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ON SOLVING NEUTROSOPHIC LINEAR COMPLEMENTARITY PROBLEM

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Abstract:
The aim of this paper is to propose a methodology for solving Linear Complementarity Problem with Single Valued Trapezoidal Neutrosophic Numbers (SVTN). The effectiveness of the proposed method is illustrated by means of a numerical example. This problem finds many applications in several areas of science, engineering and economics.

Keywords: Linear complementarity problem, Neutrosophic Set, Single Valued Trapezoidal Neutrosophic Numbers, Lemke's Algorithm.

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1. INTRODUCTION

Fuzzy systems (FSs) and Intuitionistic fuzzy systems (IFSs) cannot successfully deal with a situation where the conclusion is adequate, unacceptable and the decision maker declaration is uncertain. Therefore, some novel theories are mandatory for solving the problem with uncertainty. The neutrosophic sets (NSs) reflect on the truth membership, indeterminacy membership and falsity membership concurrently, which is more practical and adequate than FSs and IFSs in commerce, which are uncertain, incomplete and inconsistent in sequence. Single valued neutrosophic sets are an extension of NSs which were introduced by Wang and Wang [6] and further investigated by Peng and Wang in [7] where the latter authors also discussed the power aggregation operators.
Although many researchers and scientists [1-5, 8-11] have worked in the neutrosophic methods and applied it in the field of decision making, there are, however, still some viewpoints regarding defining neutrosophic numbers in different forms, and their corresponding de-impreciseness is very important.

This paper provides a new technique for solving the linear complementarity problems with fuzzy numbers. The paper is organized as follows. In section 2, Single Valued Trapezoidal Neutrosophic numbers (SVTN) and the fuzzy arithmetical operators are detailed. In section 3, the Fuzzy linear complementarity problem and an algorithm for solving a Single Valued Neutrosophic FLCP are described. In section 4, the effectiveness of the proposed method is illustrated by means of an example. Finally, the section 5 contains some concluding remarks.

2. PRELIMINARIES

2.1 Trapezoidal Fuzzy Numbers:

A trapezoidal fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ and is defined by the membership function as

$$\mu_a(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & , a_1 \le x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)} & , a_3 \le x \le a_4 \\ 0 & , \text{otherwise} \end{cases}$$

Definition 2.2:

Let E be a universe. An intuitionistic fuzzy set k over E is defined by

$$k = \{\langle x, \mu_k(x), \gamma_k(x)\rangle : x \in E\}$$

where $\mu_k: E \to [0,1]$ and $\gamma_k: E \to [0,1]$ are such that $0 \le \mu_k(x) + \gamma_k(x) \le 1$ for any $x \in E$. For each $x \in E$, the values $\mu_k(x)$ and $\gamma_k(x)$ are the degree of membership and degree of non-membership of x, respectively.

Definition 2.3:

Let E be a universe. A neutrosophic set A over E is defined by

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in E\}$$

where $T_A(x), I_A(x), F_A(x)$ are called the truth – membership function, indeterminacy membership function and falsity membership function respectively. They are respectively defined by

$$T_A : E \to]^-0, 1^+[\,,\ I_A : E \to]^-0, 1^+[\,, F_A : E \to]^-0, 1^+[$$

such that $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.4:

Let E be a universe. An single valued neutrosophic set (SVN – Set) over E is a neutrosophic set over E, but the truth – membership function, indeterminacy membership function and falsity membership function are respectively defined by

$$T_A : E \to [0,1]\,, I_A : E \to [0,1]\,, F_A : E \to [0,1]$$

such that$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2.5:

A single valued trapezoidal neutrosophic number (SVTN – Number)

$$\tilde{A} = \langle (a, b, c, d);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}}\rangle$$

is a special neutrosophic set on the real number set ℝ , whose truth – membership, indeterminacy – membership and a falsity membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a)w_{\tilde{a}}}{(b-a)} & , a \le x \le b \\ w_{\tilde{a}} & , b \le x \le c \\ \dfrac{(d-x)w_{\tilde{a}}}{(d-c)} & , c \le x \le d \\ 0 & , \text{otherwise} \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} \dfrac{\left(b - x + u_{\tilde{a}}(x-a)\right)}{(b-a)} & , a \le x \le b \\ \mu_{\tilde{a}} & , b \le x \le c \\ \dfrac{\left(x - c + u_{\tilde{a}}(d-x)\right)}{(d-c)} & , c \le x \le d \\ 0 & , \text{otherwise} \end{cases}$$

$$\lambda_a(x) = \begin{cases} \dfrac{\left(b - x + y_{\tilde{a}}(x-a)\right)}{(b-a)} & , a \le x \le b \\ y_{\tilde{a}} & , b \le x \le c \\ \dfrac{\left(x - c + y_{\tilde{a}}(d-c)\right)}{(d-c)} & , c \le x \le d \\ 0 & , \text{otherwise.} \end{cases}$$

2.5 Neutrosophic Trapezoidal Numbers:

Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (a_2, b_2, c_2, d_2);\ w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ be two single valued trapezoidal neutrosophic numbers and $\gamma \neq 0$.

2.6 Arithmetic operators :

Addition:

$$\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$$

Subtraction:

$$\tilde{a} - \tilde{b} = \langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$$

Multiplication:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & , (d_1 > 0, d_2 > 0) \\ \langle (a_1 d_2, b_1 c_2, c_1 b_2, d_1 a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & , (d_1 < 0, d_2 > 0) \\ \langle (d_1 d_2, c_1 c_2, b_1 b_2, a_1 a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & , (d_1 < 0, d_2 < 0) \end{cases}$$

Division:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & , (d_1 > 0, d_2 > 0) \\ \langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & , (d_1 < 0, d_2 > 0) \\ \langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2);\ w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle & , (d_1 < 0, d_2 < 0) \end{cases}$$

Scalar Multiplication:

$$\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & , (\gamma > 0) \\ \langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle & , (\gamma < 0) \end{cases}$$

Inverse:

$$\tilde{a}^{-1} = \langle \left(\frac{1}{d_1}, \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right);\ w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle , (\tilde{a} \neq 0)$$

3. MAIN RESULTS

3.1. Linear Complementarity Problem (LCP)

For a given vector $q \in \mathbb{R}^n$ and a given matrix $M \in \mathbb{R}^{nxn}$, the linear complementarity problem (LCP) consists in finding vectors W and Z in $\mathbb{R}^n$ such that

$$W - MZ = q \tag{1}$$

$$W_j \ge 0, Z_j \ge 0 \text{ for } j = 1, 2, \ldots, n \tag{2}$$

$$W_j Z_j = 0 \text{ for } j = 1, 2, \ldots, n \tag{3}$$

Here the pair $\left(W_j, Z_j\right)$ is said to be a pair of complementary variables.

A solution (W, Z) to the above system is called a complementary feasible solution, if (W, Z) is a basic feasible solution to (1) and (2) with one of the pair $\left(W_j, Z_j\right)$ is basic for $j = 1, \ldots, n$.

If $q \ge 0$, then we immediately see that $W = q, Z = 0$ is a solution to the linear complementarity problem.

If however, $q < 0$, we consider the related system $W - MZ - eZ_0 = q$ (4)

$$W_j \ge 0, Z_j \ge 0, Z_0 \ge 0, j = 1, \ldots, n \tag{5}$$

$$W_i Z_i = 0, j = 1, \ldots, n \tag{6}$$

where Z_0 is an artificial variable and e is an n-vector with all components equal to one.

Letting Z_0 = maximum $\{-q_i \ / \ 1 \le i \le n\}$, Z = 0, and $W = q + {}_0$, we obtain a starting solution to the above system. Lemke's algorithm attempts to drive Z_0 to zero, thus obtaining a solution to the linear complementarity problem (LCP).

The fuzzy linear complementarity problem and an algorithm for solving fuzzy linear complementarity problem are described in this section.

3.2. Fuzzy Linear Complementarity Problem (FLCP)

Assume that all parameters in (1) - (3) are fuzzy and are described by fuzzy numbers. Then, the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy numbers.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q} \quad (7)$$

$$\tilde{W}_j \ge 0, Z_j \ge 0, j = 1,\ldots,n \quad (8)$$

$$\tilde{W}_j\tilde{Z}_j = 0, j = 1,\ldots,n \quad (9)$$

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of fuzzy complementary variables.

Definition 3.1: A solution $(\tilde{W}, \tilde{Z})$ to the above system (7) - (9) is called a fuzzy complementary feasible solution, if $(\tilde{W}, \tilde{Z})$ is a fuzzy basic feasible solution to (7) and (8) with one of the pair $(\tilde{W}_j, \tilde{Z}_j)$ basic for each $j = 1,\ldots,n$.

3.2. Algorithm for Fuzzy Linear Complementarity Problem

Consider the FLCP$\left(\tilde{q}, \tilde{M}\right)$, where the fuzzy matrix $\tilde{M}$ is a positive semi definite matrix of order n. The original table for this version of the algorithm is:

$\tilde{w}$	$\tilde{Z}$	$\tilde{z}_0$	
$\tilde{I}$	$-\tilde{M}$	$-\tilde{d}$	$\tilde{q}$

(10)

This method deals only with fuzzy complementary basic vectors for (10), beginning with $\tilde{w} = \left(\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n\right)$ as the initial fuzzy complementary basic vector. All the fuzzy complementary basic vectors obtained in the method, except the terminal one, will be infeasible. When a fuzzy complementary feasible basic vector for (10) is obtained, the method terminates.

If $\tilde{q} \ge 0$, then we have the solution satisfying (7)-(9), by letting $\tilde{W} = \tilde{q}$ and $\tilde{Z} = 0$.

If $\tilde{q} < 0$, we will consider the following system

$$\tilde{W} - \tilde{M}\tilde{Z} - \tilde{e}\tilde{Z}_0 = \tilde{q} \quad (11)$$

$$\tilde{W}_j \ge 0, Z_j \ge 0, j = 1,2,3, \ldots, n \quad (12)$$

$$\tilde{W}_j\tilde{Z}_j = 0, j = 1,2,3, \ldots, n \quad (13)$$

where Z_0 is an artificial fuzzy variable and $\tilde{e}$ is an n-vector with all components equal to any constant. Letting $\tilde{Z}_0$ = maximum $\{\tilde{q}_i / 1 \le i \le n\}, \tilde{Z} = 0$ and $\tilde{W} = \tilde{q} + \tilde{e}\tilde{Z}_0$, we obtain a starting solution to the system (11)-(13). Through a sequence of pivots, we attempt to drive the fuzzy artificial variable $\tilde{Z}_0$ to level zero, thus obtaining a solution to the fuzzy linear complementarity problem (FLCP).

Step 1:

Introduce the fuzzy artificial variable $\tilde{Z}_0$ and consider the system (11)-(13).

(i) If $\tilde{q} \ge 0$, stop; then $(\tilde{W}, \tilde{Z}) = (\tilde{q}, \tilde{0})$ is a fuzzy complementary basic feasible solution.

(ii) If $\tilde{q} < 0$, display the system (11),(12) as given in the simplex method.

Let $-q_S = \text{maximum}\{-q_i / 1 \le i \le n\}$, and update the table by pivoting at row s and the $\tilde{Z}_0$ column. Thus the fuzzy basic variables $\tilde{Z}_0$ and $\tilde{W}_s$ for $j = 1,2,3,\ldots,n$ and $j \ne s$ are positive.

Let $\tilde{y}_s = \tilde{Z}_0$ and go to step 2.

Step 2:

Let $\tilde{d}_s$ be the updated column in the current table under the variable $\tilde{y}_s$.

If $\tilde{d}_s \le 0$, go to step 5, otherwise determine the index r by the following minimum ratio test, $\frac{\tilde{\tilde{q}}}{\tilde{d}_{rs}} = \min\left\{ \frac{\tilde{\tilde{q}}_i}{\tilde{d}_{is}} / \tilde{d}_{is} > 0 \right\}$, where $\tilde{\tilde{q}}$ is the updated right-hand side column denoting the values of the fuzzy basic variables.

If the fuzzy basic variable at row r is $\tilde{Z}_0$, go to step 4, otherwise, go to step 3.

Step 3:

The fuzzy basic variable at row r is either $\tilde{W}_l$ or $\tilde{Z}_l$, for some $l \ne s$. The variable $\tilde{y}_s$ enters the basis and the table is updated by pivoting at row r and the $\tilde{y}_s$ column. If $\tilde{W}_l$ leaves the basis, then let $\tilde{y}_s = \tilde{Z}_l$; and if $\tilde{Z}_l$ leaves the basis, then let $\tilde{y}_s = \tilde{W}_l$; Return to step 2.

Step 4:

Here $\tilde{y}_s$ enters the basis, and $\tilde{Z}_0$ leaves the basis. Pivot at the $\tilde{y}_s$ column and the $\tilde{Z}_0$ row, producing a fuzzy complementary basic feasible solution. Stop.

Step 5:

Stop with ray termination.

A ray $R = \left\{ (\tilde{W}, \tilde{Z}, \tilde{Z}_0) + \lambda \tilde{d} / \lambda \ge 0 \right\}$ is found such that every point in R satisfying (11), (12) and (13), where $(\tilde{W}, \tilde{Z}, \tilde{Z}_0)$ is the almost fuzzy complementary basic feasible solution, and $\tilde{d}$ is an extreme direction of the set defined by (11) and (12) having a $\tilde{1}$ in the row corresponding to $\tilde{y}_s$, $-\tilde{d}_s$ in the rows of the current basic variables and zeros everywhere else.

4. NUMERICAL EXAMPLE

Consider the fuzzy linear complementary problem

where $M = \begin{pmatrix} 3 & -1 \\ 4 & -5 \end{pmatrix}$, $q = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$ in which the fuzzy coefficients are assumed to be

$\tilde{3} = \langle (8,9,10,11); 0.5,0.7,0.5 \rangle$, $\tilde{4} = \langle (9,10,11,12); 0.5,0.7,0.4 \rangle$,

$-\tilde{1} = \langle (-4,-3,-2,-1); 0.5,0.4,0.8 \rangle$, $-\tilde{5} = \langle (-13,-12,-11,-10); 0.6,0.3,0.6 \rangle$ and

$\widetilde{10} = \langle (32,33,34,35); 0.5,0.7,0.6 \rangle$, $\tilde{5} = \langle (10,11,12,13); 0.6,0.3,0.6 \rangle$

$$M = \begin{bmatrix} \langle (8,9,10,11); 0.5,0.7,0.5 \rangle & \langle (-4,-3,-2,-1); 0.5,0.4,0.8 \rangle \\ \langle (9,10,11,12); 0.5,0.7,0.4 \rangle & \langle (-13,-12,-11,-10); 0.6,0.3,0.6 \rangle \end{bmatrix}$$

and $q = \begin{bmatrix} \langle (10,11,12,13); 0.6,0.3,0.6 \rangle \\ \langle (32,33,34,35); 0.5,0.7,0.6 \rangle \end{bmatrix}$.

Now, the fuzzy linear complementary problem is solved by the proposed algorithm and the results are tabulated in Table 4.1.

Table 4.1:

Basic Variable	W_1	W_2	Z_1	Z_2	Z_0	q
W_1	⟨(1,1,1,1); 1,0,0⟩	⟨(0,0,0,0); 1,0,0⟩	⟨(−11,−10,−9,−8); 0.5,0.7,0.5⟩	⟨(1,2,3,4); 0.5,0.4,0.8⟩	⟨(−1,−1,−1,−1); 1,0,0⟩	⟨(−13,−12,−11,−10); 0.5,0.7,0.5⟩
W_2	⟨(0,0,0,0); 1,0,0⟩	⟨(1,1,1,1); 1,0,0⟩	⟨(−12,−11,−10,−9); 0.5,0.7,0.4⟩	⟨(10,11,12,13); 0.6,0.3,0.6⟩	⟨(−1,−1,−1,−1); 1,0,0⟩	⟨(32,33,34,35); 0.5,0.7,0.6⟩
Z_0	⟨(−1,−1,−1,−1); 1,0,0⟩	⟨(0,0,0,0); 1,0,0⟩	⟨(8,9,10,11); 0.5,0.7,0.5⟩	⟨(−4,−3,−2,−1); 0.5,0.4,0.8⟩	⟨(1,1,1,1); 1,0,0⟩	⟨(10,11,12,13); 0.6,0.3,0.6⟩
W_2	⟨(−1,−1,−1,−1); 1,0,0⟩	⟨(1,1,1,1); 1,0,0⟩	⟨(−4,−2,0,2); 0.5,0.7,0.5⟩	⟨(6,8,10,12); 0.5,0.4,0.8⟩	⟨(0,0,0,0); 1,0,0⟩	⟨(42,44,46,48); 0.5,0.7,0.6⟩
Z_1	⟨(0.13,0.11,0.1,0.09); 1,0,0⟩	⟨(0,0,0,0); 0.5,0.7,0.5⟩	⟨(1,1,1,1); 0.5,0.7,0.5⟩	⟨(−0.36,−0.3,−0.22,−0.13); 0.5,0.7,0.8⟩	⟨(0.13,0.11,0.1,0.09); 0.5,0.7,0.5⟩	⟨(1.25,1.22,1.20,1.18); 0.5,0.7,0.6⟩
W_2	⟨(−0.64,−0.8,−1,−1.26); 0.5,0.7,0.5⟩	⟨(1,1,1,1); 0.5,0.7,0.5⟩	⟨(0,0,0,0); 0.5,0.7,0.5⟩	⟨(6.26,7.4,9.4,10.56); 0.5,0.7,0.8⟩	⟨(0.36,0.2,0,−0.26); 0.5,0.7,0.5⟩	⟨(46.72,46.4,46,45.5); 0.5,0.7,0.6⟩
Z_1	⟨(0.12,0.09,0.06,0.02); 0.5,0.7,0.8⟩	⟨(0.06,0.04,0.02, 0.01); 0.5,0.7,0.8⟩	⟨(1,1,1,1); 0.5,0.7,0.8⟩	⟨(0,0,0,0); 0.5,0.7,0.8⟩	⟨(0.13,0.11,0.1,0.09); 0.5,0.7,0.8⟩	⟨(3.94,3.10,2.28,1.74); 0.5,0.7,0.8⟩
Z_2	⟨(-0.06,-0.09,-0.14,-0.2); 0.5,0.7,0.8⟩	⟨(0.16,0.14,0.11, 0.09); 0.5,0.7,0.8⟩	⟨(0,0,0,0); 0.5,0.7,0.8⟩	⟨(1,1,1,1); 0.5,0.7,0.8⟩	⟨(0.03,0.02,0,−0.04); 0.5,0.7,0.8⟩	⟨(7.46,6.27,4.89,4.31); 0.5,0.7,0.8⟩

Finally we get the result of $Z_1 = \langle(3.94,3.10,2.28,1.74); 0.5,0.7,0.8\rangle$, $Z_2 = \langle(7.46,6.27,4.89,4.31); 0.5,0.7,0.8\rangle$.

5. CONCLUSION

In this paper, a new approach for solving a linear complementarity problem with fuzzy parameters is suggested. Even though we are considering for solving fuzzy Linear Complementarity Problem with Single Valued Trapezoidal Neutrosophic numbers (SVTN), this method can also be extended to non-linear and multi objective programming with fuzzy coefficients.

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