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Some Results on Neutrosophic Triplet Group and Their Applications

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Abstract: This article is based on new developments on a neutrosophic triplet group (NTG) and applications earlier introduced in 2016 by Smarandache and Ali. NTG sprang up from neutrosophic triplet set X: a collection of triplets $(b, neut(b), anti(b))$ for an $b \in X$ that obeys certain axioms (existence of neutral(s) and opposite(s)). Some results that are true in classical groups were investigated in NTG and were shown to be either universally true in NTG or true in some peculiar types of NTG. Distinguishing features between an NTG and some other algebraic structures such as: generalized group (GG), quasigroup, loop and group were investigated. Some neutrosophic triplet subgroups (NTSGs) of a neutrosophic triplet group were studied. In particular, for any arbitrarily fixed $a \in X$, the subsets $X_a = \{b \in X : neut(b) = neut(a)\}$ and $\ker f_a = \{b \in X | f(b) = neut(f(a))\}$ of X, where $f : X \to Y$ is a neutrosophic triplet group homomorphism, were shown to be NTSG and normal NTSG, respectively. Both X_a and $\ker f_a$ were shown to be a-normal NTSGs and found to partition X. Consequently, a Lagrange-like formula was found for a finite NTG X; $|X| = \sum_{a \in X} [X_a : \ker f_a] |\ker f_a|$ based on the fact that $|\ker f_a| \, | \, |X_a|$. The first isomorphism theorem $X/\ker f \cong \mathrm{Im}\, f$ was established for NTGs. Using an arbitrary non-abelian NTG X and its NTSG X_a, a Bol structure was constructed. Applications of the neutrosophic triplet set, and our results on NTG in relation to management and sports, are highlighted and discussed.

Keywords: generalized group; neutrosophic triplet set; neutrosophic triplet group; group

MSC: Primary 20N02; Secondary 20N05

1. Introduction

1.1. Generalized Group

Unified gauge theory has the algebraic structure of a generalized group abstrusely, in its physical background. It has been a challenge for physicists and mathematicians to find a desirable unified theory for twistor theory, isotopies theory, and so on. Generalized groups are instruments for constructions in unified geometric theory and electroweak theory. Completely simple semigroups are precisely generalized groups (Araujo et al. [1]). As recorded in Adeniran et al. [2], studies on the properties and structures of generalized groups have been carried out in the past, and these have been extended to smooth generalized groups and smooth generalized subgroups by Agboola [3,4], topological generalized groups by Molaei [5], Molaei and Tahmoresi [6], and quotient space of generalized groups by Maleki and Molaei [7].

Definition 1 (Generalized Group(GG)). *A generalized group X is a non-void set with a binary operation called multiplication obeying the set of rules given below.*

(i) $(ab)c = a(bc)$ *for all* $a, b, c \in X$.
(ii) *For each* $a \in X$ *there is a unique* $e(a) \in X$ *such that* $ae(a) = e(a)a = a$ *(existence and uniqueness of identity element).*
(iii) *For each* $a \in X$, *there is* $a^{-1} \in X$ *such that* $aa^{-1} = a^{-1}a = e(a)$ *(existence of inverse element).*

Definition 2. *Let X be a non-void set. Let* $(\cdot)$ *be a binary operation on X. Whenever* $a \cdot b \in X$ *for all* $a, b \in X$, *then* $(X, \cdot)$ *is called a groupoid.*

Whenever the equation $c \cdot x = d$ *(or* $y \cdot c = d$*) have unique solution with respect to x (or y) i.e., satisfies the left (or right) cancellation law, then* $(X, \cdot)$ *is called a left (or right) quasigroup. If a groupoid* $(X, \cdot)$ *is both a left quasigroup and right quasigroup, then it is called a quasigroup. If there is an element* $e \in X$ *called the identity element such that for all* $a \in X$, $a \cdot e = e \cdot a = a$, *then a quasigroup* $(X, \cdot)$ *is called a loop.*

Definition 3. *A loop is called a Bol loop whenever it satisfies the identity*

$$((ab)c)b = a((bc)b).$$

Remark 1. *One of the most studied classes of loops is the Bol loop.*

For more on quasigroups and loops, interested readers can check [8–15].

A generalized group X has the following properties:

(i) For each $a \in X$, there is a unique $a^{-1} \in X$.
(ii) $e(e(a)) = e(a)$ and $e(a^{-1}) = e(a)$ if $a \in X$.
(iii) If X is commutative, then X is a group.

1.2. Neutrosophic Triplet Group

Neutrosophy is a novel subdivision of philosophy that studies the nature, origination, and ambit of neutralities, including their interaction with ideational spectra. Florentin Smarandache [16] introduced the notion of neutrosophic logic and neutrosophic sets for the first time in 1995. As a matter of fact, the neutrosophic set is the generalization of classical sets [17], fuzzy sets [18], intuitionistic fuzzy sets [17,19], and interval valued fuzzy sets [17], to cite a few. The growth process of neutrosophic sets, fuzzy sets, and intuitionistic fuzzy sets are still evolving, with diverse applications. Some recent research findings in these directions are [20–27].

Smarandache and Ali [28] were the first to introduce the notion of the neutrosophic triplet, which they had earlier talked about at a conference. These neutrosophic triplets were used by them to introduce the neutrosophic triplet group, which differs from the classical group both in fundamental and structural properties. The distinction and comparison of the neutrosophic triplet group with the classical generalized group were given. They also drew a brief outline of the potential applications of the neutrosophic triplet group in other research fields. For discussions of results on neutrosophic triplet groups, neutrosophic quadruples, and neutrosophic duplets of algebraic structures, as well as new applications of neutrosophy, see Jaiyéọlá and Smarandache [29]. Jaiyéọlá and Smarandache [29] were the first to introduce and study inverse property neutrosophic triplet loops with applications to cryptography for the first time.

Definition 4 (Neutrosophic Triplet Set-NTS). *Let X be a non-void set together with a binary operation $\star$ defined on it. Then X is called a neutrosophic triplet set if, for any $a \in X$, there is a neutral of 'a' denoted by $neut(a)$ (not necessarily the identity element) and an opposite of 'a' denoted by $anti(a)$, with $neut(a), anti(a) \in X$ such that*

$$a \star neut(a) = neut(a) \star a = a \quad \text{and} \quad a \star anti(a) = anti(a) \star a = neut(a).$$

The elements $a, neut(a)$ and $anti(a)$ are together called neutrosophic triplet, and represented by $(a, neut(a), anti(a))$.

Remark 2. *For an $a \in X$, each of $neut(a)$ and $anti(a)$ may not be unique. In a neutrosophic triplet set $(X, \star)$, an element b (or c) is the second (or third) component of a neutrosophic triplet if $a, c \in X$ ($a, b \in X$) such that $a \star b = b \star a = a$ and $a \star c = c \star a = b$. Thus, (a, b, c) is a neutrosophic triplet.*

Example 1 (Smarandache and Ali [28]). *Consider $(\mathbb{Z}_6, \times_6)$ such that $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ and $\times_6$ is multiplication in modulo 6. $(2, 4, 2), (4, 4, 4)$, and $(0, 0, 0)$ are neutrosophic triplets, but 3 will not give rise to a neutrosophic triplet.*

Definition 5 (Neutrosophic Triplet Group—NTG). *Let $(X, \star)$ be a neutrosophic triplet set. Then $(X, \star)$ is referred to as a neutrosophic triplet group if $(X, \star)$ is a semigroup. Furthermore, if $(X, \star)$ obeys the commutativity law, then $(X, \star)$ is referred to as a commutative neutrosophic triplet group.*

Let $(X, \star)$ be a neutrosophic triplet group. Whenever $neut(ab) = neut(a)neut(b)$ for all $a, b \in X$, then X is referred to as a normal neutrosophic triplet group.

Let $(X, \star)$ be a neutrosophic triplet group and let $H \subseteq X$. H is referred to as a neutrosophic triplet subgroup (NTSG) of X if $(H, \star)$ is a neutrosophic triplet group. Whence, for any fixed $a \in X$, H is called a-normal NTSG of X, written $H \overset{a}{\lhd} X$ if $ay\, anti(a) \in H$ for all $y \in H$.

Remark 3. *An NTG is not necessarily a group. However, a group is an NTG where $neut(a) = e$, the general identity element for all $a \in X$, and $anti(a)$ is unique for each $a \in X$.*

Example 2 (Smarandache and Ali [28]). *Consider $(\mathbb{Z}_{10}, \otimes)$ such that $c \otimes d = 3cd \bmod 10$. $(\mathbb{Z}_{10}, \otimes)$ is a commutative NTG but neither a GG nor a classical group.*

Example 3 (Smarandache and Ali [28]). *Consider $(\mathbb{Z}_{10}, \star)$ such that $c \star d = 5c + d \bmod 10$. $(\mathbb{Z}_{10}, \star)$ is a non-commutative NTG but not a classical group.*

Definition 6 (Neutrosophic Triplet Group Homomorphism). *Let $f : X \to Y$ be a mapping such that X and Y are two neutrosophic triplet groups. Then f is referred to as a neutrosophic triplet group homomorphism if $f(cd) = f(c)f(d)$ for all $c, d \in X$. The kernel of f at $a \in X$ is defined by*

$$\ker f_a = \{x \in X : f(x) = neut(f(a))\}.$$

The Kernel of f is defined by

$$\ker f = \bigcup_{a \in X} \ker f_a$$

such that $f_a = f|_{X_a}$, where $X_a = \{x \in X : neut(x) = neut(a)\}$.

Remark 4. *The definition of neutrosophic triplet group homomorphism above is more general than that in Smarandache and Ali [28]. In Theorem 5, it is shown that, for an NTG homomorphism $f : X \to Y$, $f(neut(a)) = neut(f(a))$ and $f(anti(a)) = anti(f(a))$ for all $a \in X$.*

The present work is a continuation of the study of a neutrosophic triplet group (NTG) and its applications, which was introduced by Smarandache and Ali [28]. Some results that are true in classical groups were investigated in NTG and will be proved to be either generally true in NTG or true in some classes of NTG. Some applications of the neutrosophic triplet set, and our results on NTG in relation to management and sports will be discussed.

The first section introduces GG and NTG and highlights existing results that are relevant to the present study. Section 2 establishes new results on algebraic properties of NTGs and NTG homomorphisms, among which are Lagrange's Theorem and the first isomorphism theorem, and presents a method of the construction of Bol algebraic structures using an NTG. The third section describes applications of NTGs to human management and sports.

2. Main Results

We shall first establish the relationship among generalized groups, quasigroups, and loops with a neutrosophic triplet group assumed.

Lemma 1. *Let X be a neutrosophic triplet group.*

1. *X is a generalized group if it satisfies the left (or right) cancellation law or X is a left (or right) quasigroup.*
2. *X is a generalized group if and only if each element $x \in X$ has a unique $neut(x) \in X$.*
3. *Whenever X has the cancellation laws (or is a quasigroup), then X is a loop and group.*

Proof. 1. Let x have at least two neutral elements, say $neut(x), neut(x)' \in X$. Then $xx = xx \Rightarrow xx\,anti(x) = xx\,anti(x) \Rightarrow x\,neut(x) = x\,neut(x)' \overset{\text{left quasigroup}}{\underset{\text{left cancellation law}}{\Longrightarrow}} neut(x) = neut(x)'$. Therefore, X is a generalized group. Similarly, X is a generalized group if it is has the right cancellation law or if it is a right quasigroup.
2. This follows by definition.
3. This is straightforward because every associative quasigroup is a loop and group. □

2.1. Algebraic Properties of Neutrosophic Triplet Group

We now establish some new algebraic properties of NTGs.

Theorem 1. *Let X be a neutrosophic triplet group. For any $a \in X$, $anti\big(anti(a)\big) = a$.*

Proof. $anti\big(anti(a)\big)anti(a) = neut\big(anti(a)\big) = neut(a)$ by Theorem 1 ([29]). After multiplying by a, we obtain

$$\big[anti\big(anti(a)\big)anti(a)\big]a = neut(a)a = a. \tag{1}$$

$$\begin{aligned} LHS &= anti\big(anti(a)\big)\big(anti(a)a\big) = anti\big(anti(a)\big)neut(a) \\ &= anti\big(anti(a)\big)neut\big(anti(a)\big) = anti\big(anti(a)\big)neut\Big(anti\big(anti(a)\big)\Big) = anti\big(anti(a)\big). \end{aligned} \tag{2}$$

Hence, based on Equations (1) and (2), $anti\big(anti(a)\big) = a$. □

Theorem 2. *Let X be a neutrosophic triplet group such that the left cancellation law is satisfied, and $neut(a) = neut\big(a\,anti(b)\big)$ if and only if $a\,anti(b) = a$. Then X is an idempotent neutrosophic triplet group if and only if $neut(a)anti(b) = anti(b)neut(a)$ $\forall\, a, b \in X$.*

Proof. $neut(a)anti(b) = anti(b)neut(a) \Leftrightarrow (a\,neut(a))anti(b) = a\,anti(b)neut(a) \Leftrightarrow a\,anti(b) = a\,anti(b)neut(a) \Leftrightarrow neut(a) = neut\big(a\,anti(b)\big) \Leftrightarrow a\,anti(b) = a \Leftrightarrow a\,anti(b)b = ab \Leftrightarrow a\,neut(b) = ab \Leftrightarrow anti(a)a\,neut(b) = anti(a)ab \Leftrightarrow neut(a)neut(b) = neut(a)b \Leftrightarrow neut(b) = b \Leftrightarrow b = bb$. □

Theorem 3. *Let X be a normal neutrosophic triplet group in which $neut(a)anti(b) = anti(b)neut(a)\ \forall\, a, b \in X$. Then, $anti(ab) = anti(b)anti(a)\ \forall\, a, b \in X$.*

Proof. Since $anti(ab)(ab) = neut(ab)$, then by multiplying both sides of the equation on the right by $anti(b)anti(a)$, we obtain

$$\big[anti(ab)ab\big]anti(b)anti(a) = neut(ab)anti(b)anti(a). \tag{3}$$

Going by Theorem 1([29]),

$$\begin{aligned}\big[anti(ab)ab\big]anti(b)anti(a) &= anti(ab)a\big(b\ anti(b)\big)anti(a) = anti(ab)a(neut(b)anti(a))\\ &= anti(ab)(a\ anti(a))neut(b) = anti(ab)\big(neut(a)neut(b)\big)\\ &= anti(ab)neut(ab) = anti(ab)neut\big(anti(ab)\big) = anti(ab).\end{aligned} \tag{4}$$

Using Equations (3) and (4), we obtain

$$\big[anti(ab)ab\big]anti(b)anti(a) = anti(ab) \Rightarrow$$
$$neut(ab)\big(anti(b)anti(a)\big) = anti(ab) \Rightarrow anti(ab) = anti(b)anti(a).$$

□

It is worth characterizing the neutrosophic triplet subgroup of a given neutrosophic triplet group to see how a new NTG can be obtained from existing NTGs.

Lemma 2. *Let H be a non-void subset of a neutrosophic triplet group X. The following are equivalent.*

(i) *H is a neutrosophic triplet subgroup of X.*
(ii) *For all $a, b \in H$, $a\ anti(b) \in H$.*
(iii) *For all $a, b \in H$, $ab \in H$, and $anti(a) \in H$.*

Proof. **(i)⇒ (ii)** If H is an NTSG of X and $a, b \in H$, then $anti(b) \in H$. Therefore, by closure property, $a\ anti(b) \in H\ \forall\, a, b \in H$.
(ii)⇒ (iii) If $H \neq \emptyset$, and $a, b \in H$, then we have $b\ anti(b) = neut(b) \in H$, $neut(b)anti(b) = anti(b) \in H$, and $ab = a\ anti(anti(b)) \in H$, i.e., $ab \in H$.
(iii)⇒ (i) $H \subseteq X$, so H is associative since X is associative. Obviously, for any $a \in H$, $anti(a) \in H$. Let $a \in H$, then $anti(a) \in H$. Therefore, $a\ anti(a) = anti(a)a = neut(a) \in H$. Thus, H is an NTSG of X.
□

Theorem 4. *Let G and H be neutrosophic triplet groups. The direct product of G and H defined by*

$$G \times H = \{(g, h) : g \in G \text{ and } h \in H\}$$

is a neutrosophic triplet group under the binary operation $\circ$ defined by

$$(g_1, h_1) \circ (g_2, h_2) = (g_1 g_2, h_1 h_2).$$

Proof. This is simply done by checking the axioms of neutrosophic triplet group for the pair $(G \times H, \circ)$, in which case $neut(g, h) = \big(neut(g), neut(h)\big)$ and $anti(g, h) = \big(anti(g), anti(h)\big)$. □

Lemma 3. *Let $\mathcal{H} = \{H_i\}_{i\in\Omega}$ be a family of neutrosophic triplet subgroups of a neutrosophic triplet group X such that $\bigcap_{i\in\Omega} H_i \neq \emptyset$. Then $\bigcap_{i\in\Omega} H_i$ is a neutrosophic triplet subgroup of X.*

Proof. This is a routine verification using Lemma 2. □

2.2. Neutrosophic Triplet Group Homomorphism

Let us now establish results on NTG homomorphisms, its kernels, and images, as well as a Lagrange-like formula and the First Isomorphism Theorem for NTGs.

Theorem 5. *Let $f : X \to Y$ be a homomorphism where X and Y are two neutrosophic triplet groups.*

1. $f(neut(a)) = neut(f(a))$ *for all* $a \in X$.
2. $f(anti(a)) = anti(f(a))$ *for all* $a \in X$.
3. *If H is a neutrosophic triplet subgroup of X, then $f(H)$ is a neutrosophic triplet subgroup of Y.*
4. *If K is a neutrosophic triplet subgroup of Y, then $\emptyset \neq f^{-1}(K)$ is a neutrosophic triplet subgroup of X.*
5. *If X is a normal neutrosophic triplet group and the set $X_f = \{(neut(a), f(a)) : a \in X\}$ with the product*

$$(neut(a), f(a))(neut(b), f(b)) := (neut(ab), f(ab)), \text{ then}$$

X_f *is a neutrosophic triplet group.*

Proof. Since f is an homomorphism, $f(ab) = f(a)f(b)$ for all $a, b \in X$.

1. Place $b = neut(a)$ in $f(ab) = f(a)f(b)$ to obtain $f\big(a\ neut(a)\big) = f(a)f(neut(a)) \Rightarrow f(a) = f(a)f(neut(a))$. Additionally, place $b = neut(a)$ in $f(ba) = f(b)f(a)$ to obtain $f\big(neut(a)a\big) = f(neut(a))f(a) \Rightarrow f(a) = f(neut(a))f(a)$. Thus, $f(neut(a)) = neut(f(a))$ for all $a \in X$.
2. Place $b = anti(a)$ in $f(ab) = f(a)f(b)$ to obtain $f\big(a\ anti(a)\big) = f(a)f(anti(a)) \Rightarrow f(neut(a)) = f(a)f(anti(a)) \Rightarrow neut(f(a)) = f(a)f(anti(a))$. Additionally, place $b = anti(a)$ in $f(ba) = f(b)f(a)$ to obtain $f\big(anti(a)a\big) = f(anti(a))f(a) \Rightarrow f(neut(a)) = f(a)f(anti(a)) \Rightarrow neut(f(a)) = f(anti(a))f(a)$. Thus, $f(anti(a)) = anti(f(a))$ for all $a \in X$.
3. If H is an NTSG of G, then $f(H) = \{f(h) \in Y : h \in H\}$. We shall prove that $f(H)$ is an NTSG of Y by Lemma 2.
   Since $f(neut(a)) = neut(f(a)) \in f(H)$ for $a \in H$, $f(H) \neq \emptyset$. Let $a', b' \in f(H)$. Then $a' = f(a)$ and $b' = f(b)$. Thus, $a'\ anti(b') = f(a)anti(f(b)) = f(a)f(anti(b)) = f(a\ anti(b)) \in f(H)$. Therefore, $f(H)$ is an NTSG of Y.
4. If K is a neutrosophic triplet subgroup of Y, then $\emptyset \neq f^{-1}(K) = \{a \in X : f(a) \in K\}$. We shall prove that $f(H)$ is an NTSG of Y by Lemma 2.
   Let $a, b \in f^{-1}(K)$. Then $a', b' \in K$ such that $a' = f(a)$ and $b' = f(b)$. Thus, $a'\ anti(b') = f(a)anti(f(b)) = f(a)f(anti(b)) = f(a\ anti(b)) \in K \Rightarrow a\ anti(b) \in f^{-1}(K)$. Therefore, $f^{-1}(K)$ is an NTSG of X.
5. Given the neutrosophic triplet group X and the set $X_f = \{(neut(a), f(a)) : a \in X\}$ with the product $(neut(a), f(a))(neut(b), f(b)) := (neut(ab), f(ab))$. X_f is a groupoid.
   $(neut(a), f(a))(neut(b), f(b)) \cdot (neut(z), f(z)) = (neut(ab), f(ab))(neut(z), f(z)) = (neut(abz), f(abz))$
   $= (neut(a), f(a))(neut(bz), f(bz)) = (neut(a), f(a)) \cdot (neut(b), f(b))(neut(z), f(z))$.
   Therefore, X_f is a semigroup.
   For $(neut(a), f(a)) \in X_f$, let $neut(neut(a), f(a)) = \big(neut(neut(a)), neut(f(a))\big)$. Then $neut(neut(a), f(a)) = \big(neut(a), (f(neut(a))\big) \in X_f$. Additionally, let $anti(neut(a), f(a)) = \big(anti(neut(a)), anti(f(a))\big)$. Then $anti(neut(a), f(a)) = \big(neut(a), f(anti(a))\big) \in X_f$.
   Thus, $(neut(a), f(a))neut(neut(a), f(a)) = (neut(a), f(a))\big(neut(a), (f(neut(a))\big) = (neut(a), f(a))\big(neut(anti(a)), (f(neut(a))\big) = \big(neut(a\ \ anti(a)), f(a\ \ neut(a))\big) = \big(neut(neut(a)), f(a\ \ neut(a))\big) = (neut(a), f(a)) \Rightarrow (neut(a), f(a))neut(neut(a), f(a)) = (neut(a), f(a))$ and similarly, $neut(neut(a), f(a))(neut(a), f(a)) = (neut(a), f(a))$.

On the other hand, $(neut(a), f(a))anti(neut(a), f(a)) = (neut(a), f(a)) \cdot (neut(a), f(anti(a))) = (neut(a), f(a))(neut(anti(a)), (f(anti(a))) = (neut(a \ anti(a)), f(a \ anti(a))) = (neut(neut(a)), f(neut(a))) = (neut(a), (f(neut(a))) = neut(neut(a), f(a)) \Rightarrow (neut(a), f(a)) \cdot anti(neut(a), f(a)) = neut(neut(a), f(a))$ and similarly, $anti(neut(a), f(a)) \cdot (neut(a), f(a)) = neut(neut(a), f(a))$.

Therefore, X_f is a neutrosophic triplet group.

□

Theorem 6. *Let $f : X \to Y$ be a neutrosophic triplet group homomorphism.*

1. $\ker f_a \overset{a}{\triangleleft} X$.
2. $X_a \overset{a}{\triangleleft} X$.
3. *X_a is a normal neutrosophic triplet group.*
4. $anti(cd) = anti(d)anti(c) \ \forall \ c, d \in X_a$.
5. $X_a = \bigcup\limits_{c \in X_a} c \ker f_a$ *for all* $a \in X$.
6. *If X is finite,* $|X_a| = \sum_{c \in X_a} |c \ker f_a| = [X_a : \ker f_a]|\ker f_a|$ *for all* $a \in X$ *where* $[X_a : \ker f_a]$ *is the index of* $\ker f_a$ *in* X_a*, i.e., the number of distinct left cosets of* $\ker f_a$ *in* X_a.
7. $X = \bigcup\limits_{a \in X} X_a$.
8. *If X is finite,* $|X| = \sum\limits_{a \in X} [X_a : \ker f_a]|\ker f_a|$.

Proof. 1. $f(neut(a)) = neut(f(a)) = neut(neut(f(a))) = neut(f(neut(a))) \Rightarrow neut(a) \in \ker f_a \Rightarrow \ker f_a \neq \emptyset$. Let $c, d \in \ker f_a$, then $f(c) = f(d) = neut(f(a))$. We shall use Lemma 2.
$f(c \ anti(d)) = f(c)f(anti(d)) = f(c)anti(f(d)) = neut(f(a))anti(neut(f(a))) = neut(f(a))neut(f(a)) = neut(f(a)) \Rightarrow c \ anti(d) \in \ker f_a$.
Thus, $\ker f_a$ is a neutrosophic triplet subgroup of X. For the a-normality, let $d \in \ker f_a$, then $f(d) = neut(f(a))$. Therefore, $f(ad \ anti(a)) = f(a)f(d)f(anti(a)) = f(a)neut(f(a))anti(f(a)) = f(a)anti(f(a)) = neut(f(a)) \Rightarrow ad \ anti(a) \in \ker f_a$ for all $d \in \ker f_a$.
Therefore, $\ker f_a \overset{a}{\triangleleft} X$.

2. $X_a = \{c \in X : neut(c) = neut(a)\}$. $neut(neut(a)) = neut(a) \Rightarrow neut(a) \in X_a$. Therefore, $X_a \neq \emptyset$. Let $c, d \in X_a$. Then $neut(c) = neut(a) = neut(d)$. $(cd)neut(a) = c(d \ neut(a)) = c(d \ neut(d)) = cd$, and $neut(a)(cd) = (neut(a)c)d = (neut(c)c)d = cd$. Therefore, $neut(cd) = neut(a)$.
$neut(anti(c)) = anti(neut(c)) = anti(neut(a)) = neut(a) \Rightarrow anti(c) \in X_a$. Thus, X_a is a neutrosophic triplet subgroup of X.
$neut(anti(a)) = neut(a) \Rightarrow anti(a) \in X_a$. Therefore, $(ac \ anti(a))neut(a) = (ac)(anti(a)neut(a)) = ac \ anti(a)$, and $neut(a)(ac \ anti(a)) = neut(a)a(c \ anti(a)) = ac \ anti(a)$.
Thus, $neut(ac \ anti(a)) = neut(a) \Rightarrow ac \ anti(a) \in X_a$. Therefore, $X_a \overset{a}{\triangleleft} X$.

3. Let $c, d \in X_a$. Then $neut(c) = neut(a) = neut(d)$. Therefore, $neut(cd) = neut(a) = neut(a)neut(a) = neut(c)neut(d)$. Thus, X_a is a normal NTG.

4. For all $c, d \in X_a$, $neut(c)anti(d) = neut(a)anti(d) = neut(d)anti(d) = anti(d) = anti(d)neut(d) = anti(d)neut(a) = anti(d)$. Therefore, based on Point 3 and Theorem 3, $anti(cd) = anti(d)anti(c) \ \forall \ c, d \in X_a$.

5. Define a relation $\asymp$ on X_a as follows: $c \asymp d$ if $anti(c)d \in \ker f_a$ for all $c, d \in X_a$. $anti(c)c = neut(c) = neut(a) \Rightarrow anti(c)c \in \ker f_a \Rightarrow c \asymp c$. Therefore, $\asymp$ is reflexive.
$c \asymp d \Rightarrow anti(c)d \in \ker f_a \overset{\text{by 4.}}{\Rightarrow} anti(anti(c)d) \in \ker f_a \Rightarrow anti(d)c \in \ker f_a \Rightarrow d \asymp c$. Therefore, $\asymp$ is symmetric.
$c \asymp d, d \asymp z \Rightarrow anti(c)d, anti(d)z \in \ker f_a \Rightarrow anti(c)d \ anti(d)z = anti(c)neut(d)z = anti(c)neut(a)z = anti(c)z \in \ker f_a \Rightarrow c \asymp z$. Therefore, $\asymp$ is transitive and $\asymp$ is an

equivalence relation.
The equivalence class $[c]_{f_a} = \{d : anti(c)d \in \ker f_a\} = \{d : c\ anti(c)d \in c \ker f_a\} = \{d : neut(c)d \in c \ker f_a\} = \{d : neut(a)d \in c \ker f_a\} = \{d : d \in c \ker f_a\} = c \ker f_a$. Therefore, $X_a / \asymp = \{[c]_{f_a}\}_{c \in X_a} = \{c \ker f_a\}_{c \in X_a}$.
Thus, $X_a = \bigcup_{c \in X_a} c \ker f_a$ for all $a \in X$.

6. If X is finite, then $|\ker f_a| = |c \ker f_a|$ for all $c \in X_a$. Thus, $|X_a| = \sum_{c \in X_a} |c \ker f_a| = [X_a : \ker f_a] |\ker f_a|$ for all $a \in X$ where $[X_a : \ker f_a]$ is the index of $\ker f_a$ in X_a, i.e., the number of distinct left cosets of $\ker f_a$ in X_a.
7. Define a relation $\sim$ on X: $c \sim d$ if $neut(c) = neut(d)$. $\sim$ is an equivalence relation on X, so $X / \sim = \{X_c\}_{c \in X}$ and, therefore, $X = \bigcup_{a \in X} X_a$.
8. Hence, based on Point 7, if X is finite, then $|X| = \sum_{a \in X} |X_a| = \sum_{a \in X} [X_a : \ker f_a] |\ker f_a|$.

□

Theorem 7. *Let $a \in X$ and $f : X \to Y$ be a neutrosophic triplet group homomorphism. Then*

1. *f is a monomorphism if and only if $\ker f_a = \{neut(a)\}$ for all $a \in X$;*
2. *the factor set $X / \ker f = \bigcup_{a \in X} X_a / \ker f_a$ is a neutrosophic triplet group (neutrosophic triplet factor group) under the operation defined by*

$$c \ker f_a \cdot d \ker f_b = (cd) \ker f_{ab}.$$

Proof. 1. Let $\ker f_a = \{neut(a)\}$ and let $c, d \in X$. If $f(c) = f(d)$, this implies that $f(c\ anti(d)) = f(d) anti(f(d)) = f(d\ anti(f(d))) \Rightarrow f(c\ anti(d)) = neut(f(d)) \Rightarrow c\ anti(d) \in \ker f_d \Rightarrow$

$$c\ anti(d) = neut(d) = neut(anti(d)). \tag{5}$$

Similarly, $f(anti(d)c) = neut(f(d)) \Rightarrow anti(d)c \in \ker f_d \Rightarrow$

$$anti(d)c = neut(anti(d)). \tag{6}$$

Using Equations (5) and (6), $c = anti(anti(d)) = d$. Therefore, f is a monomorphism.

Conversely, if f is mono, then $f(d) = f(c) \Rightarrow d = c$. Let $k \in \ker f_a$, $a \in X$. Then $f(k) = neut(f(a)) = f(neut(a)) \Rightarrow k = neut(a)$. Therefore, $\ker f_a = \{neut(a)\}$ for all $a \in X$.

2. Let $c \ker f_a, d \ker f_b, z \ker f_c \in X / \ker f = \bigcup_{a \in X} X_a / \ker f_a$.

Groupoid: Based on the multiplication $c \ker f_a \cdot d \ker f_b = (cd) \ker f_{ab}$, the factor set $X / \ker f$ is a groupoid.

Semigroup: $(c \ker f_a \cdot d \ker f_b) \cdot z \ker f_c = (cdz) \ker f_{abc} = c \ker f_a (d \ker f_b \cdot z \ker f_c)$.

Neutrality: Let $neut(c \ker f_a) = neut(c) \ker f_{neut(a)}$. Then $c \ker f_a \cdot neut(c \ker f_a) = c \ker f_a \cdot neut(c) \ker f_{neut(a)} = (c\ neut(c)) \ker f_{a\ neut(a)} = c \ker f_a$ and similarly, $neut(c \ker f_a) \cdot c \ker f_a = c \ker f_a$.

Opposite: Let $anti(c \ker f_a) = anti(c) \ker f_{anti(a)}$. Then $c \ker f_a \cdot anti(c \ker f_a) = c \ker f_a \cdot anti(c) \ker f_{anti(a)} = (c\ anti(c)) \ker f_{a\ anti(a)} = neut(c) \ker f_{neut(a)}$. Similarly, $anti(c \ker f_a)) \cdot c \ker f_a = neut(c) \ker f_{neut(a)}$.

$\therefore$ $(X / \ker f, \cdot)$ is an NTG.

□

Theorem 8. *Let $\phi : X \to Y$ be a neutrosophic triplet group homomorphism. Then $X/\ker\phi \cong Im\ \phi$.*

Proof. Based on Theorem 6(7), $X = \bigcup_{a \in X} X_a$. Similarly, define a relation $\approx$ on $\phi(X) = \operatorname{Im}\phi$: $\phi(c) \approx \phi(d)$ if $neut(\phi(c)) = neut(\phi(d))$. $\approx$ is an equivalence relation on $\phi(X)$, so $\phi(X)/\approx = \{\phi(X_c)\}_{c \in X}$ and $\operatorname{Im}\phi = \bigcup_{c \in X} \phi(X_c)$. It should be noted that $X_a \overset{a}{\triangleleft} X$ in Theorem 6(2).

Let $\bar{\phi}_a : X_a/\ker\phi_a \to \phi(X_a)$ given by $\bar{\phi}_a(c\ker\phi_a) = \phi(c)$. It should be noted that, by Theorem 6(1), $\ker\phi_a \overset{a}{\triangleleft} X$. Therefore, $c\ker\phi_a = d\ker\phi_a \Rightarrow anti(d)c\ker\phi_a = anti(d)d\ker\phi_a = neut(d)\ker\phi_a = \ker\phi_a \Rightarrow anti(d)c\ker\phi_a = \ker\phi_a \Rightarrow \phi(anti(d)c) = neut(\phi(a)) \Rightarrow anti(\phi(d))\phi(c) = neut(\phi(a)) \Rightarrow \phi(d)anti(\phi(d))\phi(c) = \phi(d)neut(\phi(a)) \Rightarrow neut(\phi(d))\phi(c) = \phi(d)neut(\phi(a)) \Rightarrow \phi(neut(d))\phi(c) = \phi(d)\phi(neut(a)) \Rightarrow \phi(neut(d)\ c)) = \phi(d\ neut(a)) \Rightarrow \phi(neut(a)\ c)) = \phi(d\ neut(a)) \Rightarrow \phi(neut(c)\ c)) = \phi(d\ neut(c)) \Rightarrow \phi(c) = \phi(d) \Rightarrow \bar{\phi}_a(c\ker\phi_a) = \bar{\phi}_a(d\ker\phi_a)$. Thus, $\bar{\phi}_a$ is well defined.

$\bar{\phi}_a(c\ker\phi_a) = \bar{\phi}_a(d\ker\phi_a) \Rightarrow \phi(c) = \phi(d) \Rightarrow anti(\phi(d))\phi(c) = anti(\phi(d))\phi(d) = neut(\phi(d)) \Rightarrow \phi(anti(d))\phi(c) = neut(\phi(d)) = \phi(neut(d)) = \phi(neut(a)) = neut(\phi(a)) \Rightarrow \phi(anti(d)\ c) = neut(\phi(a)) \Rightarrow anti(d)\ c \in \ker\phi_a \Rightarrow d\ anti(d)\ c \in d\ker\phi_a \Rightarrow neut(d)\ c \in d\ker\phi_a \Rightarrow neut(a)\ c \in d\ker\phi_a \Rightarrow c \in d\ker\phi_a \overset{\text{Theorem 6(1)}}{\Longrightarrow} c\ker\phi_a = d\ker\phi_a$. This means that $\bar{\phi}_a$ is 1-1. $\bar{\phi}_a$ is obviously onto. Thus, $\bar{\phi}_a$ is bijective.

Now, based on the above and Theorem 7(2), we have a bijection

$$\Phi = \bigcup_{a \in X} \bar{\phi}_a : X/\ker\phi = \bigcup_{a \in X} X_a/\ker\phi_a \to \operatorname{Im}\phi = \phi(X) = \bigcup_{a \in X} \phi(X_a)$$

defined by $\Phi(c\ker\phi_a) = \phi(c)$. Thus, if $c\ker\phi_a, d\ker\phi_b \in X/\ker\phi$, then

$$\Phi\Big(c\ker\phi_a \cdot d\ker\phi_b\Big) = \Phi\big(cd\ker\phi_a b\big) = \phi(cd) = \phi(c)\phi(d) = \Phi\big(c\ker\phi_a\big)\Phi\big(d\ker\phi_b\big).$$

$\therefore\ X/\ker\phi \cong \operatorname{Im}\phi$. □

2.3. Construction of Bol Algebraic Structures

We now present a method of constructing Bol algebraic structures using an NTG.

Theorem 9. *Let X be a non-abelian neutrosophic triplet group and let $A = X_a \times X$ for any fixed $a \in X$. For $(h_1, g_1), (h_2, g_2) \in A$, define $\circ$ on A as follows:*

$$(h_1, g_1) \circ (h_2, g_2) = (h_1 h_2, h_2 g_1\ anti(h_2) g_2).$$

Then $(A, \circ)$ is a Bol groupoid.

Proof. Let $a, b, c \in A$. By checking, it is true that $a \circ (b \circ c) \neq (a \circ b) \circ c$. Therefore, $(A, \circ)$ is non-associative. X_a is a normal neutrosophic triplet group by Theorem 6(3). A is a groupoid.

Let us now verify the Bol identity:

$$((a \circ b) \circ c) \circ b = a \circ ((b \circ c) \circ b)$$

$$\text{LHS} \quad = ((a \circ b) \circ c) \circ b = \Big(h_1 h_2 h_3 h_2, h_2 h_3 h_2 g_1\ anti(h_2) g_2\ anti(h_3) g_3\ anti(h_2) g_2\Big).$$

Following Theorem 6(4),

$$\begin{aligned}\text{RHS} &= a \circ ((b \circ c) \circ b) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti\big(h_2h_3h_2\big)h_2h_3g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)\big(anti(h_3)\ anti(h_2)h_2h_3\big)g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)\big(anti(h_3)\ neut(h_2)h_3\big)g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)\big(anti(h_3)\ neut(a)h_3\big)g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)anti(h_3)h_3g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)neut(h_3)g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)neut(a)g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big) = \\ &\Big(h_1h_2h_3h_2, h_2h_3h_2g_1\ anti(h_2)g_2\ anti(h_3)g_3\ anti(h_2)g_2\Big).\end{aligned}$$

Therefore, LHS = RHS. Hence, $(A, \circ)$ is a Bol groupoid. □

Corollary 1. *Let H be a subgroup of a non-abelian neutrosophic triplet group X, and let $A = H \times X$. For $(h_1, g_1), (h_2, g_2) \in A$, define $\circ$ on A as follows:*

$$(h_1, g_1) \circ (h_2, g_2) = (h_1h_2, h_2g_1\ anti(h_2)g_2).$$

Then $(A, \circ)$ is a Bol groupoid.

Proof. A subgroup H is a normal neutrosophic triplet group. The rest of the claim follows from Theorem 9. □

Corollary 2. *Let H be a neutrosophic triplet subgroup (which obeys the cancellation law) of a non-abelian neutrosophic triplet group X, and let $A = H \times X$. For $(h_1, g_1), (h_2, g_2) \in A$, define $\circ$ on A as follows:*

$$(h_1, g_1) \circ (h_2, g_2) = (h_1h_2, h_2g_1\ anti(h_2)g_2).$$

Then $(A, \circ)$ is a Bol groupoid.

Proof. By Theorem 1(3), H is a subgroup of X. Hence, following Corollary 1, $(A, \circ)$ is a Bol groupoid. □

Corollary 3. *Let H be a neutrosophic triplet subgroup of a non-abelian neutrosophic triplet group X that has the cancellation law and let $A = H \times X$. For $(h_1, g_1), (h_2, g_2) \in A$, define $\circ$ on A as follows:*

$$(h_1, g_1) \circ (h_2, g_2) = (h_1h_2, h_2g_1\ anti(h_2)g_2).$$

Then $(A, \circ)$ is a Bol loop.

Proof. By Theorem 1(3), X is a non-abelian group and H is a subgroup of X. Hence, $(A, \circ)$ is a loop and a Bol loop by Theorem 9. □

3. Applications in Management and Sports

3.1. One-Way Management and Division of Labor

Consider a company or work place consisting of a set of people X with $|X|$ number of people. A working unit or subgroup with a leader 'a' is denoted by X_a.

$neut(x)$ for any $x \in X$ represents a co-worker (or co-workers) who has (have) a good (non-critical) working relationship with x, while $anti(x)$ represents a co-worker (or co-workers) whom x considers as his/her personal critic(s) at work.

Hence, X_a can be said to include both critics and non-critics of each worker x. It should be noted that in X_a, $neut(a) = neut(x)$ for all $x \in X_a$. This means that every worker in X_a has a good relationship with the leader 'a'.

Thus, by Theorem 6(7)—$X = \bigcup_{a \in X} X_a$ and $|X| = \Sigma_{a \in X}|X_a|$—the company or work place X can be said to have a good division of labor for effective performance and maximum output based on the composition of its various units (X_a). See Figure 1.

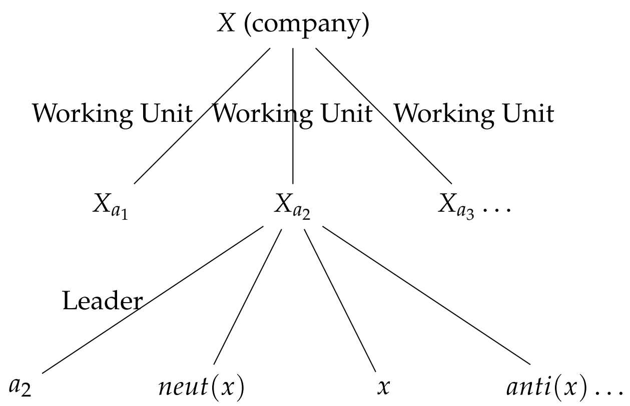

Figure 1. One-way management and division of labor.

3.2. Two-Way Management Division of Labor

Consider a company or work place consisting of a set of people X with $|X|$ number of people at a location A and another company or work place consisting of people Y with $|Y|$ number of people at another location B. Assume that both companies are owned by the same person f. Hence, $f : X \to Y$ can be considered as a movement (transfer) or working interaction between workers at A and at B. The fact that f is a neutrosophic triplet group homomorphism indicates that the working interaction between X and Y is preserved.

Let 'a' be a unit leader at A whose work correlates to another leader $f(a)$ at B. Then $Kerf_a$ represents the set of workers x in a unit at A under the leadership of 'a' such that there are other, corresponding workers $f(x)$ at B under the leadership of $f(a)$. Here, $f(x) = neut(f(a))$ means that workers $f(x)$ at B under the leadership of $f(a)$ are loyal and in a good working relationship. The mapping f_a shows that the operation of a subgroup leader (the operation is denoted by 'a') is subject to the modus operandi of the owner of the two companies, where the owner is denoted by f.

The final formula $|X| = \sum_{x \in X} [X_a : \ker f_a]|kerf_a|$ in Theorem 6(8) shows that the overall performance of the set of people X is determined by how the unit leaders 'a' at A properly harmonize with the unit leaders at B in the effective administration of $\ker f_a$ and X_a (Figure 2).

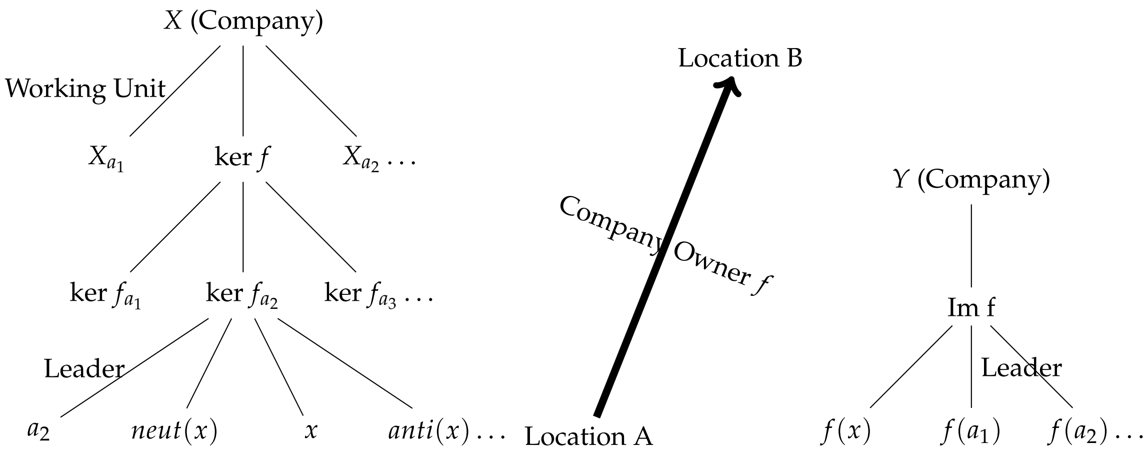

Figure 2. Two-way management division of labor.

3.3. Sports

In the composition of a team, a coach can take X_a as the set of players who play in a particular department (e.g., forward, middle field, or defence), where a is the leader of that department. Let $neut(x)$ represent player(s) whose performance is the same as that of player x, and let $anti(x)$ represent player(s) that can perform better than player x. It should be noted that the condition $neut(x) = neut(a)$ for all $x \in X_a$ means that the department X_a has player(s) who are equal in performance; i.e., those whose performance are equal to that of the departmental leader a. Hence, a neutrosophic triplet $(x, neut(x), anti(x))$ is a triple from which a coach can make a choice of his/her starting player and make a substitution. The neutrosophic triplet can also help a coach to make the best alternative choice when injuries arise. For instance, in the goal keeping department (for soccer/football), three goal keepers often make up the team for any international competition. Imagine an incomplete triplet $(x, neut(x), ?)$, i.e., no player is found to be better than x, which reduces to a duplet.

X_a can also be used for grouping teams in competitions in the preliminaries. If $x = team$, then $anti(x) = teams$ that can beat x and $neut(x) = teams$ that can play draw with x. Therefore, neutrosophic triplet $(x, neut(x), anti(x))$ is a triplet with which competition organizers can draw teams into groups for a balanced competition. The Fédération Internationale de Football Association (FIFA) often uses this template in drawing national teams into groups for its competitions. Club teams from various national leagues, to qualify for continental competitions (e.g., Union of European Football Associations (UEFA) Champions League and Confederation of African Football (CAF) Champions League), have to be among the five. This implies the application of duplets, triplets, quadruples, etc. (Figure 3).

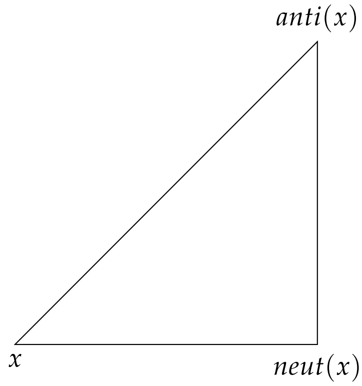

Figure 3. Sports.

Author Contributions: T.G.J. established new properties of neutrosophic triplet groups. He further presented applications of the neutrosophic triplet sets and groups to management and sports. F.S. cointroduced the neutrosophic triplet set and group, as well as their properties. He confirmed the relevance of the neutrosophic duplet and the quadruple in the applications of neutrosophic triplet set.

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