On Neutrosophic Supra *Pre*-Continuous Functions in Neutrosophic Topological Spaces

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ABSTRACT

In this paper, we introduce and investigate a new class of sets and functions between topological space called neutrosophic supra pre- continious functions. Furthermore, the concepts of neutrosophic supra pre-open maps and neutrosophic supra pre-closed maps in terms of neutrosophic supra pre-open sets and neutrosophic supra pre-closed sets, respectively, are introduced and several properties of them are investigated.

KEYWORDS: Neutrosophic supra topological spaces,neutrosophic supra *pre*-open sets and neutrosophic supra *pre*-continuous maps.

1 INTRODUCTION AND PRELIMINARIES

Intuitionistic fuzzy set is defined by Atanassov (1986) as a generalization of the concept of fuzzy set given by Zadeh (1965). Using the notation of intuitionistic fuzzy sets, Çoker

(1997) introduced the notation of intuitionistic fuzzy topological spaces. The supra topological spaces and studied s-continuous functions and s^*- continuous functions were introduced by Mashhour, Allam, Mahmoud, and Khedr (1983). El-Monsef and Ramadan (1987) introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and obtained some properties and characterizations. Min (1996) introduced fuzzy s-continuous, fuzzy s-open and fuzzy s-closed maps and established a number of characterizations. Devi, Sampathkumar, and Caldas (2008) introduced the concept of supra α-open set, and Mashhour et al. (1983) introduced, the notion of supra- semi open set, supra semi-continuous functions and studied some of the basic properties for this class of functions. Turnal (2003) introduced the concept of intuitionistic fuzzy supra topological space. After the introduction of the neutrosophic set concept (Salama & Alblowi, 2012; Smarandache, 1999). The concepts of Neutrosophic Set and Neutrosophic Topological Spaces was introduced by (Salama & Alblowi, 2012).

In this paper, we introduce and investigate a new class of sets and functions between topological space called neutrosophic supra semi-open set and neutrosophic supra semi-open continuous functions respectively.

Definition 1. Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition 2. Let X be a nonempty fixed set. A neutrosophic set [NS for short] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 1. (1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 3. Let X be a nonempty set and the neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [Complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 4. Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 5. (Dhavaseelan & Jafari, in press) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 6. (Dhavaseelan & Jafari, in press) A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (NTS) and each neutrosophic set in T is called a neutrosophic open set (NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (NCS) in X.

Definition 7. (Dhavaseelan & Jafari, in press) Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 8. Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point(in short NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$.where r denotes the degree of membership value ,t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Now we shall define the image and preimage of neutrosophic sets. Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

Definition 9. (Dhavaseelan & Jafari, in press)

(a) If $B = \{\langle y, \mu_B(y), \sigma_B(y), \gamma_B(y)\rangle : y \in Y\}$ is a neutrosophic set in Y,then the preimage of B under f, denoted by $f^{-1}(B)$, is the neutrosophic set in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(b) If $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ is a neutrosophic set in X,then the image of A under f, denoted by $f(A)$, is the neutrosophic set in Y defined by $f(A) = \{\langle y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)\rangle : y \in Y\}$. where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \sigma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise,} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\gamma_A)$ for $1 - f(1 - \gamma_A)$.

Corollary 1. (Dhavaseelan & Jafari, in press) Let A , $A_i(i \in J)$ be neutrosophic sets in X, B, $B_i(i \in K)$ be neutrosophic sets in Y and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(c) $A \subseteq f^{-1}(f(A))$ { If f is injective,then $A = f^{-1}(f(A))$ } ,

(d) $f(f^{-1}(B)) \subseteq B$ { If f is surjective,then $f(f^{-1}(B)) = B$ },

(e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$,

(f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$,

(g) $f(\bigcup A_i) = \bigcup f(A_i)$,

(h) $f(\bigcap A_i) \subseteq \bigcap f(A_i)$ { If f is injective,then $f(\bigcap A_i) = \bigcap f(A_i)$},

(i) $f^{-1}(1_N) = 1_N$,

(j) $f^{-1}(0_N) = 0_N$,

(k) $f(1_N) = 1_N$, if f is surjective

(l) $f(0_N) = 0_N$,

(m) $\overline{f(A)} \subseteq f(\overline{A})$, if f is surjective,

(n) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

2 NEUTROSOPHIC SUPRA PRE-OPEN SET.

In this section, we introduce a new class of open sets called neutrosophic supra pre-open sets and study some of their basic properties.

Definition 2.1. Let (X, τ) be an neutrosophic supra topological space. A set A is called an neutrosophic supra pre-open set (briefly NSPOS) if $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$. The complement of an neutrosophic supra pre-open set is called an neutrosophic supra pre-closed set (briefly NSPCS).

Theorem 2.2. Every neutrosophic supra-open set is neutrosophic supra pre-open.

Proof. Let A be an neutrosophic supra-open set in (X, τ). Then $A \subseteq s\text{-}Nint(A))$, we get $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$ then $s\text{-}Nint(A)) \subseteq s\text{-}Nint(s\text{-}Ncl(A))$. Hence A is neutrosophic supra pre-open in (X, τ).

The converse of the above theorem need not be true as shown by the following example.

Example 2.3.

Let
$X = \{a, b\}, A = \{x, \langle 0.5, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.3, 0.4\rangle\}, B = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}$ and $C = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.2, 0.5\rangle\}$, $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called neutrosophic supra pre-open set but it is not neutrosophic supra -open set.

Theorem 2.4. Every neutrosophic supra α-open set is neutrosophic supra pre-open

Proof. Let A be an neutrosophic supra α-open set in (X, τ). Then $A \subseteq s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A))$, it is obvious that $s\text{-}Nint(s\text{-}Ncl(s\text{-}Nint(A)) \subseteq s\text{-}Nint(s\text{-}Ncl(A))$ and $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$. Hence A is neutrosophic supra pre-open in (X, τ).

The converse of the above theorem need not be true as shown by the following example.

Example 2.5.

Let $X = \{a, b\}, A = \{x, \langle 0.3, 0.5\rangle, \langle 0.3, 0.5\rangle, \langle 0.4, 0.5\rangle\}$,
$B = \{x, \langle 0.4, 0.3\rangle, \langle 0.4, 0.3\rangle, \langle 0.5, 0.4\rangle\}$ and $C = \{x, \langle 0.4, 0.5\rangle, \langle 0.4, 0.5\rangle, \langle 0.5, 0.4\rangle\}$,

$\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called neutrosophic supra *pre*-open set but it is not neutrosophic supra α-open set.

Theorem 2.6. Every neutrosophic supra *pre*-open set is neutrosophic supra β-open
Proof. Let A be an neutrosophic supra *pre*-open set in (X, τ). It is obvious that $s\text{-}Nint(s\text{-}Ncl(A)) \subseteq s\text{-}Ncl(s\text{-}Nint(s\text{-}Ncl(A))$. Then $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$. Hence $A \subseteq s\text{-}Ncl(s\text{-}Nint(s\text{-}Ncl(A))$.
The converse of the above theorem need not be true as shown by the following example.
Example 2.7.

Let $X = \{a, b\}, A = \{x, \langle 0.2, 0.3\rangle, \langle 0.2, 0.3\rangle, \langle 0.5, 0.3\rangle\}$,
$B = \{x, \langle 0.1, 0.2\rangle, \langle 0.1, 0.2\rangle, \langle 0.6, 0.5\rangle\}$ and $C = \{x, \langle 0.2, 0.3\rangle, \langle 0.2, 0.3\rangle, \langle 0.2, 0.3\rangle\}$,
$\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called neutrosophic supra *pre*-open set but it is not neutrosophic supra *pre*-open set.

Theorem 2.8. Every neutrosophic supra *pre*-open set is neutrosophic supra b-open
Proof. Let A be an neutrosophic supra *pre*-open set in (X, τ). It is obvious that $s\text{-}Nint(s\text{-}Ncl(A)) \subseteq s\text{-}Nint(s\text{-}Ncl(A)) \cup s\text{-}Ncl(s\text{-}Nint(A)))$. Then $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$Then $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$. Hence $A \subseteq s\text{-}Nint(s\text{-}Ncl(A)) \cup s\text{-}Ncl(s\text{-}Nint(A)))$.
The converse of the above theorem need not be true as shown by the following example.
Example 2.9.

Let
$X = \{a, b\}, A = \{x, \langle 0.5, 0.2\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}, B = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}$ and $C = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.4, 0.4\rangle\}$, $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called neutrosophic supra *pre*-open set but it is not neutrosophic supra *pre*-open set.

Theorem 2.10.

(i) Arbitrary union of neutrosophic supra *pre*-open sets is always neutrosophic supra *pre*-open.

(ii) Finite intersection of neutrosophic supra *pre*-open sets may fail to be neutrosophic supra *pre*-open.

Proof.

(i) Let A and B to be neutrosophic supra *pre*-open sets. Then $A \subseteq s\text{-}Nint(s\text{-}Ncl(A))$ and $B \subseteq s\text{-}Nint(s\text{-}Ncl(B))$. Then $A \cup B \subseteq s\text{-}Nint(s\text{-}Ncl(A))$. Therefore, $A \cup B$ is neutrosophic supra *pre*-open sets.

(ii) Let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.2, 0.5\rangle\}$,

$B = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.4, 0.4\rangle\}$

and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$.

Hence A and B are neutrosophic supra pre-open but $A \cap B$ is not neutrosophic supra pre-open set.

Theorem 2.11.

(i) Arbitrary intersection of neutrosophic supra pre-closed sets is always neutrosophic supra pre-closed.

(ii) Finite union of neutrosophic supra pre-closed sets may fail to be neutrosophic supra pre-closed.

Proof.

(i) This proof immediately from Theorem 2.10

(ii) Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.3\rangle, \langle 0.2, 0.3\rangle, \langle 0.2, 0.4\rangle\}$, $B = \{x, \langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.4, 0.5\rangle\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Hence A and B are neutrosophic supra pre-closed but $A \cup B$ is not neutrosophic supra pre-closed set.

Definition 2.12. The neutrosophic supra pre-closure of a set A, denoted by s-pre-$Ncl(A)$, is the intersection of neutrosophic supra pre-closed sets including A. The neutrosophic supra pre-interior of a set A, denoted by s-pre-$Nint(A)$, is the union of neutrosophic supra pre-open sets included in A.

Remark 2. It is clear that s-pre-$Nint(A)$ is an neutrosophic supra pre-open set and s-pre-$Ncl(A)$ is an neutrosophic supra pre-closed set.

Theorem 2.14.

(i) $A \subseteq s$-pre-$Ncl(A)$; and $A =s$-pre-$Ncl(A)$ iff A is an neutrosophic supra pre-closed set;

(ii) s-pre-$Nint(A) \subseteq A$; and s-pre-$Nint(A) = A$ iff A is an neutrosophic supra pre-open set;

(iii) $X - s$-pre-$Nint(A) = s$-pre-$Ncl(X - A)$;

(iv) $X - s$-pre-$Ncl(A) = s$-pre-$Nint(X - A)$.

Proof. It is obvious.

Theorem 2.15.

(i) *s-pre-*$Nint(A) \cup$ *s-pre-*$Nint(B) \subseteq$ *s-pre-*$Nint(A \cup B)$;

(i) *s-pre-*$Ncl(A \cap B) \subseteq$ *s-pre-*$Ncl(A) \cap$ *s-pre-*$Ncl(B)$.

Proof It is obvious.
The inclusions in (i) and (ii) in Theorem 2.15 can not replaced by equalities by let $X = \{a, b\}$, $A = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.2, 0.5\rangle\}$, $B = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.4, 0.4\rangle\}$ and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$, where
s-pre-$Nint(A) = \{x, \langle 0.2, 0.5\rangle, \langle 0.2, 0.5\rangle, \langle 0.3, 0.4\rangle\}$,
s-pre-$Nint(B) = \{x, \langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.4, 0.5\rangle\}$ and
s-pre-$Nint(A \cup B) = \{x, \langle 0.5, 0.5\rangle, \langle 0.5, 0.5\rangle, \langle 0.3, 0.4\rangle\}$.
Then *s-pre-*$Ncl(A) \cap$ *s-pre-*
$Ncl(B) = \{x, \langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.2, 0.5\rangle\}$ and *s-pre-*$Ncl(A)$=*s-pre-*$Ncl(B)$=$1_\sim$.

Proposition 2.16.

(i) The intersection of an neutrosophic supra open set and an neutrosophic supra *pre*-open set is an neutrosophic supra *pre*-open set

(ii) The intersection of an neutrosophic supra α-open set and an neutrosophic supra *pre*-open set is an neutrosophic supra *pre*-open set

3 NEUTROSOPHIC SUPRA *PRE*-CONTINIOUS MAPPINGS.

In this section, we introduce a new type of continuous mapings called a neutrosophic supra *pre*-continuous mappings and obtain some of their properties and characterizations.

Definition 3.1. Let (X, τ) and (Y, σ) be the two topological sets and μ be an associated neutrosophic supra topology with τ. A map $f : (X, \tau) \to (Y, \sigma)$ is called an neutrosophic supra *pre*-continuous mapping if the inverse image of each open set in Y is an neutrosophic supra *pre*-open set in X.

Theorem 3.2. Every neutrosophic supra continuous map is an neutrosophic supra *pre*-continuous map .

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is called neutrosophic continuous map and A is an open set in Y. Then $f^{-1}(A)$ is an open set in X . Since μ is associated with τ, then $\tau \subseteq \mu$. Therefore, $f^{-1}(A)$ is an neutrosophic supra open set in X which is an neutrosophic supra *pre*-open set in X . Hence f is an neutrosophic supra *pre*-continuous map.
The converse of the above theorm is not true as shown in the following example.

Example 3.3. Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$A = \{\langle 0.5, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.3, 0.4\rangle\}$,
$B = \{\langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}$,

$C = \{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$,
$D = \{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.6, 0.5\rangle\}$. Then $\tau = \{0_{\sim},\ 1_{\sim},\ A,\ B,\ A \cup B\}$ be an neutrosophic supra topology on X.
Then the neutrosophic supra topology σ on Y is defined as follows:
$\sigma = \{0 \sim, 1 \sim, C, D, C \cup D\}$. Define a mapping $f(X, \tau) \longrightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The inverse image of the open set in Y is not an neutrosophic supra open in X but it is an neutrosophic supra *pre*-open. Then f is an neutrosophic supra *pre*-continuous map but may not be an neutrosophic supra continuous map.
The following example shows that neutrosophic supra *pre*-continuous map but may not be an neutrosophic supra α-continuous map.
Example 3.4. Let $X = \{a, b\}$ and $Y = \{u, v\}$,
$\tau = \{0_{\sim},\ 1_{\sim}, \{\langle 0.5, 0.2\rangle, \langle 05, 0.2\rangle, \langle 0.3, 0.4\rangle\}\}$,
$\{\langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}, \{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$ be a neutrosophic supra topology on X.
Then the neutrosophic supra topology σ on Y is defined as follows:
$\sigma = \{0_{\sim}, 1_{\sim}, \{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}, \{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.6, 0.5\rangle\},$
$\{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$.

Define a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ by f(a) = u and f(b) = v.The inverse image of the open set in Y is not an neutrosophic supra α-open in X but it is an neutrosophic supra *pre*-open. Then f is an neutrosophic supra *pre*-continuous map but may not be an neutrosophic supra α-continuous map.
The following example shows that neutrosophic supra b-continuous map but may not be an neutrosophic supra *pre*-continuous map.
Example 3.5. Let $X = \{a, b\}$ and $Y = \{u, v\}$,
$\tau = \{0_{\sim}, 1_{\sim}, \{\langle 0.5, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.3, 0.4\rangle\}, \{\langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}\}$,
$\{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$ be a neutrosophic supra topology on X. Then the neutrosophic supra topology σ on Y is defined as follows:
$\sigma = \{0_{\sim}, 1_{\sim}, \{, \langle 0.5, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.3, 0.4\rangle\}, \{\langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}\}$,
$\{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$.
Define a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The inverse image of the open set in Y is not an neutrosophic supra *pre*-open in X but it is an neutrosophic supra b-open. Then f is an neutrosophic supra b-continuous map but may not be an neutrosophic supra *pre*-continuous map.
The following example shows that neutrosophic supra β-continuous map but may not be an neutrosophic supra *pre*-continuous map.
Example 3.6. Let $X = \{a, b\}$ and $Y = \{u, v\}$,
$\tau = \{0_{\sim}, 1_{\sim}, \{\langle 0.5, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.3, 0.4\rangle\}, \{\langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}\}$,
$\{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$ be a neutrosophic supra topology on X. Then the neutro-

sophic supra topology σ on Y is defined as follows:
$\sigma = \{0_\sim, 1_\sim, \{\langle 0.5, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.3, 0.4\rangle\}, \{\langle 0.3, 0.4\rangle, \langle 0.3, 0.4\rangle, \langle 0.6, 0.5\rangle\}\}$,
$\{\langle 0.5, 0.4\rangle, \langle 0.5, 0.4\rangle, \langle 0.3, 0.4\rangle\}$. Define a mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The inverse image of the open set in Y is not an neutrosophic supra *pre*-open in X but it is an neutrosophic supra β-open. Then f is an neutrosophic supra β-continuous map but may not be an neutrosophic supra *pre*-continuous map.
From the above discussion we have the following diagram in which the converses of the implications need not be true (cts. is the abbrevation of continuity).

Theorem 3.7. Let (X, τ) and (Y, σ) be the two topological spaces and μ be an associated neutrosophic supra topology with τ. Let f be a map from X into Y. Then the following are equivalent:

(i) f is an neutrosophic supra *pre*-continuous map.

(ii) The inverse image of a closed sets in Y is an neutrosophic supra *pre*-closed set in X;

(iii) $s\text{-}pre\text{-}Ncl(f^{-1}(A)) \subseteq f^{-1}(Ncl(A))$ for every set A in Y;

(iv) $f(s\text{-}pre\text{-}Ncl(A)) \subseteq Ncl(f(A))$ for every set A in X;

(v) $f^{-1}(Nint(B)) \subseteq s\text{-}pre\text{-}Nint(f^{-1}(B))$ for every set B in Y.

Proof. (i)⇒(ii): Let A be a closed set in Y, then $Y - A$ is open set in Y. Then $f^{-1}(Y - A) = X - f^{-1}(A)$ is *s-pre*-open set in X. It follows that $f^{-1}(A)$ is a supra *pre*-closed subset of X.
(ii)⇒(iii): Let A be any subset of Y. Since $Ncl(A)$ is closed in Y, then it follows that $f^{-1}(Ncl(A))$ is supra *pre*-closed set in X. Therefore $s\text{-}pre\text{-}Ncl(f^{-1}(A)) \subseteq (f^{-1}(Ncl(A)))$.
(iii)⇒(iv): Let A be any subset of X. By (iii) we have $f^{-1}(Ncl(f(A))) \supseteq s\text{-}pre\text{-}Ncl(f^{-1}(f(A))) \supseteq s\text{-}pre\text{-}Ncl(A)$ and hence $f(s\text{-}pre\text{-}Ncl(A)) \subseteq Ncl(f(A))$.
(iv)⇒(v): Let B be any subset of Y. By (4) we have $f^{-1}(s\text{-}pre\text{-}Ncl(X - f^{-1}(B))) \subseteq Ncl(f(X - f^{-1}(B)))$ and $f(X - s\text{-}pre\text{-}Nint(f^{-1}(B))) \subseteq Ncl(Y - B) = Y - Nint(B))$. Therefore we have $X - s\text{-}pre\text{-}Nint(f^{-1}(B)) \subseteq f^{-1}(Y - Nint(B))$ and hence $f^{-1}(Nint(B)) \subseteq s\text{-}pre\text{-}Nint(f^{-1}(B))$.
(v)⇒(i): Let B be a open set in Y and $f^{-1}(Nint(B)) \subseteq s\text{-}pre\text{-}Nint(f^{-1}(B))$, hence $f^{-1}(B) \subseteq s\text{-}pre\text{-}Nint(f^{-1}(B))$. Then $f^{-1}(B) = s\text{-}pre\text{-}Nint(f^{-1}(B))$. But, $s\text{-}pre\text{-}Nint(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B) = s\text{-}pre\text{-}Nint(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an neutrosophic supra *pre*-open set in Y.

Theorem 3.8. If a map $f : (X, \tau) \to (Y, \sigma)$ is a *s-pre*-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is continuous, then $(g \circ f)$ is *s-pre*-continuous.

Proof. It is Obvious.

Theorem 3.9. Let $f : (X, \tau) \to (Y, \sigma)$ be an neutrosophic *s-pre*-continuous map if one of the following holds:

(i) $f^{-1}(s\text{-}pre\text{-}Nint(B)) \subseteq Nint(f^{-1}(B))$ for every set B in Y,

(ii) $Ncl(f^{-1}(A)) \subseteq f^{-1}(s\text{-}pre\text{-}Ncl(B))$ for every set B in Y,

(iii) $f(Ncl(A)) \subseteq s\text{-}pre\text{-}Ncl(f(B))$ for every A in X.

Proof. Let B be any open set of Y, if the condition (i) is satisfied, then $f^{-1}(s\text{-}pre\text{-}Nint(B)) \subseteq Nint(f^{-1}(B))$. We get, $f^{-1}(B) \subseteq Nint(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an neutrosophic open set. Every neutrosophic open set is neutrosophic supra *pre*-open set. Hence f is an neutrosophic *s-pre*-continuous .
If condition (ii) is satisfied, then we can easily prove that f is an neutrosophic supra *pre*-continuous.
Let condition (iii) is satisfied and B be any open set in Y. Then $f^{-1}(B)$ is a set in X and then we can easily prove that f is an neutrosophic *s-pre*-continuous function. If condition (iii) is satisfied, and B is any open set of Y. Then $f^{-1}(B)$ is a set in X and $f(Ncl(f^{-1}(B))) \subseteq s\text{-}pre\text{-}Ncl(f(f^{-1}(B)))$. This implies $f(Ncl(f^{-1}(B))) \subseteq s\text{-}pre\text{-}Ncl(B)$. This is nothing but condition (ii). Hence f is an neutrosophic *s-pre*-continuous.

4 NEUTROSOPHIC SUPRA *PRE*-OPEN MAPS AND NEUTROSOPHIC SUPRA *PRE*-CLOSED MAPS.

Definition 4.1.
A map $f : X \longrightarrow Y$ is called neutrosophic supra *pre*-open (res.neutrosophic supra *pre*-closed) if the image of each open (resp.closed) set in X, is neutrosophic supra *pre*-open(resp.neutrosophic supra *pre*-closed)in Y.
Theorem 4.2.

A map $f : X \longrightarrow Y$ is called an neutrosophic supra *pre*-open if and only if $f(Nint(A)) \subseteq s\text{-}pre\text{-}Nint(A)$ for every set A in X.
Proof. Suppose that f is an neutrosophic supra *pre*-open map. Since $Nint(A) \subseteq f(A)$. By hypothesis $f(Nint(A))$ is a neutrosophic supra *pre*-open set and $s\text{-}pre\text{-}Nint(f(A))$ is the largest neutrosophic supra *pre*-open set contained in $f(A)$, then $f(Nint(A)) \subseteq s\text{-}pre\text{-}Nint(f(A))$
Conversely, let A be a open set in X. Then $f(Nint(A)) \subseteq s\text{-}pre\text{-}Nint(f(A))$. Since $Nint(A) = A$,then $f(A) \subseteq s\text{-}pre\text{-}Nint(f(A))$. Therefore $f(A)$ is an neutrosophic supra *pre*-open set in Y and f is an neutrosophic supra *pre*-open.
Theorem 4.3. A map $f : X \longrightarrow Y$ is called a neutrosophic supra *pre*-closed if and only if $f(Ncl(A)) \subseteq s\text{-}pre\text{-}Ncl(A)$ for every set A in X.
Proof. Suppose that f is an neutrosophic supra *pre*-closed map. Since for each set A in X, $Ncl(A)$ is closed set in X, then $f(Ncl(A)$ is an neutrosophic supra *pre*-closed set in Y.

Also, since $f(A) \subseteq f(Ncl(A))$, then *s-pre-*$Ncl(f(A)) \subseteq f(Ncl(A))$.
Conversely, let A be a closed set in X. Since *s-pre-*$Ncl(f(A))$ is the smallest neutrosophic supra *pre*-closed set containing $f(A)$, then $f(A) \subseteq$ *s-pre-*$Ncl(f(A)) \subseteq f(Ncl(A)) = f(A)$. Thus $f(A) =$ *s-pre-*$Ncl(f(A))$. Hence $f(A)$ is an neutrosophic supra *pre*-closed set in Y. Therefore f is a neutrosophic supra *pre*-closed map.
Theorem 4.4. Let $f : X \longrightarrow Y$ and $g : y \longrightarrow Z$ be two maps.

(i) If $g \circ f$ is an neutrosophic supra *pre*-open and f is continuous surjective, then g is an neutrosophic semi-supra *pre*-open.

(ii) If $g \circ f$ is open and g is an neutrosophic supra *pre*continuous injective, then f is neutrosophic supra *pre*-open.

Theorem 4.5 Let $f : X \longrightarrow Y$ be a map. Then the following are equivalent;

(i) f is an neutrosophic supra *pre*-open map;

(ii) f is an neutrosophic supra *pre*-closed map;

(iii) f is an neutrosophic supra *pre*-continuous map.

Proof. (i)$\Longrightarrow$ (ii). Suppose B is a closed set in X. Then $X - B$ is an open set in an open set in X. By (1), $f(X-B)$ is an neutrosophic supra *pre*-open set in X. Since f is bijective, then $f(X - B) = Y - f(B)$. Hence $f(B)$ is an neutrosophic supra *pre*-closed set in Y. Therefore f is an neutrosophic supra *pre*-closed map.
(ii)$\Longrightarrow$(iii). Let f is an neutrosophic supra *pre*-closed map and B be closed set X. Since f is bijective, then $(f^{-1})^{-1}(B)$=$f(B)$ is an neutrosophic supra *pre*-closed set in Y. By Theorem 3.7 f is an neutrosophic supra *pre*-continuous map.
(iii)$\Longrightarrow$(i). Let A be an open set in X. Since f^{-1} is an neutrosophic supra *pre*-continuous map, then $(f^{-1})^{-1}(A)$=$f(A)$ is an neutrosophic supra pre-open set in Y. Hence f is an neutrosophic supra *pre*-open.

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