Shortest Path Problem under Trapezoidal Neutrosophic Information

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***Abstract*— In this research paper, a new approach is proposed for computing the shortest path length from source node to destination node in a neutrosophic environment. The edges of the network are assigned by trapezoidal fuzzy neutrosophic numbers. A numerical example is provided to show the performance of the proposed approach.**

***Keywords*— neutrosophic sets; trapezoidal neutrosophic sets; shortest path problem; score function**

I. Introduction

Smarandache [1] proposed the concept of neutrosophic sets (in short NSs) as a means of expressing the inconsistencies and indeterminacies that exists in most real-life problems. The proposed concept generalized fuzzy sets and intuitionistic fuzzy set theory [3, 4]. The notion of NSs is described with three functions: truth, an indeterminacy and a falsity, where the functions are totally independent, the three functions are inside the unit interval $]^{-}0, 1^{+}[$. To practice NSs in real life situations efficiently. A new version of NSs named Single valued Neutrosophic Sets (in short SVNSs) was defined by Smarandache in [1]. Subsequently Wang et al. [5] defined various operations and operators for the SVNS model. Additional literature on single valued neutrosophic sets can be found in [6-14, 16]. Also later on, Smarandache extended the neutrosophic set to neutrosophic overset, underset, and offset [15]. Ye [17] presented the concept of trapezoidal fuzzy neutrosophic set (in short TrFNSs) and studied some interesting results with proofs and examples. In TrFNSs, the truth, the indeterminate and the false membership degrees are expressed with Trapezoidal Fuzzy Numbers (TrFN) instead of real numbers. Smarandache and Kandasamy [25, 28-29] introduced the concept of neutrosophic graph based on the indeterminacy component (I). Later on, in [18-23, 26-27] Broumi et al. introduced different types of neutrosophic graph based on the neutrosophic values (T, I, F) including single valued neutrosophic graphs, interval valued neutrosophic graphs and bipolar neutrosophic graphs. In graph theory, the shortest path problems (in short SPP) is one of the known famous problems studied in the numerous discipline including operation research, computer science, communication network and so on. In the literature, many research papers have been focused seriously on fuzzy shortest path problems and their extensions [30-39]. Till now, few research papers deal with shortest path problems in neutrosophic environment. In [40-44], *Broumi et al.* presented some algorithms for solving the shortest path problems in neutrosophic environment. All these algorithms are based on the score functions. In this paper, the addition operation and the order relation have been given by Ye [17]. In this research paper, our main objective is to solving the shortest path problems in a network, where the edges weight are represented by trapezoidal fuzzy neutrosophic numbers.

This paper is constructed as follows: In Section 2, some basic definitions of neutrosophic sets, SVN-sets and trapezoidal fuzzy neutrosophic sets are introduced. In section 3, a new proposed algorithm for computing the trapezoidal fuzzy neutrosophic shortest path problem on a network is presented. In Section 4, a numerical example is given for computing the shortest path and shortest distance from the source node to destination node. We conclude the paper in Section 5.

II. PRELIMINARIES

In this section, some definitions related to the concept of neutrosophic sets, single valued neutrosophic and trapezoidal fuzzy neutrosophic sets are taken from [2, 5, 17]

Definition 2.1 [2] Let ζ be a universal set. The neutrosophic set A on the universal set ζ categorized into three membership functions called the true $T_A(x)$, indeterminate $I_A(x)$ and false $F_A(x)$ contained in real

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standard or non–standard subset of]⁻0, 1⁺[respectively and denoted as following

A= {<x: $T_A(x)$, $I_A(x)$, $F_A(x)$ > $x \in \zeta$ } (1)

Definition 2.2 [5] Let ζ be a universal set. The single valued neutrosophic sets (in short SVNS) A on the universal is denoted as following:

A= {<x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, $x \in \zeta$ } (2)

The function $T_A(x) \in$ [0, 1], $I_A(x)$ $\in$ [0, 1] and $I_A(x)$ $\in$ [0, 1] are named "degree of truth, indeterminacy and falsity membership of x in A", satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(\mathrm{x}) \le 3 \quad (3)$$

Definition 2.3 [17]. Let ζ be a universal set and ψ [0, 1] be *the sets of all trapezoidal fuzzy numbers on [0, 1].* The trapezoidal fuzzy neutrosophic sets (in short TrFNSs) $\breve{A}$ on the universal is denoted as following:

$$\breve{A} = \{<\mathrm{x}: \breve{T}_A(x), \breve{I}_A(x), \breve{F}_A(x)>,\ x \in \zeta\} \quad (4)$$

Where $\breve{T}_A(\mathrm{x}): \zeta \to \psi[0,1]$, $\breve{I}_A(\mathrm{x}): \zeta \to \psi[0,1]$ and $\breve{F}_A(\mathrm{x}): \zeta \to \psi[0,1]$. The trapezoidal fuzzy numbers

$\breve{T}_A(\mathrm{x}) = (\mathrm{T}_A^1(x), \mathrm{T}_A^2(x), \mathrm{T}_A^3(x), \mathrm{T}_A^4(x))$, (5)

$\breve{I}_A(x) = (I_A^1(\mathrm{x}), I_A^2(\mathrm{x}), I_A^3(\mathrm{x}), I_A^4(\mathrm{x}))$ and (6)

$\breve{F}_A(x) = (F_A^1(\mathrm{x}), F_A^2(\mathrm{x}), F_A^3(\mathrm{x}), F_A^4(\mathrm{x}))$, respectively denotes degree of truth, indeterminacy and falsity membership of x in $\breve{A}$ $\forall x \in \zeta$.

$$0 \le \mathrm{T}_A^4(x) + I_A^4(x) + F_A^4(x) \le 3. \quad (7)$$

For notational convenience, the trapezoidal fuzzy neutrosophic value (TrFNV) $\tilde{A}$ is denoted by $\breve{A} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ where,

$(\mathrm{T}_A^1(\mathrm{x}), \mathrm{T}_A^2(\mathrm{x}), \mathrm{T}_A^3(\mathrm{x}), \mathrm{T}_A^4(\mathrm{x})) = (t_1, t_2, t_3, t_4)$, (8)

$(I_A^1(\mathrm{x}), I_A^2(\mathrm{x}), I_A^3(\mathrm{x}), I_A^4(\mathrm{x})) = (i_1, i_2, i_3, i_4)$, and (9)

$(F_A^1(\mathrm{x}), F_A^2(\mathrm{x}), F_A^3(\mathrm{x}), F_A^4(\mathrm{x})) = (f_1, f_2, f_3, f_4)$ (10)

with $t_1 \le t_2 \le t_3 \le t_4$, $i_1 \le i_2 \le i_3 \le i_4$ and $f_1 \le f_2 \le f_3 \le f_4$

where, the truth membership function is given as bellow:

$$\breve{T}_A(x) = \begin{cases} \dfrac{x - t_1}{t_2 - t_1} & t_1 \le x \le t_2 \\ 1 & t_2 \le x \le t_3 \\ \dfrac{x - t_1}{t_2 - t_1} & t_3 \le x \le t_4 \\ 0 & otherwise \end{cases} \quad (11)$$

The indeterminacy membership is given as below:

$$\breve{I}_A(\mathrm{x}) = \begin{cases} \dfrac{x - i_1}{i_2 - i_1} & i_1 \le x \le i_2 \\ 1 & i_2 \le x \le i_3 \\ \dfrac{i_4 - x}{i_4 - i_3} & i_3 \le x \le i_4 \\ 0 & otherwise \end{cases} \quad (12)$$

And the falsity membership function is given as below:

$$\breve{F}_A(x) = \begin{cases} \dfrac{x - f_1}{f_2 - f_1} & f_1 \le x \le f_2 \\ 1 & f_2 \le x \le f_3 \\ \dfrac{f_4 - x}{f_4 - f_3} & f_3 \le x \le f_4 \\ 0 & otherwise \end{cases} \quad (13)$$

Definition 2.4 [17]. The trapezoidal fuzzy neutrosophic number $\breve{A} = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ is said to be trapezoidal fuzzy neutrosophic zero if and only if $(t_1, t_2, t_3, t_4) =$ (0, 0, 0, 0), (i_1, i_2, i_3, i_4) = (1, 1, 1, 1) and (f_1, f_2, f_3, f_4) =(1, 1, 1, 1) (14)

Definition 2.5 [17]. Let $\breve{A}_1$ and $\breve{A}_2$ two TrFNVs defined on the set of real numbers, denoted as :

$\breve{A}_1 = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (\mathrm{c}_1, c_2, c_3, c_4) \rangle$ and $\breve{A}_2 = \langle (e_1, e_2, e_3, e_4), (f_1, f_2, f_3, f_4), (g_1, g_2, g_3, g_4) \rangle$ and $\eta > 0$. Hence , the operations rules are defined as following:

$$\text{(i)}\ \breve{A}_1 \oplus \breve{A}_2 = \left\langle \begin{matrix} \begin{pmatrix} (a_1 + e_1 - a_1 e_1), (a_2 + e_2 - a_2 e_2), \\ (a_3 + e_3 - a_3 e_3), (a_4 + e_4 - a_4 e_4) \end{pmatrix}, \\ ((\mathrm{b}_1 f_1), (\mathrm{b}_2 f_2), (\mathrm{b}_3 f_3), (\mathrm{b}_4 f_4)), \\ (c_1 g_1), (c_2 g_2), (c_3 g_3), (c_4 g_4)) \end{matrix} \right\rangle \quad (15)$$

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$$\text{(ii) } \breve{A}_1 \otimes \breve{A}_2 = \left\langle \begin{array}{l} (a_1e_1, a_2e_2, a_3e_3, a_4e_4), \\ \begin{pmatrix} (b_1 + f_1 - b_1 f)_1, (b_2 + f_2 - b_2 f_2), \\ (b_3 + f_3 - b_3 f_3), (b_4 + f_4 - b_4 f_4) \end{pmatrix}, \\ \begin{pmatrix} (c_1 + g_1 - c_1 g_1), (c_2 + g_2 - c_2 g_2), \\ (c_3 + g_3 - c_3 g_3), (c_4 + g_4 - c_4 g_4) \end{pmatrix} \end{array} \right\rangle \quad (16)$$

$$\text{(iii) } \eta \breve{A} = \left\langle \begin{array}{l} \begin{pmatrix} (1-(1-a_1)^{\eta}), (1-(1-a_2)^{\eta}), \\ (1-(1-a_3)^{\eta}), (1-(1-a_4)^{\eta}) \end{pmatrix} \\ (b_1^{\eta}, b_2^{\eta}, b_3^{\eta}, b_4^{\eta}), (c_1^{\eta}, c_2^{\eta}, c_3^{\eta}, c_4^{\eta}) \end{array} \right\rangle \quad (17)$$

(iv)

$$\breve{A}_1^{\eta} = \left\langle \begin{array}{l} (a_1^{\eta}, a_2^{\eta}, a_3^{\eta}, a_4^{\eta}), \\ \left((1-(1-b_1)^{\eta}), (1-(1-b_2)^{\eta}), (1-(1-b_3)^{\eta}), 1-(1-b_4)^{\eta})\right), \\ \left((1-(1-c_1)^{\eta}), (1-(1-c_2)^{\eta}), (1-(1-c_3)^{\eta})), (1-(1-c_4)^{\eta})\right) \end{array} \right\rangle$$

where $\eta > 0$ (18)

Ye [17] gave the definition of score function $s(\breve{A}_1)$ and accuracy function $H(\breve{A}_1)$ to compare the grades of TrFNS. These functions shows that greater is the value, the greater is the TrFNS and by using these concept paths can be ranked.

Definition 2.6. Let $\breve{A}_1$ be a TrFNV denoted as $\breve{A}_1 = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ Hence , the score function and the accuracy function of TrFNV are denoted as below:

(i)

$$s(\breve{A}_1) = \frac{1}{12} \begin{bmatrix} 8 + (t_1 + t_2 + t_3 + t_4) - (i_1 + i_2 + i_3 + i_4) \\ -(f_1 + f_2 + f_3 + f_4) \end{bmatrix} \quad (19)$$

$$\text{(ii) } H(\breve{A}_1) = \frac{1}{4}\left[(t_1 + t_2 + t_3 + t_4) - (f_1 + f_2 + f_3 + f_4)\right] \quad (20)$$

In order to make a comparisons between two TrFNV, Ye **[17]**, presented the order relations between two TrFNVs.

Definition 2.7 Let $\breve{A}_1$ and $\breve{A}_2$ be two TrFNV defined on the set of real numbers , denoted as

$\breve{A}_1 = \langle (t_1, t_2, t_3, t_4), (i_1, i_2, i_3, i_4), (f_1, f_2, f_3, f_4) \rangle$ and

$\breve{A}_2 = \langle (p_1, p_2, p_3, p_4), (q_1, q_2, q_3, q_4), (r_1, r_2, r_3, r_4) \rangle$. Hence , the ranking method is defined as follows:

i. If $s(\breve{A}_1) \succ s(\breve{A}_2)$, then $\breve{A}_1$ is greater than $\breve{A}_2$, that is, $\breve{A}_1$ is superior to $\breve{A}_2$, denoted by $\breve{A}_1 \succ \breve{A}_2$

ii. If $s(\breve{A}_1) = s(\breve{A}_2)$, and $H(\breve{A}_1) \succ H(\breve{A}_2)$ then $\breve{A}_1$ is greater than $\breve{A}_2$, that is, $\breve{A}_1$ is superior to $\breve{A}_2$, denoted by $\breve{A}_1 \succ \breve{A}_2$.

III. TRFN- SHORTEST PATH PROBLEM

In this section, the edge length in a network is considered to be trapezoidal fuzzy neutrosophic number. To find the shortest path in a network , where the edges are characterized by trapezoidal fuzzy neutrosophic number. We present the following procedure :

Step 1 Suppose $\tilde{d}_1$ = <(0, 0, 0, 0) (1, 1, 1, 1), (1, 1, 1, 1)> and label the source node1 as [$\tilde{d}_1$ = < (0, 0, 0, 0),(1, 1, 1,1), (1, 1, 1, 1)>, -]

Let n is the destination node.

Step 2: Select $\tilde{d}_j$ = min { $\tilde{d}_i \oplus \tilde{d}_{ij}$ } for all j= 2,3,…,n.

Step 3: If the minimum provided correspond to one value of i then label node j as [$\tilde{d}_j$, i]. If the minimum provided correspond to several values of i, then it indicate that there exist more than one TrFN-path between the source node and the node j. Hence the TrFN-distance along path is $\tilde{d}_j$, so select any value of i.

Step 4: Set the destination node n be labeled as [$\tilde{d}_n$,*l*], then the TrFN-shortest path distance from source node to destination node is $\tilde{d}_n$.

Step 5: Since the destination node n is labeled [$\tilde{d}_n$, *l*]. In order to find the TrFN-shortest path connecting the source node and the destination node, identify the label of the node *l*. Set it as [$\tilde{d}_l$,*p*], Repeat step 2 and step 3 until the node 1 is obtained.

Step 6: To obtain the TrFN-shortest path, we should joining all the nodes provided by the step 5.

IV. ILLUSTRATIVE EXAMPLE

Consider a small network shown in the following figure 1 in which each edge length is represented by a trapezoidal fuzzy neutrosophic number (see table 1). This network includes 6 nodes and 8 directed edges. This problem is to compute the

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shortest path between source node and destination node in the given network.

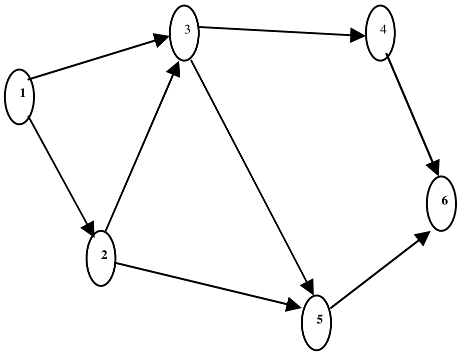

Fig.1. Trapezoidal fuzzy neutrosophic network.

The edges weight of the trapezoidal fuzzy neutrosophic network are represented by trapezoidal fuzzy neutrosophic numbers.

TABLE 1. THE EDGES WEIGHT OF THE TRAPEZOIDAL FUZZY NEUTROSOPHIC GRAPHS

Edges	Trapezoidal fuzzy neutrosophic distance
1-2	<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>
1-3	<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>
2-3	<(0.3, 0.4, 0.6, 0.7), (0.1, 0.2, 0.3, 0.5), (0.3, 0.5, 0.7, 0.9)>
2-5	<(0.1, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.7), (0.2, 0.3, 0.6, 0.7)>
3-4	<(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>
3-5	<(0.3, 0.6, 0.7, 0.8), (0.1, 0.2, 0.3, 0.4), (0.1, 0.4, 0.5, 0.6)>
4-6	<(0.4, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6), (0.1, 0.3, 0.4, 0.5)>
5-6	<(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6), (0.1, 0,3, 0.5, 0.6)>

Using the algorithm proposed in section 2, we can determine the shortest path between any two nodes. Let node 1 is the source node and node 6 is the destination node.

Suppose $\tilde{d}_1$ = <(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> and label the source node 1 as [<(0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> ,-], the value $\tilde{d}_2$, $\tilde{d}_3$, $\tilde{d}_4$, $\tilde{d}_5$ and $\tilde{d}_5$ can be computed following the iterations described below:

Iteration1: The node 2 has on predecessor, which is node 2. Following the step 2 in the proposed algorithm, we put i=1and j=2, hence the value of $\tilde{d}_2$ can be computed as follows:

$\tilde{d}_2$= min{ $\tilde{d}_1 \oplus \tilde{d}_{12}$ } = min{<(0, 0, 0), (1, 1, 1), (1, 1, 1)> ⊕ <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>= <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>

So, the minimum provided correspond to the node 1.Hence, the node 2 is labeled as
[<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>, 1]

$\tilde{d}_2$ = <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>

Iteration 2: The node 3 has two predecessors, which are node 1 and node 2. Following the step 2 in the proposed algorithm, we put i=1, 2and j=3, hence the value of $\tilde{d}_3$ can be computed as follows:

$\tilde{d}_3$=min { $\tilde{d}_1 \oplus \tilde{d}_{13}, \tilde{d}_2 \oplus \tilde{d}_{23}$ }= min {<((0, 0, 0, 0), (1, 1, 1, 1), (1, 1, 1, 1)> ⊕ <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)> ⊕ <(0.3, 0.4, 0.6, 0.7), (0.1, 0.2, 0.3, 0.5), (0.3, 0.5, 0.7, 0.9)>} = min{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15, 0.3), (0.12, 0.25, 0.42, 0.72)>}

S (<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) using Eq.19, we have

$$s(\tilde{A}_1) = \frac{1}{12}\begin{bmatrix} 8+(t_1+t_2+t_3+t_4)-(i_1+i_2+i_3+i_4) \\ -(f_1+f_2+f_3+f_4) \end{bmatrix} =0.54$$

S (<(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15,0.3), (0.12, 0.25, 0.42, 0.72)>) = 0.70
Since S (<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) < S (<(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15,0.3), (0.12, 0.25, 0.42, 0.72)>)
So, min{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> , <(0.37, 0.52, 0.72, 0.85), (0.02, 0.06, 0.15, 0.3), (0.12, 0.25, 0.42, 0.72)>}
$\tilde{d}_3$ = <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>

So, the minimum provided correspond to the node 1.Hence, the node 3 is labeled as [(<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>) , 1]
$\tilde{d}_3$= <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)>

Iteration 3: The node 4 has one predecessor, which is node 3. Following the step 2 in the proposed algorithm, we put i=1and j=4, hence the value of $\tilde{d}_4$ can be computed as follows:

$\tilde{d}_4$= min{ $\tilde{d}_3 \oplus \tilde{d}_{34}$ }= min {<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3,0.4)> ⊕ <(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>} = <(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>
So min{<(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3,0.4)> ⊕ <(0.2, 0.3, 0.5, 0.6), (0.2, 0.5, 0.6, 0.7), (0.4, 0.5, 0.6, 0.8)>}= <(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

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So, the minimum provided correspond to the node 3.Hence, the node 4 is labeled as [<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)> ,3]

$\tilde{d}_4$ =<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>

Iteration 4: The node 5 has two predecessors, which are node 2 and node 3. Following the step 2 in the proposed algorithm, we put i=2, 3 and j=5, hence the value of $\tilde{d}_5$ can be computed as follows:

$\tilde{d}_5 = \min\{ \tilde{d}_2 \oplus \tilde{d}_{25}, \tilde{d}_3 \oplus \tilde{d}_{35} \}$ = min{<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)> ⊕ <(0.1, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.7), (0.2, 0.3, 0.6, 0.7)>, <(0.2, 0.4, 0.5, 0.7), (0.3, 0.5, 0.6, 0.9), (0.1, 0.2, 0.3, 0.4)> ⊕ <(0.3, 0.6, 0.7, 0.8), (0.1, 0.2, 0.3, 0.4), (0.1, 0.4, 0.5, 0.6)>} =
min{<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> , <(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>}
S (<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>) = 0.69
S (<(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>) = 0.81
Since S (<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>) S (<(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>)
min{<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> , <(0.44, 0.76, 0.85, 0.94), (0.03, 0.1, 0.18, 0.36), (0.01, 0.08, 0.15, 0.42)>}
= <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>
So, the minimum provided correspond to the node 2.Hence, the node 5 is labeled as [<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)>, 2]

$\tilde{d}_5$ = <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>

Iteration 5: The node 6 has two predecessors, which are node 4 and node 5. Following the step 2 in the proposed algorithm, we put i=4, 5 and j=6, hence the value of $\tilde{d}_6$ can be computed as follows:

$\tilde{d}_6 = \min\{ \tilde{d}_4 \oplus \tilde{d}_{46}, \tilde{d}_5 \oplus \tilde{d}_{56} \}$= min{<(0.36, 0.58, 0.75,0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)> ⊕ <(0.4, 0.6, 0.8, 0.9), (0.2, 0.4, 0.5, 0.6), (0.1, 0.3, 0.4, 0.5)>, <(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25, 0.42), (0.02, 0.06, 0.18, 0.56)> ⊕ <(0.2, 0.3, 0.4, 0.5), (0.3, 0.4, 0.5, 0.6), (0.1, 0.5,0.3, 0.6)>} = min{<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>, <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)> }
S (<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>) = 0.87
S (<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>) = 0.81
Since S (<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>) < S (<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>)
min{<(0.616, 0.832, 0.95, 0.98), (0.012, 0.1, 0.18, 0.37), (0.004, 0.03, 0.072, 0.16)>, <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)> }= <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

$\tilde{d}_6$ = <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>
So, the minimum provided correspond to the node 5.Hence, the node 6 is labeled as [<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>, 5]
Since the destination node of the proposed network is the node 6. Hence, the TrFN- shortest distance between source node 1 and destination node is <(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>

So, the TrFN-shortest path between the source node 1 and the destination node 6 can be determined using the following method:

The node 6 takes the label [<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>, 5], which indicate that we are moving from node 5. The node 5 takes the label [<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>, 2] , which indicate that we are moving from node 2. The node 2 takes the label [<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>, 1] which indicate that we are moving from node 1.So, joining all the provided nodes, we get the TrFN-shortest path between the source node 1 and the destination node 6. Hence the TrFN-shortest path is given as follows: $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$

Following the algorithm described in section 2, the computational results for finding the TrFN-shortest path from source node 1 to destination node 6 are summarized in table 2.

TABLE II. SUMMARIZE OF TRAPEZOIDAL FUZZY NEUTROSOPHIC DISTANCE AND SHORTEST PATH.

Node	$\tilde{d}_i$	shortest path between the i-th and 1st node
2	<(0.1, 0.2, 0.3, 0.5), (0.2, 0.3, 0.5, 0.6), (0.4, 0.5, 0.6, 0.8)>	$1 \rightarrow 2$
3	<(0.2, 0.4, 0.5), (0.3, 0.5, 0.6), (0.1, 0.2, 0.3)>	$1 \rightarrow 3$
4	<(0.36, 0.58, 0.75, 0.88), (0.06, 0.25, 0.36, 0.63), (0.04, 0.1, 0.18, 0.32)>	$1 \rightarrow 3 \rightarrow 4$
5	<(0.19, 0.44, 0.58, 0.75), (0.06, 0.12, 0.25,0.42), (0.02, 0.06, 0.18, 0.56)>	$1 \rightarrow 2 \rightarrow 5$
6	<(0.352, 0.608, 0.748, 0.88), (0.018, 0.048, 0.125, 0.25), (0.002, 0.03, 0.054, 0.34)>	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$

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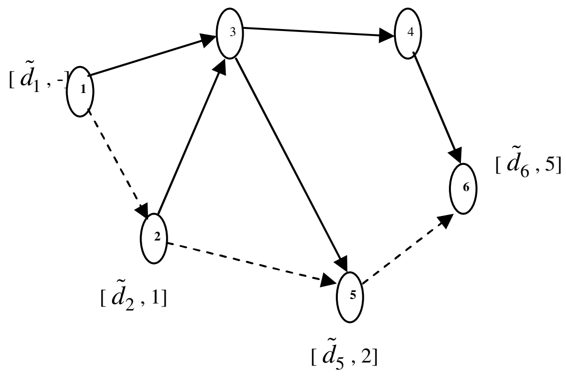

Fig. 2. TrFN-shortest path from source node 1 to destination node 6.

V. CONCLUSION

In this research paper, a new algorithm based on trapezoidal fuzzy neutrosophic numbers is presented for finding the shortest path problem in a network where the edges weight are represented by TrFNN. A numerical example is introduced to show the efficacy of the proposed algorithm. So in the next work, we plan to implement this approach practically.

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