Some weighted arithmetic operators and geometric operators with SVNSs and their application to multi-criteria decision making problems

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ABSTRACT

As a variation of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets have been developed to represent uncertain, imprecise, incomplete and inconsistent information that exists in the real world. In this paper,this article introduces an approach to handle multi-criteria decision making (MCDM) problems under the SVNSs. Therefore, we develop some new geometric and arithmetic aggregation operators, such as the single valued neutrosophic weighted arithmetic (SVNWA) operator, the single valued neutrosophic ordered weighted arithmetic (SVNOWA) operator,the single-valued neutrosophic sets hybrid ordered weighted arithmetic (SVNSHOWA) operator, the single-valued neutrosophic weighted geometric (SVNWG) operator and the single-valued neutrosophic ordered weighted geometric(SVNOWG) operator and the single-valued neutrosophic hybrid ordered weighted geometric (SVNHOG)operator, which extend the intuitionistic fuzzy weighted geometric and intuitionistic fuzzy ordered weighted geometric operators to accommodate the environment in which the given arguments are single valued neutrosophic sets which are characterized by a membership function, an indeterminacy-membership function and a non-membership function. Some numerical examples are given to illustrate the developed operators. Finally, a numerical example is used to demonstrate how to apply the proposed approach.

KEYWORDS: Neurosophic set, Single valued neutrosophic weighted geometric (SVNWG) operator, Single valued neutrosophic weighted arithmetic (SVNWA) operator.

Section1. Introduction

To handle with imprecision and uncertainty, concept of fuzzy sets and intuitionistic fuzzy sets originally introduced by Zadeh (Zadeh 1965) and Atanassov (Atanassov 1986), respectively. Then, Smarandache (Smarandache 1998) proposed concept of neutrosophic set which is generalization of fuzzy set theory and intuitionistic fuzzy sets. The neutrosophic sets may express more abundant and flexible information as compared with the fuzzy sets and intuitionistic fuzzy sets. Recently, neutrosophic sets have been researched by many scholars in different fields. For example; on multi-criteria decision making problems (Liuet al. 2017a, 2017b, 2017c, Lin 2017, Kandasamy and Smarandache 2017) etc. Also the notations such as fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets have been applied to some different fields in (Broumi et al. 2014a, 2014b,2015a, 2015b, 2015c, 2016a,2016b,2016c,2016d, He et al. 2014a, 2014b, 2014c, Sahin et al 2015,2016,Ulucay et al. 2016a, 2016b).

This paper mainly discusses extension forms of these aggregation operators with intuitionistic fuzzy sets, including the intuitionistic fuzzy weighted averaging operator, intuitionistic fuzzy OWA operator, intuitionistic fuzzy hybrid weighted averaging operator, intuitionistic fuzzy GOWA operator, intuitionistic fuzzy generalized hybrid weighted averaging operator and their applications to multi-attribute decision-making with intuitionistic fuzzy sets (Li 2011,Li 2010, Li et al. 2010a Wang et al.2009, Li et al 2010b).

In this paper, we shall develop some geometric operators andarithmetic operator, such as the single-valuedneutrosophicweighted arithmetic operator (SVNWAO), the single valued neutrosophic weighted geometric (SVNWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator and the intuitionistic fuzzy hybrid geometric (IFHG) operator. To do so, this paper is structured as follows. In Section 2, we review the weighted geometric (WG) operator and the ordered weighted geometric (OWG) operator. In Section 3, we develop the IFWG operator, the IFOWG operator, and the IFHG operator, and study their various properties. In Section 4, we give an application of the IFHG operator to multiple attribute decision making with intuitionistic fuzzy information. Concluding remarks are made in Section 5.

Section 2. Preliminaries

To facilitate the following discussion, some concepts related to neutrosophic set and single valued neutrosophic set are briefly introduced in this section.

Definition 2.1(Smarandache 1998) Let$X$be a space ofpoints (objects), with a generic element in X, denoted by x.An NS Ain $X$is characterised by a truth-membership function$T_A(x)$, an indeterminacy-membership function$I_A(x)$ and a falsity-membership function $F_A(x)$.$T_A(x)$, $I_A(x)$ and $F_A(x)$ are standard or non-standard subsets of$]0^-,1^+[$,that is, $T_A(x): X \to]0^-,1^+[$,$I_A(x): X \to]0^-,1^+[$ and $F_A(x): X \to]0^-,1^+[$.There is no restriction on the sumof $T_A(x)$, $I_A(x)$ and $F_A(x)$, therefore

$$0^- \leq sup\ T_A(x) + sup\ I_A(x) + sup\ F_A(x) \leq 3^+.$$

Definition 2.2(Smarandache 1998)The complement of a neutrosphic set $A$is denoted by $A^c$and is defined as

$$T_A^c(x) = \{1^+\} \ominus T_A(x), \qquad I_A^c(x) = \{1^+\} \ominus I_A(x), \qquad F_A^c(x) = \{1^+\} \ominus F_A(x)$$

forevery element$x$in$X$.

Definition 2.3(Smarandache 1998) A neutrosophic set $A$is contained in the other neutrosophicset B,$A \subseteq B$ if and only if

$inf\ T_A(x) \leq\ inf\ T_B(x),\ sup\ T_A(x) \leq\ sup\ T_B(x),\ inf\ I_A(x) \geq\ inf\ I_B(x),\ sup I_A(x)\ \geq$ $sup\ I_B(x),\ inf\ F_A(x) \geq\ inf\ F_B(x)\ and\ \sup F_A(x) \geq\ sup\ F_B(x)$

for every $x$in$X$.

Definition 2.4 (Smarandache 1998) The union of two neutrosophic sets $A$and $B$is a neutrosophic set C denoted by $C = A \cup B$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of $A$and $B$by

$$T_C(x) = T_A(x) \oplus T_B(x) \ominus T_A(x) \odot T_B(x)$$
$$I_C(x) = I_A(x) \odot I_B(x) \text{and}$$
$$F_C(x) = F_A(x) \odot F_B(x)$$

for any $x$in$X$.

Definition 2.5 (Smarandache 1998)The intersection of two neutrosophic sets $A$and $B$is a neutrosophicset C, denoted by $C = A \cap B$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of $A$and $B$by

$$T_C(x) = T_A(x) \odot T_B(x)$$
$$I_C(x) = I_A(x) \odot I_B(x) \text{and}$$
$$F_C(x) = F_A(x) \odot F_B(x)$$

for any $x$in$X$.

A single valued neutrosophic set (SVNS) is an instance of a neutrosophic set, which can be used in real scientific and engineering applications (Ye 2014)

Note that the set of all SVNSs on R will be denoted by Λ.

Definition 2.6 (Wang et al. 2010)Let X be a universe set, with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) Ain $X$is characterized by truth-membership function$T_A(x)$, indeterminacy-membership function $I_A(x)$and falsity-membership function $F_A(x)$. For each element $x$in$X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$. Therefore, a SVNS A can be written as follows:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}.$$

For two SVNSs A, B, Wang et al. (Wang et al. 2010)presented the following expressions:

(1) $A \subseteq B$if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$ and $F_A(x) \geq F_B(x)$ for every $x \in X$.

(2) $A = B$if and only if $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle x \in X\}$.

A SVNS $A$is usually denoted by the simplified symbol $A = \langle T_A(x), I_A(x), F_A(x)\rangle$for any $x \in X$For any two SVNSs $A$and B, the operational relations are defined by Wang et al. (Wang et al. 2010).

(1)$A \cup B = \langle \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$for every $x \in X$.

(2)$A \cup B = \langle \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$for every $x \in X$.

(3)$A \times B = \langle T_A(x) + T_B(x) - T_A(x).T_B(x), I_A(x).I_B(x), F_A(x).F_B(x)\rangle$for every $x \in X$.

For a SVNS Ain X, Ye (Ye 2014) called the triplet $\langle T_A(x), I_A(x), F_A(x)\rangle$ single valued neutrosophic number (SVNN), which is denoted by $\alpha = \langle T_A, I_A, F_A\rangle$.

Definition 2.7(Ju 2014)Let $\tilde{\alpha} = \langle T, I, F\rangle$be a $SVNN$, then the score function and the accuracyfunction of $A$are determined by Eqs. (1) and (2), respectively

$S(\tilde{\alpha}) = (T + 1 - I + 1 - F)/3$ (1)

$V(\tilde{\alpha}) = (T + F + 1 - I)/3$ (2)

Theorem 2.8(Ju 2014) Let $\tilde{\alpha} = \langle T_a, I_a, F_a \rangle$ and $\tilde{b} = \langle T_b, I_b, F_b \rangle$ be two $SVNNs$, then the comparison laws between them are shown as follows:

1-If $S(\tilde{\alpha}) > S(\tilde{b})$, then $\tilde{\alpha} > \tilde{b}$;
2-If $S(\tilde{\alpha}) < S(\tilde{b})$, then $\tilde{\alpha} < \tilde{b}$;
3-If $S(\tilde{\alpha}) = S(\tilde{b})$, then:

(1) If $(\tilde{\alpha}) > V(\tilde{b})$, then $\tilde{\alpha} > \tilde{b}$;
(2) If $(\tilde{\alpha}) < V(\tilde{b})$, then $\tilde{\alpha} < \tilde{b}$;
(3) If $(\tilde{\alpha}) = V(\tilde{b})$, then $\tilde{\alpha} = \tilde{b}$;

Definition 2.9(Chi 2013) Let $\tilde{\alpha} = \langle T, I, F \rangle$, $\tilde{\alpha}_1 = \langle T_1, I_1, F_1 \rangle$ and $\tilde{\alpha}_2 = \langle T_2, I_2, F_2 \rangle$ be any threesingle valued neutrosophic numbers, and $> (greater\ than)\ 0$, then some operational laws of the $SVNNs$ are defined as follows.
(1) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \langle T_1 + T_2 - T_1 \times T_2, I_1 \times I_2, F_1 \times F_2 \rangle$;
(2) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \langle T_1 \times T_2, I_1 + I_2 - I_1 \times I_2, F_1 + F_2 - F_1 \times F_2 \rangle$;
(3)$\lambda\tilde{\alpha} = \langle 1 - (1 - T)^{\lambda}, I^{\lambda} F^{\lambda} \rangle, \lambda > 0$;
(4)$\tilde{\alpha}^{\lambda} = \langle T^{\lambda}, 1 - (1 - I)^{\lambda}, 1 - (1 - F)^{\lambda} \rangle,\ \lambda > 0$.
Obviously, the above operational results are still $SVNNs$. Some relationships can be further established for these operations on $SVNNs$.

Section 3. Arithmetic operators and Geometric operators of the *SVNS*

3.1 Arithmetic operators of the *SVNS*

Definition 3.1.1 Let $A_j = \langle \xi_j, \varphi_j, \zeta_j \rangle \in \Lambda (j \in I_n)$. Then $SVNS$ weighted arithmetic operator,denoted by ψ_{ao}, is defined as;

$$\psi_{ao} : \Lambda^n \to \Lambda, \psi_{ao}(A_1, A_2, \dots, A_n) = \sum_{j=1}^{n} w_j A_j$$

where, $w = (w_1, w_2, \dots, w_n)^T$ is a weight vector associated with the ψ_{ao} operator, for every $j \in I_n$ such that, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 3.1.2 Let $A_j = \langle \xi_j, \varphi_j, \zeta_j \rangle \in \Lambda\ (j \in I_n)\ (w_1, w_2, \dots, w_n)^T$ be a weight vectorof A_j, for every $j \in I_n$ In such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.Then, their aggregated value by using ψ_{ao} operator is also a $SVNS$ and

$$\psi_{ao}(A_1, A_2, \ldots, A_n) = \langle \frac{\sum_{j=1}^{n} \xi_{A_j}^{w_j}}{1+\prod_{j=1}^{n} \xi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{n} \varphi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \varphi_{A_j}^{w_j} - \prod_{j=1}^{n} \varphi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \zeta_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \zeta_{A_j}^{w_j} - \prod_{j=1}^{n} \zeta_{A_j}^{w_j}\right]} \rangle$$

Proof: The proof can be made by using mathematical induction on n as; assume that,

$A_1 = \langle \xi_1, \varphi_1, \zeta_1 \rangle$and$A_2 = \langle \xi_2, \varphi_2, \zeta_2 \rangle$be two $SVNS$ then, for $n = 2$, we have

$$\psi_{ao}(A_1, A_2) = \langle \frac{\sum_{j=1}^{2} \xi_{A_j}^{w_j}}{1+\prod_{j=1}^{2} \xi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{2} \varphi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{2} \varphi_{A_j}^{w_j} - \prod_{j=1}^{2} \varphi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{2} \zeta_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{2} \zeta_{A_j}^{w_j} - \prod_{j=1}^{2} \zeta_{A_j}^{w_j}\right]} \rangle$$

$$= \langle \frac{\xi_{A_1}^{w_1} + \xi_{A_2}^{w_2}}{1+\xi_{A_1}^{w_1} . \xi_{A_1}^{w_1}}, \frac{\varphi_{A_1}^{w_1} + \varphi_{A_2}^{w_2}}{2-\left[\varphi_{A_1}^{w_1} + \varphi_{A_2}^{w_2} - \varphi_{A_1}^{w_1} . \varphi_{A_2}^{w_2}\right]}, \frac{\zeta_{A_1}^{w_1} + \zeta_{A_2}^{w_2}}{2-\left[\zeta_{A_1}^{w_1} + \zeta_{A_2}^{w_2} - \zeta_{A_1}^{w_1} . \zeta_{A_2}^{w_2}\right]} \rangle$$

If holds for $n = k$, that is

$$\psi_{ao}(A_1, A_2, \ldots, A_k) = \langle \frac{\sum_{j=1}^{k} \xi_{A_j}^{w_j}}{1+\prod_{j=1}^{k} \xi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{k} \varphi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{k} \varphi_{A_j}^{w_j} - \prod_{j=1}^{k} \varphi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{k} \zeta_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{k} \zeta_{A_j}^{w_j} - \prod_{j=1}^{k} \zeta_{A_j}^{w_j}\right]} \rangle$$

then, when $n = k + 2$; by the operational laws in Definition 2.9, I have

$$\psi_{ao}(A_1, A_2, \ldots, A_k, A_{k+1}, A_{k+2}) = \langle \frac{\sum_{j=1}^{k} \xi_{A_j}^{w_j} + \xi_{A_{k+1}}^{w_{k+1}} + \xi_{A_{k+2}}^{w_{k+2}}}{1+\left[\left(\prod_{j=1}^{k} \xi_{A_j}^{w_j}\right) . \xi_{A_{k+1}}^{w_{k+1}} . \xi_{A_{k+2}}^{w_{k+2}}\right]},$$

$$\frac{\left(\sum_{j=1}^{k} \varphi_{A_j}^{w_j}\right) + \varphi_{A_{k+1}}^{w_{k+1}} + \varphi_{A_{k+2}}^{w_{k+2}}}{2-\left[\left(\sum_{j=1}^{k} \varphi_{A_j}^{w_j}\right) + \varphi_{A_{k+1}}^{w_{k+1}} + \varphi_{A_{k+2}}^{w_{k+2}} - \left(\prod_{j=1}^{k} \varphi_{A_j}^{w_j}\right) . \varphi_{A_{k+1}}^{w_{k+1}} . \varphi_{A_{k+2}}^{w_{k+2}}\right]},$$

$$\frac{\left(\sum_{j=1}^{k} \zeta_{A_j}^{w_j}\right) + \zeta_{A_{k+1}}^{w_{k+1}} + \zeta_{A_{k+2}}^{w_{k+2}}}{2-\left[\left(\sum_{j=1}^{k} \zeta_{A_j}^{w_j}\right) + \zeta_{A_{k+1}}^{w_{k+1}} + \zeta_{A_{k+2}}^{w_{k+2}} - \left(\prod_{j=1}^{k} \zeta_{A_j}^{w_j}\right) . \zeta_{A_{k+1}}^{w_{k+1}} . \zeta_{A_{k+2}}^{w_{k+2}}\right]} \rangle$$

$$\psi_{ao}(A_1, A_2, \ldots, A_k, A_{k+1}, A_{k+2}) = \langle \frac{\sum_{j=1}^{k+2} \xi_{A_j}^{w_j}}{1+\prod_{j=1}^{k+2} \xi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{k+2} \varphi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{k+2} \varphi_{A_j}^{w_j} - \prod_{j=1}^{k+2} \varphi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{k+2} \zeta_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{k+2} \zeta_{A_j}^{w_j} - \prod_{j=1}^{k+2} \zeta_{A_j}^{w_j}\right]} \rangle$$

Example 3.1.3 $A_1 = \langle 0.3, 0.2, 0.5 \rangle$, $A_2 = \langle 0.1, 0.5, 0.7 \rangle$, $A_3 = \langle 0.7, 0.1, 0.8 \rangle$and $A_4 = \langle 0.4, 0.2, 0.5 \rangle$ $w = (0.3, 0.4, 0.1, 0.2)^T$

$$\psi_{ao}(A_1, A_2, A_3, A_4) = \langle \frac{\sum_{j=1}^{4} \xi_{A_j}^{w_j}}{1+\prod_{j=1}^{4} \xi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{4} \varphi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{4} \varphi_{A_j}^{w_j} - \prod_{j=1}^{4} \varphi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{4} \zeta_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{4} \zeta_{A_j}^{w_j} - \prod_{j=1}^{4} \zeta_{A_j}^{w_j}\right]} \rangle$$

$$= \langle \frac{0.3^{0.3} + 0.1^{0.4} + 0.7^{0.1} + 0.4^{0.2}}{1 + 0.3^{0.3} \times 0.1^{0.4} \times 0.7^{0.1} \times 0.4^{0.2}},$$

$$\frac{0.2^{0.3}+0.5^{0.4}+0.1^{0.1}+0.2^{0.2}}{2-[(0.2^{0.3}+0.5^{0.4}+0.1^{0.1}+0.2^{0.2})-(0.2^{0.3}\times 0.5^{0.4}\times 0.1^{0.1}\times 0.2^{0.2})]},$$

$$\frac{0.5^{0.3}+0.7^{0.4}+0.8^{0.1}+0.5^{0.2}}{2-[(0.5^{0.3}+0.7^{0.4}+0.8^{0.1}+0.5^{0.2})-(0.5^{0.3}\times 0.7^{0.4}\times 0.8^{0.1}\times 0.5^{0.2})]}\rangle$$

$$=\langle 2.365,-4.632,-3.800\rangle$$

Definition 3.1.4 Let$A_j=\langle \xi_j,\varphi_j,\zeta_j\rangle\in\Lambda\ (j\in I_n)\ w=(w_1,w_2,\dots,w_n)^T$.Then $SVNS$ ordered weighted arithmetic operator denoted by ψ_{oao}, is defined as;

$$\psi_{oao}:\Lambda^n\to\Lambda,\psi_{oao}(A_1,A_2,\dots,A_n)=\sum_{j=1}^{n}w_jA_j,$$

where$(w_1,w_2,\dots,w_n)^T$is a weight vector associated with the mapping ψ_{oao}; which satisfies the normalized conditions: $w_k\in[0,1]$ and $\sum_{k=1}^{n}w_k=1$; $B_j=\langle\tilde{\xi}_j,\tilde{\varphi}_j,\tilde{\zeta}_j\rangle$ is the k-th largest of the n$SVNS$which is determined through using ranking method in Definition 2.7.

It is not difficult to follows from Definition 3.1.4 that

$$\psi_{oao}(B_1,B_2,\dots,B_n)=\langle\frac{\sum_{j=1}^{n}\xi_{B_j}^{w_j}}{1+\prod_{j=1}^{n}\xi_{B_j}^{w_j}},\frac{\sum_{j=1}^{n}\tilde{\varphi}_{B_j}^{w_i}}{2-\left[\sum_{j=1}^{n}\tilde{\varphi}_{B_j}^{w_i}-\prod_{j=1}^{n}\tilde{\varphi}_{B_j}^{w_i}\right]},\frac{\sum_{j=1}^{n}\zeta_{B_j}^{w_i}}{2-\left[\sum_{j=1}^{n}\zeta_{B_j}^{w_i}-\prod_{j=1}^{n}\zeta_{B_j}^{w_i}\right]}\rangle$$

which is summarized as in Theorem 3.1.5

Theorem 3.1.5 Let$A_j=\langle\xi_j,\varphi_j,\zeta_j\rangle\in\Lambda\ (j\in I_n)$. Then$SVNS$ ordered weighted arithmetic operator denoted by ψ_{oao}, is defined as; $\psi_{oao}:\Lambda^n\to\Lambda$

$$\psi_{oao}(B_1,B_2,\dots,B_n)=\langle\frac{\sum_{j=1}^{n}\xi_{B_j}^{w_j}}{1+\prod_{j=1}^{n}\xi_{B_j}^{w_j}},\frac{\sum_{j=1}^{n}\tilde{\varphi}_{B_j}^{w_i}}{2-\left[\sum_{j=1}^{n}\tilde{\varphi}_{B_j}^{w_i}-\prod_{j=1}^{n}\tilde{\varphi}_{B_j}^{w_i}\right]},\frac{\sum_{j=1}^{n}\zeta_{B_j}^{w_i}}{2-\left[\sum_{j=1}^{n}\zeta_{B_j}^{w_i}-\prod_{j=1}^{n}\zeta_{B_j}^{w_i}\right]}\rangle$$

where$w_k\in[0,1]$and $\sum_{k=1}^{n}w_k=1$ $B_j=\langle\tilde{\xi}_j,\tilde{\varphi}_j,\tilde{\zeta}_j\rangle$is the k-th largest ofthen $SVNS$ $\tilde{A}_j(j\in I_n)$which is determined through using ranking method in Definition 2.7.

Proof: Theorem 3.1.5 can be proven in a similar way to that of Theorem 3.1.2 (omitted).

Example 3.1.6 $A_1=\langle 0.1,0.2,0.4\rangle$, $A_2=\langle 0.2,0.5,0.3\rangle$, $A_3=\langle 0.5,0.2,0.1\rangle$ *and* $A_4=\langle 0.5,0.1,0.1\rangle w=(0.1,0.4,0.3,0.2)^T$

$$s(A)=\frac{T_A+1-I_A+1-F_A}{3}$$

$$s(A_1)=\frac{(0.1+1-0.2+1-0.4)}{3}=0.5$$

$$s(A_2)=\frac{(0.2+1-0.5+1-0.3)}{3}=0.466$$

$$s(A_3) = \frac{(0.5+1-0.2+1-0.1)}{3} = 0.733$$
$$s(A_4) = \frac{(0.5+1-0.1+1-0.1)}{3} = 0.766$$

It is obvious that $s(A_4) > s(A_3) > s(A_1) > s(A_2)$. Hence, according to the above scoring function ranking method, it follows that $A_4 > A_3 > A_1 > A_2$. Thus, we have:

$$B_1 = A_4 = \langle 0.5\,, 0.4\,, 0.7\rangle$$
$$B_2 = A_3 = \langle 0.6\,, 0.4\,, 0.5\rangle$$
$$B_3 = A_1 = \langle 0.1, 0.2, 0.4\rangle$$
$$B_4 = A_2 = \langle 0.7\,, 0.2\,, 0.9\rangle$$

$$\psi_{oao}(B_1, B_2, \dots, B_n) = \left\langle \frac{\sum_{j=1}^{n} \xi_{B_j}^{w_j}}{1+\prod_{j=1}^{n} \xi_{B_j}^{w_j}}, \frac{\sum_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_i}}{2-\left[\sum_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_i} - \prod_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_i}\right]}, \frac{\sum_{j=1}^{n} \zeta_{B_j}^{w_i}}{2-\left[\sum_{j=1}^{n} \zeta_{B_j}^{w_i} - \prod_{j=1}^{n} \zeta_{B_j}^{w_i}\right]} \right\rangle$$

$$= \left\langle \frac{0.5^{0.1}+0.6^{0.4}+0.1^{0.3}+0.7^{0.2}}{1+0.5^{0.1}\times 0.6^{0.4}\times 0.1^{0.3}\times 0.7^{0.2}}, \frac{0.4^{0.1}+0.4^{0.4}+0.2^{0.3}+0.2^{0.2}}{2-[(0.4^{0.1}+0.4^{0.4}+0.2^{0.3}+0.2^{0.2})-(0.4^{0.1}\times 0.4^{0.4}\times 0.2^{0.3}\times 0.2^{0.2})]}, \right.$$
$$\left. \frac{0.7^{0.1}+0.5^{0.4}+0.4^{0.3}+0.9^{0.2}}{2-[(0.7^{0.1}+0.5^{0.4}+0.4^{0.3}+0.9^{0.2})-(0.7^{0.1}\times 0.5^{0.4}\times 0.4^{0.3}\times 0.9^{0.2})]} \right\rangle$$

$$= \langle 2.320, -4.931, -4.984\rangle$$

Definition 3.1.7 Let $A_j = \langle \xi_j, \varphi_j, \zeta_j\rangle \in \Lambda$ $(j \in I_n)$ $w = (w_1, w_2, \dots, w_n)^T$.Then $SVNS$ hybrid ordered weighted arithmetic operator denoted by ψ_{hao}, is defined as;

$$\psi_{hao} : \Lambda^n \to \Lambda,\ \psi_{hao}(A_1, A_2, \dots, A_n) = \sum_{j=1}^{n} w_j \bar{B}_j$$

where $w = (w_1, w_2, \dots, w_n)^T. w_k \in [0\,,1]and\sum_{j=1}^{n} w_j = 1$ is a weight vector associated with the mappingψ_{hgo}, $\dot{B}_k = nwA_k$ here n is regarded as a balance factor$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$is a weight vector of the $A_k \in \Lambda$ $(j \in I_n)\bar{B}_k$is the k-th largest of the n SNS $\dot{B}_k \in \Lambda$ $(j \in I_n)$ which are determined through using some ranking method such as the above scoring function ranking method.

Note that if $\omega = \left(1/n, 1/n, \dots, 1/n\right)^T$, then ψ_{hgo} degenerates to the ψ_{ogo}.

Theorem 3.1.8 Let $A_j = \langle \xi_j, \varphi_j, \zeta_j\rangle \in \Lambda$ $(j \in I_n)$ $w = (w_1, w_2, \dots, w_n)^T$be a weight vector of A_j with $w_j \in [0\,,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then their aggregated value by using ψ_{hoa} operator is also a $SVNS$ and $\psi_{hoa} : \Lambda^n \to \Lambda$

$$\psi_{hoa}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_n) = \langle \frac{\sum_{j=1}^{n} \bar{\xi}_{B_j}^{wj}}{1 + \prod_{j=1}^{n} \bar{\xi}_{B_j}^{wj}}, \frac{\sum_{j=1}^{n} \bar{\varphi}_{B_j}^{wj}}{2 - \left[\sum_{j=1}^{n} \bar{\varphi}_{B_j}^{wj} - \prod_{k=1}^{n} \bar{\varphi}_{B_j}^{wj}\right]}, \frac{\sum_{j=1}^{n} \bar{\zeta}_{B_j}^{wj}}{2 - \left[\sum_{j=1}^{n} \bar{\zeta}_{B_j}^{wj} - \prod_{j=1}^{n} \bar{\zeta}_{B_j}^{wj}\right]} \rangle$$

where $\bar{B}_j = \langle \bar{\xi}_j, \bar{\varphi}_j, \bar{\zeta}_j \rangle$ is the k-th largest of the n $SNS\ \dot{B}_j = nwA_j (j \in I_n)$ which is determined through using some ranking method such as the above scoring function ranking method.

Proof : Theorem 3.1.8 can be proven in a similar way to that of Theorem 3.1.2 (omitted).

Example 3.1.9 $A_1 = \langle 0.2, 0.1, 0.3 \rangle$, $A_2 = \langle 0.8, 0.5, 0.9 \rangle$, $A_3 = \langle 0.2, 0.4, 0.5 \rangle$ and $A_4 = \langle 0.3, 0.7, 0.2 \rangle$ $w = (0.5, 0.2, 0.1, 0.2)^T$ $\omega = (0.4, 0.1, 0.2, 0.3)$

$$\dot{B}_1 = 4 \times 0.4A_1 = \langle 1 - (1 - 0.2)^{4\times 0.4}, 0.1^{4\times 0.4}, 0.3^{4\times 0.4} \rangle = \langle 0.300, 0.025, 0.145 \rangle$$
$$\dot{B}_2 = 4 \times 0.1A_2 = \langle 1 - (1 - 0.8)^{4\times 0.1}, 0.5^{4\times 0.1}, 0.9^{4\times 0.1} \rangle = \langle 0.474, 0.757, 0.958 \rangle$$
$$\dot{B}_3 = 4 \times 0.2A_3 = \langle 1 - (1 - 0.2)^{4\times 0.2}, 0.4^{4\times 0.2}, 0.5^{4\times 0.2} \rangle = \langle 0.163, 0.480, 0.574 \rangle$$
$$\dot{B}_4 = 4 \times 0.3A_4 = \langle 1 - (1 - 0.3)^{4\times 0.3}, 0.7^{4\times 0.3}, 0.2^{4\times 0.3} \rangle = \langle 0.348, 0.651, 0.144 \rangle$$

we obtain the scores of the Simplifiedneutrosophicsets $\dot{B}_k (j = 1, 2, 3, 4)$ as follows:

$$s(A) = \frac{T_A + 1 - I_A + 1 - F_A}{3}$$

$$s(\dot{B}_1) = \frac{(0.300 + 1 - 0.025 + 1 - 0.145)}{3} = 0.709$$
$$s(\dot{B}_2) = \frac{(0.474 + 1 - 0.757 + 1 - 0.958)}{3} = 0.252$$
$$s(\dot{B}_3) = \frac{(0.163 + 1 - 0.480 + 1 - 0.574)}{3} = 0.369$$
$$s(\dot{B}_4) = \frac{(0.348 + 1 - 0.651 + 1 - 0.144)}{3} = 0.517$$

respectively. Obviously, $s(\dot{B}_1) > s(\dot{B}_4) > s(\dot{B}_3) > s(\dot{B}_2)$.Thereby, according to the above scoring function ranking method, we have:

$$\bar{B}_1 = \dot{B}_1 = \langle 0.300, 0.025, 0.145 \rangle$$
$$\bar{B}_2 = \dot{B}_4 = \langle 0.348, 0.651, 0.144 \rangle$$
$$\bar{B}_3 = \dot{B}_3 = \langle 0.163, 0.480, 0.574 \rangle$$
$$\bar{B}_4 = \dot{B}_2 = \langle 0.474, 0.757, 0.958 \rangle$$

$$\psi_{hoa}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_n) = \langle \frac{\sum_{j=1}^{n} \bar{\xi}_{B_j}^{wj}}{1 + \prod_{j=1}^{n} \bar{\xi}_{B_j}^{wj}}, \frac{\sum_{j=1}^{n} \bar{\varphi}_{B_j}^{wj}}{2 - \left[\sum_{j=1}^{n} \bar{\varphi}_{B_j}^{wj} - \prod_{k=1}^{n} \bar{\varphi}_{B_j}^{wj}\right]}, \frac{\sum_{j=1}^{n} \bar{\zeta}_{B_j}^{wj}}{2 - \left[\sum_{j=1}^{n} \bar{\zeta}_{B_j}^{wj} - \prod_{j=1}^{n} \bar{\zeta}_{B_j}^{wj}\right]} \rangle$$

$$\psi_{hoa}(\bar{B}_1, \bar{B}_2, \bar{B}_3, \bar{B}_4) = \langle \frac{0.300^{0.5} + 0.348^{0.2} + 0.163^{0.1} + 0.474^{0.2}}{1 + 0.300^{0.5} \times 0.348^{0.2} \times 0.163^{0.1} \times 0.474^{0.2}},$$

$$\frac{0.025^{0.5}+0.651^{0.2}+0.480^{0.1}+0.757^{0.2}}{2-[(0.025^{0.5}+0.651^{0.2}+0.480^{0.1}+0.757^{0.2})-(0.025^{0.5}\times 0.651^{0.2}\times 0.480^{0.1}\times 0.757^{0.2})]},$$
$$\frac{0.145^{0.5}+0.144^{0.2}+0.574^{0.1}+0.958^{0.2}}{2-[(0.145^{0.5}+0.144^{0.2}+0.574^{0.1}+0.958^{0.2})-(0.145^{0.5}\times 0.144^{0.2}\times 0.574^{0.1}\times 0.958^{0.2})]}\rangle$$

$=\langle 2.315,-3.582,-3.968\rangle$

3.2. Geometric operators of the $SVNS$

In this section, three $SVNS$ weighted geometric operator of $SVNS$ is called $SVNS$ weighted geometric operator, $SVNS$ ordered weighted geometric operator and $SVNS$hybrid ordered weighted geometric operator is given. Some of it is quoted from application in (He 2014a 2014b, 2014c, Xu and Yager 2006, Wei 2010).

Definition 3.2.1 Let$A_j=\langle \xi_j,\varphi_j,\zeta_j\rangle \in \Lambda$ $(j\in I_n)$.Then $SVNS$weighted geometric operator, denoted by $\mho_{go}$, is defined as;

$$\mho_{go}:\Lambda^n\to\Lambda,\mho_{go}(A_1,A_2,\ldots,A_n)=\prod_{j=1}^{n}A_j^{\,w_j}$$

where,$w=(w_1,w_2,\ldots,w_n)^T$ is a weight vector associated with the $\mho_{go}$operator, for every$j\in I_n$such that, $w_j\in[0,1]$and $\sum_{j=1}^{n}w_j=1$.

Theorem 3.2.2 Let $A_j=\langle \xi_j,\varphi_j,\zeta_j\rangle \in \Lambda$ $(j\in I_n)$ $(w_1,w_2,\ldots,w_n)^T$be a weight vectorof A_j, for every$j\in I_n$In such that $w_j\in[0,1]$and$\sum_{i=1}^{n}w_j=1$.Then, their aggregated value by using $\mho_{go}$operator is also a $SVNS$ and

$$\mho_{go}(A_1,A_2,\ldots,A_n)=\langle\frac{\sum_{j=1}^{n}\xi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{n}\xi_{A_j}^{w_j}-\prod_{j=1}^{n}\xi_{A_j}^{w_j}\right]},\frac{\sum_{j=1}^{n}\varphi_{A_j}^{w_j}}{1+\prod_{j=1}^{n}\varphi_{A_j}^{w_j}},\frac{\sum_{j=1}^{n}\zeta_{A_j}^{w_j}}{1+\prod_{j=1}^{n}\zeta_{A_j}^{w_j}}\rangle$$

Proof: The proof can be made by using mathematical induction on n as; assume that,

$A_1=\langle \xi_1,\varphi_1,\zeta_1\rangle$and$A_2=\langle \xi_2,\varphi_2,\zeta_2\rangle$be two SNS then,

for$n=2$, we have

$$\mho_{go}(A_1,A_2)=\langle\frac{\sum_{j=1}^{2}\xi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{2}\xi_{A_j}^{w_j}-\prod_{j=1}^{2}\xi_{A_j}^{w_j}\right]},\frac{\sum_{j=1}^{2}\varphi_{A_j}^{w_j}}{1+\prod_{j=1}^{2}\varphi_{A_j}^{w_j}},\frac{\sum_{j=1}^{2}\zeta_{A_j}^{w_j}}{1+\prod_{j=1}^{2}\zeta_{A_j}^{w_j}}\rangle$$

$$=\frac{\xi_{A_1}^{w_1}+\xi_{A_2}^{w_2}}{2-\left[\xi_{A_1}^{w_1}+\xi_{A_2}^{w_2}-\xi_{A_1}^{w_1}.\xi_{A_2}^{w_2}\right]},\frac{\varphi_{A_1}^{w_1}+\varphi_{A_2}^{w_2}}{1+\varphi_{A_1}^{w_1}.\varphi_{A_2}^{w_2}},\frac{\zeta_{A_1}^{w_1}+\zeta_{A_2}^{w_2}}{1+\zeta_{A_1}^{w_1}.\zeta_{A_2}^{w_2}}$$

If holds for $n=k$, that is

$$\mho_{go}(A_1, A_2, \dots, A_k) = \langle \frac{\sum_{j=1}^{k} \xi_{A_j}^{w_j}}{2 - \left[\sum_{j=1}^{k} \xi_{A_j}^{w_j} - \prod_{j=1}^{k} \xi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{k} \varphi_{A_j}^{w_j}}{1 + \prod_{j=1}^{k} \varphi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{k} \zeta_{A_j}^{w_j}}{1 + \prod_{i=1}^{k} \zeta_{A_j}^{w_j}} \rangle$$

then, when $n = k + 2$; by the operational laws in Definition 2.9, I have

$$\mho_{go}(A_1, A_2, \dots, A_k, A_{k+1}, A_{k+2}) = \langle \frac{\left(\sum_{j=1}^{k} \xi_{A_j}^{w_j}\right) + \xi_{A_{k+1}}^{w_{k+1}} + \xi_{A_{k+2}}^{w_{k+2}}}{2 - \left[\left(\sum_{j=1}^{k} \xi_{A_j}^{w_j}\right) + \xi_{A_{k+1}}^{w_{k+1}} + \xi_{A_{k+2}}^{w_{k+2}} - \left(\prod_{j=1}^{k} \xi_{A_j}^{w_j}\right) \times \xi_{A_{k+1}}^{w_{k+1}} \times \xi_{A_{k+2}}^{w_{k+2}}\right]},$$

$$\frac{\left(\sum_{j=1}^{k} \varphi_{A_j}^{w_j}\right) + \varphi_{A_{k+1}}^{w_{k+1}} + \varphi_{A_{k+2}}^{w_{k+2}}}{1 + \left(\prod_{j=1}^{k} \varphi_{A_j}^{w_j}\right) + \varphi_{A_{k+1}}^{w_{k+1}} + \varphi_{A_{k+2}}^{w_{k+2}}}, \frac{\left(\sum_{j=1}^{k} \zeta_{A_j}^{w_j}\right) + \zeta_{A_{k+1}}^{w_{k+1}} + \zeta_{A_{k+2}}^{w_{k+2}}}{1 + \left(\prod_{j=1}^{k} \zeta_{A_j}^{w_j}\right) + \zeta_{A_{k+1}}^{w_{k+1}} + \zeta_{A_{k+2}}^{w_{k+2}}} \rangle$$

$$\mho_{go}(A_1, A_2, \dots, A_k, A_{k+1}, A_{k+2}) = \langle \frac{\sum_{j=1}^{k+2} \xi_{A_j}^{w_j}}{2 - \left[\sum_{j=1}^{k+2} \xi_{A_j}^{w_j} - \prod_{j=1}^{k+2} \xi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{k+2} \varphi_{A_j}^{w_j}}{1 + \prod_{j=1}^{k+2} \varphi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{k+2} \zeta_{A_j}^{w_j}}{1 + \prod_{j=1}^{k+2} \zeta_{A_j}^{w_j}} \rangle$$

Example3.2.3 $A_1 = \langle 0.4, 0.5, 0.6 \rangle$, $A_2 = \langle 0.8, 0.9, 0.3 \rangle$, $A_3 = \langle 0.8, 0.5, 0.3 \rangle$ and $A_4 = \langle 0.7, 0.6, 0.5 \rangle w = (0.1, 0.5, 0.2, 0.2)^T$

$$\mho_{go}(A_1, A_2, \dots, A_n) = \langle \frac{\sum_{j=1}^{n} \xi_{A_j}^{w_j}}{2 - \left[\sum_{j=1}^{n} \xi_{A_j}^{w_j} - \prod_{j=1}^{n} \xi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \varphi_{A_j}^{w_j}}{1 + \prod_{j=1}^{n} \varphi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{n} \zeta_{A_j}^{w_j}}{1 + \prod_{j=1}^{n} \zeta_{A_j}^{w_j}} \rangle$$

$$= \langle \frac{0.4^{0.1} + 0.8^{0.5} + 0.8^{0.2} + 0.7^{0.2}}{2 - [(0.4^{0.1} + 0.8^{0.5} + 0.8^{0.2} + 0.7^{0.2}) - (0.4^{0.1} \times 0.8^{0.5} \times 0.8^{0.2} \times 0.7^{0.2})]},$$

$$\frac{0.5^{0.1} + 0.9^{0.5} + 0.5^{0.2} + 0.6^{0.2}}{1 + 0.5^{0.1} \times 0.9^{0.5} \times 0.5^{0.2} \times 0.6^{0.2}}, \frac{0.6^{0.1} + 0.3^{0.5} + 0.3^{0.2} + 0.5^{0.2}}{1 + 0.6^{0.1} \times 0.3^{0.5} \times 0.3^{0.2} \times 0.5^{0.2}} \rangle$$

$$= \langle -3.818, 2.155, 2.326 \rangle$$

Definition 3.2.4 Let $A_j = \langle \xi_j, \varphi_j, \zeta_j \rangle \in \Lambda$ $(j \in I_n)$ $w = (w_1, w_2, \dots, w_n)^T$.Then $SVNS$ ordered weighted geometric operator denoted by $\mho_{ogo}$, is defined as;

$$\mho_{ogo} : \Lambda^n \to \Lambda, \mho_{ogo}(A_1, A_2, \dots, A_n) = \prod_{j=1}^{n} B_j^{w_j}$$

where$(w_1, w_2, \dots, w_n)^T$is a weight vector associated with the mapping $\mho_{ogo}$; which satisfies the normalized conditions:$w_k \in [0, 1]$ and $\sum_{k=1}^{n} w_k = 1$; $B_k = \langle \tilde{\xi}_j, \tilde{\varphi}_j, \tilde{\zeta}_j \rangle$ is the k-th largest of the n$SVNS$which is determined through using ranking method in Definition 2.7.

It is not difficult to follows from Definition 3.2.4 that

$$\mho_{ogo}(B_1, B_2, \dots, B_n) = \langle \frac{\sum_{j=1}^{n} \xi_{B_j}^{w_j}}{2 - \left[\sum_{j=1}^{n} \xi_{B_j}^{w_j} - \prod_{j=1}^{n} \xi_{B_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_j}}{1 + \prod_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_j}}, \frac{\sum_{j=1}^{n} \zeta_{B_j}^{w_j}}{1 + \prod_{j=1}^{n} \zeta_{B_j}^{w_j}} \rangle$$

which is summarized as in Theorem 3.2.5.

Theorem 3.2.5 Let$A_j = \langle \xi_j, \varphi_j, \zeta_j \rangle \in \Lambda$ $(j \in I_n)$. Then $SVNS$ orderedweighted geometric operator denoted by$\mho_{ogo}$, is defined as; $\mho_{ogo} : \Lambda^n \rightarrow \Lambda$

$$\mho_{ogo}(B_1, B_2, \dots, B_n) = \langle \frac{\sum_{j=1}^{n} \xi_{B_j}^{w_j}}{2 - \left[\sum_{j=1}^{n} \xi_{B_j}^{w_j} - \prod_{j=1}^{n} \xi_{B_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_j}}{1 + \prod_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_j}}, \frac{\sum_{j=1}^{n} \zeta_{B_j}^{w_j}}{1 + \prod_{j=1}^{n} \zeta_{B_j}^{w_j}} \rangle$$

where$w_k \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$ $B_j = \langle \tilde{\xi}_j, \tilde{\varphi}_j, \tilde{\zeta}_j \rangle$is the k-th largest of then $SVNS$ A_j $(j \in I_n)$which is determined through using ranking method in Definition 2.7.

Proof: Theorem 3.2.5 can be proven in a similar way to that of Theorem 3.2.2 (omitted).

Example3.2.6$A_1 = \langle 0.2, 0.4, 0.3 \rangle$, $A_2 = \langle 0.7, 0.2, 0.9 \rangle$, $A_3 = \langle 0.5, 0.4, 0.7 \rangle$ and $A_4 = \langle 0.6, 0.4, 0.5 \rangle$ $w = (0.6, 0.1, 0.1, 0.2)^T$

$$s(A) = \frac{T_A + 1 - I_A + 1 - F_A}{3}$$
$$s(A_1) = \frac{(0.2 + 1 - 0.4 + 1 - 0.3)}{3} = 0.5$$
$$s(A_2) = \frac{(0.7 + 1 - 0.2 + 1 - 0.9)}{3} = 0.533$$
$$s(A_3) = \frac{(0.5 + 1 - 0.4 + 1 - 0.7)}{3} = 0.466$$
$$s(A_4) = \frac{(0.6 + 1 - 0.4 + 1 - 0.5)}{3} = 0.566$$

It is obvious that$s(A_4) > s(A_2) > s(A_1) > s(A_3)$. Hence, according to the above scoring function ranking method, it follows that $A_4 > A_2 > A_1 > A_3$. Thus, we have:

$B_1 = A_3 = \langle 0.5, 0.4, 0.7 \rangle$
$B_2 = A_4 = \langle 0.6, 0.4, 0.5 \rangle$
$B_3 = A_1 = \langle 0.2, 0.4, 0.3 \rangle$
$B_4 = A_2 = \langle 0.7, 0.2, 0.9 \rangle$

$$\mho_{ogo}(B_1, B_2, \dots, B_n) = \langle \frac{\sum_{j=1}^{n} \xi_{B_j}^{w_j}}{2 - \left[\sum_{j=1}^{n} \xi_{B_j}^{w_j} - \prod_{j=1}^{n} \xi_{B_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_j}}{1 + \prod_{j=1}^{n} \tilde{\varphi}_{B_j}^{w_j}}, \frac{\sum_{j=1}^{n} \zeta_{B_j}^{w_j}}{1 + \prod_{j=1}^{n} \zeta_{B_j}^{w_j}} \rangle$$

$$= \langle \frac{0.5^{0.6} + 0.6^{0.1} + 0.2^{0.1} + 0.7^{0.2}}{2 - [(0.5^{0.6} + 0.6^{0.1} + 0.2^{0.1} + 0.7^{0.2}) - (0.5^{0.6} \times 0.6^{0.1} \times 0.2^{0.1} \times 0.7^{0.2})]},$$

$$\frac{0.4^{0.6}+0.4^{0.1}+0.4^{0.1}+0.2^{0.2}}{1+0.4^{0.6}\times 0.4^{0.1}\times 0.4^{0.1}\times 0.2^{0.2}},\frac{0.7^{0.6}+0.5^{0.1}+0.3^{0.1}+0.9^{0.2}}{1+0.7^{0.6}\times 0.5^{0.1}\times 0.3^{0.1}\times 0.9^{0.2}}\rangle$$

$$=\langle -3.818, 2.310, 2.252\rangle$$

Definition 3.2.7 Let $A_j = \langle \xi_j, \varphi_j, \zeta_j\rangle \in \Lambda (j\in I_n) w = (w_1, w_2, \dots, w_n)^T$.Then$SVNS$hybrid ordered weighted geometric operator denoted by $\mho_{hgo}$, is defined as;

$$\mho_{hog} : \Lambda^n \rightarrow \Lambda\ \mho_{hog}(A_1, A_2, \dots, A_n) = \prod_{j=1}^{n} \bar{B}_j^{\,w_j}$$

where $w = (w_1, w_2, \dots, w_n)^T . w_j \in [0,1]and\sum_{j=1}^{n} w_j = 1$ is a weight vector associated with the mapping$\mho_{hog}$, $\dot{B}_j = nwA_j$ here n is regarded as a balance factor$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$is a weight vector of the$A_j \in \Lambda\ (j \in I_n) \bar{B}_j$is the k-th largest of the n$SVNS\ \dot{B}_j \in \Lambda\ (j \in I_n)$which are determined through using some ranking method such as the above scoring function ranking method.

Note that if $\omega = \left(1/n, 1/n, \dots, 1/n\right)^T$, then $\mho_{hog}$ degenerates to the $\mho_{hog}$:

Theorem 3.2.8 Let$A_j = \langle \xi_j, \varphi_j, \zeta_j\rangle \in \Lambda (j\in I_n) w = (w_1, w_2, \dots, w_n)^T$be a weight vector of$A_j$with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then their aggregated value by using $\mho_{hog}$ operator is also a $SVNS$ and $\mho_{hog} : \Lambda^n \rightarrow \Lambda$

$$\mho_{hog}(\bar{B}_1, \bar{B}_2, \dots, \bar{B}_n) = \langle \frac{\sum_{j=1}^{n} \xi_{\bar{B}_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \xi_{\bar{B}_j}^{w_j} - \prod_{j=1}^{n} \xi_{\bar{B}_j}^{w_j}\right]}, \frac{\sum_{k=1}^{n} \varphi_{\bar{B}_j}^{w_j}}{1+\prod_{k=1}^{n} \varphi_{\bar{B}_j}^{w_j}}, \frac{\sum_{k=1}^{n} \zeta_{\bar{B}_j}^{w_j}}{1+\prod_{k=1}^{n} \zeta_{\bar{B}_j}^{w_j}}\rangle$$

where$\bar{B}_j = \langle \bar{\xi}_j, \bar{\varphi}_j, \bar{\zeta}_j\rangle$ is the k-th largest of the n $SVNS\ \dot{B}_j = nwA_j (j \in I_n)$ which is determined through using some ranking method such as the above scoring function ranking method.

Proof : Theorem 3.2.8 can be proven in a similar way to that of Theorem 3.2.2 (omitted).

Example 3.2.9 $A_1 = \langle 0.2, 0.7, 0.6\rangle$, $A_2 = \langle 0.3, 0.5, 0.8\rangle$, $A_3 = \langle 0.2, 0.9, 0.6\rangle$and $A_4 = \langle 0.2, 0.2, 0.4\rangle$ $w = (0.1, 0.5, 0.1, 0.3)^T$ $\omega = (0.4, 0.2, 0.2, 0.2)$

$$\dot{B}_1 = 4\times 0.4A_1 = \langle 1-(1-0.2)^{4\times 0.4}, 0.7^{4\times 0.4}, 0.6^{4\times 0.4}\rangle = \langle 0.300, 0.565, 0.441\rangle$$
$$\dot{B}_2 = 4\times 0.2A_2 = \langle 1-(1-0.3)^{4\times 0.2}, 0.5^{4\times 0.2}, 0.8^{4\times 0.2}\rangle = \langle 0.248, 0.574, 0.836\rangle$$
$$\dot{B}_3 = 4\times 0.2A_3 = \langle 1-(1-0.2)^{4\times 0.2}, 0.9^{4\times 0.2}, 0.6^{4\times 0.2}\rangle = \langle 0.163, 0.919, 0.664\rangle$$
$$\dot{B}_4 = 4\times 0.2A_4 = \langle 1-(1-0.2)^{4\times 0.2}, 0.2^{4\times 0.2}, 0.4^{4\times 0.2}\rangle = \langle 0.163, 0.275, 0.480\rangle$$

we obtain the scores of the single-valuedneutrosophicsets$\dot{B}_k (j = 1, 2, 3, 4\ )$as follows:

$$s(A) = (T_A + 1 - I_A + 1 - F_A)/3$$

$$s(\dot{B}_1)=\frac{(0.300+1-0.565+1-0.441)}{3}=0.431$$
$$s(\dot{B}_2)=\frac{(0.248+1-0.574+1-0.836)}{3}=0.279$$
$$s(\dot{B}_3)=\frac{(0.163+1-0.919+1-0.664)}{3}=0.193$$
$$s(\dot{B}_4)=\frac{(0.163+1-0.275+1-0.480)}{3}=0.469$$

respectively. Obviously, $s(\dot{B}_4)>s(\dot{B}_1)>s(\dot{B}_2)>s(\dot{B}_3)$.Thereby, according to the above scoring function ranking method, we have:

$$\bar{B}_1=\dot{B}_4=\langle 0.163, 0.275, 0.480\rangle$$
$$\bar{B}_2=\dot{B}_1=\langle 0.300, 0.565, 0.441\rangle$$
$$\bar{B}_3=\dot{B}_2=\langle 0.248, 0.574, 0.836\rangle$$
$$\bar{B}_4=\dot{B}_3=\langle 0.163, 0.919, 0.664\rangle$$

$$\mho_{hog}(\bar{B}_1,\bar{B}_2,\dots,\bar{B}_n)=\langle\frac{\sum_{j=1}^{n}\xi_{\bar{B}_j}^{w_j}}{2-\left[\sum_{j=1}^{n}\xi_{\bar{B}_j}^{w_j}-\prod_{j=1}^{n}\xi_{\bar{B}_j}^{w_j}\right]},\frac{\sum_{k=1}^{n}\varphi_{\bar{B}_j}^{w_j}}{1+\prod_{k=1}^{n}\varphi_{\bar{B}_j}^{w_j}},\frac{\sum_{k=1}^{n}\zeta_{\bar{B}_j}^{w_j}}{1+\prod_{k=1}^{n}\zeta_{\bar{B}_j}^{w_j}}\rangle$$

$$\mho_{hog}(\bar{B}_1,\bar{B}_2,\bar{B}_3,\bar{B}_4)$$
$$=\langle\frac{0.163^{0.1}+0.300^{0.5}+0.248^{0.1}+0.163^{0.3}}{2-[(0.163^{0.1}+0.300^{0.5}+0.248^{0.1}+0.163^{0.3})-(0.163^{0.1}\times0.300^{0.5}\times0.248^{0.1}\times0.163^{0.3})]},$$
$$\frac{0.275^{0.1}+0.565^{0.5}+0.574^{0.1}+0.919^{0.3}}{1+0.275^{0.1}\times0.565^{0.5}\times0.574^{0.1}\times0.919^{0.3}},\frac{0.480^{0.1}+0.441^{0.5}+0.836^{0.1}+0.664^{0.3}}{1+0.480^{0.1}\times0.441^{0.5}\times0.836^{0.1}\times0.664^{0.3}}\rangle$$
$$=\langle -4.705, 2.206, 2.252\rangle$$

Section4. Single valued neutrosophic sets and their applications in multi-criteria groupdecision-making problems

There is a panel with four possible alternatives to invest the money (adapted from Herrera 2000): (1) $x_1$is a car company; (2) $x_2$is a food company; (3) $x_3$is a computer company; (4) $x_4$is a television company. TheinvestmentCompany must take a decision according to the following three criteria: (1) $u_1$is the risk analysis;(2) $u_2$is the growth analysis; (3) $u_3$is the environmental impact analysis; (4) $u_4$social political impactanalysis. The four possible alternatives are to be evaluated under the above three criteria by correspondingto linguistic values of SVNSs for linguistic terms (adapted from Ye 2011), as shown in Table 1.

$$\begin{array}{c} \\ x_1\\ x_2\\ x_3\\ x_4\end{array}\begin{array}{c}\begin{array}{cccc} u_1 & u_2 & u_3 & u_4\end{array}\\ \begin{pmatrix}\langle 0.3,0.2,0.5\rangle & \langle 0.1,0.5,0.7\rangle & \langle 0.7,0.1,0.8\rangle & \langle 0.4,0.2,0.5\rangle\\ \langle 0.2,0.7,0.6\rangle & \langle 0.3,0.5,0.8\rangle & \langle 0.2,0.9,0.6\rangle & \langle 0.2,0.2,0.4\rangle\\ \langle 0.4,0.5,0.6\rangle & \langle 0.8,0.9,0.3\rangle & \langle 0.8,0.5,0.3\rangle & \langle 0.7,0.6,0.5\rangle\\ \langle 0.2,0.4,0.3\rangle & \langle 0.7,0.2,0.9\rangle & \langle 0.5,0.4,0.7\rangle & \langle 0.6,0.4,0.5\rangle\end{pmatrix}\end{array}$$

Definition 4.1 :Let $X = (x_1, x_2, \ldots, x_m)$ be a set of alternatives, $U = (u_1, u_2, \ldots, u_n)$be the set of attributes. If $a_{ij} = \langle \xi_{ij}, \ \varphi_{ij}, \ \zeta_{ij} \rangle \in \Lambda$, then

$$[a_{ij}]_{m\times n} = \begin{matrix} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \begin{matrix} u_1 & u_2 & \cdots & u_n \\ \left(\begin{matrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{matrix} \right. & \begin{matrix} a_{12} \\ a_{22} \\ \vdots \\ a_{n2} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \vdots \\ \cdots \end{matrix} & \left. \begin{matrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{nn} \end{matrix} \right) \end{matrix}$$

is called an SVNS-multi-criteria decision-making matrix of the decision maker.

Now, we can give an algorithm of the $SVNSs$-multi-criteria decision-making method as follows;

Algorithm:

Step 1. Construct the decision-making matrix $[a_{ij}]_{m\times n}$ for decision;

Step 2. Compute the $SVNS$s $\psi_{ao}(A_1, A_2, \ldots, A_n) = \langle \frac{\sum_{j=1}^{n} \xi_{A_j}^{w_j}}{1+\prod_{j=1}^{n} \xi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{n} \varphi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \varphi_{A_j}^{w_j} - \prod_{j=1}^{n} \varphi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \zeta_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \zeta_{A_j}^{w_j} - \prod_{j=1}^{n} \zeta_{A_j}^{w_j}\right]} \rangle$and write the decision-making matrix$[A_{ij}]_{m\times n}$; and obtain the scores of the $SVNSs$ $\psi_{ao}(A_1, A_2, \ldots, A_n)$;

Step 3. Compute the $SVNS$s$\psi_{oao}(B_1, B_2, \ldots, B_n) = \langle \frac{\sum_{j=1}^{n} \xi_{B_j}^{w_j}}{1+\prod_{j=1}^{n} \xi_{B_j}^{w_j}}, \frac{\sum_{j=1}^{n} \varphi_{B_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \varphi_{B_j}^{w_j} - \prod_{j=1}^{n} \varphi_{B_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \zeta_{B_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \zeta_{B_j}^{w_j} - \prod_{j=1}^{n} \zeta_{B_j}^{w_j}\right]} \rangle$and write the decision-making matrix$[A_{ij}]_{m\times n}$; and obtain the scores of the $SVNS$s $\psi_{oao}(B_1, B_2, \ldots, B_n)$;

Step 4. Compute the $SVNS$s$\psi_{hoa}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_n) = \langle \frac{\sum_{j=1}^{n} \xi_{\bar{B}_j}^{w_j}}{1+\prod_{j=1}^{n} \xi_{\bar{B}_j}^{w_j}}, \frac{\sum_{j=1}^{n} \varphi_{\bar{B}_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \varphi_{\bar{B}_j}^{w_j} - \prod_{k=1}^{n} \varphi_{\bar{B}_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \zeta_{\bar{B}_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \zeta_{\bar{B}_j}^{w_j} - \prod_{k=1}^{n} \zeta_{\bar{B}_j}^{w_j}\right]} \rangle$and write the decision-making matrix$[A_{ij}]_{m\times n}$; and obtain the scores of the $SVNS$s$\psi_{hoa}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_n)$;

Step 5. Compute the $SVNS$s$\mho_{go}(A_1, A_2, \ldots, A_n) = \langle \frac{\sum_{j=1}^{n} \xi_{A_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \xi_{A_j}^{w_j} - \prod_{j=1}^{n} \xi_{A_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \varphi_{A_j}^{w_j}}{1+\prod_{j=1}^{n} \varphi_{A_j}^{w_j}}, \frac{\sum_{j=1}^{n} \zeta_{A_j}^{w_j}}{1+\prod_{j=1}^{n} \zeta_{A_j}^{w_j}} \rangle$then, write the decision-making matrix$[A_{ij}]_{m\times n}$; and obtain the scores of the $SVNS$s$\mho_{go}(A_1, A_2, \ldots, A_n)$;

Step 6. Compute the $SVNS$s $\mho_{ogo}(B_1, B_2, \ldots, B_n) = \langle \frac{\sum_{j=1}^{n} \xi_{B_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \xi_{B_j}^{w_j} - \prod_{j=1}^{n} \xi_{B_j}^{w_j}\right]}, \frac{\sum_{j=1}^{n} \varphi_{B_j}^{w_j}}{1+\prod_{j=1}^{n} \varphi_{B_j}^{w_j}}, \frac{\sum_{j=1}^{n} \zeta_{B_j}^{w_j}}{1+\prod_{j=1}^{n} \zeta_{B_j}^{w_j}} \rangle$

then, write the decision-making matrix$[A_{ij}]_{m\times n}$; and obtain the scores of the $SVNS$s$\mho_{ogo}(B_1, B_2, \ldots, B_n)$;

Step 7. Compute the $SVNS$s$\mho_{hog}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_n) = \langle \frac{\sum_{j=1}^{n} \xi_{\bar{B}_j}^{w_j}}{2-\left[\sum_{j=1}^{n} \xi_{\bar{B}_j}^{w_j} - \prod_{j=1}^{n} \xi_{\bar{B}_j}^{w_j}\right]}, \frac{\sum_{k=1}^{n} \varphi_{\bar{B}_j}^{w_j}}{1+\prod_{k=1}^{n} \varphi_{\bar{B}_j}^{w_j}}, \frac{\sum_{k=1}^{n} \zeta_{\bar{B}_j}^{w_j}}{1+\prod_{k=1}^{n} \zeta_{\bar{B}_j}^{w_j}} \rangle$

then, write the decision-making matrix$[A_{ij}]_{m\times n}$; and obtain the scores of the $SVNS$s $\mho_{hog}(\bar{B}_1, \bar{B}_2, \ldots, \bar{B}_n)$;

Step 8. Rank all alternatives x_i by using the ranking method of $SVNS$s and determine the best alternative.

Section5. Application

In this section, we give an application for the $SVNS$s -multi-criteria decision-making method, by using the G_{hgo}operator. Some of it is quoted from application in (Deli 2015, Herrera 2000, Ye 2011).

Example 5.1 Let us consider the decision-making problem adapted from (Ye 2015). There is an investment company, which wants to invest a sum of money in the best option. There is a panel with the set of the four alternatives is denoted by $X = \{x_1 = \text{ car company}, x_2 = \text{food company}, x_3 = \text{computer company }, x_4 = \text{television company }\}$ to invest the money. The investment company must take a decision according to the set of the four attributes is denotedby$U$=$\{u_1$=risk analysis, u_2=growth analysis, u_3=environmental impact analysis, u_4= social political impact analysis$\}$.Then, the weight vector of the attributes is $\omega = (0.1, 0.2, 0.3, 0.4)^T$and the position weight vector is $w = (0.3, 0.4, 0.1, 0.2)^T$by using the weight determination based on the normal distribution. For the evaluation of an alternative x_i $(i = 1,2,3,4)$ with respect to a criterion u_j $(j = 1,2,3,4)$, it is obtained from the questionnaire of a domain expert. Then,

Step 1. The decision maker construct the decision matrix $[a_{ij}]_{4\times 4}$as follows:

$$[a_{ij}]_{m\times n} = \begin{matrix} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} \begin{matrix} u_1 & u_2 & u_3 & u_4 \\ \end{matrix} \begin{pmatrix} 0.3\ 0.2\ 0.5 & 0.1\ 0.5\ 0.7 & 0.7\ 0.1\ 0.8 & 0.4\ 0.2\ 0.5 \\ 0.2\ 0.7\ 0.6 & 0.3\ 0.5\ 0.8 & 0.2\ 0.9\ 0.6 & 0.2\ 0.2\ 0.4 \\ 0.4\ 0.5\ 0.6 & 0.8\ 0.9\ 0.3 & 0.8\ 0.5\ 0.3 & 0.7\ 0.6\ 0.5 \\ 0.2\ 0.4\ 0.3 & 0.7\ 0.2\ 0.9 & 0.5\ 0.4\ 0.7 & 0.6\ 0.4\ 0.5 \end{pmatrix}$$

Step 2. The values of$\psi_{oao}(A_1, A_2, \dots, A_n)$are compute with the help of single-valued neutrosophic weighted arithmetic operator.

$$[A_{ij}]_{4\times 4} = \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} \begin{pmatrix} 2.365 & -4.632 & -3.800 \\ 2.275 & -3.820 & -3.804 \\ 2.194 & -3.793 & -3.924 \\ 2.288 & -4.168 & -3.754 \end{pmatrix}$$

The score function values of x_1, x_2, x_3 and x_4 are calculated.

$$s(A) = \frac{T_A + 1 - I_A + 1 - F_A}{3}$$

$$s(x_1) = \frac{2.365 + 1 - (-4.632) + 1 - (-3.800)}{3} = 4.265$$

$$s(x_2) = \frac{2.275 + 1 - (-3.820) + 1 - (-3.804)}{3} = 3.966$$
$$s(x_3) = \frac{2.194 + 1 - (-3.793) + 1 - (-3.924)}{3} = 3.970$$
$$s(x_4) = \frac{2.288 + 1 - (-4.168) + 1 - (-3.754)}{3} = 4.070$$

$$s(x_1) > s(x_4) > s(x_3) > s(x_2)$$

Step 3.The values of $\psi_{oao}(A_1, A_2, \ldots, A_n)$ are compute with the help of single-valued neutrosophic ordered weighted arithmetic operator.

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} \begin{pmatrix} 2.305 & -4.967 & -3.778 \\ 2.275 & -3.783 & -3.787 \\ 2.176 & -3.802 & -3.949 \\ 2.221 & -4.168 & -3.794 \end{pmatrix}$$

The score function values of x_1, x_2, x_3 *and* x_4 are calculated.

$$s(x_1) = 4.350$$
$$s(x_2) = 3.949$$
$$s(x_3) = 3.976$$
$$s(x_4) = 4.061$$

$$s(x_1) > s(x_4) > s(x_3) > s(x_2)$$

Step 4. The values of $\psi_{oao}(A_1, A_2, \ldots, A_n)$ are compute with the help ofthe Single-valued neutrosophic sets hybrid ordered weighted arithmetic operator(SVNSHOWA).

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} \begin{pmatrix} 2.301 & -5.279 & -3.754 \\ 2.267 & -3.715 & -3.720 \\ 2.208 & -3.774 & -3.970 \\ 2.299 & -4.086 & -3.779 \end{pmatrix}$$

The score function values of x_1, x_2, x_3 *and* x_4 are calculated.

$$s(x_1) = 4.445$$
$$s(x_2) = 3.901$$
$$s(x_3) = 3.984$$
$$s(x_4) = 4.055$$

$$s(x_1) > s(x_4) > s(x_3) > s(x_2)$$

Step 5. The values of $\mho_{ogo}(A_1, A_2, \ldots, A_n)$are compute with the help of single-valued neutrosophic weighted geometric operator.

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}\begin{pmatrix} -4.319 & 2.280 & 2.205 \\ -4.882 & 2.264 & 2.193 \\ -3.769 & 2.178 & 2.294 \\ -3.819 & 2.325 & 2.236 \end{pmatrix}$$

The score function values of x_1, x_2, x_3 *and* x_4 are calculated.

$$s(x_1) = -2.268$$
$$s(x_2) = -2.446$$
$$s(x_3) = -2.080$$
$$s(x_4) = -2.126$$

$$s(x_3) > s(x_4) > s(x_1) > s(x_2)$$

Step 6. The values of $U_{ogo}(A_1, A_2, \dots, A_n)$ are compute with the help of single-valued neutrosophic ordered weighted geometric operator.

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}\begin{pmatrix} -4.086 & 2.299 & 2.201 \\ -4.882 & 2.301 & 2.210 \\ -3.792 & 2.230 & 2.280 \\ -3.848 & 2.325 & 2.184 \end{pmatrix}$$

The score function values of x_1, x_2, x_3 *and* x_4 are calculated.

$$s(x_1) = -2.195$$
$$s(x_2) = -2.464$$
$$s(x_3) = -2.097$$
$$s(x_4) = -2.119$$

$$s(x_3) > s(x_4) > s(x_1) > s(x_2)$$

Step 7. The values of$U_{ogo}(A_1, A_2, \dots, A_n)$are compute with the help of single-valued neutrosophic hybrid ordered weighted geometric operator.

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}\begin{pmatrix} -4.014 & 2.326 & 2.219 \\ -5.132 & 2.377 & 2.254 \\ -3.739 & 2.251 & 2.317 \\ -3.837 & 2.312 & 2.240 \end{pmatrix}$$

The score function values of x_1, x_2, x_3 *and* x_4 are calculated.

$$s(x_1) = -2.187$$
$$s(x_2) = -2.588$$
$$s(x_3) = -2.102$$
$$s(x_4) = -2.130$$

$$s(x_3) > s(x_4) > s(x_1) > s(x_2)$$

Step 8.

Operators	
$\psi_{ao}(A_1, A_2, A_3, A_4)$	$s(x_1) > s(x_4) > s(x_3) > s(x_2)$
$\psi_{oao}(A_1, A_2, A_3, A_4)$	$s(x_1) > s(x_4) > s(x_3) > s(x_2)$
$\psi_{hoa}(A_1, A_2, A_3, A_4)$	$s(x_1) > s(x_3) > s(x_2) > s(x_4)$
$\mho_{go}(A_1, A_2, A_3, A_4)$	$s(x_3) > s(x_4) > s(x_1) > s(x_2)$
$\mho_{ogo}(A_1, A_2, A_3, A_4)$	$s(x_3) > s(x_4) > s(x_1) > s(x_2)$
$\mho_{hog}(A_1, A_2, A_3, A_4)$	$s(x_3) > s(x_1) > s(x_4) > s(x_2)$

Section 6. FUTURE RESEARCH DIRECTIONS

In this paper, this article introduces an approach to handle multi-criteria decision making (MCDM) problems under the SVNSs. Using this concept we can extend our work in (1) More effective approaches for SVNSs (2) How to determine the weight vectors for SVNSs (3) An approach of multi-criteria decision making with weight expressed by SVNSs.

Section 7. Conclusion

This paper proposes six operator are called the single valued neutrosophic weighted geometric (SVNWG) operator, the single valued neutrosophic ordered weighted geometric (SVNOWG) operator, the single-valued neutrosophic sets hybrid ordered weighted arithmetic (SVNSHOWA) operator, the single-valued neutrosophic weighted geometric (SVNWG) operator , the single-valued neutrosophic ordered weighted geometric(SVNOWG) operator and the single-valued neutrosophic hybrid ordered weighted geometric (SVNHOG)operator. Then an approach is developed to solve more general multi-criteria decision making problems as straightforward manner.

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