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Section of Fuzzy Neutrosophic Soft Matrix

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Abstract

This article expose the notion of $(\alpha, \alpha', \alpha'')$-cut of a fuzzy neutrosophic soft matrix (FNSM) and decompose a fuzzy neutrosophic soft matrix using its $(\alpha, \alpha', \alpha'')$-cut.
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1 Introduction

Fuzzy sets , theory of intuitionistic fuzzy sets , theory of vague sets, theory of interval Mathematics and theory of rough sets which can be considered as Mathematical tools for dealing with uncertainities. But all these theries have their inherent difficulties as pointed out in [4]. The reason for these difficulties is, possibly, the inadequency of the parametrization tools of the theories. Consequently, Molodtsov [4] initiated the concept of soft set theory as a Mathematical tool for dealing with uncertainties which is free from the above difficulties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work [4].

Smarandache [5] introduced the concept of neutrosophic set which is a Mathematical tool for handling problems involving imprecise, indeterminancy and inconsistent data. Maji et al.,[3] extended soft set to intuitionistic fuzzy soft set and neutrosophic soft sets. Arokiarani and sumathi[1] introduced fuzzy neutrosophic soft matrix of the fuzzy neutrosophic soft set. K.H. Kim and W.Roush [2] introduced the concept of section($\alpha - cut$) of fuzzy matrix. F.I Sidky and E.G. Emam [6] discussed the relation between a fuzzy matrix and its section. Consequently Sriram and Murugadas [7] introduced $(\alpha, \alpha^{'})$-cut of intuitionistic fuzzy matrix.

Uma et al.,[8] introduced two types of fuzzy neutrosophic soft matrices. In this paper $(\alpha, \alpha^{'}, \alpha^{''})$-cut of fuzzy neutrosophic soft matrices has been introduced and fuzzy neutrosophic soft matrix has been decomposed by means of its sections $(\alpha, \alpha^{'}, \alpha^{''}$-cut).

2 Preliminaries

Let $\mathcal{N}_{m\times n}$ denotes fuzzy neutrosophic soft matrix of order $m \times n$ and $\mathcal{N}_n$ denotes fuzzy neutrosophic soft matrix of order n.

Definition 1. [8][Type-I] Let $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle)$,
$B = (\langle b_{ij}^T, b_{ij}^I, b_{ij}^F\rangle) \in \mathcal{N}_{m\times n}$ the componentwise addition and componentwise multiplication is defined as
$A \oplus B = \ (sup\left\{a_{ij}^T, b_{ij}^T\right\},\ \ sup\left\{a_{ij}^I, b_{ij}^I\right\},\ \ inf\left\{a_{ij}^F, b_{ij}^F\right\})$.
$A \odot B = (inf\{a_{ij}^T, b_{ij}^T\},\ \ inf\{a_{ij}^I, b_{ij}^I\},\ \ sup\{a_{ij}^F, b_{ij}^F\})$.

Definition 2. [8] Let $A \in \mathcal{N}_{m\times n}, B \in \mathcal{F}_{n\times p}$, the composition of A and B is defined as

$$A \circ B = \left(\sum_{k=1}^{n}(a_{ik}^T \wedge\ b_{kj}^T),\quad \sum_{k=1}^{n}(a_{ik}^I \wedge\ b_{kj}^I),\quad \prod_{k=1}^{n}(a_{ik}^F \vee b_{kj}^F\right)$$

equivalently we can write the same as

$$= \left(\bigvee_{k=1}^{n}(a_{ik}^T \wedge b_{kj}^T),\quad \bigvee_{k=1}^{n}(a_{ik}^I \wedge b_{kj}^I),\quad \bigwedge_{k=1}^{n}(a_{ik}^F \vee b_{kj}^F)\right).$$

The product $A \circ B$ is defined if and only if the number of columns of A is same as the number of rows of B. A and B are said to be conformable for multiplication. We shall use AB instead of $A \circ B$.

Definition 3. [8] Let $A = (\langle a_{ij}^T,\ a_{ij}^I,\ a_{ij}^F\rangle)$, $B = (\langle b_{ij}^T,\ b_{ij}^I,\ b_{ij}^F\rangle)$

be two FNSMs of same dimensions. We write $A \leq B$ if $a_{ij}^T \leq b_{ij}^T,\ a_{ij}^I \leq b_{ij}^I,\ a_{ij}^F \geq b_{ij}^F$ for all i, j and we say that A is dominated by B (or) B dominates A. A and B are said to be comparable, if either $A \leq B$ (or) $B \leq A$. $A < B$ if $a_{ij}^T < b_{ij}^T,\ a_{ij}^I < b_{ij}^I,\ a_{ij}^F > b_{ij}^F$.

3 Resolution Of Fuzzy Neutrosophic Soft Matrix

Definition 4. Let $A \in FNSS$. For any two comparable elements $\langle a, a^{'}, a^{''}\rangle, \langle b, b^{'}, b^{''}\rangle \in A$ define

$$\langle a, a^{'}, a^{''}\rangle \leftarrow \langle b, b^{'}, b^{''}\rangle = \begin{cases} \langle 1,1,0\rangle & \text{if } \langle a, a^{'}, a^{''}\rangle \geq \langle b, b^{'}, b^{''}\rangle \\ \langle a, a^{'}, a^{''}\rangle & \text{if } \langle a, a^{'}, a^{''}\rangle < \langle b, b^{'}, b^{''}\rangle. \end{cases}$$

and $\langle a, a^{'}, a^{''}\rangle \leftarrow \langle b, b^{'}, b^{''}\rangle = \langle b, b^{'}, b^{''}\rangle \rightarrow \langle a, a^{'}, a^{''}\rangle$

Definition 5. Let $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle) \in N_{mn}$ and $B = (\langle b_{ij}^T, b_{ij}^I, b_{ij}^F\rangle) \in N_{nq}$ be FNSMs,define

$$A \leftarrow B = (\bigwedge_{k=1}^{n} (\langle a_{ik}^T, a_{ik}^I, a_{ik}^F\rangle \leftarrow \langle b_{kj}^T, b_{kj}^I, b_{kj}^F\rangle))$$

Definition 6. For $A \in \mathcal{N}_n$, if
(i) $A^2 \leq A$, then A is called transitive. (ii)$I_n \leq A$, then A is called reflexive. (iii)$A = A^T$ then A is called symmetric. (iv) $A^2 = A$, then A is called idempotent. (v)$A \wedge A^T \leq I_n$, then A is called antisymmetric.
If A has a similarity relation if and only if it is reflexive,symmetric and transitive. If A is reflexive and transitive, then A is a matrix representing an neutrosophic preorder.

In this part we have discussed $\langle a, a^{'}, a^{''}\rangle$- cut of fuzzy neutrosophic soft matrix and some of its properties. Through out this section FNSM means FNSM in which the entries are comparable.

Definition 7. Let $A \in \mathcal{N}_{m\times n}$. Define for $\langle \alpha, \alpha^{'}, \alpha^{''}\rangle \in [0,1]$ with $\alpha + \alpha^{''} + \alpha^{''} \leq 3$ the section $A_{\langle \alpha, \alpha^{'}, \alpha^{''}\rangle}$ (or) $\langle \alpha, \alpha^{'}, \alpha^{''}\rangle$- level cut=$(\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle_{\langle \alpha, \alpha^{'}, \alpha^{''}\rangle})$, where

$$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle_{\langle \alpha, \alpha^{'}, \alpha^{''}\rangle} = \begin{cases} (1,1,0) & \text{if } \langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle \geq \langle \alpha, \alpha^{'}, \alpha^{''}\rangle \\ \langle 0,0,1\rangle & otherwise. \end{cases}$$

If ψ_A= the set of all nonzero entries of A, then for each $\langle \alpha, \alpha^{'}, \alpha^{''} \rangle \in \psi_A$, the term $A_{\langle \alpha, \alpha', \alpha'' \rangle}$ are called the zero patterns of A (or) $\langle \alpha, \alpha^{'}, \alpha^{''} \rangle$ level cuts of A.

Proposition 8. *Let $A \in \mathcal{N}_{m\times n}, \psi_A$ be the set of all non -zero patterns of A. Then for $\langle a, a^{'}, a^{''} \rangle, \langle b, b^{'}, b^{''} \rangle \in \psi_A$ with $\langle a, a^{'}, a^{''} \rangle \geq \langle b, b^{'}, b^{''} \rangle$ the zero patterns $A_{\langle a,a',a'' \rangle} \leq A_{\langle b,b',b'' \rangle}$.*

Proof. Let $A_{\langle a,a',a'' \rangle} = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle a,a',a'' \rangle})$ and
$A_{\langle b,b',b'' \rangle} = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle b,b',b'' \rangle})$.
Now $\langle a, a^{'}, a^{''} \rangle \geq \langle b, b^{'}, b^{''} \rangle$ implies $a \geq b, a^{'} \geq b^{'}, a^{''} \leq b^{''}$
Case(i). If $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle a,a',a'' \rangle} = \langle 0, 0, 1 \rangle$ for all i,j, then obviously
$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle a,a',a'' \rangle} \leq \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle b,b',b'' \rangle}$
Case(ii). If $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle a,a',a'' \rangle} = \langle 1, 1, 0 \rangle$ for all i, j, then
$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle \geq \langle a, a^{'}, a^{''} \rangle \geq \langle b, b^{'}, b^{''} \rangle$ for all i,j.
Therefore $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle b,b',b'' \rangle} = \langle 1, 1, 0 \rangle$ for all i,j.
Thus $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle a,a',a'' \rangle} \leq \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle b,b',b'' \rangle}$ for all i,j.
Hence $A_{\langle a,a',a'' \rangle} \leq A_{\langle b,b',b'' \rangle}$. □

Theorem 9. *Let $A \in \mathcal{N}_{m\times n}$ and ψ_A be the set of all non-zero elements of A. Then $A = max_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle \in \psi_A} \{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle A_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle}\}$; that is A can be expressed as a fuzzy neutrosophic linear combination of its zero patterns. This expression is called resolution of A.*

Proof. Let $\langle b_{ij}^T, b_{ij}^I, b_{ij}^F \rangle = max_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle \in \psi_A} \{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle \times i, j^{th}$ entry of
$A_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle}\} = max_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle \in \psi_A} \{min\{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle, \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle}\}\}$
$= max_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle \in \psi_A} \{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle\} = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$
since$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle_{\langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle} = \langle 1, 1, 0 \rangle (or) \langle 0, 0, 1 \rangle$ according as
$\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle \geq \langle \alpha_k, \alpha_k^{'}, \alpha_k^{''} \rangle$ or not. Therefore $\langle b_{ij}^T, b_{ij}^I, b_{ij}^F \rangle = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle$ for all i,j. □

Definition 10. For $\langle a^T, a^I, a^F \rangle, \langle b^T, b^I, b^F \rangle \in FNSS$, with $a^T + a^I + a^F \leq 3, b^T + b^I + b^F \leq 3$ define

$$\langle a^T, a^I, a^F \rangle \ominus \langle b^T, b^I, b^F \rangle = \begin{cases} \langle a^T, a^I, a^F \rangle & \text{if } \langle a^T, a^I, a^F \rangle > \langle b^T, b^I, b^F \rangle \\ \langle 0, 0, 1 \rangle & \text{if } \langle a^T, a^I, a^F \rangle \leq \langle b^T, b^I, b^F \rangle. \end{cases}$$

For $A, B \in \mathcal{N}_{m\times n}, P \in \mathcal{N}_{n\times m}, D \in FNSM_n$,

define $A \ominus B = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle \ominus \langle b_{ij}^T, b_{ij}^I, b_{ij}^F \rangle)$,
$A \setminus P = A \ominus AP, \Delta S = S - S^T$ and $\nabla S = S \wedge S^T$for all i,j.

Definition 11. Let $A, B \in \mathcal{N}_{m \times n}$, then

$$A \ominus B = \begin{cases} \langle a_{ij}^T, a_{ij}^T, a_{ij}^T \rangle & \text{if } \langle a_{ij}^T, a_{ij}^T, a_{ij}^T \rangle > \langle b_{ij}^T, b_{ij}^T, b_{ij}^T \rangle \\ \langle 0, 0, 1 \rangle & \text{if } \langle a_{ij}^T, a_{ij}^T, a_{ij}^T \rangle \le \langle b_{ij}^T, b_{ij}^T, b_{ij}^T \rangle \end{cases}$$

Lemma 12. *For $\langle a^T, a^I, a^F \rangle, \langle b^T, b^I, b^F \rangle$ and $\langle \alpha, \alpha^{'}, \alpha^{''} \rangle \in [0,1]$ with $\langle a^T + a^I + a^F \rangle \le 3, \langle b^T + b^I + b^F \rangle \le 3, \langle \alpha + \alpha^{'} + \alpha^{''} \rangle \le 3$ we have the following.*
(i)$\langle a^T, a^I, a^F \rangle \ge \langle b^T, b^I, b^F \rangle \Rightarrow$
$\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} \ge \langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$
(ii)$(\langle a^T, a^I, a^F \rangle \langle b^T, b^I, b^F \rangle)_{\langle \alpha, \alpha', \alpha'' \rangle} =$
$\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} \langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$
(iii)$(\langle a^T, a^I, a^F \rangle + \langle b^T, b^I, b^F \rangle)_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} +$
$\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$
(iv)$(\langle a^T, a^I, a^F \rangle \to \langle b^T, b^I, b^F \rangle)_{\langle \alpha, \alpha', \alpha'' \rangle} \le \langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} \to$
$\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$
(v)$(\langle a^T, a^I, a^F \rangle \ominus \langle b^T, b^I, b^F \rangle)_{\langle \alpha, \alpha', \alpha'' \rangle} \ge \langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} \ominus$
$\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$

Proof. **(i)Case 1**. If $\langle a^T, a^I, a^F \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$ and
$\langle b^T, b^I, b^F \rangle \le \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$, then $\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$ and
$\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 0, 0, 1 \rangle$.
Therefore $\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} \ge \langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$.
Case 2.If $\langle a^T, a^I, a^F \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$ and $\langle b^T, b^I, b^F \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$,
then $\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$ and $\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$.
Case 3. If $\langle a^T, a^I, a^F \rangle \le \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$ and $\langle b^T, b^I, b^F \rangle \le \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$,
then $\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 0, 0, 1 \rangle$ and $\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 0, 0, 1 \rangle$.
Hence in all the above three cases,
$\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} \ge \langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle}$.
(ii)**Case 1**. If $\langle a^T, a^I, a^F \rangle \langle b^T, b^I, b^F \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$, then
$\langle a^T, a^I, a^F \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$ and $\langle b^T, b^I, b^F \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$. Therefore
$\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$ and $\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$.
Thus $(\langle a^T, a^I, a^F \rangle \langle b^T, b^I, b^F \rangle)_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle \ge \langle \alpha, \alpha^{'}, \alpha^{''} \rangle$.
Hence $(\langle a^T, a^I, a^F \rangle \langle b^T, b^I, b^F \rangle)_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$ Also
$\langle a^T, a^I, a^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$ and $\langle b^T, b^I, b^F \rangle_{\langle \alpha, \alpha', \alpha'' \rangle} = \langle 1, 1, 0 \rangle$

gives $\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}\langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 1,1,0\rangle$

Case 2. If $\langle a^T, a^I, a^F\rangle\langle b^T, b^I, b^F\rangle \leq \langle\alpha,\alpha^{'},\alpha^{''}\rangle$, then $(\langle a^T, a^I, a^F\rangle\langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 0,0,1\rangle$. Also, either $\langle a^T, a^I, a^F\rangle \leq \langle\alpha,\alpha^{'},\alpha^{''}\rangle$ or $\langle b^T, b^I, b^F\rangle \leq \langle\alpha,\alpha^{'},\alpha^{''}\rangle$. Therefore $\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 0,0,1\rangle$ or $\langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 0,0,1\rangle$.
Thus $\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}\langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 0,0,1\rangle$.
Hence $(\langle a^T, a^I, a^F\rangle\langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle} = \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}$ $\langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}$.

(iii) Case 1. If $(\langle a^T, a^I, a^F\rangle + \langle b^T, b^I, b^F\rangle) \geq \langle\alpha,\alpha^{'},\alpha^{''}\rangle$, then either $\langle a^T, a^I, a^F\rangle \geq \langle\alpha,\alpha^{'},\alpha^{''}\rangle$ or $\langle b^T, b^I, b^F\rangle \geq \langle\alpha,\alpha^{'},\alpha^{''}\rangle$ or both. Therefore, $(\langle a^T, a^I, a^F\rangle + \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 1,1,0\rangle$ and $\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} + \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 1,1,0\rangle$.

Case 2. If $(\langle a^T, a^I, a^F\rangle + \langle b^T, b^I, b^F\rangle) < \langle\alpha,\alpha^{'},\alpha^{''}\rangle$, then $\langle a^T, a^I, a^F\rangle < \langle\alpha,\alpha^{'},\alpha^{''}\rangle$ and $\langle b^T, b^I, b^F\rangle < \langle\alpha,\alpha^{'},\alpha^{''}\rangle$. Therefore $(\langle a^T, a^I, a^F\rangle + \langle b^T, b^I, b^F\rangle) = \langle 0,0,1\rangle = \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} + \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 0,0,1\rangle + \langle 0,0,1\rangle = \langle 0,0,1\rangle$. Hence (iii) holds.

(iv) Case 1.If $\langle b^T, b^I, b^F\rangle \geq \langle a^T, a^I, a^F\rangle$, then $(\langle a^T, a^I, a^F\rangle \rightarrow \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 1,1,0\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 1,1,0\rangle$ and $\langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \geq \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}$by (i)
Therefore,$\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \rightarrow \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 1,1,0\rangle$.

Case 2. If $\langle b^T, b^I, b^F\rangle < \langle a^T, a^I, a^F\rangle$, then
$(\langle a^T, a^I, a^F\rangle \rightarrow \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle}$
$= \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}$ and $\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \rightarrow \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle}$

$$= \begin{cases} \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} & \text{if } \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} > \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \\ \langle 1,1,0\rangle & \text{otherwise} \end{cases}$$

Therefore (iv) holds.

(v) Case 1. If $\langle b^T, b^I, b^F\rangle \geq \langle a^T, a^I, a^F\rangle$, then $(\langle a^T, a^I, a^F\rangle \ominus \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle} =$

$$\langle 0,0,1\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \begin{cases} \langle 0,0,1\rangle & \text{if } \langle 0,0,1\rangle < \langle\alpha,\alpha^{'},\alpha^{''}\rangle \\ \langle 1,1,0\rangle & \text{otherwise} \end{cases}$$

and $\langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \ominus \langle b^T, b^I, b^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} = \langle 0,0,1\rangle$ Lemma12
Therefore, $(\langle a^T, a^I, a^F\rangle \ominus \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle} \geq \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \ominus \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle}$

Case 2. If $\langle b^T, b^I, b^F\rangle < \langle a^T, a^I, a^F\rangle$, then
$(\langle a^T, a^I, a^F\rangle \ominus$
$\langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle}$

$= \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \geq \langle a^T, a^I, a^F\rangle_{\langle\alpha,\alpha',\alpha''\rangle} \ominus \langle b^T, b^I, b^F\rangle)_{\langle\alpha,\alpha',\alpha''\rangle}$. □

Proposition 13. *Let $A, B \in N_{m\times n}, R \in N_n$ and $C \in N_{n\times p}$. Then we have the following.*
(i) $A \geq B \Rightarrow A_{\langle\alpha,\alpha',\alpha''\rangle} \geq B_{\langle\alpha,\alpha',\alpha''\rangle}$
(ii)$(A \wedge B)_{\langle\alpha,\alpha',\alpha''\rangle} = A_{\langle\alpha,\alpha',\alpha''\rangle} \wedge B_{\langle\alpha,\alpha',\alpha''\rangle}$
(iii)$(A + B)_{\langle\alpha,\alpha',\alpha''\rangle} = A_{\langle\alpha,\alpha',\alpha''\rangle} + B_{\langle\alpha,\alpha',\alpha''\rangle}$
(iv)$(A \rightarrow B)_{\langle\alpha,\alpha',\alpha''\rangle} \leq A_{\langle\alpha,\alpha',\alpha''\rangle} \rightarrow B_{\langle\alpha,\alpha',\alpha''\rangle}$
(v)$(A \ominus B)_{\langle\alpha,\alpha',\alpha''\rangle} \geq A_{\langle\alpha,\alpha',\alpha''\rangle} \ominus B_{\langle\alpha,\alpha',\alpha''\rangle}$.
(vi)$(AB)_{\langle\alpha,\alpha',\alpha''\rangle} = A_{\langle\alpha,\alpha',\alpha''\rangle}B_{\langle\alpha,\alpha',\alpha''\rangle}$
(vii)$(A \setminus R)_{\langle\alpha,\alpha',\alpha''\rangle} \geq A_{\langle\alpha,\alpha',\alpha''\rangle} \setminus R_{\langle\alpha,\alpha',\alpha''\rangle}$
(viii)$(A^T)_{\langle\alpha,\alpha',\alpha''\rangle} = (A_{\langle\alpha,\alpha',\alpha''\rangle})^T$.

Proof. The proof of the proposition is evident from the above lemma 12. □

Proposition 14. *Let $A, B \in N_{m\times n}$ and for $a_1, a_2, a_3, b_1, b_2, b_3 \in [0, 1]$ satisfying $a_1 + a_2 + a_3 \leq 3, b_1 + b_2 + b_3 \leq 3$ with $\langle a_1, a_2, a_3\rangle \leq \langle b_1, b_2, b_3\rangle$ we have*
(i)$(A + B)_{\langle b_1,b_2,b_3\rangle} \leq A_{\langle a_1,a_2,a_3\rangle} + B_{\langle b_1,b_2,b_3\rangle} \leq (A + B)_{\langle a_1,a_2,a_3\rangle}$
(ii)$(A \wedge B)_{\langle b_1,b_2,b_3\rangle} \leq A_{\langle a_1,a_2,a_3\rangle} \wedge B_{\langle b_1,b_2,b_3\rangle} \leq (A \wedge B)_{\langle a_1,a_2,a_3\rangle}$

Proof. (i)$(A + B)_{\langle b_1,b_2,b_3\rangle} = A_{\langle b_1,b_2,b_3\rangle} + B_{\langle b_1,b_2,b_3\rangle}$
by Proposition 13(iii)$\leq A_{\langle a_1,a_2,a_3\rangle}+B_{\langle a_1,a_2,a_3\rangle} \leq A_{\langle a_1,a_2,a_3\rangle}+B_{\langle a_1,a_2,a_3\rangle}$
by Proposition 13 (i) $= (A+B)_{\langle a_1,a_2,a_3\rangle}$ again by Proposition 13(iii).
(ii)$(A \wedge B)_{\langle b_1,b_2,b_3\rangle} = A_{\langle b_1,b_2,b_3\rangle} \wedge B_{\langle b_1,b_2,b_3\rangle} \leq A_{\langle a_1,a_2,a_3\rangle} \wedge B_{\langle a_1,a_2,a_3\rangle}$
$= (A \wedge B)_{\langle a_1,a_2,a_3\rangle}$
Remark . The generalization of the above proposition to a finite number n.

$$(\sum_{i=1}^{n} A_i)_{\{\max_{i=1}^{n}\langle a_i^T,a_i^I,a_i^F\rangle\}} \leq \sum_{i=1}^{n}(A_i)_{\langle a_i^T,a_i^I,a_i^F\rangle} \leq (\sum_{i=1}^{n} A_i)_{\{min_{i=1}^{n}\langle a_i^T,a_i^I,a_i^F\rangle\}}$$

and

$$(\bigwedge_{i=1}^{n} A_i)_{\{\max_{i=1}^{n}\langle a_i^T,a_i^I,a_i^F\rangle\}} \leq \bigwedge_{i=1}^{n}(A_i)_{\{\max_{i=1}^{n}\langle a_i^T,a_i^I,a_i^F\rangle\}} \leq (\bigwedge_{i=1}^{n} A_i)_{\{\min_{i=1}^{n}\langle a_i^T,a_i^I,a_i^F\rangle\}}.$$

□

Proposition 15. *For $A \in N_n$ we have*
(i)$\Delta A_{\langle\alpha,\alpha',\alpha''\rangle} \leq (\Delta A)_{\langle\alpha,\alpha',\alpha''\rangle}(ii)\nabla A_{\langle\alpha,\alpha',\alpha''\rangle} = (\nabla A)_{\langle\alpha,\alpha',\alpha''\rangle}$.

Proof. (i)$\Delta A_{\langle\alpha,\alpha',\alpha''\rangle} = A_{\langle\alpha,\alpha',\alpha''\rangle} \ominus (A_{\langle\alpha,\alpha',\alpha''\rangle})^T$

$= A_{\langle\alpha,\alpha',\alpha''\rangle} \ominus (A^T)_{\langle\alpha,\alpha',\alpha''\rangle} \leq (A \ominus A^T)_{\langle\alpha,\alpha',\alpha''\rangle} = (\Delta A)_{\langle\alpha,\alpha',\alpha''\rangle}$
$(ii)\nabla A_{\langle\alpha,\alpha',\alpha''\rangle} = A_{\langle\alpha,\alpha',\alpha''\rangle} \wedge (A_{\langle\alpha,\alpha',\alpha''\rangle})^T = A_{\langle\alpha,\alpha',\alpha''\rangle} \wedge (A^T)_{\langle\alpha,\alpha',\alpha''\rangle}$
$= (A \wedge A^T)_{\langle\alpha,\alpha',\alpha''\rangle} = (\nabla A)_{\langle\alpha,\alpha',\alpha''\rangle}$. □

Conclusion:
In this paper a fuzzy neutrosphic soft matrix is decomposed by means of its section of fuzzy neutrosphic soft matrix of Type-I.

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