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Quadri Partitioned Neutrosophic Soft Set

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Abstract

The aim of this paper is to introduce the new concept of quadri partitioned neutrosophic soft set and discussed some of its properties. And the focus of this paper is to propose a new notion of quadri partitioned neutrosophic soft set and to study some basic operations and results in quadri partitioned neutrosophic soft set. Further we develop a systematic study on quadri partitioned neutrosophic soft set and obtain various properties induced by them. Some equivalent characterization and inter-relations among them are discussed with counter example.

Keywords: Soft set, quadri partitioned, Neutrosophic set, quadric partitioned neutrosophic set.

1. Introduction

The fuzzy set was introduced by Zadeh [19] in 1965. F. Smarandache introduced the idea of the Neutrosophic set. It is a mathematical method for handling issues involving unreliable, indeterminate and inconsistent details.

A neutrosophic set [13] is proposed by F. Smarandache. The indeterminacy membership function walks along independently of the membership of the truth or the membership of falsity in neutrosophic sets. Neutrosophic theory has been extensively discussed in the treatment of real life conditions involving uncertainty by researchers for application purposes. While the hesitation margin of neutrosophical theory is independent of membership in truth or falsehood, it still seems more general than intuitionist fuzzy sets. Recently, the relationships between inconsistent intuitionistic fuzzy sets, image fuzzy sets, neutrosophic sets, and intuitionistic fuzzy sets have been examined in Atanassov et al.[3]; however, it remains doubtful whether the indeterminacy associated with a particular element exists due to the element's ownership or non-belongingness. Chatterjee et al.[4] have pointed out this while implementing a more general structure of neutrosophic set viz.quadri partitioned single valued neutrosophic set (QSVNS). "In fact, the concept of QSVNS is extended from the four numerical-valued neutrosophical logic of Smarandache and the four valued logic of Belnap, where indeterminacy is split into two parts, namely "unknown" and "contradiction. However, in the sense of neutrosophic science, the QSVNS seems very rational.[1-8] Chatterjee[4] et al. also studied a real-life instance in their analysis for a better understanding of a QSVNS setting and showed that such circumstances occur very naturally.[9-15]. Molodtsov[7] first proposed the idea of Soft Sets as an entirely new mathematical method to solve problems dealing with uncertainties. A soft set is defined by Molodtsov[7] as a parameterized family of universe set subsets where each member is regarded as a set of approximate elements of the soft set. In the past few years, different researchers have researched the foundations of soft set theory.

In this paper, we have to introduce the concept of quadripartitioned neutrosophic soft set and topological space and establish some of its properties. [16-20].

2. Preliminaries

Definition: 2.1 [13]

Let U be a universe. A Neutrosophic set A on U can be defined as follows:

$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$

Where $T_A, I_A, F_A: U \to [0,1]$ and
$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$
Here $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminancy and $F_A(x)$ is the degree of non-membership.

Definition: 2.2 [7]

Let U be an initial universe set and E be a set of parameters or attributes with respect to U. Let P(U) denote the power set of U and A ⊆ U. A pair (F, A) is called a soft set over U, where F is a mapping given by F : A → P(U).

In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For e ∈ A, F(e) may be considered as the set of e-elements or e-approximate elements of the sets (F, A). Thus (F, A) is defined as (F, A) $=\{F(e) \in P(X): e \in E, Fe=\emptyset \ if e \notin A$.

Definition: 2.3 [4]

Let U be a universe. A quadri partitioned neutrosophic set A on U is defined as

$A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >: x \in U\}$

Where $T_A, F_A, C_A, U_A : X \longrightarrow [0,1]and$
$0 \le T_A(x) + C_A(x) + U_A(x) + F_A(x) \le 4$
Here $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

3. Quadri Partitioned Neutrosophic Soft Set (QNSS)

Definition: 3.1

Let X is an initial universe set and E is a set of parameters. Consider a non-empty set A and A ⊆ E. Let P(X) denote the set of all quadri partitioned neutrosophic sets of X. The collection (F, A) is termed to be the quadri partitioned neutrosophic soft set (QNSS) over X, where F is a mapping given by F : A⟶ P(X)

Where

$A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >: x \in U\}$

Where $T_A, F_A, C_A, U_A : X \longrightarrow [0,1]$ and
$0 \le T_A(x) + C_A(x) + U_A(x) + F_A(x) \le 4$
Here $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

Example :

X = Set of Mobile Phones = {M_1, M_2, M_3, M_4}

A= {e_1,e_2,e_3,e_4}= { expensive, battery, memory capacity, camera quality}

F(expensive)=
$\{< M_1, 0.5,0.6,0.7,0.2 > < M_2, 0.7,0.5,0.4,0.1 >$
$< M_3, 0.6,0.5,0.4,0.3 > < M_4, 0.3,0.2,0.6,0.1 >\}$

F(battery) =
$\{< M_1, 0.4,0.3,0.2,0.6 >, < M_2, 0.8,0.6,0.5,0.4 >$
$< M_3, 0.2,0.3,0.5,0.6 > < M_4, 0.6,0.7,0.8,0.2 >$

F(memory capacity)=
$\{< M_1, 0.2,0.3,0.5,0.6 >, < M_2, 0.6,0.5,0.4,0.3 >$
$< M_3, 0.5,0.6,0.7,0.2 > < M_4, 0.3,0.2,0.6,0.1 >$

F(camera quality) =
$\{< M_1, 0.7,0.5,0.4,0.3 >, < M_2, 0.2,0.3,0.5,0.6 >$
$< M_3, 0.6,0.5,0.4,0.3 > < M_4, 0.5,0.4,0.2,0.3 >$

Describe the attraction by customer for mobile phones, F(e_1)={M_2,M_3}, F(e_2)={M_1,M_2,M_3}, F(e_3) = {M_2,M_4}, F(e_4)= {M_1,M_4}

Then the set (F, A) = { F(e_1), F(e_2),F(e_3), F(e_4)}is a quadri partitioned neutrosophic soft set.

Definition: 3.2

A quadri partitioned neutrosophic soft set A is contained in another quadri partitioned neutrosophic soft set B (A$\subseteq B$) if $T_A(x) \le T_B(x)$, $C_A(x) \le C_B(x)$, $U_A(x) \ge U_B(x)$ and $F_A(x) \ge F_B(x)$.

Definition: 3.3

The complement of a quadri partitioned neutrosophic soft set (F, A) is denoted by$(F, A)^c$ and is defined as

F^c(x)=$\{< x, F_A(x), U_A(x), C_A(x), T_A(x) >: x \in X\}$

Definition: 3.4

Let X be a non-empty set,

A = $< x, T_A(x), C_A(x), U_A(x), F_A(x) >$ and

B= $< x, T_B(x), C_A(x), U_B(x), F_B(x) >$ are quadri partitioned neutrosophic soft sets. Then

A∪ B=
$< x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), min(U_A(x), U_B(x)), min(F_A(x), F_B(x)) >$

A∩ B=
$< x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), max(U_A(x), U_B(x)), max(F_A(x), F_B(x)) >$

Definition: 3.5

A quadri partitioned neutrosophic soft set (F, A) over the universe X is said to be empty neutrosophic soft set with respect to the parameter A if $T_{F(e)} = 0$, $C_{F(e)} = 0$, $U_{F(e)} = 1$, $FFe=1, \forall x \in X, \forall e \in A$. It is denoted by $0N$

Definition: 3.6

A quadri partitioned neutrosophic soft set (F, A) over the universe X is said to be universe neutrosophic soft set with respect to the parameter A if $T_{F(e)} = 1$, $C_{F(e)} = 1$, $U_{F(e)} = 0$, $F_{F(e)}$=0, $\forall x \in X, \forall e \in A$. It is denoted by 1_N

Remark: $0_N^c = 1_N$ and $1_N^c = 0_N$

Definition: 3.7

Let A and B be two quadri partitioned neutrosophic soft sets then A\B may be defined as

A\B=

$< x, min(T_A(x), F_B(x)), min(C_A(x), U_B(x)),$
$max(U_A(x), C_B(x)), max(F_A(x), T_B(x)) >$

Definition: 3.8

The set F_E is called absolute quadri partitioned neutrosophic soft set over X if F(e) = 1_N for any $e \in E$. We denote it by X_E

Definition: 3.9

The set F_E is called relative null quadri partitioned neutrosophic soft set over X if F(e) = 0_N for any $e \in E$. We denote it by $\emptyset_E$

Obviously $\emptyset_E = X_E^c$ and $X_E = \emptyset_E^c$

Definition: 3.10

The complement of a quadri partitioned neutrosophic soft set (F, A) can also be defined as $(F, A)^c = X_E \backslash F(e)$ for all $e \in A$.

Note: We denote X_E by X in the proofs of proposition.

Definition: 3.11

If (F, A) and (G, B) be two quadri partitioned neutrosophic soft set then "(F,A) AND (G,B)" is a denoted by (F,A) ∧(G, B) and is defined by (F,A) ∧ (G,B) = (H, A ×B)

Where H (a, b) = F(a) ∩ G(b) $\forall a \in A$ and $\forall b \in B$, where ∩ is the operation intersection of quadri partitioned neutrosophic soft set.

Definition: 3.12

If (F, A) and (G, B) be two quadri partitioned neutrosophic soft set then "(F,A) OR (G,B)" is a denoted by (F, A) ∨(G, B) and is defined by (F,A) ∨ (G,B) = (K, A ×B) where K (a, b) = F(a) ∪G(b) $\forall a \in A$ and $\forall b \in B$, where ∪ is the operation union of quadri partitioned neutrosophic soft set.

Theorem : 3.13

Let (F, A) and (G, A) be two QNSS over the universe X. Then the following are true.

(i) (F,A)⊆ (G,A) iff (F,A) ∩ (G,A) = (F,A)

(ii) (F,A)⊆ (G,A) iff (F,A) ∪ (G,A) = (F,A)

Proof:

(i)Suppose that (F, A) ⊆ (G, A), then F(e) ⊆G(e) for all e∈ A. Let (F, A) ∩ (G, A) = (H, A).

Since H(e) = F(e) ∩ G(e) = F(e) for all e∈ A, by definition (H, A)=(F, A).

Suppose that (F, A) ∩ (G, A) = (F,A).

Let (F, A) ∩ (G, A) = (H, A).

Since H(e) = F(e) ∩ G(e) = F(e) for all e∈ A,we know that F(e) ⊆ G(e) for all e∈ A.

Hence (F,A) ⊆ (G,A).

(ii) The proof is similar to (i).

Theorem : 3.14

Let (F, A), (G, A), (H, A), and (S, A) are QNSS over the universe X. Then the following are true.

(i) If (F, A) ∩ (G, A) =$\emptyset_A$, then (F,A)⊆ (G,A)^c

(ii) If (F,A)⊆ (G,A) and (G,A) ⊆ (H,A) then (F,A)⊆ (H,A)

(iii) If (F,A)⊆ (G,A) and (H,A)⊆ (S,A) then (F, A) ∩ (H, A)⊆(G, A) ∩ (S, A)

(iv) (F, A)⊆ (G, A) iff (G,A)^c ⊆ (F,A)^c

Proof:

(i)Suppose that (F, A) ∩ (G, A) =$\emptyset_A$.

Then F(e)∩ G(e) = ∅.

So, F(e)⊆X\G(e)=$G^c(e)$ for all e∈ A.

Therefore, we have (F,A)⊆ (G,A)^c

Proof of (ii) and (iii) are obvious.

(iv) (F,A)⊆ (G,A)

⟺F(e)⊆ G(e) for all e∈ A.

⟺(G(e))^c ⊆(F(e))^c for all e∈ A.

$\Leftrightarrow G^c(e) \subseteq F^c(e)\ for\ all\ e \in A.$

⟺(G,A)^c ⊆ (F,A)^c

Definition: 3.15

Let I be an arbitrary index $\{(F_i, A)\}_{i\in I}$ be a subfamily of quadri partitioned neutrosophic soft set over the universe X.

(i) The union of these quadri partitioned neutrosophic soft set is the quadri partitioned neutrosophic soft set (H,A) where H(e) = $\bigcup_{i\in I} F_i(e)$ for each e∈ A.

$We\ write\ \bigcup_{i\in I}(F_i, A)$ = (H, A)

(ii) The intersection of these quadri partitioned neutrosophic soft set is the quadri partitioned neutrosophic soft set (M,A) where M(e)= $\bigcap_{i\in I} F_i(e)$ for each e∈ A.

$We\ write\ \bigcap_{i\in I}(F_i, A)$=(M,A)

Theorem: 3.16

Let I be an arbitrary index set and $\{(F_i, A)\}_{i\in I}$be a subfamily of QNSS over the universe X. Then

(i) $(\bigcup_{i\in I}(F_i, A))^C = \bigcap_{i\in I}(F_i, A)^C$

(ii) $(\bigcap_{i\in I}(F_i, A))^C = \bigcup_{i\in I}(F_i, A)^C$

Proof:

(i) $(\bigcup_{i\in I}(F_i, A))^C$ = (H,A)^C, By definition $H^C(e)=X_E\backslash H(e)=X_E\backslash\bigcup_{i\in I} F_i(e) = \bigcap_{i\in I}(X_E\backslash F_i(e))$ for all e∈ A.

On the other hand, $(\bigcap_{i\in I}(F_i, A))^C$ =(K,A).

By definition, K(e)=$\bigcap_{i\in I} F_i^C(e)$= $\bigcap_{i\in I}(X - F_i(e))$for all e$\in A$.

(ii) Proof is similar to (i).

Note: We denote $\emptyset_E$ by $\emptyset$ and X_E by X.

Theorem: 3.17

Let (F, A) be QNSS over the universe X. Then the following are true.

(i) $(\emptyset, A)^c$ = (X, A)

(ii) $(X, A)^c$ =$(\emptyset, A)$

Proof:

(i) Let ($\emptyset$, A) =(F, A)

Then F(e) = $\{< x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x) >: x \in X\}$ = {(x,0,0,1,1): $x \in X$} ∀e∈ A,

$(\emptyset, A)^c = (F, A)^c$

Then ∀e∈ A,

$(F(e))^c = \{< x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x): x \in X\}^c$

$= \{< x, F_{F(e)}(x), U_{F(e)}(x), C_{F(e)}(x), T_{F(e)}(x) >: x \in X\}$

= {(x,1,1,0,0): $x \in X$} = X

Thus $(\emptyset, A)^c$ = (X, A)

(ii) Proof is similar to (i)

Theorem: 3.18

Let (F, A) be QNSS over the universe X. Then the following are true.

(i) (F, A)∪ ($\emptyset$, A) = (F, A)

(ii) (F, A)∪ (X, A) = (X, A)

Proof:

(i) (F, A)

= {e, $\left(x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x)\right)$: $x \in X$} ∀e∈ A

($\emptyset$, A) = {e,(x,0,0,1,1):x ∈X} ∀e∈ A

(F, A)∪ ($\emptyset$, A) = {e,$\left(x, max(T_{F(e)}(x), 0\right)$, $max(C_{F(e)}(x)$,0)$, min\left(U_{F(e)}(x), 1\right)$, $min(F_{F(e)}(x)$,1)): $x \in X$} e∈ A

={e, $x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x)$): $x \in X$} ∀e∈ A

= (F, A)

(ii) Proof is similar to (i).

Theorem: 3.19

Let (F, A) be QNSS over the universe X. Then the following are true.

(i) (F, A)∩ ($\emptyset$, A) = ($\emptyset$, A)

(ii) (F, A)∩ (U, A) = (F, A)

Proof:

(i) (F, A)

= {e,$\left(x, T_{F(e)}(x), C_{F(e)}(x), U_{F(e)}(x), F_{F(e)}(x)\right)$: $x \in X$}∀e∈ A

($\emptyset$, A) = {e, (x, 0, 0, 1, 1): x ∈X} ∀e∈ A

(F,A)∩($\emptyset$,A)

={e, ($x, min(T_{F(e)}(x)$,0)$, min(C_{F(e)}(x)$,0), $max(U_{F(e)}(x)$,1)$max(F_{F(e)}(x)$,1)): $x \in X$} ∀e∈ A

= {e, , (x, 0,0,1,1): $x \in X$}∀e∈ A

= ($\emptyset$, A)

(ii) Proof is similar to (i).

Note: We denote T_F(x), $C_F(x), U_F(x)$ and $F_F(x)$ by $T_F C_F, U_F and\ F_F$

Theorem: 3.20

Let (F, A) and (G, A) be two QNSS set over the universe X. Then the following are true.

(i) (F, A)∪ ($\emptyset$, B) = (F, A) iff B⊆A

(ii) (F, A)∪ (X, B) = (X, A) iff A⊆ B

Proof:

(i) We have for (F, A),

F(e) = $\{(x, T_F, C_F, U_F, F_F): x \in X\}$∀e∈ A

Also let ($\emptyset$, B) = (G, B) then

G(e) = {(x,0,0,1,1): x ∈X} ∀e∈B

Let (F, A)∪ ($\emptyset$, B) = (F, A)∪ (G, B) = (H,C) where C = A ∪B and for all e ∈ C

H(e) may be defined as

$$\begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\} if\ e \in A - B \\ \{(x, 0,0,1,1): x \in X\} if\ e \in B - A \\ \{(x, max(T_{F(e)}, 0), max(C_{F(e)}, 0), min(U_{F(e)}, 1), min(F_{F(e)}, 1)): x \in X\}\ if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\} if\ e \in A - B \\ \{(x, 0,0,1,1): x \in U\}\ if\ e \in B - A \\ \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\} if\ e \in A \cap B \end{cases}$$

Let B⊆A

Then H(e) =

$$\begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}(x)): x \in X\} if\ e \in A - B \\ \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\} if\ e \in A \cap B \end{cases}$$

= F(e) $\forall e \in A$

Conversely Let $(F, A) \cup (\emptyset, B) = (F, A)$

Then A = A ∪B$\Longrightarrow B \subseteq A$

(ii) Proof is similar to (i)

Theorem: 3.21

Let (F, A) and (G, B) be two QNSS over the

universe X. Then the following are true.
(i) (F, A)∩ (∅, B) = (∅, A∩B)
(ii)(F, A)∩ (U, B) = (F, A∩B)

Proof:

(i) We have for (F,A)
F(e) = $\{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\}$ ∀e∈ A
Also let (∅, B) = (G,B) then
G(e) = {(x,0,0,1,1): x ∈X} ∀e∈B
Let (F,A) ∩ (∅,B)=(F, A) ∩(G,B) =(H,C) where C = A ∩B and ∀e∈C

$$H(e) = \{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), max(U_{F(e)}, U_{G(e)}) max(F_{F(e)}, F_{G(e)})): x \in X\}$$
$$= \{(x, min(T_{F(e)}, 0), min(C_{F(e)}, 0), max(U_{F(e)}, 1), max(F_{F(e)}, 1)): x \in X\}$$
$$= \{(x, 0,0,1,1): x \in X\}$$
= (G,B) = (∅, B)

Thus (F, A)∩ (∅, B) = (∅, B) =(∅, A∩B)
(ii) Proof is similar to (i).

Theorem: 3.22

Let (F, A) and (G, B) be two QNSS over the universe X. Then the following are true.
(i) ((F, A)∪ (G, B)) ^{C} ⊆ (F,A) ^{C}∪(G,B) ^{C}
(ii) (F, A)^C∩(G,B)^C ⊆((F, A)∩ (G, B))^C

Proof:

Let (F,A)∪(G,B) = (H,C) Where C = A∪B, ∀ e∈ C
H(e) may be defined as

$$\begin{cases} \{(x, T_{F(e)}, C_{F(e)}, U_{F(e)}, F_{F(e)}): x \in X\} if\ e \in A - B \\ \{(x, T_{G(e)}, C_{G(e)}, U_{G(e)}, F_{G(e)}): x \in X\}\ if\ e \in B - A \\ \{(x, max(T_{F(e)}, T_{G(e)}), max(C_{F(e)}, C_{G(e)}), min(U_{F(e)}, U_{G(e)}), min(F_{F(e)}, F_{G(e)})): x \in X\}\ if\ e \in A \cap B \end{cases}$$

Thus (F,A)∪(G,B))^C = (H,C)^C Where C = A∪B and∀ e∈ C

$$(H(e))^C = \begin{cases} (F(e))^C\ if\ e \in A - B \\ (G(e))^C\ if\ e \in B - A \\ (F(e) \cup G(e))^C if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}, U_{F(e)}, C_{F(e)}, T_{F(e)}): x \in X\}\ if\ e \in A - B \\ \{(x, F_{G(e)}, U_{G(e)}, C_{G(e)}, T_{G(e)}): x \in X\}\ if\ e \in B - A \\ \{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)})): x \in X\}\ if e \in A \cap B \end{cases}$$

Again (F,A)^C∪(G,B)^C=(I,J) say J = A∪B and ∀ e∈J

$$I(e) = \begin{cases} (F(e))^C\ if\ e \in A - B \\ (G(e))^C\ if\ e \in B - A \\ (F(e) \cup G(e))^C if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}, U_{F(e)}, C_{F(e)}, T_{F(e)}): x \in X\} if e \in A - B \\ \{(x, F_{G(e)}, U_{G(e)}, C_{G(e)}, T_{G(e)}): x \in X\} if e \in B - A \\ \{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)})) : x \in X\}\ if\ e \in A \cap B \end{cases}$$

So, C⊆ J∀ e∈ J,
(H(e))^C ⊆I(e)
Thus (F,A)∪(G,B))^C⊆(F,A)^C∪(G,B)^C
(ii) Let (F,A)∩(G,B) = (H,C) Where C = A∩B and ∀ e∈ C
H(e) = F(e)∩G(e)

$$=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), max(U_{F(e)}(x), U_{G(e)}(x)), max(F_{F(e)}, F_{G(e)}))\}$$

Thus((F,A)∩(G,B))^C = (H,C)^C Where C = A∩B and ∀ e∈ C

$$(H(e))^C=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}), max(U_{F(e)}, U_{G(e)}), max(F_{F(e)}, F_{G(e)}))\}^C$$
$$=\{(x, max(F_{F(e)}, F_{G(e)}), max(U_{F(e)}, U_{G(e)}), min(C_{F(e)}, C_{G(e)}), min(T_{F(e)}, T_{G(e)}))\}$$

Again (F,A)^C∩ (G,B)^C = (I,J) say where J= A∩ B and ∀ e∈ J I(e) = (F(e))^C∩ (G(e))^C

$$=\{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)}))\}$$

We see that C = J and ∀ e∈ J, I(e) ⊆(H(e))^C
Thus (F,A)^C ∩(G,B)^C ⊆ ((F, A)∩ (G, B))^C

Theorem: 3.23

Let (F, A) and (G, A) are two QNSS over the same universe X. We have the following
(i)((F, A)∪ (G, A))^C= (F,A)^C ∩(G,A)^C
(ii) ((F, A)∩(G, A))^C = (F,A)^C∪(G,A)^C

Proof:

(i) Let (F,A) ∪(G,A) =(H,A) ∀ e∈A
H(e) = F(e)∪G(e)

$$= \{(x, max(T_{F(e)}, T_{G(e)}), max(C_{F(e)}, C_{G(e)}), min(U_{F(e)}, U_{G(e)}), min(F_{F(e)}, F_{G(e)}))\}$$

Thus (F,A) ∪ (G,A))^C =(H,A)^C∀ e∈ A
(H(e))^C = (F(e)∪G(e))^C

$$=\{(x, max(T_{F(e)}, T_{G(e)}), max(C_{F(e)}, C_{G(e)}), min(U_{F(e)}, U_{G(e)}), min(F_{F(e)}, F_{G(e)}))\}^C$$
$$=\{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)}))\}$$

Again (F, A)^C∩ (G, A)^C =(I,A) where∀ $e \in A$
I(e) = (F(e))^C∩ (G(e))^C

$$=\{(x, min(F_{F(e)}, F_{G(e)}), min(U_{F(e)}, U_{G(e)}), max(C_{F(e)}, C_{G(e)}), max(T_{F(e)}, T_{G(e)}))\}$$

Thus ((F, A)∪ (G, A))^C = (F,A)^C ∩(G,A)^C

(ii) Let (F,A) ∩ (G,A) =(H,A) ∀ e∈ A

H(e) = F(e)∩G(e)

$=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}),$ $max(U_{F(e)}, U_{G(e)}) max(F_{F(e)}, F_{G(e)}))\}$∀ e∈ A

Thus (F,A) ∩ (G,A))^C =(H,A)^C

(H(e))^C = (F(e)∩G(e))^C

$=\{(x, min(T_{F(e)}, T_{G(e)}), min(C_{F(e)}, C_{G(e)}),$ $max(U_{F(e)}, U_{G(e)}), max(F_{F(e)}, F_{G(e)}))\}^C$

$=\{(x, max(F_{F(e)}, F_{G(e)}), max(U_{F(e)}, U_{G(e)}),$ $min(C_{F(e)}, C_{G(e)}), min(T_{F(e)}, T_{G(e)}))\} \forall e \in A$

Again (F, A)^C∪ (G, A)^C =(I,A) where$\forall e \in A$

I(e) = (F(e))^C∪ (G(e))^C

$=\{(x, max(F_{F(e)}(x), F_{G(e)}(x)), max(U_{F(e)}(x),$ $U_{G(e)}(x)), min(C_{F(e)}(x), C_{G(e)}(x)),$ $min(T_{F(e)}(x), T_{G(e)}(x))\}$

Thus ((F, A) ∩ (G, A))^C = (F,A)^C∪(G,A)^C

Theorem: 3.24

Let (F, A) and (G, A) are two QNSS over the same universe X. We have the following

(i) ((F, A)∧ (G, A))^C= (F,A)^C ∨(G, A)^C

(ii) ((F, A)∨ (G, A))^C = (F,A)^C∧(G,A)^C

Proof:

Let (F, A) ∧(G,B)=(H,A ×B) where

H(a, b) = F(a) ∩G(b) ∀ $a \in A$ and ∀ $b \in B$

where ∩ is the operation intersection of QNSS.

Thus H (a ,b)=F(a) ∩G(b)

$=\{(x, min(T_{F(a)}, T_{G(b)}, min(C_{F(a)}, C_{G(b)}),$ $max(U_{F(a)}, U_{G(b)}), max(F_{F(a)}, F_{G(b)})\}$

((F, A) ∧ (G,B))^C=(H,A ×B)^C $\forall (a, b) \in A \times B$

Thus (H(a, b))^C

$=\{(x, min(T_{F(a)}, T_{G(b)}), min(C_{F(a)}, C_{G(b)}),$ $max(U_{F(a)}, U_{G(b)}), max(F_{F(a)}, F_{G(b)})\}^C$

$=\{(x, max(F_{F(a)}, F_{G(b)}), max(U_{F(a)}, U_{G(b)}),$ $min(C_{F(a)}, C_{G(b)}), min(T_{F(a)}, T_{G(b)})\}$

Let (F,A)^C ∨(G, A)^C =(R,A ×B) where

R(a,b)=(F(a))^C∪ (G(b))^C ∀ $a \in A$ and ∀ $b \in B$

where ∪is the operation union of quadri partitioned neutrosophic soft set.

R(a,b)=$\{(x, max(F_{F(a)}, F_{G(b)}), max(U_{F(a)}, U_{G(b)}),$ $min(C_{F(a)}, C_{G(b)}), min(T_{F(a)}, T_{G(b)})\}$

Hence ((F, A)∧ (G, A))^C= (F,A)^C ∨(G, A)^C

Similarly we can prove (ii).

Conclusion

In this paper, we have studied the quadri partitioned neutrosophic set and soft set. Based on their definition and properties, we have newly introduced the quadric partitioned neutrosophic soft set and some basic definition on this topic also we have discussed some of its properties and theorems.

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