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Some Relation Properties of Rough Neutrosophic Multisets

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ABSTRACT

Rough neutrosophic multisets are an improved model of generalization pf neutrosophic multisets represented within the Pawlak's boundary set of information: lower and upper approximation. The concepts of rough neutrosophic multisets can be easily extended to a relation, mainly since a relation is also a set, i.e. a subset of a Cartesian product. This paper establishes an axiomatic definition of rough neutrosophic multisets relation of Cartesian product over a universal set. Some of the operations and properties of rough neutrosophic multisets, such as max, min, the composition of two rough neutrosophic multisets relation and inverse rough neutrosophic multisets relation, are studied with a proven condition. An algorithm of rough neutrosophic multisets relation is also presented as a step followed to obtain the rough neutrosophic multisets relation. Successful analysis using rough neutrosophic multisets relation theory is represented by the illustrative example of expert opinion about automobile popularity. In conclusion, with a specified condition in uncertainty information, rough neutrosophic multiset relations are generalized in terms of the relation properties of a rough fuzzy relation, rough intuitionistic fuzzy relation, and rough neutrosophic relation over universal. Subsequently, their properties could also be examined.

Keywords: Max, min, neutrosophic multisets, relation, rough neutrosophic multisets, rough set, uncertainty

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INTRODUCTION

The notion of relation plays a significant role in describing the relationship of strength and weakness between the element theory or variable presented in a universal set. For example, in the mathematical formulation of uncertainty information such as win and loss, and accept and reject conditions, the relation between these variables is defined to check whether there is a significant relationship among them. A bigger value for the relation indicates a stronger relationship between the information. This relation can be defined in a universal set or more than one universal set.

Uncertainty information always involves inconsistent information due to the results from opinion experts. The data set for uncertainty information is not suitable to be expressed by a standard mathematical data set. For example, an expert may give a different value for a person's intelligence, such as quite intelligent or more intelligent. These different values assigned to person intelligent is more valuable if there is a definition added by how many percentages the intelligent belong to that person. Inspired from that, fuzzy sets (FS) as defined by (Zadeh, 1965) formulated uncertainty information for inconsistent information. Zadeh introduced membership degree, assigned as (T_A (x)) to replace the inconsistent information with the membership degree value [0, 1]. By FS theory, the membership degree for quite intelligent was assigned as 0.4, and membership degree for more intelligent was assigned as 0.6. For this situation, especially for uncertain information, different experts might assign different values of a membership degree according to their judgement.

Later, Atanassov (1986) generalized FS to intuitionistic fuzzy set (IFS) where the uncertainty information involving incomplete and inconsistent information was overcome. He introduced a pair of value set memberships $(T_A(x), F_A(x))$, namely membership degree ($T_A(x)$) and non-membership degree ($F_A(x)$) in an uncertain information environment. The values of the membership degree and non-membership degree are also in the interval [0, 1]. FS and IFS theory are modelled based on the successful mathematical formulation for incomplete and inconsistent information but are still lacking in formulating indeterminate information. Occasionally, when dealing with uncertain information, there is still a need for indeterminate information. For example, in a voting system, the percentage for who have voted and who has not voted can be represented by IFS, but in voting, we must also consider the result for those who have not voted at all (indeterminate).

In IFS theory, this indeterminate information is represented by 1-T_A (x)-F_A (x). But it is still not a concrete value for indeterminate value, since the value can be zero. Therefore, Smarandache (1998) initiated the theory of neutrosophic set (NS) and neutrosophic logic (NL) by generalizing FS, IFS, fuzzy logic (FL) and intuitionistic fuzzy logic (IFL). Compared to FS and IFS, NS can express the indeterminate information by introduced the membership degree of indeterminacy (I_A (x)) and adding in incomplete and inconsistent information. The triple set member (T_A (x),I_A (x),F_A (x)) is defined for NS. The value for membership degree, indeterminacy degree and non-membership degree also lies between [0,1].

In general, more than one or repeated uncertainty information also take into consideration in formulating the uncertainty model. For example, in medical analysis, the different time is taken (morning, noon, evening) of the symptoms such as temperature and headache of the patient will help to get an accurate diagnosis. Later, for the first time, Smarandache (2013) refined NS to a neutrosophic refined set (NRS). The element set of NS in NRS is allowed for repeated with different or same elements; (T1, T2, …, T_m and I1, I2, …, In and F1, F2, …, Fr). Then, single valued neutrosophic multisets (SVNM) (Ye & Ye, 2014) inspired by NRS (Smarandache, 2013) was published with operation and properties of NS number. Instead of one-time for each element, the NRS and SVNM also allowed an element to occur more than once with possibly the same or different truth membership sequences, indeterminacy membership sequences, and falsity membership sequences $({T_A}^i(x), {F_A}^i(x), {I_A}^i(x))$. The combination idea of the NRS and SVNM is used in our research.

Recently, Alias et al. (2017) introduced the concept of rough neutrosophic multiset (RNM) which was a hybrid of the rough set (RS) and SVNM theory for solving multiple uncertain information in Pawlak's approximation. The RNM deals with truth membership sequence, indeterminate membership sequence, and falsity membership sequence. The thoery is a generalization of the RNS and from interest of researchers to formulate the uncertain and imprecise information. It has become a valuable component in investigating decision-making analysis, pattern and selection recognition, and the control issue. For example, in selection system for university acceptance (accept, neutral, reject) of multiple choice of candidate, there must be multiple phases of the selection process in university before the best candidate is chosen. All the processes in the selection system will be formulated by RNM theory.

This paper introduces the RNM relation by first giving their basic operations and properties of relation set based on RNM theory. The RNM relation is studied to verify the interaction between variables used in the RNM theory, such as types of car with the relation of popularities of car in making the right decision to buy or sold it. RNM relation is another new tool in dealing with uncertain and imprecise information.

This paper is structured by first presenting the axiomatic definition of RNM relation in the universe set as a novel notion. Before that, important preliminaries related to RNM relation is presented. Next, some relation of RNM relation such as max, min, the composition between two RNM relation, and the inverse of RNM relation are studied. Subsequently, their properties is also examined. The illustrative example with algorithm is presented to verify the RNM relation application. Then, the results and discussions section is given. Lastly, the conclusion section provides an overall summary of RNM relation performance.

MATERIAL AND METHOD

Generalization of Rough Neutrosophic Multisets Relation

This section recalls the important definition to define the rough neutrosophic multisets (RNM) relation. All the proofs of operations and properties may be obtained from (Alias et al., 2017; Nguyen et al., 2014; Pawlak, 1982; Ye & Ye, 2014). Some basic concepts of uncertainty information and hybrid uncertainty information are also discussed in this section.

Some Basic Concepts of Uncertainty Information and Hybrid Uncertainty Information

Many theories were later introduced with the same direction of a fuzzy sets, such as soft set (SS) (Molodtsov, 1999) and rough set (RS) (Pawlak, 1982). The hybrid mathematical formulation of the neutrosophic set (NS) and neutrosophic multisets (NM) for solving uncertainty information has been widely explored. Mandal (2015) combined the theory of NS with SS and introduced the Neutrosophic Soft Set (NSS) to solve the uncertainty involving soft set condition. Then, Alkhazaleh (2016) generalized NSS by introducing the concept of time-neutrosophic soft set and study some of its properties. Ali & Smarandache, (2017) introduced the Complex Neutrosophic Set (CNS) by hybridization of complex set and NS, and successfully introduced the relation and operation of CNS. Broumi et al. (2014) combined the RS and NS by introducing the Rough Neutrosophic Set (RNS). The theory of RNS is well discussed and applied in decision making (Mondal & Pramanik, 2015; Mondal et al., 2016; Pramanik & Mondal, 2015; Yang et al., 2017). Then, Broumi and Smarandache (2014) introduced interval valued Neutrosophic Rough Set (IVNRS) to overcome the interval set of RNS. In any case, every part of these theories has its specialities and difficulties.

Definition 1 (Rough set). Let R be an equivalence relation on the universal set V. Then, the pair (V,R) is called a Pawlak's approximation space. An equivalence class of R containing element set $\varkappa$ will be denoted by $[\varkappa]_R$. Now, for $X \subseteq V$, the upper and lower approximation of X with the respect to (U,R) are denoted by, respectively A1 ($\varkappa$) and A2 ($\varkappa$), and are defined as:

$$A_1(\varkappa) = \{\varkappa : [\varkappa]_{\mathcal{R}} \subseteq X\} \text{ and } A_2(\varkappa) = \{\varkappa : [\varkappa]_{\mathcal{R}} \cap X = \emptyset\} \quad (1)$$

Now, if $A_1(\varkappa)=A_2(\varkappa)$ then X is called definable; otherwise, the pair $A(X) = (A_1(\varkappa), A_2(\varkappa))$ is called the rough set of X in U.

Some concepts of Single Valued Neutrosophic Multisets (SVNM) and Rough Neutrosophic Multisets (RNM)

Single valued neutrosophic multisets (SVNM) (Ye & Ye, 2014) inspired from NRS (Smarandache, 2013), and generalization of Fuzzy multiset (FM) (Yager, 1986), and Intuitionistic Fuzzy Multisets (Shinoj & John, 2012) was published with operation and properties of neutrosophic number.

Definition 2 (Single valued Neutrosophic Multisets). Let E be a universe. A neutrosophic multisets NM(H) on E can be defined in Equation 2 as:

$$NM(\mathcal{H}) = \{<\varkappa, \left(T_{\mathcal{H}}^{1}(\varkappa), T_{\mathcal{H}}^{2}(\varkappa), \ldots, T_{\mathcal{H}}^{p}(\varkappa)\right), \left(I_{\mathcal{H}}^{1}(\varkappa), I_{\mathcal{H}}^{2}(\varkappa), \ldots, I_{\mathcal{H}}^{p}(\varkappa)\right),$$

$$(F_{\mathcal{H}}^{1}(\varkappa), F_{\mathcal{H}}^{2}(\varkappa), \ldots, F_{\mathcal{H}}^{p}(\varkappa))>: \varkappa \in E\} \quad (2)$$

where, $T_{\mathcal{H}}^{1}(\varkappa), T_{\mathcal{H}}^{2}(\varkappa), \ldots, T_{\mathcal{H}}^{p}(\varkappa)$ is the truth membership sequence, $\left(I_{\mathcal{H}}^{1}(\varkappa), I_{\mathcal{H}}^{2}(\varkappa), \ldots, I_{\mathcal{H}}^{p}(\varkappa)\right.$ is the indeterminacy membership sequence, $\left(F_{\mathcal{H}}^{1}(\varkappa), F_{\mathcal{H}}^{2}(\varkappa), \ldots, F_{\mathcal{H}}^{p}(\varkappa)\right.$ is the falsity membership sequence, *p* is dimension of the *NM(H)*, $\varkappa$ is element set in universe *E*, and *H* is NM element in universe *E*.

The triple membership sequences of NM may be in a decreasing or increasing order. For convenience, the *NM(H)* can be denoted as:

$$NM(\mathcal{H}) = \{<\varkappa, \left(T_{\mathcal{H}}^{i}(\varkappa), I_{\mathcal{H}}^{i}(\varkappa), F_{\mathcal{H}}^{i}(\varkappa)\right)>: \varkappa \in E, i = 1, 2, \ldots, p\} \quad (3)$$

where i=1,2,…,p is NM sequence order, $T_{\mathcal{H}}^{i}(\varkappa)$ is the truth membership sequence, $I_{\mathcal{H}}^{i}(\varkappa)$ is the indeterminacy membership sequence, and $F_{\mathcal{H}}^{i}(\varkappa)$ is the falsity membership sequence. Also, $T_{\mathcal{H}}^{i}(\varkappa), I_{\mathcal{H}}^{i}(\varkappa), F_{\mathcal{H}}^{i}(\varkappa) \in [0,1]$ satisfies the condition

$$0 \leq T_{\mathcal{H}}^{i}(\varkappa) + I_{\mathcal{H}}^{i}(\varkappa) + F_{\mathcal{H}}^{i}(\varkappa) \leq 3 \text{ for any } \varkappa \in E \text{ and } i = 1, 2, \ldots, p.$$

Definition 3 (Rough Neutrosophic Multisets). Let E be a non-null set and R be an equivalence relation on E. Let H be neutrosophic multisets (NM) in E with the truth-membership sequence $(T_{\mathcal{H}}^{i})$, indeterminacy-membership sequences $(I_{\mathcal{H}}^{i})$ and falsity-membership sequences (F_H^i). The lower and the upper approximations of H in the approximation (E,R) denoted by Nm(H) and $\overline{\boldsymbol{Nm}}(\mathcal{H})$ are respectively defined in Equation 4 as:

$$\underline{\boldsymbol{Nm}}(\mathcal{H}) = \left\{\langle \boldsymbol{x}, \left(\boldsymbol{T}_{\underline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}), \boldsymbol{I}_{\underline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}), \boldsymbol{F}_{\underline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}), \right)\rangle \middle| \boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}, \boldsymbol{x} \in \boldsymbol{E}\right\}, \text{ and}$$

$$\overline{\boldsymbol{Nm}}(\mathcal{H}) = \left\{\langle \boldsymbol{x}, \left(\boldsymbol{T}_{\overline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}), \boldsymbol{I}_{\overline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}), \boldsymbol{F}_{\overline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}), \right)\rangle \middle| \boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}, \boldsymbol{x} \in \boldsymbol{E}\right\} \quad (4)$$

where $\boldsymbol{T}_{\underline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}) = \wedge_{\boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}} \boldsymbol{T}_{\mathcal{H}}^{\boldsymbol{i}}(\boldsymbol{y})$, $\boldsymbol{I}_{\underline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}) = \vee_{\boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}} \boldsymbol{I}_{\mathcal{H}}^{\boldsymbol{i}}(\boldsymbol{y})$, $\boldsymbol{F}_{\underline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}) = \vee_{\boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}} \boldsymbol{F}_{\mathcal{H}}^{\boldsymbol{i}}(\boldsymbol{y})$,

$\boldsymbol{T}_{\overline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}) = \vee_{\boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}} \boldsymbol{T}_{\mathcal{H}}^{\boldsymbol{i}}(\boldsymbol{y})$, $\boldsymbol{I}_{\overline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}) = \wedge_{\boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}} \boldsymbol{I}_{\mathcal{H}}^{\boldsymbol{i}}(\boldsymbol{y})$, $\boldsymbol{F}_{\overline{\boldsymbol{Nm}}(\mathcal{H})}^{\boldsymbol{i}}(\boldsymbol{x}) = \wedge_{\boldsymbol{y} \in [\boldsymbol{x}]_{\boldsymbol{R}}} \boldsymbol{F}_{\mathcal{H}}^{\boldsymbol{i}}(\boldsymbol{y})$.

Here i=1,2,…,p and positive integers are NM sequence order, ∧ and ∨ denote "min" and "max" operators, respectively; [x]_R is the equivalence class of the x, $T_{\underline{Nm}(\mathcal{H})}^{i}(x)$ is a lower approximation of NM truth membership sequence, $I_{\underline{Nm}(\mathcal{H})}^{i}(x)$ is the lower approximation of the NM indeterminacy membership sequence, $F_{\underline{Nm}(\mathcal{H})}^{i}(x)$ is the approximation of the NM falsity membership sequence, $T_{\overline{Nm}(\mathcal{H})}^{i}(x)$ is the upper approximation of NM truth membership sequence, $I^{i}_{\overline{Nm}(\mathcal{H})}(x)$ is the approximation of NM indeterminacy membership sequence, $F^{i}_{\overline{Nm}(\mathcal{H})}(x)$ is the upper approximation of NM falsity membership sequence, $\boldsymbol{T^{i}_{\mathcal{H}}(y), I^{i}_{\mathcal{H}}(y)}$ **and** $\boldsymbol{F^{i}_{\mathcal{H}}(y)}$ and $F^{i}_{\overline{Nm}(\mathcal{H})}(x)$ are the membership sequences, indeterminacy sequences, and non-membership sequences of y with respect to H.

It can be said that $\boldsymbol{T^{i}_{\underline{Nm}(\mathcal{H})}(x), I^{i}_{\underline{Nm}(\mathcal{H})}(x), F^{i}_{\underline{Nm}(\mathcal{H})}(x)} \in [0,1]$ and $\boldsymbol{0 \leq T^{i}_{\underline{Nm}(\mathcal{H})}(x) + I^{i}_{\underline{Nm}(\mathcal{H})}(x) + F^{i}_{\underline{Nm}(\mathcal{H})}(x) \leq 3}$. Then, $\underline{Nm}(H)$ is a NM. Similarly, we have $\boldsymbol{T^{i}_{\overline{Nm}(\mathcal{H})}(x), I^{i}_{\overline{Nm}(\mathcal{H})}(x), F^{i}_{\overline{Nm}(\mathcal{H})}(x)} \in [0,1]$ and $0 \leq \boldsymbol{T^{i}_{\overline{Nm}(\mathcal{H})}(x) + I^{i}_{\overline{Nm}(\mathcal{H})}(x) + F^{i}_{\overline{Nm}(\mathcal{H})}(x)} \leq 3$. Then, $\overline{Nm}(\mathcal{H})$ is NM. Since $\underline{Nm}$ (*H*) and $\overline{Nm}(\mathcal{H})$ are two NMs in E, the NM mappings $\underline{Nm}, \overline{Nm}: Nm(E) \rightarrow Nm(E)$ are respectively referred to as the lower and upper NM approximation operators, while the pair of $\left(\underline{Nm}(\mathcal{H}), \overline{Nm}(\mathcal{H})\right)$ are called the rough neutrosophic multisets (RNM) in (U,R), respectively.

Rough Neutrosophic Multisets Relation

In this section, the Cartesian product of two rough neutrosophic multisets (RNM) over a universe is defined for RNM theory. Then, the RNM relation is examined for their desired properties. Lastly, the illustrative example followed the algorithm given is presented.

Assumptions and Notations

The RNM relation has been developed based on the following assumptions and notations.

Assumption. In the following section, we have considered only the case of the same repeated numbers of components 1,2,…,p for truth membership sequences $(T_{\mathcal{H}}{}^{i})$, indeterminacy membership sequences $(I_{\mathcal{H}}{}^{i})$, and falsity membership sequences $(F_{\mathcal{H}}{}^{i})$ of NM H. We use NM numbers in the example of RNM relation.

Notations. The following notations as shown in Table 1 are used in the RNM relation model.

Table 1
Notations and descriptions used in the rough neutrosophic multiset relation model

Notation		Description
Q	:	Non-empty/ universe set
T	:	Truth membership degree
I	:	Indeterminacy membership degree
F	:	Falsity membership degree

Table 1 *(Continued)*

Notation		Description
X, Y, Z, Z'	:	Rough neutrosophic multiset in Q
h	:	Element set X in universe Q
g	:	Element set Y in universe Q
$\imath$	:	Element set Z in universe Q
t	:	Element set Z' in universe Q
$\varkappa, \hbar$	:	Element set in universe Q
i	:	Integer iteration number for neutrosophic multisets
p	:	Integer sequence for neutrosophic multisets
$Q \times Q$	:	Cartesian product of element set in universe Q
$X \times Y$	:	Cartesian product of X and Y in universe Q
$Y \times Z$	:	Cartesian product of Y and Z in universe Q
$X \times Z$	:	Cartesian product of X and Z in universe Q
$Z \times Z'$	:	Cartesian product of Z and Z' in universe Q
$Y \times X$	:	Cartesian product of Y and X in universe Q
(h, g)	:	A pair of elements set h and g in the universe relation $Q \times Q$
$(h, \imath)$	:	A pair of elements set h and $\imath$ in the universe relation $Q \times Q$
$(\imath, h)$	:	A pair of elements set $\imath$ and h in the universe relation $Q \times Q$
$(g, \imath)$	:	A pair of elements set g and $\imath$ in the universe relation $Q \times Q$
$(\imath, g)$	:	A pair of elements set $\imath$ and g in the universe relation $Q \times Q$
(h, t)	:	A pair of elements set h and t in the universe relation $Q \times Q$
(g, t)	:	A pair of elements set g and t in the universe relation $Q \times Q$
$(\imath, t)$	:	A pair of elements set $\imath$ and t in the universe relation $Q \times Q$
(g, h)	:	A pair of elements set g and h in the universe relation $Q \times Q$
$T_X^i(h)$	:	Truth membership sequence for X
$T_Y^i(g)$	:	Truth membership sequence for Y
$I_X^i(h)$	:	Indeterminacy membership sequence for X
$I_Y^i(g)$	:	Indeterminacy membership sequence for Y
$F_X^i(h)$	:	Falsity membership sequence for X
$F_Y^i(g)$	:	Falsity membership sequence for Y
$T_{X\times Y}^i(h, g)$	:	Truth membership sequence for cartesian product of X and Y
$I_{X\times Y}^i(h, g)$	:	Indeterminacy membership sequence for cartesian product of X and Y

Table 1 *(Continued)*

Notation		**Description**
$F^{i}_{X\times Y}(h,g)$	:	Falsity membership sequence for cartesian product of X and Y
$\mathfrak{R}$	:	Rough neutrosophic multisets relation on $Q \times Q$ based on the $X \times Y$
$\mathbf{T}^{i}_{\mathfrak{R}}(h,g)$	:	Truth-membership sequence relation for (h,g)
$\boldsymbol{I}^{i}_{\mathfrak{R}}(h,g)$	:	Indeterminacy-membership sequence relation for (h,g)
$\boldsymbol{F}^{i}_{\mathfrak{R}}(h,g)$	:	Falsity-membership sequence relation for (h,g)
$\mathcal{R}$	:	Equivalence relation of neutrosophic multisets
$\mathcal{R}_Q$	:	Equivalence relation of neutrosophic multisets in universe Q
$\underline{X \times Y}$	:	Lower approximation of neutrosophic multisets $X \times Y$ in universe Q
$\overline{X \times Y}$	:	Upper approximation of neutrosophic multisets $X \times Y$ in universe Q
$\underline{X \times Z}$	:	Lower approximation of neutrosophic multisets $X \times Z$ in universe Q
$\overline{X \times Z}$	:	Lower approximation of neutrosophic multisets $X \times Z$ in universe Q
$\underline{\mathcal{R}_Q}(X)$	:	Equivalence relation of lower approximation neutrosophic multisets X in universe Q
$\underline{\mathcal{R}_Q}(Y)$	:	Equivalence relation of lower approximation neutrosophic multisets Y in universe Q
$\underline{\mathcal{R}_Q}(X) \times \underline{\mathcal{R}_Q}(Y)$	:	Cartesian product of lower approximation $\underline{\mathcal{R}_Q}(X)$ and $\underline{\mathcal{R}_Q}(Y)$ in universe Q
$\overline{\mathcal{R}}_Q(X)$	:	Equivalence relation of upper approximation neutrosophic multisets X in universe Q
$\overline{\mathcal{R}}_Q(Y)$	:	Equivalence relation of upper approximation neutrosophic multisets Y in universe Q
$\overline{\mathcal{R}}_Q(X) \times \overline{\mathcal{R}}_Q(Y)$	:	Cartesian product of upper approximation $\underline{\mathcal{R}_Q}(X)$ and $\underline{\mathcal{R}_Q}(Y)$ in universe Q
a, b, c, d	:	Constant value for rough neutrosophic multisets relation in between [0, 1]
$M(\mathfrak{R})$	:	Matrix form for rough neutrosophic multisets relation
$M(\mathfrak{R})^t$	:	Transpose matrix form for rough neutrosophic multisets relation
$\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3$	:	Rough neutrosophic multisets relation on $Q \times Q$ based on the $X \times Y$
$\mathfrak{R}^{-1}$	:	Inverse rough neutrosophic multisets relation on $Q \times Q$ based on the $X \times Y$
$T^{i}_{\mathfrak{R}_1}(h,g)$, $T^{i}_{\mathfrak{R}_2}(h,g)$	:	Truth membership sequence for rough neutrosophic multisets relation of (h,g)

Table 1 *(Continued)*

Notation		Description
$I^i_{\Re_1\wedge\Re_2}(h,g)$	:	Indeterminacy membership sequence for intersection operation of rough neutrosophic multisets relation of (h,g)
$F^i_{\Re_1\wedge\Re_2}(h,g)$	:	Falsity membership sequence for intersection operation of rough neutrosophic multisets relation of (h,g)
$\Re_1 \vee \Re_2$	:	Union operator of two rough neutrosophic multisets relation
$T^i_{\Re_1\vee\Re_2}(h,g)$	:	Truth membership sequence for union operation of rough neutrosophic multisets relation of (h,g)
$I^i_{\Re_1\vee\Re_2}(h,g)$	:	Indeterminacy membership sequence for union operation of rough neutrosophic multisets relation of (h,g)
$F^i_{\Re_1\vee\Re_2}(h,g)$	:	Falsity membership sequence for union operation of rough neutrosophic multisets relation of (h,g)
$\Re_1 \otimes \Re_2$	:	Multiply operator of two rough neutrosophic multisets relation
$T^i_{\Re_1\otimes\Re_2}(h,g)$	:	Truth membership sequence for multiply operation of rough neutrosophic multisets relation of (h,g)
$I^i_{\Re_1\otimes\Re_2}(h,g)$	:	Indeterminacy membership sequence for multiply operation of rough neutrosophic multisets relation of (h,g)
$F^i_{\Re_1\otimes\Re_2}(h,g)$	:	Falsity membership sequence for multiply operation of rough neutrosophic multisets relation of (h,g)
$\Re_1 \oplus \Re_2$	:	Addition operator of two rough neutrosophic multisets relation
$T^i_{\Re_1\oplus\Re_2}(h,g)$	:	Truth membership sequence for addition operation of rough neutrosophic multisets relation of (h,g)
$I^i_{\Re_1\oplus\Re_2}(h,g)$	:	Indeterminacy membership sequence for addition operation of rough neutrosophic multisets relation of (h,g)
$F^i_{\Re_1\oplus\Re_2}(h,g)$	:	Falsity membership sequence for addition operation of rough neutrosophic multisets relation of (h,g)
$\Re_1 \circ \Re_2$	:	Composition operator of two rough neutrosophic multisets relation
$\Re_1 \circ (\Re_2 \circ \Re_3)$	:	Composition operator of three rough neutrosophic multisets relation
$T^i_{\Re_1\circ\Re_2}(h,\imath)$	:	Truth membership sequence for composition operation of rough neutrosophic multisets relation of $(h,\imath)$
$I^i_{\Re_1\circ\Re_2}(h,\imath)$	:	Indeterminacy membership sequence for composition operation of rough neutrosophic multisets relation of $(h,\imath)$
$F^i_{\Re_1\circ\Re_2}(h,\imath)$	:	Falsity membership sequence for composition operation of rough neutrosophic multisets relation of $(h,\imath)$

Table 1 *(Continued)*

Notation		Description
$T^{i}_{\Re_2}(g, \imath)$	:	Truth membership sequence for rough neutrosophic multisets relation of $(g, \imath)$
$I^{i}_{\Re_2}(g, \imath)$	:	Indeterminacy membership sequence for rough neutrosophic multisets relation of $(g, \imath)$
$F^{i}_{\Re_2}(g, \imath)$	:	Falsity membership sequence for rough neutrosophic multisets relation of $(g, \imath)$
$T^{i}_{\Re_1\circ(\Re_2\circ\Re_3)}(h, t)$	:	Truth membership sequence for composition operation of rough neutrosophic multisets relation of (h, t)
$T^{i}_{(\Re_2\circ\Re_3)}(g, t)$	:	Truth membership sequence for composition operation of rough neutrosophic multisets relation of (g, t)
$T^{i}_{\Re_3}(\imath, t)$	:	Truth membership sequence for composition operation of rough neutrosophic multisets relation of $(\imath, t)$
$T^{i}_{\Re^{-1}}(g, h)$	:	Truth membership sequence for inverse rough neutrosophic multisets relation of (g, h)
$I^{i}_{\Re^{-1}}(g, h)$	:	Indeterminacy membership sequence for inverse rough neutrosophic multisets relation of (g, h)
$F^{i}_{\Re^{-1}}(g, h)$	:	Falsity membership sequence for inverse rough neutrosophic multisets relation of (g, h)
$(\Re_1\circ\ \Re_2)^{-1}$	:	Inverse composition operator for rough neutrosophic multisets relation
${\Re_1}^{-1}, {\Re_2}^{-1}$	:	Inverse relation for rough neutrosophic multisets relation
$T^{i}_{(\Re_1\circ\Re_2)^{-1}}(\imath, h)$	:	Truth membership sequence for inverse composition operation of rough neutrosophic multisets relation of $(\imath, h)$
$T^{i}_{(\Re_2)^{-1}}(\imath, g)$	:	Truth membership sequence for inverse operation of rough neutrosophic multisets relation of $(\imath, g)$
$h\mathcal{R}_Q g$	:	Equivalence relation of (h, g) in universe Q

Phase 1: Development of Operation and Properties for Rough Neutrosophic Multisets Relation Theory

Definition 4. Let Q be a non-empty set and X and Y be the rough neutrosophic multiset (RNM) in Q. Then, the Cartesian product of X and Y is RNM in $Q\times Q$, denoted by $X\times Y$, is defined as:

$$X\times Y=\left\{< (h,g),\left(T^{i}_{X\times Y}(h,g)\right)\left(I^{i}_{X\times Y}(h,g)\right),\left(F^{i}_{X\times Y}(h,g)\right)>:(h,g)\in Q\times Q\right\} \quad (5)$$

where

$T^i_{X\times Y}(h,g) = \min\{T^i_X(h), T^i_Y(g)\}$, $I^i_{X\times Y}(h,g) = \max\{I^i_X(h), I^i_Y(g)\}$,

$F^i_{X\times Y}(h,g) = \max\{F^i_X(h), F^i_Y(g)\}$, $T^i_{X\times Y}, I^i_{X\times Y}, F^i_{X\times Y}: Q \to [0,1]$, and $i = 1, 2, \ldots, p$.

Definition 5. Let Q be a non-empty set and X and Y be the rough neutrosophic multisets (RNM) in Q. We call $\Re \subseteq Q \times Q$ is RNM relation on Q×Q based on the X×Y, where X×Y is characterized by the truth-membership sequence $\mathbf{T}^i_{\Re}$, the indeterminacy-membership sequences $\mathbf{I}^i_{\Re}$ and the falsity-membership sequences $\mathrm{F}^i_{\Re}$, defined as:

$$\Re = \left\{< (h,g), \left(T^i_{\Re}(h,g)\right)\left(I^i_{\Re}(h,g)\right), \left(F^i_{\Re}(h,g)\right) >: (h,g) \in Q \times Q\right\} \quad (6)$$

with conditions if satisfied:

(1) i) $T^i_{\Re}(h,g) = 1$ for all $(h,g) \in \underline{X \times Y}$ where $\underline{X \times Y} = \underline{\mathcal{R}}_Q(X) \times \underline{\mathcal{R}}_Q(Y)$,

ii) $T^i_{\Re}(h,g) = 0$, for all $(h,g) \in Q \times Q - \overline{X \times Y}$ where $\overline{X \times Y} = \overline{\mathcal{R}}_Q(X) \times \overline{\mathcal{R}}_Q(Y)$,

iii) $0 < T^i_{\Re}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$.

(2) i) $I^i_{\Re}(h,g) = 0$, for all $(h,g) \in \underline{X \times Y}$ where $\underline{X \times Y} = \underline{\mathcal{R}}_Q(X) \times \underline{\mathcal{R}}_Q(Y)$,

ii) $I^i_{\Re}(h,g) = 1$, for all $(h,g) \in Q \times Q - \overline{X \times Y}$ where $\overline{X \times Y} = \overline{\mathcal{R}}_Q(X) \times \overline{\mathcal{R}}_Q(Y)$,

iii) $0 < I^i_{\Re}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$.

(3) i) $F^i_{\Re}(h,g) = 0$, for all $(h,g) \in \underline{X \times Y}$ where $\underline{X \times Y} = \underline{\mathcal{R}}_Q(X) \times \underline{\mathcal{R}}_Q(Y)$,

ii) $F^i_{\Re}(h,g) = 1$, for all $(h,g) \in Q \times Q - \overline{X \times Y}$ where $\overline{X \times Y} = \overline{\mathcal{R}}_Q(X) \times \overline{\mathcal{R}}_Q(Y)$,

iii) $0 < F^i_{\Re}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$.

The RNM relation can be presented by relational tables and matrices like FS and IFS relation representation. Since the triple (T^i_A, I^i_A, F^i_A) has values within the interval [0,1], the elements of the neutrosophic matrix also have values within [0,1].

Now, we can consider some properties of RNM relation. All the properties must satisfy the RNM definition and RNM relation condition in Definitions 4 and 5, respectively.

Proposition 1. Let $\Re_1, \Re_2$ be two rough neutrosophic multisets relation on Q×Q based on the X×Y. Then $\Re_1 \wedge \Re_2$, where

$$T^i_{\Re_1\wedge\Re_2}(h,g) = \min\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\},$$
$$I^i_{\Re_1\wedge\Re_2}(h,g) = \max\{I^i_{\Re_1}(h,g), I^i_{\Re_2}(h,g)\},$$
$$F^i_{\Re_1\wedge\Re_2}(h,g) = \max\{F^i_{\Re_1}(h,g), F^i_{\Re_2}(h,g)\}$$

for all $(h,g) \in Q \times Q$, is a rough neutrosophic multisets on Q×Q based on the X×Y and i=1,2,…,p.

Proof We show that $\Re_1 \wedge \Re_2$ satisfies Definition 5.

1) i) Since $T^i_{\Re_1}(h,g) = T^i_{\Re_2}(h,g) = 1$ for all $(h,g) \in \underline{X \times Y}$ then

$T^i_{\Re_1 \wedge \Re_2}(h,g) = \min\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\} = 1$ for all $(h,g) \in \underline{X \times Y}$.

ii) Since $T^i_{\Re_1}(h,g) = T^i_{\Re_2}(h,g) = 0$ for all $(h,g) \in Q \times Q - \overline{X \times Y}$

then $T^i_{\Re_1 \wedge \Re_2}(h,g) = \min\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\} = 0$ for all

$(h,g) \in Q \times Q - \overline{X \times Y}$.

iii) Since $0 < T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$ then

$0 < T^i_{\Re_1 \wedge \Re_2}(h,g) = \min\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\} < 1$ for all

$(h,g) \in \overline{X \times Y} - \underline{X \times Y}$.

A similar proofing step is satisfied for conditions 2 and 3 in definition 5.

Proposition 2. Let $\Re_1, \Re_2$ be two rough neutrosophic multisets relation on *Q*×*Q* based on the X×Y. Then $\Re_1 \vee \Re_2$, where

$$T^i_{\Re_1 \vee \Re_2}(h,g) = \max\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\},$$

$$I^i_{\Re_1 \vee \Re_2}(h,g) = \min\{I^i_{\Re_1}(h,g), I^i_{\Re_2}(h,g)\},$$

$$F^i_{\Re_1 \vee \Re_2}(h,g) = \min\{F^i_{\Re_1}(h,g), F^i_{\Re_2}(h,g)\}$$

for all $(h,g) \in Q \times Q$, is a rough neutrosophic multisets on Q×Q based on the *X*×*Y* and i=1,2,…,p.

Proof We show that $\Re_1 \vee \Re_2$, satisfies definition 5.

(1) i) Since $T^i_{\Re_1}(h,g) = T^i_{\Re_2}(h,g) = 1$ **for all** $(h,g) \in \underline{X \times Y}$ **then**

$T^i_{\Re_1 \vee \Re_2}(h,g) = \max\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\} = 1$ **for all** $(h,g) \in \underline{X \times Y}$**.**

ii) Since $T^i_{\Re_1}(h,g) = T^i_{\Re_2}(h,g) = 0$ for all $(h,g) \in Q \times Q - \overline{X \times Y}$

then $T^i_{\Re_1 \vee \Re_2}(h,g) = \max\{T^i_{\Re_1}(h,g), T^i_{\Re_2}(h,g)\} = 0$ for all $(h,g) \in Q \times Q - \overline{X \times Y}$.

iii) Since $0 < T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$ then

$0 < T^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(h,g) = \max\{T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(h,g)\} < 1$ for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$.

A similar proofing step is satisfied for conditions 2 and 3 in definition 5.

Lemma 1. If $0<\varkappa,\hbar<1$, then

$0<\varkappa\hbar<1$ (Obvious)

$0<\varkappa+\hbar-\varkappa\hbar<1$.

Since $0<\varkappa,\hbar<1$ then $\varkappa+\hbar\geq 2\sqrt{\varkappa\hbar}>2\varkappa\hbar>\varkappa\hbar>0$, therefore $\varkappa+\hbar-\varkappa\hbar>0$. On the other hand, $1-(\varkappa+\hbar-\varkappa\hbar)=(1-\varkappa)(1-\hbar)>0$ then $\varkappa+\hbar-\varkappa\hbar<1$.

The following properties of RNM relation are obtained using these algebraic results (Lemma 1)

Proposition 3. Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relations on Q×Q based on the X×Y. Then $\mathfrak{R}_1 \otimes \mathfrak{R}_2$, where,

$$T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(h,g) = T^i_{\mathfrak{R}_1}(h,g) \cdot T^i_{\mathfrak{R}_2}(h,g),$$

$$I^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(h,g) = I^i_{\mathfrak{R}_1}(h,g) + I^i_{\mathfrak{R}_2}(h,g) - I^i_{\mathfrak{R}_1}(h,g) \cdot I^i_{\mathfrak{R}_2}(h,g)$$

$$F^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(h,g) = F^i_{\mathfrak{R}_1}(h,g) + F^i_{\mathfrak{R}_2}(h,g) - F^i_{\mathfrak{R}_1}(h,g) \cdot F^i_{\mathfrak{R}_2}(h,g)$$

for all $(h,g) \in Q \times Q$, is a rough neutrosophic multisets on Q×Q based on the X×Y and i=1,2,…,p.

Proof The relation $\mathfrak{R}_1 \otimes \mathfrak{R}_2$, satisfies definition 5.

(1) i) Since $T^i_{\mathfrak{R}_1}(h,g) = T^i_{\mathfrak{R}_2}(h,g) = 1$ **for all** $(h,g) \in \underline{X \times Y}$ **then**

$T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(h,g) = T^i_{\mathfrak{R}_1}(h,g) \cdot T^i_{\mathfrak{R}_2}(h,g) = 1$ **for all** $(h,g) \in \underline{X \times Y}$**.**

ii) Since $T^i_{\mathfrak{R}_1}(h,g) = T^i_{\mathfrak{R}_2}(h,g) = 0$ for all $(h,g) \in Q \times Q - \overline{X \times Y}$

then $T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(h,g) = T^i_{\mathfrak{R}_1}(h,g) \cdot T^i_{\mathfrak{R}_2}(h,g) = 0$ for all

$(h,g) \in Q \times Q - \overline{X \times Y}$.

iii) Since $0 < T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$ then

$0 < T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(h,g) = T^i_{\mathfrak{R}_1}(h,g) \cdot T^i_{\mathfrak{R}_2}(h,g) < 1$ for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$

(Lemma 1 (i)).

The proof is also true for conditions 2 and 3 by following Lemma 1 (ii).

Proposition 4. Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relations on *Q*×*Q* based on the *X*×*Y*. Then $\mathfrak{R}_1 \otimes \mathfrak{R}_2$, where,

$$T^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(h, g) = T^i_{\mathfrak{R}_1}(h, g) + T^i_{\mathfrak{R}_2}(h, g) - T^i_{\mathfrak{R}_1}(h, g) \cdot T^i_{\mathfrak{R}_2}(h, g),$$

$$I^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(h, g) = I^i_{\mathfrak{R}_1}(h, g) \cdot I^i_{\mathfrak{R}_2}(h, g),$$

$$F^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(h, g) = F^i_{\mathfrak{R}_1}(h, g) \cdot F^i_{\mathfrak{R}_2}(h, g)$$

for all $(h, g) \in Q \times Q$, *is a rough neutrosophic multisets on Q × Q based on the X × Y and i=1,2,…,p.*

Proof The relation satisfies $\mathfrak{R}_1 \otimes \mathfrak{R}_2$, definition 5 and Lemma 1. All the proving steps are similar to Proposition 3.

Phase 2: Development of the Composition of Two Rough Neutrosophic Multisets Relation Theory

The composition of two relations is essential for applications in real life. This relation can be computed over a universe with useful significance. For example, in the same group of medical diagnoses, the relation between patient and symptoms, and the relation between disease and symptoms, will be composited to find the relation between patient and disease. Some of the applications of the composition of two relation sets were discussed by Guo et al. (2017) and Yang et al. (2016).

Definition 6. Let *Q* be a non-empty set and X, Y and Z be the RNM in *Q*. Let $\mathfrak{R}_1, \mathfrak{R}_2$ are two RNM relations on *Q*×*Q*, based on *X*×*Y*,*Y*×*Z*, respectively. The composition of $\mathfrak{R}_1, \mathfrak{R}_2$ denoted as $\mathfrak{R}_1 \circ \mathfrak{R}_2$ as defined by Q×Q based on X×Z, where,

$$T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(h, \imath) = \max_{g \in Q}\{\min[T^i_{\mathfrak{R}_1}(h, g), T^i_{\mathfrak{R}_2}(g, \imath)]\},$$

$$I^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(h, \imath) = \min_{g \in Q}\{\max[I^i_{\mathfrak{R}_1}(h, g), I^i_{\mathfrak{R}_2}(g, \imath)]\},$$

$$F^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(h, \imath) = \min_{g \in Q}\{\max[F^i_{\mathfrak{R}_1}(h, g), F^i_{\mathfrak{R}_2}(g, \imath)]\}$$

for all $(h, \imath), (g, \imath) \in Q \times Q$ and i=1,2,…,p.

All the propositions must satisfy the condition in definition 4 and 5, respectively.

Proposition 5. $\mathfrak{R}_1 \circ \mathfrak{R}_2$ is a rough neutrosophic multisets relation on Q×Q based on X×Z.

Proof Since R_1,R_2 are two rough neutrosophic multisets relations on Q×Q based on X×Y,Y×Z, respectively:

Similar proving steps follow for conditions 2 and 3.

(1) i) Then $T^i_{\mathfrak{R}_1}(h,\imath) = 1 = T^i_{\mathfrak{R}_2}(h,\imath)$ **for all** $(h,\imath) \in \underline{X \times Z}$**. Let** $(h,\imath) \in \underline{X \times Z}$**, now**

$T^i_{\mathfrak{R}_1\circ\mathfrak{R}_2}(h,\imath) = \max_{g\in Q}\{\min[T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(g,\imath)]\} = 1$**. This holds for all** $(h,\imath) \in \underline{X \times Z}$**.**

ii) Let $(h,\imath) \in Q \times Q - \overline{X \times Z}$. So, $T^i_{\mathfrak{R}_1}(h,\imath) = 0 = T^i_{\mathfrak{R}_2}(h,\imath)$ for all $(h,\imath) \in Q \times Q - \overline{X \times Z}$. Then

$T^i_{\mathfrak{R}_1\circ\mathfrak{R}_2}(h,\imath) = \max_{g\in Q}\{\min[T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(g,\imath)]\} = 0$ for all $(h,\imath) \in Q \times Q - \overline{X \times Z}$.

iii) Again, since $0 < T^i_{\mathfrak{R}_1}(h,\imath), T^i_{\mathfrak{R}_2}(h,\imath) < 1$, for all $(h,\imath) \in \overline{X \times Z} - \underline{X \times Z}$,

then $0 < max_{g\in Q}\{\min[T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(g,\imath)]\} < 1$ such that $0 < T^i_{\mathfrak{R}_1\circ\mathfrak{R}_2}(h,\imath) < 1$ for all $(h,\imath) \in$

$\overline{X \times Z} - \underline{X \times Z}$.

Proposition 6. Let Q be a non-empty set. R_1,R_2,R_3 are rough neutrosophic multisets relations on Q×Q based on X×Y,Y×Z,Z×Z', respectively. Then $(\mathfrak{R}_1 \circ \mathfrak{R}_2) \circ \mathfrak{R}_3 = \mathfrak{R}_1 \circ (\mathfrak{R}_2 \circ \mathfrak{R}_3)$. The proof is similar to indeterminacy function and falsity function.

Proof For all $h, g, \imath, t \in Q$ we have

$$T^i_{\mathfrak{R}_1\circ(\mathfrak{R}_2\circ\mathfrak{R}_3)}(h,t) = \max_{g\in Q}\{\min[T^i_{\mathfrak{R}_1}(h,g), T^i_{(\mathfrak{R}_2\circ\mathfrak{R}_3)}(g,t)]\}$$

$$= \max_{g\in Q}\{\min[T^i_{\mathfrak{R}_1}(h,g), \max_{\imath\in Q}\{\min[T^i_{\mathfrak{R}_2}(g,\imath), T^i_{\mathfrak{R}_3}(\imath,t)]]\}\}$$

$$= \max_{\imath\in Q}\{\min\{\max_{g\in Q}\{\min[T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(g,\imath)], T^i_{\mathfrak{R}_3}(\imath,t)]\}\}$$

$$= \max_{\imath\in Q}\{\min[T^i_{(\mathfrak{R}_1\circ\mathfrak{R}_2)}(h,g), T^i_{\mathfrak{R}_3}(g,t)]\}$$

$$= T^i_{(\mathfrak{R}_1\circ\mathfrak{R}_2)\circ\mathfrak{R}_3}(h,t);$$

Note that $\mathfrak{R}_1 \circ \mathfrak{R}_2 \neq \mathfrak{R}_2 \circ \mathfrak{R}_1$, since the composition of two RNM relations $\mathfrak{R}_1, \mathfrak{R}_2$ exists, the composition of two RNM relations R_2,R_1 does not necessarily exist.

Phase 3: Development of Inverse Rough Neutrosophic Multisets Relation Theory

The inverse relation is formed by changing the element set of each of the ordered pairs in the given relation. Since we realize that relations are regularly indicated by relation and function, therefore, we prove that the RNM relation theory has an inverse relation.

Definition 7. Let Q be a non-empty set and X and Y be the rough neutrosophic multisets (RNM) in Q. $\mathfrak{R} \subseteq Q \times Q$ is RNM relation on Q×Q based on the X×Y. Then, we define $\mathfrak{R}^{-1} \subseteq Q \times Q$ as the RNM relation on Q×Q based on Y×X defined in Equation 7 as:

$$\mathfrak{R}^{-1} = \left\{< (g,h), \left(T^i_{\mathfrak{R}^{-1}}(g,h)\right)\left(I^i_{\mathfrak{R}^{-1}}(g,h)\right), \left(F^i_{\mathfrak{R}^{-1}}(g,h)\right) >: (g,h) \in Q \times Q\right\} \quad (7)$$

where

$$T^i_{\mathfrak{R}^{-1}}(g,h) = T^i_{\mathfrak{R}}(h,g), I^i_{\mathfrak{R}^{-1}}(g,h) = I^i_{\mathfrak{R}}(h,g), F^i_{\mathfrak{R}^{-1}}(g,h) = F^i_{\mathfrak{R}}(h,g)$$

for all $(g,h) \in Q \times Q$ and $i = 1, 2, \ldots, p$.

The relation R^{-1} is called the inverse rough neutrosophic multisets relation of R.

Proposition 7. $(\mathfrak{R}^{-1})^{-1} = \mathfrak{R}$.

Proof Since all the membership degree is true for all condition in definition 5; we only show the proving for truth membership sequence.

(1) i) $T^i_{(\mathfrak{R}^{-1})^{-1}}(h,g) = T^i_{\mathfrak{R}^{-1}}(g,h) = T^i_{\mathfrak{R}}(h,g) = 1$ **for all** $(h,g) \in \underline{X \times Y}$

ii) $T^i_{(\mathfrak{R}^{-1})^{-1}}(h,g) = T^i_{\mathfrak{R}^{-1}}(g,h) = T^i_{\mathfrak{R}}(h,g) = 0$ for all $(h,g) \in Q \times Q - \overline{X \times Y}$

iii) $0 < T^i_{(\mathfrak{R}^{-1})^{-1}}(h,g) = T^i_{\mathfrak{R}^{-1}}(g,h) = T^i_{\mathfrak{R}}(h,g) < 1$ for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$

Thus $(\mathfrak{R}^{-1})^{-1} = \mathfrak{R}$.

Proposition 8. Let R_1,R_2 be two rough neutrosophic multisets relations on $Q \times Q$, based on $X \times Y$, $Y \times Z$, respectively. Then $(\mathfrak{R}_1 \circ \mathfrak{R}_2)^{-1} = {\mathfrak{R}_2}^{-1} \circ {\mathfrak{R}_1}^{-1}$.

Proof For all $h, g, \imath \in Q$, we have:

$$T^i_{(\mathfrak{R}_1 \circ \mathfrak{R}_2)^{-1}}(\imath, h) = T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(h, \imath)$$

$$= \max_{g \in Q}\left\{\min\left[T^i_{\mathfrak{R}_1}(h,g), T^i_{\mathfrak{R}_2}(g,\imath)\right]\right\}$$

$$= \max_{g \in Q}\left\{\min\left[T^i_{(\mathfrak{R}_1)^{-1}}(g,h), T^i_{(\mathfrak{R}_2)^{-1}}(\imath,g)\right]\right\}$$

$$= \max_{g \in Q}\left\{\min\left[T^i_{(\mathfrak{R}_2)^{-1}}(\imath,g), T^i_{(\mathfrak{R}_1)^{-1}}(g,h)\right]\right\} = T^i_{(\mathfrak{R}_2)^{-1} \circ (\mathfrak{R}_1)^{-1}}(\imath, h);$$

The proof is similar to indeterminacy function and falsity function.

Thus, it can be concluded that $(\mathfrak{R}_1 \circ \mathfrak{R}_2)^{-1} = {\mathfrak{R}_2}^{-1} \circ {\mathfrak{R}_1}^{-1}$.

Phase 4: Verifying Section by Illustrative Example of Rough Neutrosophic Multisets Relation Theory

In this section, the relation between three types of cars represented by the popularity of each car depending on expert opinion was studied. The definition of rough neutrosophic multiset (RNM) relation theory (definition 5) was used to get the best solution of the relation analysis. The flowchart of the algorithm used in solving RNM relation is shown in Figure 1 and an algorithm for each step is presented in Table 2.

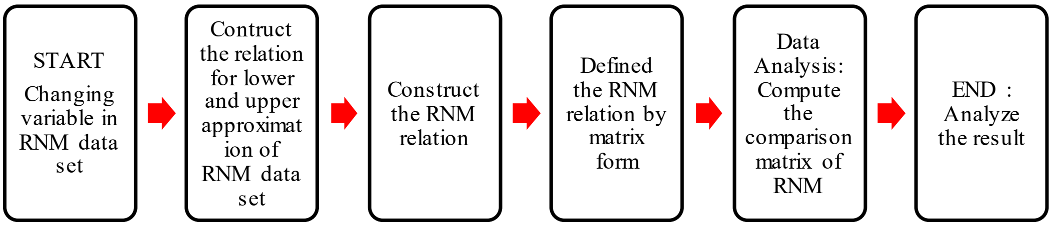

Figure 1. The flowchart of an algorithm in solving RNM relation

Table 2
An algorithm of steps to defined rough neutrosophic multisets relation.

Step		Algorithm
Step 1	:	Changing the variable in RNM data set: Compute lower and upper approximation values for rough neutrosophic multisets ($\underline{\mathcal{R}_Q}(X)$, $\underline{\mathcal{R}_Q}(Y)$, $\overline{\mathcal{R}_Q}(X)$, $\overline{\mathcal{R}_Q}(Y)$ by definition 3.
Step 2	:	Construct the relation of $\underline{X \times Y} = \underline{\mathcal{R}_Q}(X) \times \underline{\mathcal{R}_Q}(Y)$, relation of $\overline{X \times Y} = \overline{\mathcal{R}_Q}(X) \times \overline{\mathcal{R}_Q}(Y)$, and relation of $Q \times Q$. Note that $T^i_{\mathfrak{R}}(h,g) = 1$, $I^i_{\mathfrak{R}}(h,g) = 0$ and $F^i_{\mathfrak{R}}(h,g) = 0$ for all $(h,g) \in \underline{X \times Y}$ by definition 4 and 5.
Step 3	:	Construct a rough neutrosophic multisets relation $\mathfrak{R}$. Note that, $T^i_{\mathfrak{R}}(h,g) = 0$, $I^i_{\mathfrak{R}}(h,g) = 1$ and $F^i_{\mathfrak{R}}(h,g) = 1$ for all $(h,g) \in Q \times Q - \overline{X \times Y}$ by definition 5.
Step 4	:	Defined the RNM relation by matrix form: (i) Compute the unknown possible values by considering the condition of $0 < T^i_{\mathfrak{R}}(h,g) < 1$, for all $(h,g) \in \overline{X \times Y} - \underline{X \times Y}$ and neutrosophic multi relation of $T^i_{\mathfrak{R}}(h,g) \leq T^i_{X \times Y}(h,g)\ \forall\ (h,g) \in Q \times Q$. (ii) Defined $\mathfrak{R} \subseteq Q \times Q$ as a rough neutrosophic multisets relation on $Q \times Q$ based on the $X \times Y$ by a matrix form. We can have different values for unknown value based on different cases.
Step 5	:	Data Analysis: Compute the comparison matrix of RNM
Step 6	:	Analysis of the result

Consider the following example

Example 1. Assume that Q={1,2,3} be a universal set of types of car available for purchase in shop Z. R_Q be an equivalent relation on Q as hR_Q g if and only if h,g indicated the same popularities of car. hR_Q g is defined by R_Q={{1,3},{2}}. This relation justified that the car types 1 and 3 are in the same contingent and type 2 is in different contingent, but all the cars are still in the same universe. All the cars effected the profit of shop Z. Now, we try to get the opinion from two independent experts about the level of popularities of the car whether the car comprised "high", a level of indeterminacy with respect to the experts which was "not popular at all" and whether they felt that the car comprised "low level". Two – phases of the meeting were done in order to get the best result and the data was represented in neutrosophic multisets number. By satisfying all the condition in definition 3, we define the relation of rough neutrosophic multisets R on popularities of cars $Q \times Q$ based on experts' opinion $X \times Y$ as follows:

X=((1,0.3),(0.4,0.7),(0.6,0.8))/1+((0.5,0.7),(0.1,0.3),(0.4,0.5))/2+ ((1,0.6),(0.4,0.5),(0.6,0.7))/3,

Y=((0.4,0.6),(0.3,0.5),(0.1,0.7))/1+((0.5,0.4),(0.1,0.7),(0.3,0.8))/2+((1,0.7),(0.2,0.5),(0.1,0.7))/3

By following the algorithm presented (step 1 – 4), here we define a rough neutrosophic multisets relation R by a matrix.

We defined $\mathfrak{R} \subseteq Q \times Q$ as a rough neutrosophic multisets relation on $Q \times Q$ based on the X×Y by a matrix form:

$$M(\mathfrak{R}) = \begin{bmatrix} (0.4,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.6,0),(1,1),(1,1) \\ (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) \\ (0.5,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \end{bmatrix}$$

Consider the RNM relation M(R) in example 1; then the inverse RNM relation R^{-1} of RNM relation R by using matrix is defined as:

Next, we compute the comparison matrix using the formula $D_{\mathfrak{R}}^{i} = T_{\mathfrak{R}}^{i} + I_{\mathfrak{R}}^{i} - F_{\mathfrak{R}}^{i}$ for all i, and select the maximum value for the comparison table. The result is shown in Table 3.

Table 3

Comparison matrix of rough neutrosophic multi relation R.

$\mathfrak{R}$	1	2	3
1	(**0.4**, 0.0)	(**0.0**, 0.0)	(**0.6**, 0.0)
2	(**0.0**, 0.0)	(**0.0**, 0.0)	(**0.0**, 0.0)
3	(**0.5**, 0.0)	(**0.0**, 0.0)	(**0.9**, 0.0)

By using row-sum and column-sum, the score for popularities of the car was computed as shown in Table 4.

RESULT AND DISCUSSION

The expert's opinion to decide the relationship between types of car and their popularities was successfully determined by rough neutrosophic multisets theory. According to the finding represented in Table 4, the experts agreed that the car type 1 was highly popular compared to car type 2 and type 3. This result was accurate since we had considered all uncertainty information level which level of popularities of the car was whether the car

Table 4
Popularity score for three types of car

	Row sum	Column sum	Popularity Score
1	1.0	0.9	**0.1**
2	0.0	0	0.0
3	1.3	1.5	(0.2)

comprised of "high", a level of indeterminacy with respect to the experts which is "not popular at all" and whether they feel that the car comprised of "low level". The repeated phase which was two – phases of the meeting was done in order to get the best result. Compared to others mathematical set data for uncertainty information such as fuzzy set, intuitionistic fuzzy set, neutrosophic set and rough neutrosophic set, our set data was more applicable, and it was already discussed in the literature. Therefore, the RNM relation theory could be used to represent the relation between the specified uncertainty information within the same universe.

The rough neutrosophic multisets (RNM) relation is a relation on neutrosophic multisets (NM), so we can consider it to be RNM relation over the universe. The RNM relation follows the condition of relation on NM, which is

Therefore, over universe Q, the RNM relation generalized relation for the rough neutrosophic set (RNS), rough intuitionistic fuzzy set (RIFS) and rough fuzzy set (RFS).

$T^i_{\Re}(h,g) \leq T^i_{X\times Y}(h,g), I^i_{\Re}(h,g) \geq I^i_{X\times Y}(h,g), F^i_{\Re}(h,g) \geq F^i_{X\times Y}(h,g)$ for all $(h,g) \in Q \times Q$, and

$0 \leq T^i_{\Re}(h,g) + I^i_{\Re}(h,g) + F^i_{\Re}(h,g) \leq 3.$

All the properties are true for RNS, RIFS and RFS.

1. The relation for RNS over universe Q is obtained when i=1 for all element T,I,F in definition 5 for Eq. (1),

R={<(*h*,*g*),(T_R (*h*,*g*))(I_R (*h*,*g*)),(F_R (*h*,*g*))>:(*h*,*g*) ∈ $Q\times Q$}.

2. The relation for RIFS over universe Q is obtained when i=1 for element T and F and condition (2) in definition 5 for Eq. (1) is omitted,

R={<(*h*,*g*),(T_R (*h*,*g*)),(F_R (*h*,*g*))>:(*h*,*g*) ∈ $Q\times Q$}.

3. The relation for RFS over universe Q is obtained when i=1 for element T and condition (2) and (3) in definition 5 is omitted, R={<(*h*,*g*),(T_R (*h*,*g*))>:(*h*,*g*) ∈ $Q\times Q$}.

CONCLUSION

Nowadays, there is much mathematical formulation developed to overcome the problem related to uncertainty information. Since the real data in the form of classical data is not suitable to represent the solution of uncertainty data, Zadeh (1965) introduced the fuzzy set data to overcome the inconsistent information in uncertainty problem. Then, Atanassov (1986) overcome the situation of inconsistent and incomplete information in the uncertainty problem. Next, Smarandache (1998) solved the issue of indeterminate information in uncertainty problem. The situation changed from single problem to repeated problem, and then a multiset approach was introduced for each of the set theory. All the fundamental theory approach has the same research direction to overcome the problem in uncertainty information that consists of inconsistent, incomplete and indeterminate information.

A consequence of that, this paper presents a new notion for uncertainty information by introduced rough neutrosophic multisets (RNM) relation into the universe. As the development of the RNM relation in RNM theory, the relationship developing RNM relation has primary consequences for both theories and application which is rough set and single valued neutrosophic multisets.

The important properties for fundamental theory was presented in this paper. We had computed the operation of min, max, the composition of two RNM and inverse relation for RNM. An algorithm with an illustrative example of relation analysis of popularities of cars used by different expert opinions is presented to help other researchers better understand the RNM relation theory procedure. The result shows a significant relationship for cars used based on popularity. The discussion has indicated that RNM relation also allows for the generalization of the RNS relation, RIFS relation, and RFS relation. This generalization helps other researchers to elaborate more on relations involving uncertainty information.

RNM relation is studied to verify the value of the interactions between variable use in neutrosophic multiset with rough approximation, such as the relationship between patient and symptoms in the different time taken to diagnose the right patient disease. RNM relation is a useful tool in dealing with multiple phases of inconsistent, incomplete and indeterminate information.

In future work, it would be meaningful to explore further RNM theory properties such as distance and similarity measure and their applications in solving uncertainty information.

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