_{i1}} \cdot I_{D_{i2}})(xy_1)(xy_2) = \max\{I_{B_1}(x), I_{D_{i2}}(y_1y_2)\}$$

$$\tilde{F}_{C_{i1} \cdot C_{i2}}(xy_1)(xy_2) = (\tilde{F}_{C_{i1}} \cdot \tilde{F}_{C_{i2}})(xy_1)(xy_2) = rmax\{\tilde{F}_{A_1}(x), \tilde{F}_{C_{i2}}(y_1y_2)\}$$
$$F_{D_{i1} \cdot D_{i2}}(xy_1)(xy_2) = (F_{D_{i1}} \cdot F_{D_{i2}})(xy_1)(xy_2) = \min\{F_{B_1}(x), F_{D_{i2}}(y_1y_2)\}$$

(iii) if $x_1x_2 \in E_{i1}$ and $y_1y_2 \in E_{i2}, i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1} \cdot C_{i2}}(x_1y_1)(x_2y_2) = (\tilde{T}_{C_{i1}} \cdot \tilde{T}_{C_{i2}})(x_1y_1)(x_2y_2)$$
$$= rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$T_{D_{i1} \cdot D_{i2}}(x_1y_1)(x_2y_2) = (T_{D_{i1}} \cdot T_{D_{i2}})(x_1y_1)(x_2y_2)$$
$$= \max\{T_{D_{i1}}(x_1x_2), T_{D_{i2}}(y_1y_2)\}$$

$$\tilde{I}_{C_{i1} \cdot C_{i2}}(x_1y_1)(x_2y_2) = (\tilde{I}_{C_{i1}} \cdot \tilde{I}_{C_{i2}})(x_1y_1)(x_2y_2)$$
$$= rmin\{\tilde{I}_{C_{i1}}(x_1x_2), \tilde{I}_{C_{i2}}(y_1y_2)\}$$
$$I_{D_{i1} \cdot D_{i2}}(x_1y_1)(x_2y_2) = (I_{D_{i1}} \cdot I_{D_{i2}})(x_1y_1)(x_2y_2)$$
$$= \max\{I_{D_{i1}}(x_1x_2), I_{D_{i2}}(y_1y_2)\}$$

$$\tilde{F}_{C_{i1} \cdot C_{i2}}(x_1y_1)(x_2y_2) = (\tilde{F}_{C_{i1}} \cdot \tilde{F}_{C_{i2}})(x_1y_1)(x_2y_2)$$
$$= rmax\{\tilde{F}_{C_{i1}}(x_1x_2), \tilde{F}_{C_{i2}}(y_1y_2)\}$$
$$F_{D_{i1} \cdot D_{i2}}(x_1y_1)(x_2y_2) = (F_{D_{i1}} \cdot F_{D_{i2}})(x_1y_1)(x_2y_2)$$
$$= \min\{F_{D_{i1}}(x_1x_2), F_{D_{i2}}(y_1y_2)\}$$

Example: Let $\breve{G}_{s1}$ and $\breve{G}_{s2}$be two neutrosophic cubic graph structures as shown in figure:13. Then their lexicographic product will be

$lM_1 \cdot M_2$

$$\left\{\begin{array}{l} \{(a,x), ([0.2,0.3],0.5), ([0.3,0.4],0.6), ([0.6,0.7],0.4)\}, \\ \{(a,y), ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.6,0.7],0.5)\}, \\ \{(b,x), ([0.2,0.3],0.6), ([0.4,0.5],0.2), ([0.5,0.6],0.3)\}, \\ \{(b,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.1,0.2],0.3)\}, \\ \{(c,x), ([0.2,0.3],0.5), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\}, \\ \{(c,y), ([0.4,0.6],0.3), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\} \end{array}\right.$$

$$= \left\langle\begin{array}{l} \{(a,y), ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.6,0.7],0.5)\}, \\ \{(b,x), ([0.2,0.3],0.6), ([0.4,0.5],0.2), ([0.5,0.6],0.3)\}, \\ \{(b,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.1,0.2],0.3)\}, \\ \{(c,x), ([0.2,0.3],0.5), ([0.5,0.6],0.3), ([0.7,0.8],0.2)\}, \end{array}\right\rangle N_{11} \cdot N_{12}$$

$$= \left\langle\begin{array}{l} \{(a,x)(b,y), ([0.2,0.3],0.6), ([0.3,0.4],0.6), ([0.6,0.7],0.3)\}, \\ \{(a,y)(b,x), ([0.2,0.3],0.6), ([0.3,0.4],0.6), ([0.6,0.7],0.3)\}, \\ \{(a,x)(a,y), ([0.2,0.3],0.5), ([0.3,0.4],0.6), ([0.6,0.7],0.4)\} \end{array}\right\rangle N_{21} \cdot N_{12}$$

$$= \left\langle\begin{array}{l} \{(b,x)(c,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), ([0.7,0.8],0.2)\}, \\ \{(b,x)(b,y), ([0.2,0.3],0.6), ([0.4,0.5],0.3), (0.5,0.6],0.3)\} \end{array}\right\rangle$$

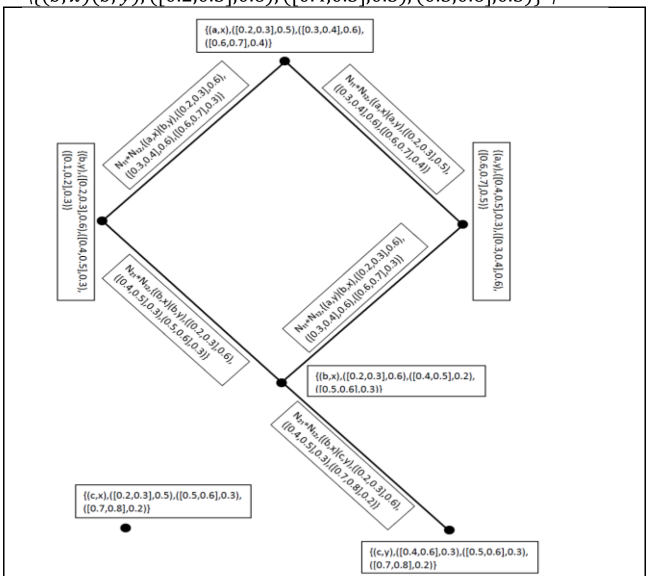

Figure:15 Laxicographic product of two neutrosophic cubic graph structures

Proposition 3.18 *The lexicographic product of two neutrosophic cubic graph structures is again a neutrosophic cubic graph structure.*

Proof. Let $\breve{G}_{S1} = (M_1, N_{11}, N_{21}, \ldots, N_{n1})$ and $\breve{G}_{S2} = (M_2, N_{12}, N_{22}, \ldots, N_{n2})$ be two neutrosophic cubic graph structures defined on $\breve{G}_1^* = (V_1, E_{11}, E_{12}, \ldots, E_{1n})$ and $\breve{G}_2^* = (V_2, E_{21}, E_{22}, \ldots, E_{2n})$ respectively. Condition is obvious for $M_1 \cdot M_2$. Therefore we verify for $N_{n1} \cdot N_{n2}; n = 1,2,\ldots,n$, where

$$N_{n1} \cdot N_{n2} = \{((\tilde{T}_{C_{n1} \cdot C_{n2}}, T_{D_{n1} \cdot D_{n2}}), (\tilde{I}_{C_{n1} \cdot C_{n2}}, I_{D_{n1} \cdot D_{n2}}), (\tilde{F}_{C_{n1} \cdot C_{n2}}, F_{D_{n1} \cdot D_{n2}}))\}$$

(i) Let $x \in V_1$ and $y_1y_2 \in E_{i2}; i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1} \cdot C_{i2}}(xy_1)(xy_2) = rmin\{\tilde{T}_{A_1}(x), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$\preceq rmin\{(\tilde{T}_{A_1}(x), rmin((\tilde{T}_{A_2}(y_1), (\tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{rmin(\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(y_1)), rmin(\tilde{T}_{A_1}(x), \tilde{T}_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{T}_{A_1} \cdot \tilde{T}_{A_2})(x, y_1), ((\tilde{T}_{A_1} \cdot \tilde{T}_{A_2})(x, y_2)\}$$
$$T_{D_{n1} \cdot D_{n2}}((xy_1)(xy_2)) = \max\{(T_{B_1}(x), T_{D_{n2}}(y_1y_2))\}$$
$$\le \max\{(T_{B_1}(x), \max((T_{B_2}(y_1), (T_{B_2}(y_2))\}$$
$$= \max\{\max((T_{B_1}(x), (T_{B_2}(y_1)), \max((T_{B_1}(x), (T_{B_2}(y_2))\}$$
$$= \max\{(T_{B_1} \cdot T_{B_2})(x, y_1), (T_{B_1} \cdot T_{B_2})(x, y_2)\}$$

(ii) Let $x_1x_2 \in E_{i1}$ and $y_1y_2 \in E_{i2}; i = 1,2,\ldots,n$

$$\tilde{T}_{C_{i1} \cdot C_{i2}}(x_1y_1)(x_2y_2) = rmin\{\tilde{T}_{C_{i1}}(x_1x_2), \tilde{T}_{C_{i2}}(y_1y_2)\}$$
$$\preceq rmin\{rmin(\tilde{T}_{A_1}(x_1), \tilde{T}_{A_1}(x_2)), rmin(\tilde{T}_{A_2}(y_1), \tilde{T}_{A_2}(y_2))\}$$

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$$=$$
$$rmin\{rmin(\tilde{T}_{A_1}(x_1),\tilde{T}_{A_2}(y_1)),rmin(\tilde{T}_{A_1}(x_2),T'_{A_2}(y_2))\}$$
$$= rmin\{(\tilde{T}_{A_1}\cdot\tilde{T}_{A_2})(x_1y_1),(\tilde{T}_{A_1}\cdot\tilde{T}_{A_2})(x_2y_2)$$
$$T_{D_{i1}\cdot D_{i2}}(x_1y_1)(x_2y_2) = \max\{T_{D_{i1}}(x_1x_2),T_{D_{i2}}(y_1y_2)\}$$
$$\leq \max\{\max(T_{B_1}(x_1),T_{B_1}(x_2)),(T_{B_2}(y_1),T_{B_2}(y_2))\}$$
$$= \max\{\max(T_{B_1}(x_1),T_{B_2}(y_1)),(T_{B_1}(x_2),T_{B_2}(y_2))\}$$
$$= \max\{(T_{B_1}\cdot T_{B_2})(x_1y_1),(T_{B_1}\cdot T_{B_2})(x_2y_2)\}$$

Similarly we can show it for $(\tilde{I}_{C_{n1}\cdot C_{n2}},I_{D_{n1}\cdot D_{n2}})$ and $(\tilde{F}_{C_{n1}\cdot C_{n2}},F_{D_{n1}\cdot D_{n2}})$. This completes the proof.

IV. Application in Multiple Attribute Group Decision Making Problem

In this section we discuss a multiple attribute group decision making problem and developed an algorithm.

Graphs are very important in daily life and allow us to study the behavior of something quickly. Graphs allow us to make a mental image of the data, so we can say that graphs help us to build a bridge between the abstract and the real. For too long we as humans have taken too much work upon our shoulders, its time to simplify our life and to use the best tool for the job. Graphing is one of these tools that might be used in such circumstances. Graphs are used in everyday life, from the local newspaper to the magazine stand. In computer science graphs are used to represent the flow of computation, used to measure the trafficking to a site, also used in fraud detection etc. So it is one of these skills that you simply cannot do without the help of graphs. Graphs can help us and make our life simpler from student to professionals. Fuzzy graph theory has been used in the world of Mathematics due to its effective applications.

We first provide an algorithm and then we discuss an example.

Algorithm:

1. Select the set $V=\{A_1,A_2,\dots,A_n\}$ of alternatives as a vertex set from the problem which is under study and select the membership grade for each element in the vertex set based on certain attributes.
2. Select the set $E=\{E_{11},E_{21},E_{31},\dots,E_{n1}\}$ of attributes or criteria as the set of edges.
3. Use the Definition 3.1 of neutrosophic cubic graphs structures for finding the membership grade of each E_{i1} for $i=1,2,3\dots.n$.
4. After having the values of V and E, draw the graph.
5. Find the strength of each edge using the following definition and comapre them,

Definition 4.1 *Let $E=\{N_{11}\}$ be a edge having neutrosophic cubic value and we define strength of edge as*

$$S(E)=[\{(T_{11}^{-}+I_{11}^{-}-F_{11}^{-})+(T_{11}^{+}+I_{11}^{+}-F_{11}^{+})\}+T_{11}+I_{11}-F_{11}]$$

where $S\in[-3,3]$.

This is same as the score of a neutrosophic cubic numbers. Here we used it for the graphs instead of numbers in terms of neutrosophic cubic sets.

Example: Neutrosophic cubic graphs have vast applications in industries as discussed in [24]. Neutrosophic cubic graph structures have more vast applications in daily life, industries, economy and in foreign policy etc. The foreign policy of a country is influenced by so many factors. Some of them are listed as, "Geography, Size, Culture and History, Economics Development, Technology, Social Structure, Public Mood, Political Organization, Role of Press, Political Accountability,Leadership, Military Relation, Economic and trade policy, Diplomacy, Alliance, Membership of International Institute in Country, Religious Relation, Religious Festivals, Intelligence agencies and Boundaries etc". Here we discuss some of the above mentioned factors effecting the foreign policy for presenting an application of our developed mathematical procedure. We apply the algorithm as under:

1. Let us consider a vertex set $V=\{A,B,C,D\}$ of countries. Find the membership grade of each element of V using the Neutrosophic cubic sets as under:

$$M=\left\langle\begin{matrix}\{A,([0.3,0.4],0.6),([0.1,0.2],0.1),([0.6,0.7],0.3)\},\\ \{B,([0.4,0.5],0.2),([0.7,0.8],0.4),([0.1,0.2],0.5)\},\\ \{C,([0.5,0.6],0.4),([0.2,0.3],0.6),([0.4,0.5],0.2)\},\\ \{D,([0.1,0.2],0.5),([0.3,0.4],0.9),([0.5,0.6],0.4)\}\end{matrix}\right\rangle$$

2. These countries are interlinked with each other by some relations given by

$$E=\{E_{11},E_{21},E_{31}\}$$
$$=\{\text{religiousrelations},\text{traderelations},\text{securityrelations}\}$$

where

$$E_{11}=(C_{11},D_{11})$$
$$=\{(\tilde{T}_{C_{11}},T_{D_{11}}),(\tilde{I}_{C_{11}},I_{D_{11}}),(\tilde{F}_{C_{11}},F_{D_{11}})\}$$
$$=\{\text{religious beliefs},\text{religious festivals},\text{effects of religion on society}\}$$

$$E_{21}=(C_{21},D_{21})$$
$$=\{(\tilde{T}_{C_{21}},T_{D_{21}}),(\tilde{I}_{C_{21}},I_{D_{21}}),(\tilde{F}_{C_{21}},F_{D_{21}})\}$$
$$=\{\text{import},\text{export},\text{exchange}\}$$

$$E_{31}=(C_{31},D_{31})$$
$$=\{(\tilde{T}_{C_{31}},T_{D_{31}}),(\tilde{I}_{C_{31}},I_{D_{31}}),(\tilde{F}_{C_{31}},F_{D_{31}})\}$$
$$=\{\text{Army},\text{boundaries},\text{intelligenceagencies}\}$$

As the above given factors highly effect the relations among countries. These factors are responsible for the peace or war between two countries.

3. Using the Definition 3.1 we have

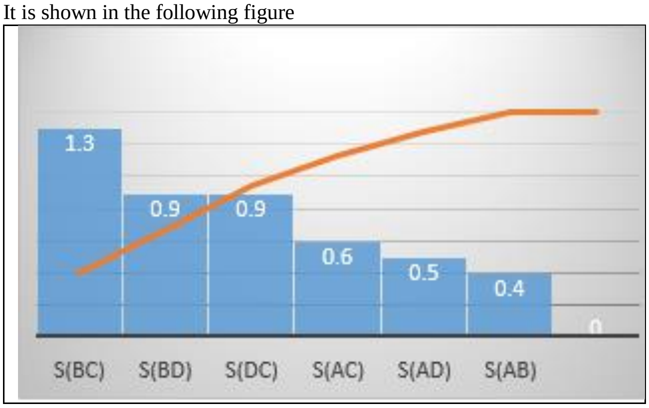
$$lN_{11}=\left\langle\begin{matrix}\{AB,([0.3,0.4],0.6),([0.1,0.2],0.4),([0.6,0.7],0.3)\},\\ \{CD,([0.1,0.2],0.5),([0.2,0.3],0.9),([0.5,0.6],0.2)\}\end{matrix}\right\rangle N_{21}$$
$$=\left\langle\begin{matrix}\{AD,([0.1,0.2],0.6),([0.1,0.2],0.9),([0.6,0.7],0.3)\},\\ \{BC,([0.4,0.5],0.4),([0.2,0.3],0.6),([0.4,0.5],0.2)\}\end{matrix}\right\rangle N_{31}$$
$$=\left\langle\begin{matrix}\{AC,([0.3,0.4],0.6),([0.1,0.2],0.6),([0.6,0.7],0.2)\},\\ \{BD,([0.1,0.2],0.5),([0.3,0.4],0.9),([0.5,0.6],0.4)\}\end{matrix}\right\rangle$$

4.Draw the graph as under;

5. Strength of edges is as under using the Definition 4.1, we have

$$S(AB)=0.4,$$
$$S(AC)=0.6,$$
$$S(AD)=0.5,$$
$$S(BD)=0.9,$$
$$S(DC)=0.9,$$
$$S(BC)=1.3.$$

It is shown in the following figure



$$S(BC)>S(DC)=S(BD)>S(AC)>S(AD)>S(AB).$$

Thus we can concluded that the countries $\boldsymbol{B}$ and $\boldsymbol{C}$ have strong relations between each other.

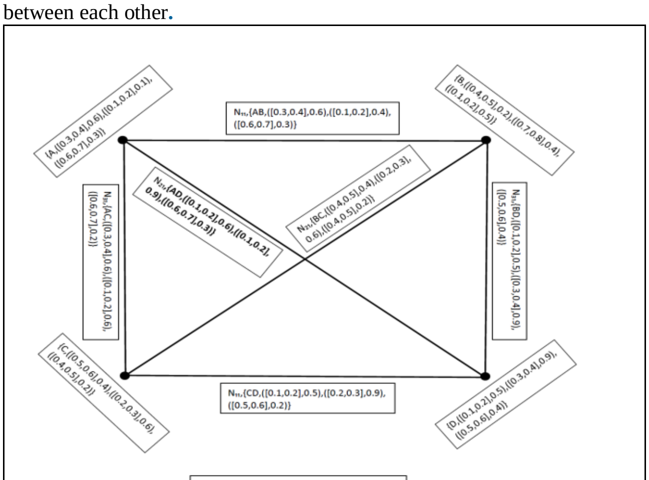


Figure: 16 Neutrosophic cubic graph structure

V. Comparative Analysis and Conclusions:

All versions of neutrosophic sets like, single valued neutrosophic set, interval valued neutrosophic set and neutrosophic cubic set are used in literature so far for the applications of neutrosophic sets. But neutrosophic cubic sets are a more generalized tool to handle imprecision and vagueness and all other versions of neutrosophic sets are the special cases of it. On the other sides we have the comparison between the different types of graphs as shown the following table:

Type of Graph	Advantages and Limitations
Crisp Graphs	These can handle only exact information
Fuzzy Graphs	These can handle imprecise and vague information but only can handle only the positive aspects.
Intuitionistic Fuzzy Graphs	These can handle both positive and negative aspects, but it is not always possible to assign a single membership and non-membership value.
Single values Neutrosophic Graphs	These can handle positive, negative and hesitant information's in a much better way as compared to previous ones. But like intutionistic fuzzy graphs it is not always possible to assign a single membership and non-membership value.
Interval-valued Neutrosophic Graphs	It can handle many problems as compared to previous. Yet have some limitations which can be handle through the hybrid version of neutrosophic cubic graphs.
Neutrosophic Cubic Graphs	This is the most generalized version of fuzzy graphs and it can handle many imprecise and vague problems. But in Neutrosophic Cubic Graphs the number of the set of edges is the only one. When the number of edges is more than one then we need the concept of neutrosophic cubic structures.

So, we used the concept of neutrosophic cubic sets in this paper with the concept of neutrosophic cubic structures.

We have observed that by increasing the set of edges we can find more insight of the problem which is not possible through a single set of edges. In this paper we discussed the idea of neutrosophic cubic graph structures, and different operations on it such as Cartesian product, composition, P-union, R-union, P-join, R-join, cross product, strong product and lexicographic product. We provided different examples and results related to these operations. We also observed that R-union of two neutrosophic cubic graph structures may not be a neutrosophic cubic graph structure. Further we provided applications of neutrosophic cubic graph structures. In future we will try to different kinds of neutrosophic cubic graphs structures and will explore more results related with the application in real life.

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Dr. Muhammad Gulistan got his MPhil degree from Quaide Azam University islamabad in 2011 and PhD degree from Hazara University in 2016. Now he is working as an assistant professor in the department of mathematics and statistics of Hazara university. His area of research is "Cubic sets and their generalizations, Non-associative hyperstructures, neutrosophic cubic sets, neutrosophic cubic graphs, decision making". He published more than 50 research papers in different well reputed journals. He supervised many M.Phil. and PhD research students.

Mr. Mumtaz Ali currently works at the School of Agricultural, Computational and Environmental Sciences, University of Southern Queensland, Springfield Campus, Australia. Mumtaz does research in Environmental and agricultural modelling using statistical and machine learning models, Decision support systems, Medical image segmentation and diagnosis, fuzzy set, Algebra and Logic and Foundations of Mathematics. Their current project is 'New Trends in Neutrosophic Theory and Applications'. He has published many research papers in neutrosophic sets.

Dr. Muhammad Azhar got his Mphil degree from Hazara University in 2015 and got very recently his PhD degree from the department of mathematics and statistics of Hazara university under the supervision of Dr. Muhammad Gulistan. He has recently published many good papers.

Seungmin Rho received his M.S. and Ph. D Degrees in Computer Science from Ajou University, Korea in 2003 and 2008, respectively. He visited Multimedia Systems and Networking Lab. in Univ. of Texas at Dallas from Dec. 2003 to March 2004. Before he joined the Computer Sciences Department of Ajou University, he spent two years in industry. In 2008–2009, he was a Postdoctoral Research Fellow at the Computer Music Lab of the School of Computer Science in Carnegie Mellon University. He had been working as a Research Professor at School of Electrical Engineering in Korea University during 2009–2011. In 2012, he was an assistant professor at Division of Information and Communication in Baekseok University. Now he is currently an assistant professor at Department of Media Software at Sungkyul University. His current research interests include database, big data analysis, music retrieval, multimedia systems, machine learning, knowledge management as well as computational intelligence. He has published 100 papers in refereed journals and conference proceedings in these areas. He has been involved in more than 20 conferences and workshops as various chairs and more than 30 conferences/workshops as a program committee member. He has edited a number of international journal special issues as a guest editor, such as Multimedia Systems, Information Fusion, Engineering Applications of Artificial Intelligence, New Review of Hypermedia and Multimedia, Multimedia Tools and Applications, Personal and Ubiquitous Computing, Telecommunication Systems, Ad Hoc & Sensor Wireless Networks and etc. He has received a few awards including Who's Who in America, Who's Who in Science and Engineering, and Who's Who in the World in 2007 and 2008, respectively.

SEIFEDINE KADRY received the bachelor's degree in applied mathematics from Lebanese University, in 1999, the M.S. degree in computation from Reims University, France, and EPFL, Lausanne, in 2002, the Ph.D. degree in applied statistics from Blaise Pascal University, France, in 2007, and the HDR degree from Rouen University, in 2017. He is currently an ABET Program Evaluator. His research interests include education using technology, system prognostics, stochastic systems, and probability and reliability analyses.