On $\alpha\omega$-closed sets and its connectedness in terms of neutrosophic topological spaces

[1]M. Parimala, [2]M. Karthika, [3]Florentin Smarandache and [4]Said Broumi
[1,2]Bannari Amman Institute of Technology, Sathyamangalam, India
[3]University of New Mexico, Gallup, USA
[4]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco
rishwanthpari@gmail.com[1], Karthikamuthsamy1991@gmail.com[2], fsmarandache@gmail.com [3], broumisaid78@gmail.com[4]

Abstract

The aim of this paper is to introduce the notion of neutrosophic $\alpha\omega$-closed sets and study some of the properties of neutrosophic $\alpha\omega$-closed sets. Further, we investigated neutrosophic $\alpha\omega$- continuity, neutrosophic $\alpha\omega$-irresoluteness, neutrosophic $\alpha\omega$ connectedness and neutrosophic contra $\alpha\omega$ continuity along with examples.

Keywords: neutrosophic topology, neutrosophic $\alpha\omega$-closed set, neutrosophic $\alpha\omega$-continuous function and neutrosophic contra $\alpha\omega$-continuous mappings.

1 Introduction

Zadeh [19] introduced truth (t) or the degree of membership of an object in fuzzy set theory. The falsehood (f) or the degree of non-membership of an object along with membership of an object introduced by Atanassov [4,5,6] in intuitionistic fuzzy set. Neutrosophic (i) or the degree of indeterminacy of an object along with membership and non-membership of an objects for incomplete, imprecise, indeterminate information is introduced by Smarandache [16,17] in 1998. The neutrosophic triplet set consist of three components $(t, f, i) = (truth, falsehood, indeterminacy)$. The neutrosophic topological spaces introduced and developed by Salama et al., [15]. This leads to many investigation among researchers in the field of neutrosophic topology and their application in decision making algorithms [8,11,12,13,14]. Arokiarani et al.,[3] introduced and studied α- open sets in neutrosophic topological spaces. Devi et al., [7,9,10] introduced $\alpha\omega$-closed sets in general topology, fuzzy topology and intuitionistic fuzzy topology. In this article, we introduce neutrosophic $\alpha\omega$-closed sets in neutrosophic topological spaces. Also, we introduce and investigate neutrosophic $\alpha\omega$-continuous,neutrosophic $\alpha\omega$-irresoluteness, neutrosophic $\alpha\omega$ connectedness and neutrosophic contra $\alpha\omega$-continuous mappings .

2 Preliminaries

Let (X, τ) be the neutrosophic topological space(NTS). Each neutrosophic set(NS) in (X, τ) is called a neutrosophic open set(NOS) and its complement is called a neutrosophic closed set (NCS).

We provide some of the basic definitions in neutrosophic sets. These are very useful in the sequel.

Definition 2.1. [17] A neutrosophic set (NS) A is an object of the following form

$$U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u)\rangle : u \in X\}$$

where the mappings $\mu_U : X \to I$, $\nu_U : X \to I$ and $\omega_U : X \to I$ denote the degree of membership (namely $\mu_U(u)$), the degree of indeterminacy (namely $\nu_U(u)$) and the degree of nonmembership (namely $\omega_U(u)$) for

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each element $u \in X$ to the set U, respectively and $0 \leq \mu_U(u) + \nu_U(u) + \omega_U(u) \leq 3$ for each $u \in X$.

Definition 2.2. [17] Let U and V be NSs of the form $U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u)\rangle : u \in X\}$ and $V = \{\langle u, \mu_V(u), \nu_V(u), \omega_V(u)\rangle : u \in X\}$. Then

(i) $U \subseteq V$ if and only if $\mu_U(u) \leq \mu_V(u)$, $\nu_U(u) \geq \nu_V(u)$ and $\omega_U(u) \geq \omega_V(u)$;

(ii) $\overline{U} = \{\langle u, \nu_U(u), \mu_U(u), \omega_U(u)\rangle : u \in X\}$;

(iii) $U \cap V = \{\langle u, \mu_U(u) \wedge \mu_V(u), \nu_U(u) \vee \nu_V(u), \omega_U(u) \vee \omega_V(u)\rangle : u \in X\}$;

(iv) $U \cup V = \{\langle u, \mu_U(u) \vee \mu_V(u), \nu_U(u) \wedge \nu_V(u), \omega_U(u) \wedge \omega_V(u)\rangle : u \in X\}$.

We will use the notation $U = \langle u, \mu_U, \nu_U, \omega_U\rangle$ instead of $U = \{\langle u, \mu_U(u), \nu_U(u), \omega_U(u)\rangle : u \in X\}$. The NSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{\langle u, \underline{0}, \underline{1}, \underline{1}\rangle : u \in X\}$ and $1_\sim = \{\langle u, \underline{1}, \underline{0}, \underline{0}\rangle : u \in X\}$.

Let $r, s, t \in [0, 1]$ such that $0 \leq r + s + t \leq 3$. A neutrosophic point (NP) $p_{(r,s,t)}$ is neutrosophic set defined by

$$p_{(r,s,t)}(u) = \begin{cases} (r, s, t)(x) & if\ u = p \\ (0, 1, 1) & otherwise \end{cases}$$

Let f be a mapping from an ordinary set X into an ordinary set Y, If $V = \{\langle y, \mu_V(y), \nu_V(y), \omega_V(y)\rangle : y \in Y\}$ is a NS in Y , then the inverse image of V under f is a NS defined by

$$f^{-1}(V) = \{\langle u, f^{-1}(\mu_V)(u), f^{-1}(\nu_V)(u), f^{-1}(\omega_V)(u)\rangle : u \in X\}$$

The image of NS $U = \{\langle v, \mu_U(v), \nu_U(v), \omega_U(v)\rangle : v \in Y\}$ under f is a NS defined by $f(U) = \{\langle v, f(\mu_U)(v), f(\nu_U)(v), f(\omega_U)$ $v \in Y\}$ where

$$f(\mu_U)(v) = \begin{cases} \sup\limits_{u \in f^{-1}(v)} \mu_U(u), & if\ f^{-1}(v) \neq 0 \\ 0 & otherwise, \end{cases}$$

$$f(\nu_U)(v) = \begin{cases} \inf\limits_{u \in f^{-1}(v)} \nu_U(u), & if\ f^{-1}(v) \neq 0 \\ 1 & otherwise, \end{cases}$$

$$f(\omega_U)(v) = \begin{cases} \inf\limits_{u \in f^{-1}(v)} \omega_U(u), & if\ f^{-1}(v) \neq 0 \\ 1 & otherwise, \end{cases}$$

for each $v \in Y$.

Definition 2.3. [15] A neutrosophic topology (NT) in a nonempty set X is a family τ of NSs in X satisfying the following axioms:

(NT1) $0_\sim, 1_\sim \in \tau$;

(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(NT3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

Definition 2.4. [15] Let U be a NS in NTS X. Then
Nint(U) = $\cup\{O : O$ is an NOS in X and O $\subseteq U\}$ is called a neutrosophic interior of U;
Ncl(U) = $\cap\{O : O$ is an NCS in X and O $\supseteq U\}$ is called a neutrosophic closure of U.

Definition 2.5. [15] Let $p_{(r,s,t)}$ be a NP in NTS X. A NS U in X is called a neutrosophic neighborhood (NN) of $p_{(r,s,t)}$ if there exists a NOS V in X such that $p_{(r,s,t)} \in V \subseteq U$.

Definition 2.6. [3] A subset U of a neutrosophic space (X, τ) is called

1. a neutrosophic pre-open set if $U \subseteq Nint(Ncl(U))$ and a neutrosophic pre-closed set if $Ncl(Nint(U)) \subseteq U$,

2. a neutrosophic semi-open set if $U \subseteq Ncl(Nint(U))$ and a neutrosophic semi-closed set if $Nint(Ncl(U)) \subseteq U$,

3. a neutrosophic α-open set if $U \subseteq Nint(Ncl(Nint(U)))$ and a neutrosophic α-closed set if $Ncl(Nint(Ncl(U))) \subseteq U$,

The pre-closure (resp. semi-closure, α-closure) of a subset U of a neutrosophic space (X,τ) is the intersection of all pre-closed (resp. semi-closed, α-closed) sets that contain U and is denoted by $Npcl(U)$ (resp. $Nscl(U)$, $N\alpha cl(U)$).

3 On neutrosophic $\alpha\omega$-closed sets

Definition 3.1. A subset A of a neutrosophic topological space (X,τ) is called

1. a neutrosophic $N\omega$-closed set if $Ncl(U) \subseteq G$ whenever $U \subseteq G$ and G is neutrosophic semi-open in (X,τ).
2. a neutrosophic $\alpha\omega$-closed ($N\alpha\omega$-closed) set if $N\omega cl(U) \subseteq G$ whenever $U \subseteq G$ and G is an $N\alpha$-open set in (X,τ). Its complement is called a neutrosophic $\alpha\omega$-open ($N\alpha\omega$-open) set.

Definition 3.2. Let U be a NS in NTS X. Then
$N\alpha\omega int(U) = \cup\{O : O$ is an $N\alpha\omega$OS in X and O $\subseteq U\}$ is said to be a neutrosophic $\alpha\omega$-interior of U;
$N\alpha\omega cl(U) = \cap\{O : O$ is an $N\alpha\omega$CS in X and O $\supseteq U\}$ is said to be a neutrosophic $\alpha\omega$-closure of U.

Theorem 3.3. *Every $N\alpha$-closed set and N-closed set are $N\alpha\omega$-closed set.*
Proof. Let U be an $N\alpha$-closed set, then $U = N\alpha cl(U)$. Let $U \subseteq G$, G is $N\alpha$-open. Since U is $N\alpha$-closed, $N\omega cl(U) \subseteq N\alpha cl(U) \subseteq G$. Thus U is $N\alpha\omega$-closed.

Theorem 3.4. *Every neutrosophic semi-closed set in a neutrosophic set is an $N\alpha\omega$-closed.*
Proof. Let U be a Nsemi-closed set in (X,τ), then $U = Nscl(U)$. Let $U \subseteq G$, G is $N\alpha$-open in (X,τ). Since U is Nsemi-closed, $N\omega cl(U) \subseteq Nscl(U) \subseteq G$. This shows that U is $N\alpha\omega$-closed set.
The converses of the above theorems are not true as explained in Example 3.5.

Example 3.5. Let $X = \{u, v, w\}$ and neutrosophic sets A, B, C be defined by:

$$A = \langle(0.1, 0.4, 0.7), (0.9, 0.6, 0.3), (0.9, 0.6, 0.3)\rangle$$
$$B = \langle(0.6, 0.6, 0.4), (0.2, 0.7, 0.8), (1, 0.6, 0.5)\rangle$$
$$C = \langle(0.1, 0.4, 0.8), (0.2, 0.6, 0.4), (0.6, 0.5, 0.9)\rangle$$

Let $\tau = \{0_\sim, A, 1_\sim\}$. Then B is $N\alpha\omega$-closed in (X,τ) but not $N\alpha$-closed and thus it is not N-closed and C is $N\alpha\omega$-closed in (X,τ) but not $Nsemi$-closed.

Theorem 3.6. Let (X,τ) be a NTS and let $U \in NS(X)$. If U is $N\alpha\omega$-closed set and $U \subseteq V \subseteq N\omega cl(U)$, then V is $N\alpha\omega$-closed set.
Proof. Let G be a $N\alpha$-open set such that $V \subseteq G$. Since $U \subseteq V$, then $U \subseteq G$. But U is $N\alpha\omega$-closed, so $N\omega cl(U) \subseteq G$. Since $V \subseteq N\omega cl(U)$. Since $N\omega cl(V) \subseteq N\omega cl(U)$ and hence $N\omega cl(V) \subseteq G$. Therefore V is a $N\alpha\omega$-closed set.

Theorem 3.7. Let U be a $N\alpha\omega$-open set in X and $N\omega int(U) \subseteq V \subseteq U$, then V is $N\alpha\omega$-open.
Proof. Suppose U is $N\alpha\omega$-open in X and $N\omega int(U) \subseteq V \subseteq U$. Then $\overline{U}$ is $N\alpha\omega$-closed and $\overline{U} \subseteq \overline{V} \subseteq N\omega cl(\overline{U})$. Then $\overline{U}$ is a $N\alpha\omega$-closed set by theorem 3.5. Hence V is a $N\alpha\omega$-open set in X.

Theorem 3.8. A NS U in a NTS (X,τ) is a $N\alpha\omega$-open set if and only if $V \subseteq N\omega int(U)$ whenever V is a $N\alpha$-closed set and $V \subseteq U$.
Proof. Let U be a $N\alpha\omega$-open set and let V be a $N\alpha$-closed set such that $V \subseteq U$. Then $\overline{U} \subseteq \overline{V}$ and hence $N\omega cl(\overline{U}) \subseteq \overline{V}$, since $\overline{U}$ is $N\alpha\omega$-closed. But $N\omega cl(\overline{U}) = \overline{N\omega int(U)}$, thus $V \subseteq N\omega int(U)$.
Conversely, suppose that the condition is satisfied, then $\overline{N\omega int(U)} \subseteq \overline{V}$ whenever $\overline{V}$ is $N\alpha$-open set and $\overline{U} \subseteq \overline{V}$. This implies that $N\omega cl(\overline{U}) \subseteq \overline{V} = G$ where G is $N\alpha$-open set and $\overline{U} \subseteq G$. Therefore $\overline{U}$ is $N\alpha\omega$-closed set and hence U is $N\alpha\omega$-open.

Theorem 3.9. Let U be a $N\alpha\omega$-closed subset of (X,τ). Then $N\omega cl(U) - U$ does not contain any non-empty $N\alpha\omega$-closed set.
Proof. Assume that U is a $N\alpha\omega$-closed set. Let F be a non-empty $N\alpha\omega$-closed set, such that $F \subseteq$

$N\omega cl(U) - U = N\omega cl(U) \cap \overline{U}$. i.e., $F \subseteq N\omega cl(U)$ and $F \subseteq \overline{U}$. Therefore, $U \subseteq \overline{F}$. Since $\overline{F}$ is a $N\alpha\omega$-open set, $N\omega cl(U) \subseteq \overline{F} \Rightarrow F \subseteq (N\omega cl(U) - U) \cap (\overline{N\omega cl(U)}) \subseteq N\omega cl(U) \cap \overline{N\omega cl(U)}$. i.e., $F \subseteq \phi$. Therefore F is empty.

Corollary 3.10. Let U be a $N\alpha\omega$-closed set of (X, τ). Then $N\omega cl(U) - U$ does not contain no non-empty N-closed set.
Proof. The proof follows from the Theorem 3.9.

Theorem 3.11. If U is both $N\omega$-open and $N\alpha\omega$-closed set, then U is a $N\omega$-closed set.
Proof. Since U is both $N\omega$-open and $N\alpha\omega$-closed set in X, then $N\omega cl(U) \subseteq U$. Also we have $U \subseteq N\omega cl(U)$. This gives that $N\omega cl(U) = U$. Therefore U is a $N\omega$-closed set in X.

4 On neutrosophic $\alpha\omega$-continuity, connectedness and contra-continuity

Definition 4.1. Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces.

1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be a neutrosophic $\alpha\omega$-continuous (briefly, $N\alpha\omega$-continuous) function if the inverse image of every open set in Y is a $N\alpha\omega$-open set in X.
   Equivalently, if the inverse image of every open set in (Y, σ) is $N\alpha\omega$-open in (X, τ);

2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be a neutrosophic $\alpha\omega$-irresolute (briefly, $N\alpha\omega$-irresolute) function if the inverse image of every $N\alpha\omega$-open set in Y is a $N\alpha\omega$-open set in X.
   Equivalently, if the inverse image of every $N\alpha\omega$-open set in (Y, σ) is $N\alpha\omega$-open in (X, τ);

Definition 4.2. A NTS (X, τ) is said to be neutrosophic-$\alpha\omega T_{1/2}$($N\alpha\omega T_{1/2}$ in short) space if every $N\alpha\omega C$ in X is an NC in X.

Definition 4.3. Let (X, τ) be any neutrosophic topological space. (X, τ) is said to be neutrosophic $\alpha\omega$-disconnected (in shortly $N\alpha\omega$-disconnected) if there exists a $N\alpha\omega$-open and $N\alpha\omega$-closed set $\overline{F}$ such that $\overline{F} \neq 0_\sim$ and $\overline{F} \neq 1_\sim$. (X, τ) is said to be neutrosophic $\alpha\omega$-connected if it is not neutrosophic $\alpha\omega$-disconnected.

Theorem 4.4. Every $N\alpha\omega$-connected space is neutrosophic connected.
Proof. For a $N\alpha\omega$-connected (X, τ) space and let (X, τ) not be neutrosophic connected. Hence, there exists a proper neutrosophic set, $\overline{F} =< \mu_{\overline{F}(x)}, \sigma_{\overline{F}(x)}, \nu_{\overline{F}(x)} >$, $\overline{F} \neq 0_\sim$ and $\overline{F} \neq 1_\sim$, such that $\overline{F}$ is both neutrosophic open and neutrosophic closed in (X, τ). Since every neutrosophic open set is $N\alpha\omega$-open and neutrosophic closed set is $N\alpha\omega$-closed, X is not $N\alpha\omega$-connected. Therefore, (X, τ) is neutrosophic connected.
However, the converse is not true.

Example 4.5. Let $X = \{u, v, w\}$ and neutrosophic sets A, B and C be defined by:

$$A = \langle (0.4, 0.5, 0.5), (0.4, 0.5, 0.5), (0.5, 0.5, 0.5) \rangle$$
$$B = \langle (0.7, 0.6, 0.5), (0.7, 0.6, 0.5), (0.3, 0.4, 0.5) \rangle$$
$$C = \langle (0.5, 0.6, 0.5), (0.5, 0.6, 0.5), (0.5, 0.6, 0.5) \rangle$$

Let $\tau = \{0_\sim, A, B, 1_\sim\}$. It is obvious that (X, τ) is NTS. Now, (X, τ) is neutrosophic connected. However, it is not a $N\alpha\omega$-connected.

Theorem 4.6. Let (X, τ) be a neutrosophic $\alpha\omega T_{1/2}$ space. (X, τ) is neutrosophic connected iff (X, τ) is $N\alpha\omega$-connected.

Proof. Let (X, τ) is neutrosophic connected. Suppose that (X, τ) is not $N\alpha\omega$-connected, and there exists a neutrosophic set $\overline{F}$ which is both $N\alpha\omega$-open and $N\alpha\omega$-closed. Since (X, τ) is neutrosophic $\alpha\omega T_{1/2}$, $\overline{F}$ is both neutrosophic open and neutrosophic closed. Therefore, (X, τ) is not a neutrosophic connected which is contradiction to our hypothesis. Hence, (X, τ) is $N\alpha\omega$-connected.
Conversely, let (X, τ) is $N\alpha\omega$-connected. Suppose that (X, τ) is not neutrosophic connected, and there exists a neutrosophic set $\overline{F}$ such that $\overline{F}$ is both NCs and NOs $\in (X, \tau)$. Since the neutrosophic open set is $N\alpha\omega$-open and the neutrosophic closed set is $N\alpha\omega$-closed, (X, τ) is not $N\alpha\omega$-connected. Hence, (X, τ) is neutrosophic connected.

Theorem 4.7. Suppose (X, τ) and (Y, σ) are any two NTSs. If $g : (X, \tau) \to (Y, \sigma)$ is $N\alpha\omega$-continuous surjection and (X, τ) is $N\alpha\omega$-connected, then (Y, σ) is neutrosophic connected.
Proof. Suppose that (Y, σ) is not neutrosophic connected, such that the neutrosophic set $\overline{F}$ is both neutrosophic open and neutrosophic closed in (Y, σ). Since g is $N\alpha\omega$-continuous, $g^{-1}(\overline{F})$ is $N\alpha\omega$-open and $N\alpha\omega$-closed in (Y, σ). Thus, (Y, σ) is not $N\alpha\omega$-connected. Hence, (Y, σ) is neutrosophic connected.

Theorem 4.8. Let $g : (X, \tau) \to (Y, \sigma)$ be a function. Then the following conditions are equivalent.

(i) g is $N\alpha\omega$-continuous;

(ii) The inverse $f^{-1}(U)$ of each N-open set U in Y is $N\alpha\omega$-open set in X.

Proof. It is clear, since $g^{-1}(\overline{U}) = \overline{g^{-1}(U)}$ for each N-open set U of Y.

Theorem 4.9. If $g : (X, \tau) \to (Y, \sigma)$ be a $N\alpha\omega$-continuous mapping, then the following statements holds:

(i) $g(N\alpha\omega Ncl(U)) \subseteq Ncl(g(U))$, for all neutrosophic set U in X;

(ii) $N\alpha\omega Ncl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$, for all neutrosophic set V in Y.

Proof.

(i) Since $Ncl(g(U))$ is neutrosophic closed set in Y and g is $N\alpha\omega$-continuous, then $g^{-1}(Ncl(g(U)))$ is $N\alpha\omega$-closed in X. Now, since $U \subseteq g^{-1}(Ncl(g(U)))$. So, $N\alpha\omega cl(U) \subseteq g^{-1}(Ncl(g(U)))$. Therefore, $g(N\alpha\omega Ncl(U)) \subseteq Ncl(g(U))$.

(ii) By replacing U with V in (i), we obtain $g(N\alpha\omega cl(g^{-1}(V))) \subseteq Ncl(g(g^{-1}(V))) \subseteq Ncl(V)$. Hence $N\alpha\omega cl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$.

Theorem 4.10. Let g be a function from a NTS (X, τ) to a NTS (Y, σ). Then the following statements are equivalent.

(i) g is a neutrosophic $\alpha\omega$-continuous function.

(ii) For every NP $p_{(r,s,t)} \in X$ and each NN U of $g(p_{(r,s,t)})$, there exists a $N\alpha\omega$-open set V such that $p_{(r,s,t)} \in V \subseteq g^{-1}(U)$.

(iii) For every NP $p_{(r,s,t)} \in X$ and each NN U of $g(p_{(r,s,t)})$, there exists a $N\alpha\omega$-open set V such that $p_{(r,s,t)} \in V$ and $g(V) \subseteq U$.

Proof. $(i) \Rightarrow (ii)$. If $p_{(r,s,t)}$ is a NP in X and also if U be a NN of $g(p_{(r,s,t)})$, then there exists a NOS W in Y such that $g(p_{(r,s,t)}) \in W \subset U$. we have g is neutrosophic $\alpha\omega$-continuous, $V = g^{-1}(W)$ is an $N\alpha\omega OS$ and

$$p_{(r,s,t)} \in g^{-1}(g(p_{(r,s,t)})) \subseteq g^{-1}(W) = V \subseteq g^{-1}(U).$$

Thus (ii) is a valid statement.
$(ii) \Rightarrow (iii)$. Let $p_{(r,s,t)}$ be a NP in X and take U be a NN of $g(p_{(r,s,t)})$. Then there exists a $N\alpha\omega OS\ U$ such that $p_{(r,s,t)} \in V \subseteq g^{-1}(U)$ by (ii). Thus, we have $p_{(r,s,t)} \in V$ and $g(V) \subseteq g(g^{-1}(U)) \subseteq U$. Hence (iii) is valid.
$(iii) \Rightarrow (i)$. Let V be a NOS in Y and let $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in g(g^{-1}(V)) \subset V$. Since V is a NOS, it follows that V is a NN of $g(p_{(r,s,t)})$ so from (iii), there exists a $N\alpha\omega OS\ U$ such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. This implies that

$$p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V).$$

Then, we know that $g^{-1}(V)$ is a $N\alpha\omega OS$ in X. Thus g is neutrosophic $\alpha\omega$-continuous.

Definition 4.11. A function is said to be a neutrosophic contra $\alpha\omega$-continuous function if the inverse image of each NOS V in Y is a N$\alpha\omega$CS in X.

Theorem 4.12. Let $g : (X, \tau) \to (Y, \sigma)$ be a function. Then, the following assertions are equivalent:

(i) g is a neutrosophic contra $\alpha\omega$-continuous function;

(ii) $g^{-1}(V)$ is a N$\alpha\omega$ CS in X, for each NOS V in Y.

Proof. $(i) \Rightarrow (ii)$ Let g be any neutrosophic contra $\alpha\omega$-continuous function and let V be any NOS in Y. Then, $\overline{V}$ is a NCS in Y. By the assumption $g^{-1}(\overline{V})$ is a $N\alpha\omega OS$ in X. Hence, we get that $g^{-1}(V)$ is a $N\alpha\omega CS$ in X.

The converse of the theorem can be done in the same sense.

Theorem 4.13. Let $g : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an NTS X into an NTS Y. The mapping g is neutrosophic contra $\alpha\omega$-continuous if $Ncl(g(U)) \subseteq g(N\alpha\omega int(U))$, for each NS U in X.
Proof. Let V be any NCS in X. Then, $Ncl(V) = V$, and also g is onto, by assumption, it shows that $g(N\alpha\omega int(g^{-1}(V))) \supseteq Ncl(g(g^{-1}(V))) = Ncl(V) = V$. Hence $g^{-1}(g(N\alpha\omega int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Since g is an into mapping, we have $N\alpha\omega int(g^{-1}(V)) = g^{-1}(g(N\alpha\omega int(g^{-1}(V)))) \supseteq g^{-1}(V)$. Therefore $N\alpha\omega int(g^{-1}(V))$
$= g^{-1}(V)$, so $g^{-1}(V)$ is a $N\alpha\omega$OS in X. Hence g is a neutrosophic contra $\alpha\omega$-continuous mapping.

Theorem 4.14. Let $g : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent:

(i) g is a neutrosophic contra $\alpha\omega$-continuous mapping;

(ii) for each NP $p_{(r,s,t)}$ in X and NCS V containing $g(p_{(r,s,t)})$ there exists $N\alpha\omega OS\ U$ in X containing $p_{(r,s,t)}$ such that $A \subseteq f^{-1}(B)$;

(iii) for each NP $p_{(r,s,t)}$ in X and NCS V containing $p_{(r,s,t)}$ there exists $N\alpha\omega OS\ U\ in\ X$ containing $p_{(r,s,t)}$ such that $g(U) \subseteq V$.

Proof. $(i) \Rightarrow (ii)$ Let g be an neutrosophic contra $\alpha\omega$-continuous mapping, let V be any NCS in Y and let $p_{(r,s,t)}$ be a NP in X and such that $g(p_{(r,s,t)}) \in V$. Then $p_{(r,s,t)} \in g^{-1}(V) = N\alpha\omega int(g^{-1}(V))$. Let $U = N\alpha\omega int(g^{-1}(V))$. Then U is an $N\alpha\omega OS$ and $U = N\alpha\omega int(g^{-1}(V)) \subseteq g^{-1}(V)$.
$(ii) \Rightarrow (iii)$ The results follows from the evident relations $g(U) \subseteq g(g^{-1}(V)) \subseteq V$.
$(iii) \Rightarrow (i)$ Let V be any NCS in Y and let $p_{(r,s,t)}$ be a NP in X such that $p_{(r,s,t)} \in g^{-1}(V)$. Then $g(p_{(r,s,t)}) \in V$. According to the assumption, there exists an $N\alpha\omega OS\ U$ in X such that $p_{(r,s,t)} \in U$ and $g(U) \subseteq V$. Hence $p_{(r,s,t)} \in U \subseteq g^{-1}(g(U)) \subseteq g^{-1}(V)$. Therefore $p_{(r,s,t)} \in U = \alpha\omega int(U) \subseteq N\alpha\omega int(g^{-1}(V))$. Since, $p_{(r,s,t)}$ is an arbitrary NP and $g^{-1}(V)$ is the union of all NPs in $g^{-1}(V)$, we obtain that $g^{-1}(V) \subseteq N\alpha\omega int(g^{-1}(V))$. Thus g is a neutrosophic contra $N\alpha\omega$-continuous mapping.

Corollary 4.15. Let $X,\ X_1\ and\ X_2$ be NTSs, $p_1 : X \to X_1 \times X_2\ (i = 1,\ 2)$ and $p_2 : X \to X_1 \times X_2$ are the projections of $X_1 \times X_2$ onto X_i, $(i = 1,\ 2)$. If $g : X \to X_1 \times X_2$ is a neutrosophic contra $\alpha\omega$-continuous, then $p_i g$ are also neutrosophic contra $\alpha\omega$-continuous mapping.
Proof. The proof follows from the fact that the projections are all neutrosophic continuous functions.

Theorem 4.16. Let $g : (X_1, \tau) \to (Y_1, \sigma)$ be a function. If the graph $h : X_1 \to X_1 \times Y_1$ of g is neutrosophic contra $\alpha\omega$-continuous, then g is neutrosophic contra $\alpha\omega$-continuous.
Proof. For every NOS V in Y_1 holds $g^{-1}(V) = 1 \wedge g^{-1}(V) = h^{-1}(1 \times V)$. Since h is a neutrosophic contra $\alpha\omega$-continuous mapping and $1 \times V$ is a NOS in $X_1 \times Y_1$, $g^{-1}(V)$ is a $N\alpha\omega CS$ in X_1, so g is a neutrosophic contra $\alpha\omega$-continuous mapping.

5 Conclusions

In this paper, we introduced and investigated the neutrosophic $\alpha\omega$ closed sets and its properties. Also, we investigated the continuity, irresolute, connectedness and contra-continuity in terms of neutrosophic $\alpha\omega$ closed sets.

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