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PERIODICITY OF INTERVAL FUZZY NEUTROSOPHIC SOFT MATRICES

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ABSTRACT. In this article the out-to-out description of d-periodic interval fuzzy neutrosophic soft matrices (d-PIFNSMs) over fuzzy neutrosophic soft algebra (FNSA) is furnished and d-peroidicity properties are proved. Delineation of the d-periodicity of interval valued fuzzy soft neutrosohic soft matrices(IFNSM) is interpreted.

1. INTRODUCTION

In genuine world, we face such a large number of unpredictability in varying background fields. Anyway the greater part of the current mathematical instruments for formal displaying, thinking and registering are fresh deterministic and exact in character. There are hypothesis viz., theory of likelihood, proof, fuzzy set, interval fuzzy set, neutrosophic fuzzy set, vague set, interval mathematics, rough set for managing unreliabilities. In genuine world, we face such a significant number of vulnerabilities in varying backgrounds fields. This hypotheses have their own challenges as called attention to by Molodtsove [13]. In 1999, Molodtsove [13] started a novel idea of soft set hypothesis, which is totally new approch for displaying ambiguity and unpredictability. Soft set hypothesis has a

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rich potential for application in tackling useful issues in financial aspects, sociology, clinical science ect. Later on Maji et.al, [11] have proposed the hypothesis of fuzzy soft set. Maji et.al, [10] stretched out soft sets to intuitionistics fuzzy soft sets.

Intuitionistic fuzzy matrix sums up the fuzzy matrix presented by Thomsom [15]. In [2] Borah et.al, extented fuzzy soft matrix hypothesis and its application. Likewise, [14], Rajarajeswari et.al, proposed new definitions for intuitionistic fuzzy soft matrices and its sort. Sumathi and Arokiarani [1] presented new procedure on fuzzy neutrosophic soft matrices. First time Kavitha et.al, [3–6] presented the idea of one of a unique solvability of max-min operation through FNSM condition $Ax = b$ and clarified strong regularity of FNSMs over FNSA and processing the greatest X-eigenvector of FNSM. They additionally presented on the power of FNSM framework. In Murugadas [8, 9] et.al, presented Monotone interval fuzzy neutrosophic soft eigenproblem and Monotone fuzzy neutrosophic soft eigenspace structure in max-min algebra. Uma et.al, [16], presented two kinds of fuzzy neutrosophic soft matrices.

The point of this paper is to portray purported d-periodicity of FNSMs with vague information (IFNSMs))and to produce algorithms for checking the corresponding properties of IFNSMs.

2. Preliminaries and Backgrounds

For basic definition and illustrations about neutrosophic set, neutrosophic soft set, FNSMs see [5, 6].

The FNSA $\mathcal{N}$ is the triplet $(\mathcal{N}, \oplus, \otimes)$, where $(\mathcal{N}, \leq)$ is a bounded linearly ordered set with binary operations maximum and minimum, denoted by $\otimes, \oplus$. Throughout the paper J represents an index set.

The least element in $\mathcal{N}$ will be denoted by $\langle 0, 0, 1\rangle$, the greatest one by $\langle 1, 1, 0\rangle$.

For $\leq$ relation, strongly connected component $\mathcal{K}$, threshold digraph (TDG), period $\mathcal{K}$ and period for FNSM see [7].

For any $n \in J$, $\mathcal{N}_{(n,n)}$ denotes the set of all square matrices of order n and $\mathcal{N}_{(n)}$ the set of all n-dimensional column vectors over FNSA.

For a FNSM $A \in \mathcal{N}_{(n,n)}$ the symbol $G(A) = (N, E_G)$ stands for a complete, arc-weighted digraph associated with A, i.e., the node set of $G(A)$ is N, and the capacity of any are (i, j) is $\langle a^T_{ij}, a^I_{ij}, a^F_{ij}\rangle$. Let $\phi \neq \tilde{N} \subset N$. $G/\tilde{N}$ stands for the subdigraph of digraph $G(A) = (N, E_G)$ with the node set $\tilde{N}$ and arc set $E_{G/\tilde{N}} = \{(i,j) \in E_G; i,j \in \tilde{N}\}$. If $\mathcal{K}$ is trivial, then the period $\mathcal{P}(\mathcal{K}) = 1$. By $\mathcal{S}^*(G)$ we denote the set of all non-trivial strongly connected components of G. The set of all strongly connected components of G is denoted by $\mathcal{S}(G)$. $\mathcal{P}(A)$ denote the period FNSM A

Theorem 2.1. *[7] Let* $A, C \in \mathcal{N}_{(n,n)}$.
Let $\langle h^T, h^I, h^F\rangle, \langle h_1^T, h_1^I, h_1^F\rangle$, $\langle h_2^T, h_2^I, h_2^F\rangle \in \mathcal{N}$.
(i) *If* $A \leq C$ *then* $G(A, \langle h^T, h^I, h^F\rangle) \subseteq G(C, \langle h^T, h^I, h^F\rangle)$,
(ii) *if* $\langle h_1^T, h_1^I, h_1^F\rangle < \langle h_2^T, h_2^I, h_2^F\rangle$ *then* $G(A, \langle h_2^T, h_2^I, h_2^F\rangle) \subseteq G(A, \langle h_1^T, h_1^I, h_1^F\rangle)$.

Theorem 2.2. *[7] Let* $A \in \mathcal{N}_{(n,n)}, d \in \mathbb{N}$. *Then*
(i) $\mathcal{P}(A) \mid d \Leftrightarrow (\forall \langle h^T, h^I, h^F\rangle \in \mathcal{N})(\forall \mathcal{K} \in \mathcal{S}^*(G(A, \langle h^T, h^I, h^F\rangle)))per\ \mathcal{K} \mid d$,
(ii) $per\ A = lcm\{per \mathcal{K}); \ \mathcal{K} \in \mathcal{S}^*(A)\}$.

3. PERIODICITY OF INTERVAL FNSMS

In this section we shall deal with FNSMs with interval elements. Similarly to [8] we define an IFNSM **A**.

Definition 3.1. *Let* $\underline{A}, \overline{A} \in \mathcal{N}_{(n,n)}, \ \underline{A} \leq \overline{A}$. *An IFNSM **A** with bounds* $\underline{A}$ *and* $\overline{A}$ *is defined as follows*

$$\boldsymbol{A} = [\underline{A}, \overline{A}] = \{A \in \mathcal{N}_{(n,n)}; \ \underline{A} \leq A \leq \overline{A}\}.$$

Definition 3.2. *An IFNSM **A** is called Possibly d-periodic if there exists FNSM* $A \in \boldsymbol{A}$ *such that* $\mathcal{P}(A) \mid d$, *Universally d-periodic if for each FNSM* $A \in \boldsymbol{A}$ $\mathcal{P}(A) \mid d$ *holds.*

3.1. Possible d-periodic(PDP).

In this part we will prove a sufficient and necessary condition for an IFNSM to be possibly d-periodic and find the FNSM $A \in \mathbf{A}$ such that $\mathcal{P}(A) \mid d$ in positive case.

Theorem 3.1. *Let* $G' \subseteq G, \mathcal{K} \in \mathcal{S}^*(G)$ *and* $\mathcal{K}' \in \mathcal{S}^*(G'/N_{\mathcal{K}})$. *Then* $\mathcal{P}(\mathcal{K}) \mid \mathcal{P}(\mathcal{K}')$.

Proof. Since $\{l(c); c\ is\ a\ cycle\ from\ \mathcal{K}'\} \subseteq \{l(\tilde{c});\ \tilde{c}\ is\ a\ cycle\ from\ \mathcal{K})\}$ we have $\mathcal{P}(\mathcal{K}) = gcd\{l(c);\ c\ is\ a\ cycle\ from\ \mathcal{K}\} \mid gcd\{l(\tilde{c};\ \tilde{c}\ is\ a\ cycle\ from\ \mathcal{K}')\} = \mathcal{P}(\mathcal{K}')$. □

Denote $H = \{\langle \overline{a}_{ij}^T, \overline{a}_{ij}^I, \overline{a}_{ij}^F\rangle;\ i,j \in J\} = \{\langle h^T, h^I, h^F\rangle^1, \langle h^T, h^I, h^F\rangle^2, \dots, \langle h^T, h^I, h^F\rangle^r\}$ where $\langle h^T, h^I, h^F\rangle^1 > \langle h^T, h^I, h^F\rangle^2 > \dots > \langle h^T, h^I, h^F\rangle^r$.

Lemma 3.1. *For all $\langle h^T, h^I, h^F\rangle \in H$ and for every $\mathcal{K} \in \mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F\rangle))$ such that $\mathcal{P}(\mathcal{K}) \nmid d$ the digraph $G(\underline{A}, \langle h^T, h^I, h^F\rangle)/N_{\mathcal{K}}$ be acyclic. Then for all $k, l \in J$ such that $l \leq k \leq r$ holds $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^{(k)}, \langle h^T, h^I, h^F\rangle^{(l)}))$.*

Proof. By mathematical induction on k.

(i) For $k = 1$ we show that $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^{(1)}, \langle h^T, h^I, h^F\rangle)^{(1)})$. Let $\mathcal{K}^1, \mathcal{K}^2, \dots, \mathcal{K}^m \in \mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(1)}))$ such that $\mathcal{P}(\mathcal{K}^s) \mid d, s = 1, 2, \dots, m$. We have, $\mathcal{K}^s \in \mathcal{S}^*(G(A^{(1)}, \langle h^T, h^I, h^F\rangle^{(1)}))$ for $s \leq m$.

Moreover $\mathcal{S}^*(G(A^{(1)}, \langle h^T, h^I, h^F\rangle^{(1)})) = \{\mathcal{K}^1, \mathcal{K}^2, \dots, \mathcal{K}^m\}$. Necessarily $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^{(1)}, \langle h^T, h^I, h^F\rangle^{(1)}))$.

(ii) If $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^{(k)}, \langle h^T, h^I, h^F\rangle^{(l)})), l \leq k$. It is obvious that $G(A^{(k+1)}, \langle h^T, h^I, h^F\rangle^{(l)})$ and $G(A^{(k)}, \langle h^T, h^I, h^F\rangle^{(l)})$ are same for all $l \in J, l \leq k$.

Inevitably, $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^{(k+1)}, \langle h^T, h^I, h^F\rangle^{(l)})), l \leq k$. The proof of $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^{(k+1)}, \langle h^T, h^I, h^F\rangle^{(l)}))$ follows from the fact that:

$$\mathcal{S}^*(G(A^{(k)}, \langle h^T, h^I, h^F\rangle^{(k+1)})) = \{\mathcal{K} \in (G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(k+1)}));\ \mathcal{P}(\mathcal{K}) \mid d\}.$$

□

Theorem 3.2. *An IFNSM **A** is PDP if and only if for each $\langle h^T, h^I, h^F\rangle \in H$ and for each $\mathcal{K} \in \mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F\rangle))$ such that $\mathcal{P}(\mathcal{K}) \nmid d$ the digraph $G(\underline{A}, \langle h^T, h^I, h^F\rangle)/N_{\mathcal{K}}$ is acyclic.*

Proof. If $\langle h^T, h^I, h^F\rangle \in H$ and $\mathcal{K} \in \mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F\rangle))$ such that $\mathcal{P}(\mathcal{K}) \nmid d$ and the digraph $G(\underline{A}, \langle h^T, h^I, h^F\rangle)/N_{\mathcal{K}} \subseteq \mathcal{K}$ has a cycle c. Let $A \in$ **A** be fixed. As $G(\underline{A}, \langle h^T, h^I, h^F\rangle \subseteq G(A, \langle h^T, h^I, h^F\rangle))$, there exists $\mathcal{K}' \in \mathcal{S}^*(G(A, \langle h^T, h^I, h^F\rangle))$ such that $c \in \mathcal{K}'$.

Since

$$G(A, \langle h^T, h^I, h^F\rangle) \subseteq G(\overline{A}, \langle h^T, h^I, h^F\rangle)\,,$$

and $\mathcal{K}' \in \mathcal{S}^*(G(A, \langle h^T, h^I, h^F\rangle/N_{\mathcal{K}}))$ by Theorem 2.2 we get $\mathcal{P}(\mathcal{K}) \mid \mathcal{P}(\mathcal{K}')$ and so $\mathcal{P}(\mathcal{K}') \nmid d$. From Theorem 2.2, $\mathcal{P}(A) \nmid d$. Consequently the IFNSM **A** is not PDP.

For the reverse part, if $G(\underline{A}, \langle h^T, h^I, h^F \rangle)/N_{\mathcal{K}}$ is acyclic for all $\langle h^T, h^I, h^F \rangle \in H$ and for every $\mathcal{K} \in \mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F \rangle))$ such that $\mathcal{P}(\mathcal{K}) \nmid d$. We shall setup a FNSM $A^* \in \mathbf{A}$ such that $\mathcal{P}(A^*) \mid d$.

Initially raise an auxiliary sequence of FNSMs $\{A^{(k)}\}_{k=0}^{r} = \{(\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^{(k)})\}_{k=0}^{r}$ recurrently as follows.

$$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^{(0)} = \langle \underline{a}_{ij}^T, \underline{a}_{ij}^I, \underline{a}_{ij}^F \rangle^{(s)} \text{ for each } i, j \in N,$$

$$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^{(k+1)} = \begin{cases} \langle h^T, h^I, h^F \rangle^{(k+1)}, (i,j) \in U_{s \in M} E_{\mathcal{K}^s} \\ \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^{(k)} < \langle h^T, h^I, h^F \rangle^{(k+1)}, \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^{(k)}, \text{otherwise}, \end{cases}$$

for each $k \in J$, where $\mathcal{K}^1, \mathcal{K}^2, \dots, \mathcal{K}^m \in \mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F \rangle^{(k+1)}))$ are such that $\mathcal{P}(\mathcal{K}^s) \mid d$ for $s = 1, 2, .., m$.

By the previous lemma 3.1. for $k = r$, we get $\mathcal{P}(\mathcal{K}) \mid d$ for each $\mathcal{K} \in \mathcal{S}^*(G(A^{(r)}, \langle h^T, h^I, h^F \rangle^{(l)}))$ and $l \leq r$. Let $A^* = A^{(r)}$. Since $G(\overline{A}, \langle h^T, h^I, h^F \rangle^{(r)})$ is complete, $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^* \geq \langle h^T, h^I, h^F \rangle^{(r)}$ for each $i, j \in J$, then $\mathcal{P}(A^*) \mid d$ if we show that $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^*, \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^*))$, for $i, j \in N$ such that $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^* > \langle h^T, h^I, h^F \rangle^{(r)}$, $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^* \notin H$.

Evidently $G(A^*, \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^*) = G(A^*, \langle h^T, h^I, h^F \rangle^{(p)})$ where $p \in J$ is such that $\langle h^T, h^I, h^F \rangle^{(p+1)} < \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^* < \langle h^T, h^I, h^F \rangle^{(p)}$. Thus $\mathcal{P}(\mathcal{K}') \mid d$ for each $\mathcal{K}' \in \mathcal{S}^*(G(A^*, \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle^*)), i, j \in N$. Accordingly $\mathcal{P}(A^*) \mid d$ from Theorem 2.2. Thus the IFNSM **A** is a PDP. □

Let us consider the following IFNSM to illustrate the Possible d-periodicity .

Example 1. Let $O = \langle 0, 0, 1 \rangle,\ I = \langle 1, 1, 0 \rangle$ and $\mathbf{A} = [\underline{A}, \overline{A}]$

$$\overline{A} = \begin{bmatrix} \langle 0,0,1 \rangle & \langle 0.4\ 0.3\ 0.6 \rangle & \langle 0,0,1 \rangle & \langle 0.5,0.4,0.6 \rangle & \langle 0,0,1 \rangle \\ \langle 0.2,0.1,0.8 \rangle & \langle 0,0,1 \rangle & \langle 0.3,0.2,0.7 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle] \\ \langle 0.3,0.2,0.7 \rangle & \langle 0,0,1 \rangle & \langle 0.2,0.1,0.8 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle \\ \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0.1,0.1,0.9 \rangle & \langle 0.2,0.1,0.8 \rangle \\ \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0.1,0.1,0.9 \rangle & \langle 0.2,0.1,0.8 \rangle & \langle 0,0,1 \rangle] \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} \langle 0,0,1\rangle & \langle 0.3\ 0.2\ 0.7\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.1,0.1,0.9\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.1,0.1,0.9\rangle & \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle \end{bmatrix}$$

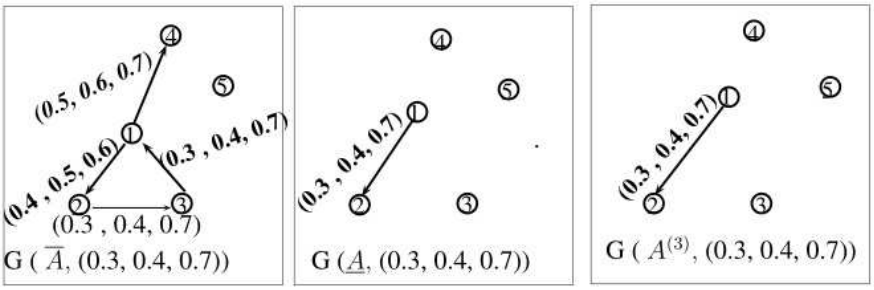

FIGURE 1. Threshold digraphs $\langle h^T, h^I, h^F\rangle^{(3)} = \langle 0.3\ 0.2\ 0.7\rangle$

For $\langle h^T, h^I, h^F\rangle^{(1)} = \langle 0.5, 0.4, 0.6\rangle$ and $\langle h^T, h^I, h^F\rangle^{(2)} = \langle 0.4, 0.3, 0.6\rangle$,

$$G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(1)}), \qquad G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(2)})$$

are acyclic, so $A^1 = A^2 = \underline{A}$.

For $\langle h^T, h^I, h^F\rangle^{(3)} = \langle 0.3\ 0.2\ 0.7\rangle$, $G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(3)})$ are presented in Figure 1.

Certainly $G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(3)})$ has exactly one element in $\mathcal{S}^*$ with period $\mathcal{P}(\mathcal{K}) = 3 \nmid 4$. As $G(\underline{A}, 3)/N_{\mathcal{K}}$ is acyclic, which satisfy the Theorem 2.2 and $A^{(3)} = \underline{A}$. For $\langle h^T, h^I, h^F\rangle^{(4)} = \langle 0.2, 0.1, 0.8\rangle$ there are two elements in $\mathcal{S}^*(G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(4)}))$: $\mathcal{K}_1$ with $N_{\mathcal{K}(1)} = \{1,2,3\}, \mathcal{P}(\mathcal{K}_1) = 1$ and $\mathcal{K}_2$ with $N_{\mathcal{K}_2} = \{4,5\}, \mathcal{P}(\mathcal{K}_2) = 2$ (see Figure 2). As $\mathcal{P}(\mathcal{K}_1) \mid 4$ and $\mathcal{P}(\mathcal{K}_2) \mid 4$ we calculate the FNSM $A^{(4)}$ from $A^{(3)}$ by increasing elements $\langle a^T_{33}, a^I_{33}, a^F_{33}\rangle^{(3)}$ and $\langle a^T_{45}, a^I_{45}, a^F_{45}\rangle^{(3)}$ to $\langle 0.2, 0.1, 0.8\rangle$. In Figure 3 we see for $\langle h^T, h^I, h^F\rangle^{(5)} = 1$ the $G(\overline{A}, 1)$ is strongly connected with period equal to one, so we compute the FNSM $A^{(5)}$ from $A^{(4)}$ by increasing elements $\langle a^T_{14}, a^I_{14}, a^F_{14}\rangle^{(4)}$ and $\langle a^T_{44}, a^I_{44}, a^F_{44}\rangle^{(4)}$ to $\langle 0.1, 0.1, 0.9\rangle$.

We get

$$A^{(4)} = \begin{bmatrix} \langle 0,0,1\rangle & \langle 0.3\ 0.2\ 0.7\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle] \\ \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle \mathbf{0.2,0.1,0.8}\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle \mathbf{0.2,0.1,0.8}\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.1,0.1,0.9\rangle & \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle \end{bmatrix}$$

$$A^{(5)} = \begin{bmatrix} \langle 0,0,1\rangle & \langle 0.3,0.2,0.7\rangle & \langle 0,0,1\rangle & \langle \mathbf{0.1,0.1,0.9}\rangle & \langle 0,0,1\rangle \\ \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0.2,0.1,0.1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle] \\ \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle \mathbf{0.1,0.1,0.9}\rangle & \langle 0.2,0.1,0.8\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.1,0.1,0.9\rangle & \langle 0.2,0.1,0.8\rangle & \langle 0,0,1\rangle \end{bmatrix}.$$

For $\langle h^T, h^I, h^F\rangle^{(6)} = \langle 0,0,1\rangle$ the $G(\overline{A}, \langle 0,0,1\rangle)$ is complete and $A^{(6)} = A^{(5)}$. Since $\mathcal{P}(A^{(5)}) = 2 \mid 4$, the FNSM $A^{(5)}$ is d-periodic and so IFNSM **A** is possibly d-periodic.

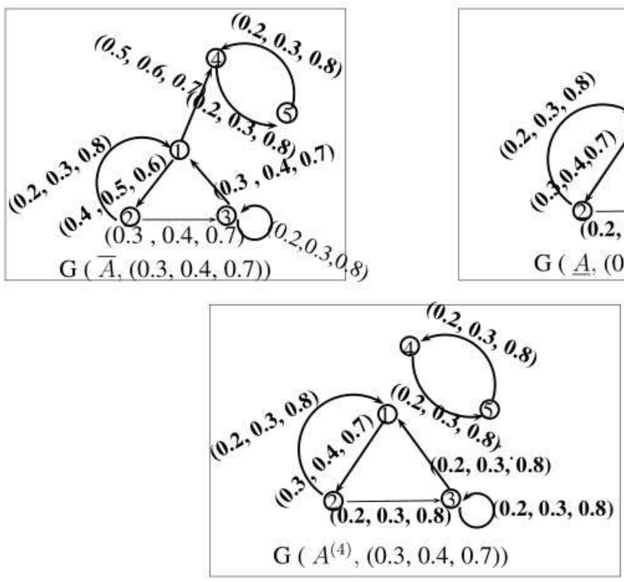

FIGURE 2. Threshold digraphs for $\langle h^T, h^I, h^F\rangle^{(4)} = \langle 0.2, 0.3, 0.8\rangle$

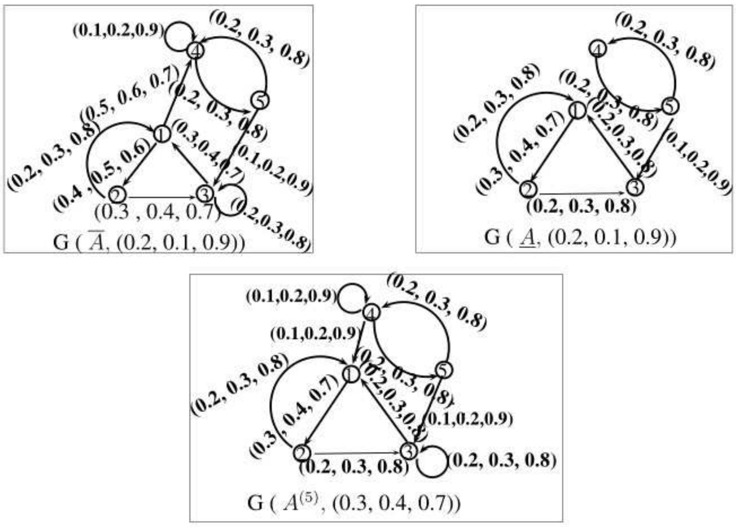

FIGURE 3. Threshold digraphs for $\langle h^T, h^I, h^F\rangle^5 = \langle 0.1, 0.2, 0.9\rangle$

4. UNIVERSAL D-PERIODICITY

In this part we concentrate on if and only if constraints for an IFNSM to be universally d-periodic (UDP).

For a given $\langle h^T, h^I, h^F\rangle \in H$ let us denote $N^h = N\setminus \bigcup_{j=1}^{s} N_{\mathcal{K}^j}$, if

$$\mathcal{S}^*(G(\underline{A}, \langle h^T, h^I, h^F\rangle)) = \{\mathcal{K}^1, \ldots, \mathcal{K}^s\}.$$

Theorem 4.1. *Let* **A** *be an IFNSM. Then* **A** *is UDP if and only if*

$$\mathcal{P}(\underline{A}) \mid d \textit{ and } (\forall \langle h^T, h^I, h^F\rangle \in H)(\forall c \in G(\overline{A}, \langle h^T, h^I, h^F\rangle)/N^h)[l(c) \mid d].$$

Proof. For $\mathcal{P}(\underline{A}) \nmid d$ or there exist $\langle h^T, h^I, h^F\rangle \in H$ and $c \in G(\overline{A}, \langle h^T, h^I, h^F\rangle)/N^h$ such that $l(c) \nmid d.$

If $\mathcal{P}(\underline{A}) \nmid d$ then **A** is not universally d-periodic.

As the second case we setup the FNSM $\tilde{A} = (\langle \tilde{a}^T_{ij}, \tilde{a}^I_{ij}, \tilde{a}^F_{ij}\rangle)$ as follows:

$$\langle \tilde{a}^T_{ij}, \tilde{a}^I_{ij}, \tilde{a}^F_{ij}\rangle = \begin{cases} \langle \overline{a}^T_{ij}, \overline{a}^I_{ij}, \overline{a}^F_{ij}\rangle \ if \ (i,j) \in c, \\ \langle \underline{a}^T_{ij}, \underline{a}^I_{ij}, \underline{a}^F_{ij}\rangle \ otherwise. \end{cases}$$

There exists $\mathcal{K}^* \in \mathcal{S}^*G(\tilde{A}, \langle h^T, h^I, h^F\rangle)$ contain a cycle c only. Since $\mathcal{P}(\mathcal{K}^* = l(c)) \nmid d$ by Theorem 2.2, $\mathcal{P}(\tilde{A}) \nmid d.$ Thus an IFNSM **A** is not UDP.

Conversely if **A** is not UDP and $\mathcal{P}(\underline{A}) \mid d$. We show that there exists $\langle h^T, h^I, h^F\rangle \in H$ and $c \in G(\overline{A}, \langle h^T, h^I, h^F\rangle)/N^h$ such that $l(c) \nmid d$.

If **A** is not UDP then there exists $A \in \mathbf{A}, \langle h^T, h^I, h^F\rangle \in \mathcal{N}$ and $\mathcal{K} \in \mathcal{S}^*(G(A, \langle h^T, h^I, h^F\rangle))$ such that $\mathcal{P}(\mathcal{K}) \nmid d$. Further $\mathcal{P}(\underline{A}) \mid d$ implies $N_{\mathcal{K}} \subseteq N^h$. From $\mathcal{P}(\mathcal{K}) \nmid d$ implies that there exists a cycle $c \in \mathcal{K}$ such that $l(c) \nmid d$. From $G(A, \langle h^T, h^I, h^F\rangle)/N^h \subseteq G(\overline{A}, \langle h^T, h^I, h^F\rangle)/N^h$ it follows that $c \in G(\overline{A}, \langle h^T, h^I, h^F\rangle)N^h$. Define $\langle \tilde{h}^T, \tilde{h}^I, \tilde{h}^F\rangle$ as follows:

$$\langle \tilde{h}^T, \tilde{h}^I, \tilde{h}^F\rangle = \begin{cases} \langle h^T, h^I, h^F\rangle^{(r)}, \ if \ \langle h^T, h^I, h^F\rangle \leq \min\limits_{i,j\in N} \langle \overline{a}^T_{ij}, \overline{a}^I_{ij}, \overline{a}^F_{ij}\rangle = \langle h^T, h^I, h^F\rangle^{(r)}, \\ \langle h^T, h^I, h^F\rangle^{(k)} \ if \ \langle h^T, h^I, h^F\rangle^{(k)} \geq \langle h^T, h^I, h^F\rangle > \langle h^T, h^I, h^F\rangle^{(k+1)}. \end{cases}$$

Since $N^h \subseteq N^{\tilde{h}}$ and $G(\overline{A}, \langle h^T, h^I, h^F\rangle^{(r)}) = G(\overline{A}, \langle \tilde{h}^T, \tilde{h}^I, \tilde{h}^F\rangle)$ implies

$$c \in G(\overline{A}, \langle \tilde{h}^T, \tilde{h}^I, \tilde{h}^F\rangle)/N^{\tilde{h}}.$$

Thus a cycle $c \in G(\overline{A}, \langle \tilde{h}^T, \tilde{h}^I, \tilde{h}^F\rangle)/N^{\tilde{h}}$ exists such that $l(c) \nmid d$. □

Notice that Theorem 2.2 implies that the computational complexity of a procedure based on checking all cycles in $G(\overline{A}, \langle h^T, h^I, h^F\rangle)/\tilde{N}$ can be exponentially large. The efficient algorithm for the interval circulant matrices is suggested in [12].

5. Conclusion

In this paper the authors presented d-periodicity of fuzzy neutrosophic soft matrices (FNSMs) with interval fuzzy neutrosophic soft matrices (IFNSMs).

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