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Strong Interval-valued Neutrosophic Intuitionistic Fuzzy Graph

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Abstract

In this paper, the strong interval-valued neutrosophic intuitionistic fuzzy graphs are suggest. Cartesian product, composition and clamp of two strong interval-valued neutrosophic intuitionstic fuzzy graphs defined. Some propositions involving strong interval-valued neutrosophic intuitionistic fuzzy graphs are stated and proved. We introduce the opinion of product of two interval-valued intuitionistic fuzzy graphs and investigate some of their properties. We discuss some propositions on Cartesian product and define some properties on it. Strong interval-valued neutrosophic intuitionistic fuzzy graphs allows attaching the truth-membership(t), indeterminacy-membership(i) and falsity –membership degrees (f) both to vertices and edges. Combining the strong interval valued neutrosophic set with graph theory, a new graph model emerges, called strong interval neutrosophic intuitionistic fuzzy graph.

Keywords: Intuitionstic fuzzy graph, Interval valued intuitionstic fuzzy graph.

1. Introduction

In 1965, Zadeh [21] inaugurated the conception of fuzzy set as a method of finding uncertainty. In 1986, Atanassov proposed Intuitionistic Fuzzy set (IFS) [3] which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After three years Atanassov and Gargov [4] introduced Interval-Valued Intuitionistic Fuzzy Set (IVIFS) which is helpful to model the problem precisely.

In 1975, Rosenfeld [14] introduced the concept of fuzzy graphs. Yeh and Bang [8] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graphs. It has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, transportation, ect. Interval-Valued Fuzzy Graphs (IVFG) are defined by Akram and Dudec [1] in 2011. Atanassov [4] introduced the concept of intuitionistic fuzzy relations and Intutionistic Fuzzy Graph (IFG).In fact, interval-valued fuzzy graphs and interval valued intuitionistic fuzzy graphs are two different models that extend theory of fuzzy graph. S.N.Mishra and A.Pal [9] introduced the product of interval valued intuitionistic fuzzy graph. Akram and Bijan Davvaz [1] introduced Strong Intuitionistic Fuzzy Graphs(SIFG). In this paper, The opinion of Strong Interval-Valued Intuitionistic Fuzzy Graphs (SIVIFG) are introduced.

Many works on fuzzy graphs, intuitionistic fuzzy graphs and interval valued intuitionistic fuzzy graphs (Antonios K et al. 2014; Bhattacharya 1987; Mishra and pal 2013; Nagoor Gani and shajitha Begum 2010; Nagoor Gani and Latha 2012; Shannon and atanassov 1994) have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and/or intuitionistic fuzzy sets.Further on, Broumi et al. (2016b) introduced a new neutrosophic graph model, This model allows attaching the membership (t), indeterminacy(i) and non-membership degrees (f) both to vertices and edges.

2. Preliminaries

In this section, the authors mainly recall some notions related to neutrosophic graphs, strong interval valued neutrosophic sets, intuitionistic fuzzy graphs, Strong interval valued neutrosophic intuitionistic fuzzy graphs, proper to the present work. The readers are consulted for further details to (Broumi et al.2016b; Mishra and Pal 2013; Nagoor Gani and Basheer Ahamed 2003;Parvathi and Karunambigai 2006;Smarandache 2006;wang et al.2010; Wang et al. 2005a).

3.Interval-Valued Intuitionistic Fuzzy Graph

In this section, we introduce the Interval valued intuitionstic fuzzy graph and the conception of Strong interval valued intuitionstic fuzzy graph.

Definition 3.1

A fuzzy set V is a mapping σ from V to [0,1]. A fuzzy graph G is a pair of functions G= (σ,μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, i.e. $\mu(uv)\leq \sigma(u)\wedge\sigma(v)$. The underlying crisp graph of G=(σ,μ) is denoted by $G^*=(V,E)$ where $E\subseteq V\times V$.

Let D[0,1] be the set of all closed subintervals of the interval [0,1] and element of this set are denoted by uppercase letters. If $M\in D[0,1]$ then it can be represented as $M=[M_L,M_U]$,where M_L and M_U are the lower and upper limits of M.

Definition 3.2

An intuitionistic fuzzy graph with underlying set V is defined to be a pair G $=(A,B)$ where

(i) the functions $\mu_A : V \to [0,1]$ and $\gamma_A : V \to [0,1]$ denote the degree of membership and non membership of the element $x \in V$ respectively, such that $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for all $x \in V$.

(ii) the function $\mu_B : E \subseteq V \times V \to [0,1]$ and $\gamma_B : E \subseteq V \times V \to [0,1]$ are defined by $\mu_B((x,y)) \le \min(\mu_A(x), \mu_A(y))$ and $\gamma_B((x,y)) \ge \max(\gamma_A(x), \gamma_A(y))$ such that $0 \le \mu_B((x,y)) + \gamma_B((x,y)) \le 1$, $\forall (x,y) \in E$.

Definition 3.3

An intuitionistic fuzzy graph $G=(A,B)$ is called strong intuitionistic fuzzy graph if $\mu_B(xy) = \min(\mu_A(x), \mu_A(y))$ and $\gamma_B(xy) = \max(\gamma_A(x), \gamma_A(y))$, for all $xy \in E$.

Definition 3.4

An interval valued intuitionistic fuzzy graph with underlying set V is defined to be pair G $=(A,B)$ where

1) The functions $M_A : V \to D[0,1]$ and $N_A : V \to D[0,1]$ denote the degree of membership and non membership of the element $x \in V$, respectively such that $0 \le M_A(x) + N_A(x) \le 1$ for all $x \in V$.

2) The functions $M_B : E \subseteq V \times V \to D[0,1]$ and $N_B : E \subseteq V \times V \to D[0,1]$ are defined by $M_{BL}((x,y)) \le \min(M_{AL}(x), M_{AL}(y))$ and $N_{BL}((x,y)) \ge \max(N_{AL}(x), N_{AL}(y))$, and $M_{BU}((x,y)) \le \min(M_{AU}(x), M_{AU}(y))$ and $N_{BU}((x,y)) \ge \max(N_{AU}(x), N_{AU}(y))$ such that $0 \le M_{BU}((x,y)) + N_{BU}((x,y)) \le 1$ $\forall (x,y) \in E$.

Strong Interval-Valued Intuitionistic Fuzzy Graph

Definition 3.5

An interval valued intuitionistic fuzzy graph G=(A,B) is called strong interval valued intuitionistic fuzzy graph if $M_{BL}(xy) = \min(M_{AL}(x), M_{AL}(y))$ and $N_{BL}(xy) = \max(N_{AL}(x), N_{AL}(y))$, if $M_{BU}(xy) = \min(M_{AU}(x), M_{AU}(y))$ and $N_{BU}(xy) = \max(N_{AU}(x), N_{AU}(y))$, $\forall xy \in E$.

Example 3.1

Figure 1 is an SIVIFG G=(A,B) where A=$\{\langle x,[0.4,0.6],[0.2,0.3]\rangle,\langle y,[0.5,0.6],[0.1,0.3]\rangle,\langle z,[0.7,0.5],[0.3,0.1]\rangle\}$ B=$\{\langle xy,[0.4,0.6],[0.2,0.3]\rangle,\langle yz,[0.5,0.6]\rangle,\langle xz,[0.4,0.6],[0.3,0.1]\rangle\}$.

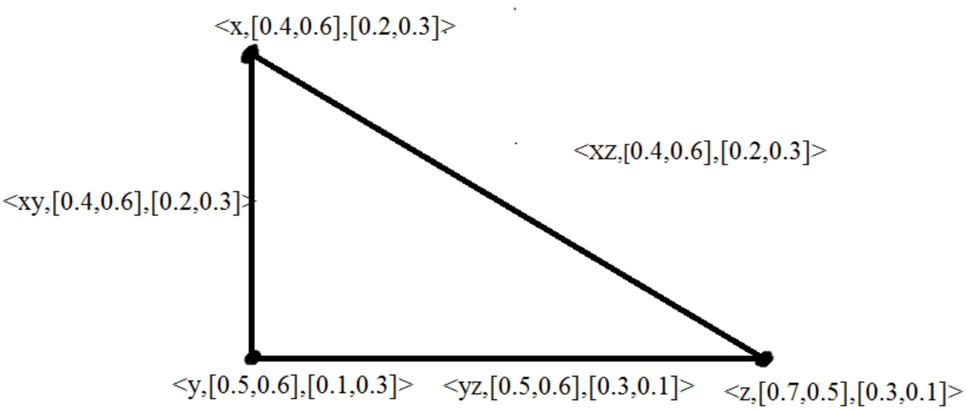

Figure-1 Strong Interval Valued Intuitionistic Fuzzy Graph

Definition 3.6

Let A_1 and A_2 be interval-valued intuitionistic fuzzy subsets of V_1 and V_2 respectively. Let B_1 and B_2 interval-valued intuitionistic fuzzy subsets of E_1 and E_2 respectively. The **Cartesian product** of two SIVIFG_S G_1 and G_2 is denoted by $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ and is defined as follows:

1) $(M_{A_1L} \times M_{A_2L})(x_1, x_2) = \min(M_{A_1L}(x_1), M_{A_2L}(x_2))$

$(M_{A_1U} \times M_{A_2U})(x_1, x_2) = \min(M_{A_1U}(x_1), M_{A_2U}(x_2))$

$(N_{A_1L} \times N_{A_2L})(x_1, x_2) = \max(N_{A_1L}(x_1), N_{A_2L}(x_2))$

$(N_{A_1U} \times N_{A_2U})(x_1, x_2) = \max(N_{A_1U}(x_1), N_{A_2U}(x_2))$, $\forall x_1 \in V_1, x_2 \in V_2$

2) $(M_{B_1L} \times M_{B_2L})((x, x_2)(x, y_2)) = \min(M_{A_1L}(x), M_{B_2L}(x_2 y_2))$

$(M_{B_1U} \times M_{B_2U})((x, x_2)(x, y_2)) = \min(M_{A_1U}(x), M_{B_2U}(x_2 y_2))$

$(N_{B_1L} \times N_{B_2L})((x, x_2)(x, y_2)) = \max(N_{A_1L}(x), N_{B_2L}(x_2 y_2))$

$(N_{B_1U} \times N_{B_2U})((x, x_2)(x, y_2)) = \max(N_{A_1U}(x), N_{B_2U}(x_2 y_2))$ $\forall x_1 \in V_1, x_2 y_2 \in E_2$

3) $(M_{B_1L} \times M_{B_2L})((x_1, z)(y_1, z)) = \min(M_{B_1L}(x_1y_1), M_{A_2L}(z))$

$(M_{B_1U} \times M_{B_2U})((x_1, z)(y_1, z)) = \min(M_{B_1U}(x_1y_1), M_{A_2U}(z))$

$(N_{B_1L} \times N_{B_2L})((x_1, z)(y_1, z)) = \max(N_{B_1L}(x_1y_1), N_{A_2L}(z))$

$(N_{B_1U} \times N_{B_2U})((x_1, z)(y_1, z)) = \max(N_{B_1U}(x_1y_1), N_{A_2U}(z)) \qquad \forall z \in V_2, x_1y_1 \in E_1$

Theorem 3.1

Union of two strong interval valued intuitionistic fuzzy graphs is always SIVIFG.

Proof:

Let G_1 and G_2 are SIVIFG, there exist $x_iy_i \in E_i$, i=1,2 such that

$M_{B_iL}((x_i, y_i)) = \min(M_{A_iL}(x_i), M_{A_iL}(y_i))$, i=1,2.

$M_{B_iU}((x_i, y_i)) = \min(M_{A_iU}(x_i), M_{A_iU}(y_i))$, i=1,2.

$N_{B_iL}((x_i, y_i)) = \max(N_{A_iL}(x_i), N_{A_iL}(y_i))$, i=1,2.

$N_{B_iU}((x_i, y_i)) = \max(N_{A_iU}(x_i), N_{A_iU}(y_i))$, i=1,2.

Let $E = \{(x, x_2)(x, y_2) / x_1 \in v_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in v_2, x_1y_1 \in E_1\}$

Consider, $(x, x_2)(x, y_2) \in E$, we have

$(M_{B_1L} \times M_{B_2L})((x, x_2)(x, y_2)) = \min(M_{A_1L}(x), M_{B_2L}(x_2y_2))$

$= \min(M_{A_1L}(x), M_{A_2L}(x_2), M_{A_{2L}}(y_2)).$

Similarly,

$(M_{B_1U} \times M_{B_2U})(x_1, x_2) = \min(M_{A_1U}(x), M_{B_2U}(x_2y_2))$

$= \min(M_{A_1U}(x), M_{A_2U}(x_2), M_{A_2U}(y_2)).$

$(M_{A_1L} \times M_{A_2L})(x_1, x_2) = \min(M_{A_1L}(x_1), M_{A_2L}(x_2))$

$(M_{A_1U} \times M_{A_2U})(x_1, x_2) = \min(M_{A_1U}(x_1), M_{A_2U}(x_2))$

$(M_{A_1L} \times M_{A_2L})(x_1, y_2) = \min(M_{A_1L}(x_1), M_{A_2L}(y_2))$

$(M_{A_1U} \times M_{A_2U})(x_1, y_2) = \min(M_{A_1U}(x_1), M_{A_2U}(y_2))$

$\min((M_{A_1U} \times M_{A_2U})(x, x_2), (M_{A_1U} \times M_{A_2U})(x, y_2)) = \min(\min(M_{A_1U}(x), M_{A_2U}(x_2)),$

$\min(M_{A_1U}(x), M_{A_2U}(y_2)))$

$= \min(M_{A_1U}(x), M_{A_2U}(x_2), M_{A_2U}(y_2))$

Hence,

$(M_{B_1L} \times M_{B_2L})((x, x_2)(x, y_2)) = \min((M_{A_1L} \times M_{A_2L})(x, x_2), (M_{A_1L} \times M_{A_2L})(x, y_2))$

$(M_{B_1U} \times M_{B_2U})((x, x_2)(x, y_2)) = \min((M_{A_1U} \times M_{A_2U})(x, x_2), (M_{A_1U} \times M_{A_2U})(x, y_2))$

Similarly , we can show that

$(N_{B_1L} \times N_{B_2L})\ ((x,x_2)(x,y_2)) = \max((N_{A_1L} \times N_{A_2L})(x,x_2),(N_{A_1L} \times N_{A_2L})(x,y_2))$

Example:3.2

Consider the fuzzy graph with five vertices given in figure

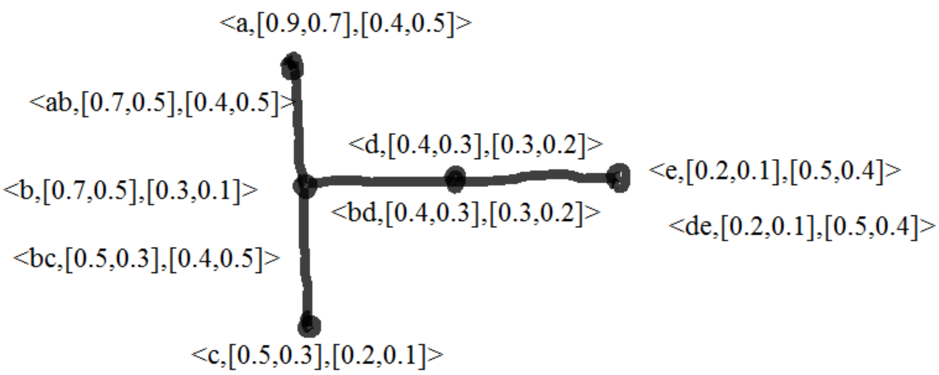

Figure-2 Strong Interval Valued Intuitionistic Fuzzy Graph

A={ $\langle a,[0.9,0.7],[0.4,0.5]\rangle$, $\langle b,[0.7,0.5],[0.3,0.1]\rangle$, $\langle c,[0.5,0.3],[0.2,0.1]\rangle$, $\langle d,[0.4,0.3],[0.3,0.2]\rangle$, $\langle e,[0.2,0.1],[0.5,0.4]\rangle$ }

B={ $\langle ab,[0.7,0.5],[0.4,0.5]\rangle$, $\langle bc,[0.5,0.3],[0.3,0.1]\rangle$, $\langle bd,[0.4,0.3],[0.3,0.2]\rangle$, $\langle de,[0.2,0.1],[0.5,0.4]\rangle$ }.

4. Operation on Strong Interval-Valued Neutrosophic Intuitionistic Fuzzy Graph

Definition 4.1

By an Strong Interval-Valued Neutrosophic Intuitionistic Fuzzy Graph G^* =(V,E) one means a pair G=(A,B), Where A=< $[T_{AL},T_{AL}]$, $[T_{AU},T_{AU}]$, $[I_{AL},I_{AL}]$, $[I_{AU},I_{AU}]$,$[F_{AL},F_{AL}]$, $[F_{AU},F_{AU}]$ > is an Strong Interval-Valued Neutrosophic Intuitionistic set on V and B=< $[T_{BL},T_{BL}]$, $[T_{BU},T_{BU}]$, $[I_{BL},I_{BL}]$, $[I_{BU},I_{BU}]$,$[F_{BL},F_{BL}]$,$[F_{BU},F_{BU}]$ > is an Strong Interval-Valued Neutrosophic Intuitionistic relation on E satisfying the following condition:

1. V={v_1,v_2,…..v_n} such that T_{AL}:V$\to$[0,1], T_{AU}:V$\to$[0,1], I_{AL}:V$\to$[0,1], I_{AU}:V$\to$[0,1], and F_{AL}:V$\to$[0,1], F_{AU} :V$\to$[0,1] denote the degree of truth-membership, the degree of inderminancy membership and falsity-membership of the element $y\in V$, respectively, and $0\le T_A(v_i)+I_A(v_i)+F_A(v_i)\le 1$, for every $v_i\in V$.

2.The functions T_{BL}:V$\times V\to$[0,1], T_{BU}:V$\times V\to$[0,1], I_{BL}:V$\times V\to$[0,1], I_{BU}:V$\times V\to$[0,1] and F_{BL}:V$\times V\to$[0,1], F_{BU}:V$\times V\to$[0,1], such that

$$T_{BL}(v_i,v_j)\le \min[T_{AL}(v_i),T_{AL}(v_j)]$$

$$T_{BU}(v_i,v_j)\le \min[T_{AU}(v_i),T_{AU}(v_j)]$$

$$I_{BL}(v_i,v_j)\ge \max[I_{BL}(v_i),I_{BL}(v_j)]$$

$$I_{BU}(v_i,v_j)\ge \max[I_{BU}(v_i),I_{BU}(v_j)]$$

And

$$F_{BL}(v_i,v_j)\ge \max[F_{BL}(v_i),F_{BL}(v_j)]$$

$$F_{BU}(v_i,v_j)\ge \max[F_{BU}(v_i),F_{BU}(v_j)]$$

Denote the degree of truth-membership, the degree of inderminancy membership and falsity-membership of the degree $(v_i,v_j)\in E$ respectively, where

$$0\le T_B(v_i,v_j)+I_B(v_i,v_j)+F_B(v_i,v_j)\le 1 \text{ for all } (v_i,v_j)\in E$$

Example 4.1

Figure 3 is an example for SIVNIFG, G=(A,B) defined on a graph G^*=(V,E) such that V={x,y,z}, E={xy,yz,zx}, A is an Strong Interval Valued Neutrosophic Intuitionistic Fuzzy set of V

A={$\langle x,[0.2,0.4],[0.3,0.2],[0.4,0.3]\rangle,\langle y,[0.5,0.3],[0.2,0.1],[0.3,0.5]\rangle,$
$\langle z,[0.4,0.3],[0.2,0.4],[0.3,0.1]\rangle$ } and B an Strong Interval Valued Neutrosophic Intuitionistic Fuzzy set of $E\subseteq V\times V$.

B={$\langle xy,[0.2,0.4],[0.3,0.2],[0.4,0.3]\rangle,\langle yz,[0.4,0.3],[0.2,0.4],[0.3,0.5]\rangle,$
$\langle zx,[0.2,0.4],[0.3,0.2],[0.4,0.3]\rangle$ }.

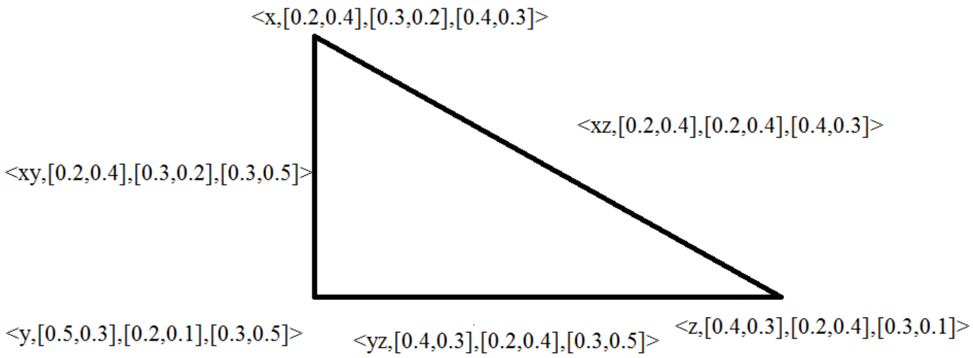

Figure-3 Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph

By routine computations, it is easy to see that G=(A,B) is an Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph of G^*.

Definition 4.2

Let $G^* = G_1^* \times G_2^* = (V, E)$ be the Cartesian product of two graphs where $V = V_1 \times V_2$ and $E = \{(x, x_2)(x, y_2) / x_1 \in v_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in v_2, x_1y_1 \in E_1\}$; then, the Cartesian product $G = G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ is an Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph defined by

1) $(T_{A_1L} \times T_{A_2L})(x_1, x_2) = \min(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \times T_{A_2U})(x_1, x_2) = \min(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(I_{A_1L} \times I_{A_2L})(x_1, x_2) = \max(I_{A_1L}(x_1), I_{A_2L}(x_2))$

$(I_{A_1U} \times I_{A_2U})(x_1, x_2) = \max(I_{A_1U}(x_1), I_{A_2U}(x_2))$

$(F_{A_1L} \times F_{A_2L})(x_1, x_2) = \max(F_{A_1L}(x_1), F_{A_2L}(x_2))$

$(F_{A_1U} \times F_{A_2U})(x_1, x_2) = \max(F_{A_1U}(x_1), F_{A_2U}(x_2))$ for all $(x_1, x_2) \in V$.

2) $(T_{B_1L} \times T_{B_2L})(x, x_2)(x, y_2) = \min(T_{A_1L}(x_1), T_{B_2L}(x_2y_2))$

$(T_{B_1U} \times T_{B_2U})(x, x_2)(x, y_2) = \min(T_{A_1U}(x_1), T_{B_2U}(x_2y_2))$

$(I_{B_1L} \times I_{B_2L})(x, x_2)(x, y_2) = \max(I_{A_1L}(x_1), I_{B_2L}(x_2y_2))$

$(I_{B_1U} \times I_{B_2U})(x, x_2)(x, y_2) = \max(I_{A_1U}(x_1), I_{B_2U}(x_2y_2))$

$$(F_{B_1L} \times F_{B_2L})(x, x_2)(x, y_2) = \max(F_{A_1L}(x_1), F_{B_2L}(x_2y_2))$$
$$(F_{B_1U} \times F_{B_2U})(x, x_2)(x, y_2) = \max(F_{A_1U}(x_1), F_{B_2U}(x_2y_2)), \qquad \forall x \in V_1, \forall x_2y_2 \in E_2$$

3) $(T_{B_1L} \times T_{B_2L})(x_1, z)(y_1, z) = \min(T_{B_1L}(x_1y_1), T_{A_2L}(z))$
$$(T_{B_1U} \times T_{B_2U})(x_1, z)(y_1, z) = \min(T_{B_1U}(x_1y_1), T_{A_2U}(z))$$
$$(I_{B_1L} \times I_{B_2L})(x_1, z)(y_1, z) = \max(I_{B_1L}(x_1y_1), I_{A_2L}(z))$$
$$(I_{B_1U} \times I_{B_2U})(x_1, z)(y_1, z) = \max(I_{B_1U}(x_1y_1), I_{A_2U}(z))$$
$$(F_{B_1L} \times F_{B_2L})(x_1, z)(y_1, z) = \max(F_{B_1L}(x_1y_1), F_{A_2L}(z))$$
$$(F_{B_1U} \times F_{B_2U})(x_1, z)(y_1, z) = \max(F_{B_1U}(x_1y_1), F_{A_2U}(z)), \qquad \forall z \in V_2, \forall x_1y_1 \in E_1$$

Example 4.2

Let $G_1^* = (A_1, B_1)$ and $G_2^* = (A_2, B_2)$ be two graphs where V_1={a,b}, V_2={c,d}, E_1={a,b} and E_2={c,d}. Consider two Strong Interval Valued Neutrosophic Intuitionistic Graphs:

$A_1 = \{< a, [0.3, 0.4], [0.5, 0.1], [0.2, 0.4] >, < b, [0.5, 0.1], [0.2, 0.4], [0.1, 0.3] > \}$,
B_1={<ab,[0.3,0.4],[0.5,0.1],[0.2,0.4]>};

$A_2 = \{< c, [0.2, 0.5], [0.3, 0.2], [0.5, 0.2] >, < d, [0.3, 0.2], [0.4, 0.1], [0.2, 0.4] > \}$,
B_2={<cd,[0.2,0.5],[0.4,0.1],[0.5,0.2]>};

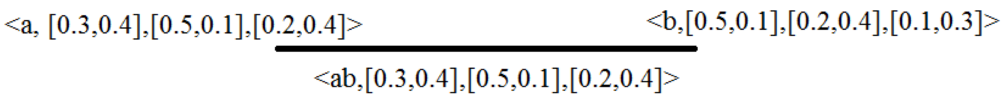

Figure-4 Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph G_1

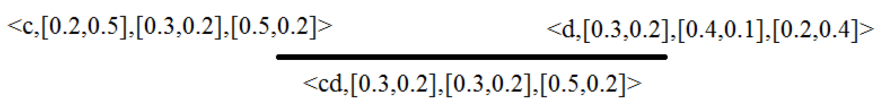

Figure-5 Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph G_2

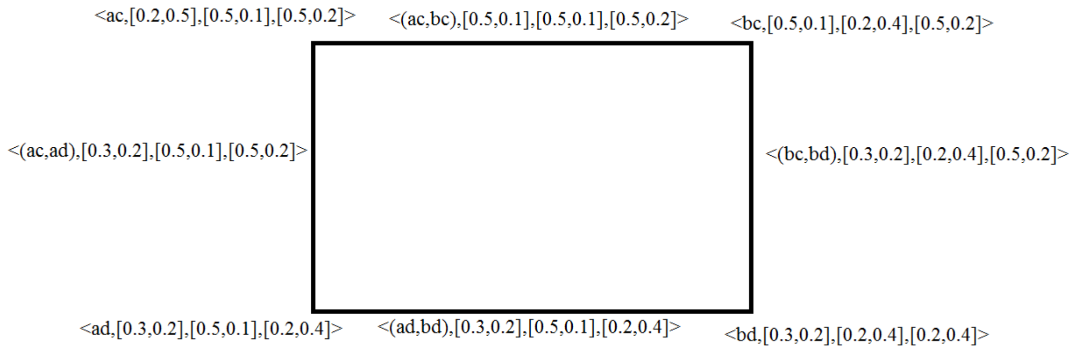

Figure-6 Cartesian product of Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph

Proposition 4.1

If G_1 and G_2 are the Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graphs then the Cartesian product $G_1 \times G_2$ is a Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graphs.

Proof:

Let $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$

Verifying only conditions for $B_1 \times B_2$, because conditions for $A_1 \times A_2$ are obvious.

Let $E = \{(x, x_2)(x, y_2) / x_1 \in v_1, x_2 y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in v_2, x_1 y_1 \in E_1\}$

Considering $(x,x_2)(x,y_2) \in E$ we have

$$(T_{B_1L} \times T_{B_2L})((x, x_2)(x, y_2)) = \min(T_{A_1L}(x), T_{B_2L}(x_2 y_2))$$
$$\leq \min(T_{A_1L}(x), \min(T_{A_2L}(x_2), T_{A_2L}(y_2)))$$
$$= \min(\min(T_{A_1L}(x), T_{A_2L}(x_2))) ,\min(\min(T_{A_1L}(x), T_{A_2L}(y_2)))$$
$$= \min ((T_{A_1L} \times T_{A_2L})(x, x_2), (T_{A_1L} \times T_{A_2L})(x, y_2)),$$
$$(T_{B_1U} \times T_{B_2U})((x, x_2)(x, y_2)) = \min(T_{A_1U}(x), T_{B_2U}(x_2 y_2))$$
$$\leq \min(T_{A_1U}(x), \min(T_{A_2U}(x_2), T_{A_2U}(y_2)))$$
$$= \min(\min(T_{A_1U}(x), T_{A_2U}(x_2))) ,\min(\min(T_{A_1U}(x), T_{A_2U}(y_2)))$$
$$= \min ((T_{A_1U} \times T_{A_2U})(x, x_2), (T_{A_1U} \times T_{A_2U})(x, y_2)),$$
$$(I_{B_1L} \times I_{B_2L})((x, x_2)(x, y_2)) = \max(I_{A_1L}(x), I_{B_2L}(x_2 y_2))$$
$$\geq \max(I_{A_1L}(x), \max(I_{A_2L}(x_2), I_{A_2L}(y_2)))$$
$$= \max(\max(I_{A_1L}(x), I_{A_2L}(x_2))) ,\max(\max(I_{A_1L}(x), I_{A_2L}(y_2)))$$

$$= \max((I_{A_1L} \times I_{A_2L})(x, x_2), (I_{A_1L} \times I_{A_2L})(x, y_2)),$$

$$(I_{B_1U} \times I_{B_2U})((x, x_2)(x, y_2)) = \max(I_{A_1U}(x), I_{B_2U}(x_2 y_2))$$
$$\geq \max(I_{A_1U}(x), \max(I_{A_2U}(x_2), I_{A_2U}(y_2)))$$
$$= \max(\max(I_{A_1U}(x), I_{A_2U}(x_2))), \max(\max(I_{A_1U}(x), I_{A_2U}(y_2)))$$
$$= \max((I_{A_1U} \times I_{A_2U})(x, x_2), (I_{A_1U} \times I_{A_2U})(x, y_2)),$$

$$(F_{B_1L} \times F_{B_2L})((x, x_2)(x, y_2)) = \max(F_{A_1L}(x), F_{B_2L}(x_2 y_2))$$
$$\geq \max(F_{A_1L}(x), \max(F_{A_2L}(x_2), F_{A_2L}(y_2)))$$
$$= \max(\max(F_{A_1L}(x), F_{A_2L}(x_2))), \max(\max(F_{A_1L}(x), F_{A_2L}(y_2)))$$
$$= \max((F_{A_1L} \times F_{A_2L})(x, x_2), (F_{A_1L} \times F_{A_2L})(x, y_2)),$$

$$(F_{B_1U} \times F_{B_2U})((x, x_2)(x, y_2)) = \max(F_{A_1U}(x), F_{B_2U}(x_2 y_2))$$
$$\geq \max(F_{A_1U}(x), \max(F_{A_2U}(x_2), F_{A_2U}(y_2)))$$
$$= \max(\max(F_{A_1U}(x), F_{A_2U}(x_2))), \max(\max(F_{A_1U}(x), F_{A_2U}(y_2)))$$
$$= \max((F_{A_1U} \times F_{A_2U})(x, x_2), (F_{A_1U} \times F_{A_2U})(x, y_2)).$$

Similarly, we prove $((x_1,z)(y_1,z)) \in E$

Hence

$$(T_{B_1L} \times T_{B_2L})((x_1, z)(y_1, z)) = \min((T_{A_1L} \times T_{A_2L})(x_1, z), (T_{A_1L} \times T_{A_2L})(y_1, z)),$$
$$(T_{B_1U} \times T_{B_2U})((x_1, z)(y_1, z)) = \min((T_{A_1U} \times T_{A_2U})(x_1, z), (T_{A_1U} \times T_{A_2U})(y_1, z)),$$
$$(I_{B_1L} \times I_{B_2L})((x_1, z)(y_1, z)) = \max((I_{A_1L} \times I_{A_2L})(x_1, z), (I_{A_1L} \times I_{A_2L})(y_1, z)),$$
$$(I_{B_1U} \times I_{B_2U})((x_1, z)(y_1, z)) = \max((I_{A_1U} \times I_{A_2U})(x_1, z), (I_{A_1U} \times I_{A_2U})(y_1, z)),$$
$$(F_{B_1L} \times F_{B_2L})((x_1, z)(y_1, z)) = \max((F_{A_1L} \times F_{A_2L})(x_1, z), (F_{A_1L} \times F_{A_2L})(y_1, z)),$$
$$(F_{B_1U} \times F_{B_2U})((x_1, z)(y_1, z)) = \max((F_{A_1U} \times F_{A_2U})(x_1, z), (F_{A_1U} \times F_{A_2U})(y_1, z)).$$

Hence proved

Definition 4.3

Let $G^* = G_1^* \times G_2^* = (V_1 \times V_2, E)$ be the composition of two graphs where $E = \{(x, x_2)(x, y_2) / x \in V_1, x_2 y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in V_2, x_1 y_1 \in E_1\} \cup \{(x_1, x_2)(y_1, y_2) / x_1 y_1 \in E_1, x_2 \neq y_2\}$; then, the composition of Strong Interval Valued Neutrosophic Intuitionistic Fuzzy graphs $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ is an SIVNIFG defined by

1) $(T_{A_1L} \circ T_{A_2L})(x_1, x_2) = \min(T_{A_1L}(x_1), T_{A_2L}(x_2))$
$(T_{A_1U} \circ T_{A_2U})(x_1, x_2) = \min(T_{A_1U}(x_1), T_{A_2U}(x_2))$
$(I_{A_1L} \circ I_{A_2L})(x_1, x_2) = \max(I_{A_1L}(x_1), I_{A_2L}(x_2))$
$(I_{A_1U} \circ I_{A_2U})(x_1, x_2) = \max(I_{A_1U}(x_1), I_{A_2U}(x_2))$
$(F_{A_1L} \circ F_{A_2L})(x_1, x_2) = \max(F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \circ F_{A_2U})(x_1, x_2) = \max(F_{A_1U}(x_1), F_{A_2U}(x_2)) \qquad \forall x_1 \in V_1, x_2 \in V_2;$

2) $(T_{B_1L} \circ T_{B_2L})(x, x_2)(x, y_2) = \min(T_{A_1L}(x_1), T_{B_2L}(x_2 y_2))$
$(T_{B_1U} \circ T_{B_2U})(x, x_2)(x, y_2) = \min(T_{A_1U}(x_1), T_{B_2U}(x_2 y_2))$
$(I_{B_1L} \circ I_{B_2L})(x, x_2)(x, y_2) = \max(I_{A_1L}(x_1), I_{B_2L}(x_2 y_2))$
$(I_{B_1U} \circ I_{B_2U})(x, x_2)(x, y_2) = \max(I_{A_1U}(x_1), I_{B_2U}(x_2 y_2))$
$(F_{B_1L} \circ F_{B_2L})(x, x_2)(x, y_2) = \max(F_{A_1L}(x_1), F_{B_2L}(x_2 y_2))$
$(F_{B_1U} \circ F_{B_2U})(x, x_2)(x, y_2) = \max(F_{A_1U}(x_1), F_{B_2U}(x_2 y_2)), \qquad \forall x \in V_1, \forall x_2 y_2 \in E_2;$

3) $(T_{B_1L} \circ T_{B_2L})(x_1, z)(y_1, z) = \min(T_{B_1L}(x_1 y_1), T_{A_2L}(z))$
$(T_{B_1U} \circ T_{B_2U})(x_1, z)(y_1, z) = \min(T_{B_1U}(x_1 y_1), T_{A_2U}(z))$
$(I_{B_1L} \circ I_{B_2L})(x_1, z)(y_1, z) = \max(I_{B_1L}(x_1 y_1), I_{A_2L}(z))$
$(I_{B_1U} \circ I_{B_2U})(x_1, z)(y_1, z) = \max(I_{B_1U}(x_1 y_1), I_{A_2U}(z))$
$(F_{B_1L} \circ F_{B_2L})(x_1, z)(y_1, z) = \max(F_{B_1L}(x_1 y_1), F_{A_2L}(z))$
$(F_{B_1U} \circ F_{B_2U})(x_1, z)(y_1, z) = \max(F_{B_1U}(x_1 y_1), F_{A_2U}(z)), \qquad \forall z \in V_2, \forall x_1 y_1 \in E_1$

4) $(T_{B_1L} \circ T_{B_2L})((x_1, x_2)(y_1, y_2)) = \min(T_{A_2L}(x_2), T_{A_2L}(y_2), T_{B_1L}(x_1 y_1))$
$(T_{B_1U} \circ T_{B_2U})((x_1, x_2)(y_1, y_2)) = \min(T_{A_2U}(x_2), T_{A_2U}(y_2), T_{B_1U}(x_1 y_1))$
$(I_{B_1L} \circ I_{B_2L})((x_1, x_2)(y_1, y_2)) = \max(I_{A_2L}(x_2), I_{A_2L}(y_2), I_{B_1L}(x_1 y_1))$
$(I_{B_1U} \circ I_{B_2U})((x_1, x_2)(y_1, y_2)) = \max(I_{A_2U}(x_2), I_{A_2U}(y_2), I_{B_1U}(x_1 y_1))$
$(F_{B_1L} \circ F_{B_2L})((x_1, x_2)(y_1, y_2)) = \max(F_{A_2L}(x_2), F_{A_2L}(y_2), F_{B_1L}(x_1 y_1))$
$(F_{B_1U} \circ F_{B_2U})((x_1, x_2)(y_1, y_2)) = \max(F_{A_2U}(x_2), F_{A_2U}(y_2), F_{B_1U}(x_1 y_1))$
$\forall (x_1, x_2)(y_1, y_2) \in E^{\circ} - E$, where $E^0 = E \cup \{(x_1, x_2)(y_1, y_2) / x_1 y_1 \in E_1, x_2 \neq y_2\}$.

Example 4.3

Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two graphs where V_1={a,b}, V_2={c,d}, E_1={a,b} and E_2={c,d}. Consider two Strong Interval Valued Neutrosophic Intuitionistic Graphs:

$A_1 = \{< a, [0.5,0.3], [0.3,0.2], [0.1,0.3] >, < b, [0.3,0.4], [0.5,0.2], [0.1,0.2] > \}$,
B_1={<ab,[0.3,0.4],[0.5,0.2],[0.1,0.3]>};

$A_2 = \{< c, [0.3,0.2], [0.5,0.2], [0.1,0.4] >, < d, [0.1,0.3], [0.3,0.2], [0.4,0.3] > \}$,
B_1={<cd,[0.1,0.3],[0.5,0.2],[0.4,0.3]>};

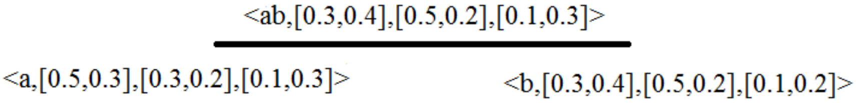

Figure-7 Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph G_1

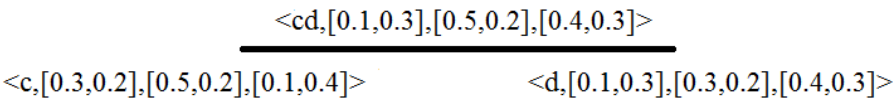

Figure-8 Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph G_2

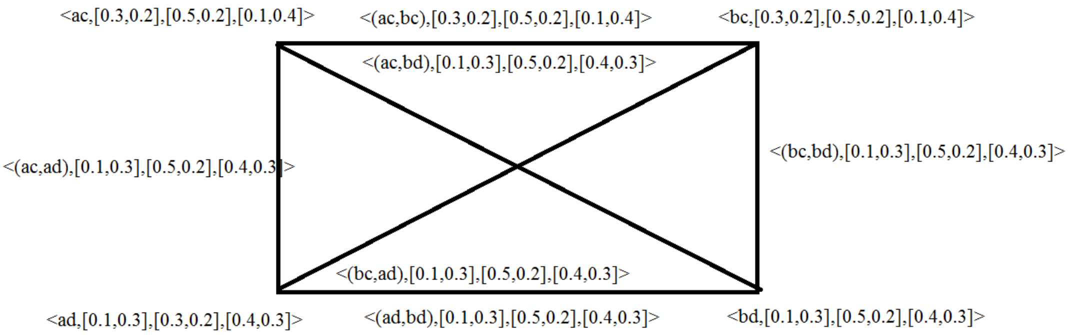

Figure-9 Composition of Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph

Proposition 4.2

If G_1 and G_2 are the Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graphs then the Composition $G_1 \circ G_2$ is a Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graphs.

Proof:

Let $G_1 \circ G_2 = (A_1 \circ A_2, B_1 \circ B_2)$

Verifying only conditions for $B_1 \circ B_2$, because conditions for $A_1 \circ A_2$ are obvious.

Let $E = \{(x, x_2)(x, y_2) / x_1 \in v_1, x_2 y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in v_2, x_1 y_1 \in E_1\}$

Considering $(x,x_2)(x,y_2) \in E$ we have

$$(T_{B_1L} \circ T_{B_2L})((x, x_2)(x, y_2)) = \min(T_{A_1L}(x), T_{B_2L}(x_2 y_2))$$
$$\leq \min(T_{A_1L}(x), \min(T_{A_2L}(x_2), T_{A_2L}(y_2)))$$
$$= \min(\min(T_{A_1L}(x), T_{A_2L}(x_2))), \min(\min(T_{A_1L}(x), T_{A_2L}(y_2)))$$
$$= \min((T_{A_1L} \circ T_{A_2L})(x, x_2), (T_{A_1L} \circ T_{A_2L})(x, y_2)),$$

$$(T_{B_1U} \circ T_{B_2U})((x, x_2)(x, y_2)) = \min(T_{A_1U}(x), T_{B_2U}(x_2 y_2))$$
$$\leq \min(T_{A_1U}(x), \min(T_{A_2U}(x_2), T_{A_2U}(y_2)))$$
$$= \min(\min(T_{A_1U}(x), T_{A_2U}(x_2))), \min(\min(T_{A_1U}(x), T_{A_2U}(y_2)))$$
$$= \min((T_{A_1U} \circ T_{A_2U})(x, x_2), (T_{A_1U} \circ T_{A_2U})(x, y_2)),$$

$$(I_{B_1L} \circ I_{B_2L})((x, x_2)(x, y_2)) = \max(I_{A_1L}(x), I_{B_2L}(x_2 y_2))$$
$$\geq \max(I_{A_1L}(x), \max(I_{A_2L}(x_2), I_{A_2L}(y_2)))$$
$$= \max(\max(I_{A_1L}(x), I_{A_2L}(x_2))), \max(\max(I_{A_1L}(x), I_{A_2L}(y_2)))$$
$$= \max((I_{A_1L} \circ I_{A_2L})(x, x_2), (I_{A_1L} \circ I_{A_2L})(x, y_2)),$$

$$(I_{B_1U} \circ I_{B_2U})((x, x_2)(x, y_2)) = \max(I_{A_1U}(x), I_{B_2U}(x_2 y_2))$$
$$\geq \max(I_{A_1U}(x), \max(I_{A_2U}(x_2), I_{A_2U}(y_2)))$$
$$= \max(\max(I_{A_1U}(x), I_{A_2U}(x_2))), \max(\max(I_{A_1U}(x), I_{A_2U}(y_2)))$$
$$= \max((I_{A_1U} \circ I_{A_2U})(x, x_2), (I_{A_1U} \circ I_{A_2U})(x, y_2)),$$

$$(F_{B_1L} \circ F_{B_2L})((x, x_2)(x, y_2)) = \max(F_{A_1L}(x), F_{B_2L}(x_2 y_2))$$
$$\geq \max(F_{A_1L}(x), \max(F_{A_2L}(x_2), F_{A_2L}(y_2)))$$
$$= \max(\max(F_{A_1L}(x), F_{A_2L}(x_2))), \max(\max(F_{A_1L}(x), F_{A_2L}(y_2)))$$
$$= \max((F_{A_1L} \circ F_{A_2L})(x, x_2), (F_{A_1L} \circ F_{A_2L})(x, y_2)),$$

$$(F_{B_1U}\circ F_{B_2U})((x,x_2)(x,y_2)) = \max(F_{A_1U}(x), F_{B_2U}(x_2y_2))$$
$$\geq \max(F_{A_1U}(x), \max(F_{A_2U}(x_2), F_{A_2U}(y_2)))$$
$$= \max(\max(F_{A_1U}(x), F_{A_2U}(x_2))), \max(\max(F_{A_1U}(x), F_{A_2U}(y_2)))$$
$$= \max((F_{A_1U}\circ F_{A_2U})(x,x_2), (F_{A_1U}\circ F_{A_2U})(x,y_2)).$$

Consider $(x_1,z)(y_1,z)\in E$

$$(T_{B_1L}\circ T_{B_2L})((x_1,z)(y_1,z)) = \min(T_{B_1L}(x_1y_1), T_{A_2L}(z))$$
$$\leq \min(T_{B_1L}(x_1), (T_{B_1L}(y_1), \min(T_{A_2L}(z)))$$
$$= \min(\min(T_{B_1L}(x_1), T_{A_2L}(z))), \min(\min(T_{B_1L}(y_1), T_{A_2L}(z)))$$
$$= \min((T_{B_1L}\circ T_{A_2L})(x_1,z), (T_{B_1L}\circ T_{A_2L})(y_1,z)),$$

$$(T_{B_1U}\circ T_{B_2U})((x_1,z)(y_1,z)) = \min(T_{B_1U}(x_1y_1), T_{A_2U}(z))$$
$$\leq \min(T_{B_1U}(x_1), (T_{B_1U}(y_1), \min(T_{A_2U}(z)))$$
$$= \min(\min(T_{B_1U}(x_1), T_{A_2U}(z))), \min(\min(T_{B_1U}(y_1), T_{A_2U}(z)))$$
$$= \min((T_{B_1U}\circ T_{A_2U})(x_1,z), (T_{B_1U}\circ T_{A_2U})(y_1,z)),$$

$$(I_{B_1L}\circ I_{B_2L})((x_1,z)(y_1,z)) = \max(I_{B_1L}(x_1y_1), I_{A_2L}(z))$$
$$\geq \max(I_{B_1L}(x_1), (I_{B_1L}(y_1), \max(I_{A_2L}(z)))$$
$$= \max(\max(I_{B_1L}(x_1), I_{A_2L}(z))), \max(\max(I_{B_1L}(y_1), I_{A_2L}(z)))$$
$$= \max((I_{B_1L}\circ I_{A_2L})(x_1,z), (I_{B_1L}\circ I_{A_2L})(y_1,z)),$$

$$(I_{B_1U}\circ I_{B_2U})((x_1,z)(y_1,z)) = \max(I_{B_1U}(x_1y_1), I_{A_2U}(z))$$
$$\geq \max(I_{B_1U}(x_1), (I_{B_1U}(y_1), \max(I_{A_2U}(z)))$$
$$= \max(\max(I_{B_1U}(x_1), I_{A_2U}(z))), \max(\max(I_{B_1U}(y_1), I_{A_2U}(z)))$$
$$= \max((I_{B_1U}\circ I_{A_2U})(x_1,z), (I_{B_1U}\circ I_{A_2U})(y_1,z)),$$

$$(F_{B_1L}\circ F_{B_2L})((x_1,z)(y_1,z)) = \max(F_{B_1L}(x_1y_1), F_{A_2L}(z))$$
$$\geq \max(F_{B_1L}(x_1), (F_{B_1L}(y_1), \max(F_{A_2L}(z)))$$
$$= \max(\max(F_{B_1L}(x_1), F_{A_2L}(z))), \max(\max(F_{B_1L}(y_1), F_{A_2L}(z)))$$
$$= \max((F_{B_1L}\circ F_{A_2L})(x_1,z), (F_{B_1L}\circ F_{A_2L})(y_1,z)),$$

$$(F_{B_1U}\circ F_{B_2U})((x_1,z)(y_1,z)) = \max(F_{B_1U}(x_1y_1), F_{A_2U}(z))$$
$$\geq \max(F_{B_1U}(x_1), (F_{B_1U}(y_1), \max(F_{A_2U}(z)))$$
$$= \max(\max(F_{B_1U}(x_1), F_{A_2U}(z))), \max(\max(F_{B_1U}(y_1), F_{A_2U}(z)))$$

$= \max\ ((F_{B_1U} \circ F_{A_2U})(x_1, z), (F_{B_1U} \circ F_{A_2U})(y_1, z))$**,**

Consider $(x_1, x_2)(y_1, y_2) \in E^0 - E$

$$(T_{B_1L} \circ T_{B_2L})((x_1, x_2)(y_1, y_2)) = \min(T_{A_2L}(x_2), T_{A_2L}(y_2), T_{B_1L}(x_1y_1))$$
$$\leq \min(T_{A_2L}(x_2), T_{A_2L}(y_2), \min(T_{A_1L}(x_1), T_{A_2L}(y_1)))$$
$$= \min(\min(T_{A_1L}(x_1), T_{A_2L}(x_2))), \min(\min(T_{A_1L}(y_1), T_{A_2L}(y_2)))$$
$$= \min\ ((T_{A_1L} \circ T_{A_2L})(x_1, x_2), (T_{A_1L} \circ T_{A_2L})(y_1, y_2)),$$

$$(T_{B_1U} \circ T_{B_2U})((x_1, x_2)(y_1, y_2)) = \min(T_{A_2U}(x_2), T_{A_2U}(y_2), T_{B_1U}(x_1y_1))$$
$$\leq \min(T_{A_2U}(x_2), T_{A_2U}(y_2), \min(T_{A_1U}(x_1), T_{A_2U}(y_1)))$$
$$= \min(\min(T_{A_1U}(x_1), T_{A_2U}(x_2))), \min(\min(T_{A_1U}(y_1), T_{A_2U}(y_2)))$$
$$= \min\ ((T_{A_1U} \circ T_{A_2U})(x_1, x_2), (T_{A_1U} \circ T_{A_2U})(y_1, y_2)),$$

$$(I_{B_1L} \circ I_{B_2L})((x_1, x_2)(y_1, y_2)) = \max(I_{A_2L}(x_2), I_{A_2L}(y_2), I_{B_1L}(x_1y_1))$$
$$\geq \max(I_{A_2L}(x_2), I_{A_2L}(y_2), \max(I_{A_1L}(x_1), I_{A_2L}(y_1)))$$
$$= \max(\max(I_{A_1L}(x_1), I_{A_2L}(x_2))), \max(\max(I_{A_1L}(y_1), I_{A_2L}(y_2)))$$
$$= \max\ ((I_{A_1L} \circ I_{A_2L})(x_1, x_2), (I_{A_1L} \circ I_{A_2L})(y_1, y_2)),$$

$$(I_{B_1U} \circ I_{B_2U})((x_1, x_2)(y_1, y_2)) = \max(I_{A_2U}(x_2), I_{A_2U}(y_2), I_{B_1U}(x_1y_1))$$
$$\geq \max(I_{A_2U}(x_2), I_{A_2U}(y_2), \max(I_{A_1U}(x_1), I_{A_2U}(y_1)))$$
$$= \max(\max(I_{A_1U}(x_1), I_{A_2U}(x_2))), \max(\max(I_{A_1U}(y_1), I_{A_2U}(y_2)))$$
$$= \max\ ((I_{A_1U} \circ I_{A_2U})(x_1, x_2), (I_{A_1U} \circ I_{A_2U})(y_1, y_2)),$$

$$(F_{B_1L} \circ F_{B_2L})((x_1, x_2)(y_1, y_2)) = \max(F_{A_2L}(x_2), F_{A_2L}(y_2), F_{B_1L}(x_1y_1))$$
$$\geq \max(F_{A_2L}(x_2), F_{A_2L}(y_2), \max(F_{A_1L}(x_1), F_{A_2L}(y_1)))$$
$$= \max(\max(F_{A_1L}(x_1), F_{A_2L}(x_2))), \max(\max(F_{A_1L}(y_1), F_{A_2L}(y_2)))$$
$$= \max\ ((F_{A_1L} \circ F_{A_2L})(x_1, x_2), (F_{A_1L} \circ F_{A_2L})(y_1, y_2)),$$

$$(F_{B_1U} \circ F_{B_2U})((x_1, x_2)(y_1, y_2)) = \max(F_{A_2U}(x_2), F_{A_2U}(y_2), F_{B_1U}(x_1y_1))$$
$$\geq \max(F_{A_2U}(x_2), F_{A_2U}(y_2), \max(F_{A_1U}(x_1), F_{A_2U}(y_1)))$$
$$= \max(\max(F_{A_1U}(x_1), F_{A_2U}(x_2))), \max(\max(F_{A_1U}(y_1), F_{A_2U}(y_2)))$$
$$= \max\ ((F_{A_1U} \circ F_{A_2U})(x_1, x_2), (F_{A_1U} \circ F_{A_2U})(y_1, y_2)).$$

This completes the proof.

Proposition 4.3

If $G_1 \times G_2$ is Strong Interval Valued Netrosophic Intuitionistic Fuzzy Graph then at least G_1 or G_2 must be strong .

Proof:

Suppose that G_1 and G_2 are not SIVNIFG, there exist $x_i y_i \in E_i$, i=1,2 such that

1) $(T_{A_1L} \circ T_{A_2L})(x_1, x_2) \leq \min(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \circ T_{A_2U})(x_1, x_2) \leq \min(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(I_{A_1L} \circ I_{A_2L})(x_1, x_2) \geq \max(I_{A_1L}(x_1), I_{A_2L}(x_2))$

$(I_{A_1U} \circ I_{A_2U})(x_1, x_2) \geq \max(I_{A_1U}(x_1), I_{A_2U}(x_2))$

$(F_{A_1L} \circ F_{A_2L})(x_1, x_2) \geq \max(F_{A_1L}(x_1), F_{A_2L}(x_2))$

$(F_{A_1U} \circ F_{A_2U})(x_1, x_2) \geq \max(F_{A_1U}(x_1), F_{A_2U}(x_2))$.

$$E = \{(x, x_2)(x, y_2) / x_1 \in v_1, x_2 y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in v_2, x_1 y_1 \in E_1\}$$

Consider, $(x, x_2)(x, y_2) \in E,$ we have

$$(T_{B_1L} \circ T_{B_2L})((x, x_2)(x, y_2)) = \min(T_{A_1L}(x), T_{B_2L}(x_2 y_2))$$
$$\leq \min(T_{A_1L}(x), \min(T_{A_2L}(x_2), T_{A_2L}(y_2)))$$
$$= \min(\min(T_{A_1L}(x), T_{A_2L}(x_2))), \min(\min(T_{A_1L}(x), T_{A_2L}(y_2)))$$
$$= \min((T_{A_1L} \circ T_{A_2L})(x, x_2), (T_{A_1L} \circ T_{A_2L})(x, y_2)),$$

$$(T_{B_1U} \circ T_{B_2U})((x, x_2)(x, y_2)) = \min(T_{A_1U}(x), T_{B_2U}(x_2 y_2))$$
$$\leq \min(T_{A_1U}(x), \min(T_{A_2U}(x_2), T_{A_2U}(y_2)))$$
$$= \min(\min(T_{A_1U}(x), T_{A_2U}(x_2))), \min(\min(T_{A_1U}(x), T_{A_2U}(y_2)))$$
$$= \min((T_{A_1U} \circ T_{A_2U})(x, x_2), (T_{A_1U} \circ T_{A_2U})(x, y_2))$$

$(T_{A_1L} \circ T_{A_2L})(x_1, x_2) = \min(T_{A_1L}(x_1), T_{A_2L}(x_2))$

$(T_{A_1U} \circ T_{A_2U})(x_1, x_2) = \min(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(I_{A_1L} \circ I_{A_2L})(x_1, x_2) = \max(I_{A_1L}(x_1), I_{A_2L}(x_2))$

$(I_{A_1U} \circ I_{A_2U})(x_1, x_2) = \max(I_{A_1U}(x_1), I_{A_2U}(x_2))$

$(F_{A_1L} \circ F_{A_2L})(x_1, x_2) = \max(F_{A_1L}(x_1), F_{A_2L}(x_2))$

$(F_{A_1U} \circ F_{A_2U})(x_1, x_2) = \max(F_{A_1U}(x_1), F_{A_2U}(x_2))$

$\min((T_{A_1U} \circ T_{A_2U})(x, x_2), (T_{A_1U} \circ T_{A_2U})(x, y_2)) =$

$\min(\min(T_{A_1U}(x), T_{A_2U}(x_2)), \min(T_{A_1U}(x), T_{A_2U}(y_2)))$

$= \min(T_{A_1U}(x), T_{A_2U}(x_2), T_{A_2U}(y_2))$

$\max((I_{A_1U} \circ I_{A_2U})(x, x_2), (I_{A_1U} \circ I_{A_2U})(x, y_2)) =$

$\max(\max(I_{A_1U}(x), I_{A_2U}(x_2)), \max(I_{A_1U}(x), I_{A_2U}(y_2)))$

$= \max(I_{A_1U}(x), I_{A_2U}(x_2), I_{A_2U}(y_2))$

Hence,

$(T_{B_1L} \circ T_{B_2L})((x, x_2)(x, y_2)) \leq \min((T_{A_1L} \circ T_{A_2L})(x, x_2), (T_{A_1L} \circ T_{A_2L})(x, y_2))$**,**

$(T_{B_1U} \circ T_{B_2U})((x, x_2)(x, y_2)) \leq \min((T_{A_1U} \circ T_{A_2U})(x, x_2), (T_{A_1U} \circ T_{A_2U})(x, y_2))$**,**

$(I_{B_1L} \circ I_{B_2L})((x, x_2)(x, y_2)) \geq \max((I_{A_1L} \circ I_{A_2L})(x, x_2), (I_{A_1L} \circ I_{A_2L})(x, y_2))$**,**

$(I_{B_1U} \circ I_{B_2U})((x, x_2)(x, y_2)) \geq \max((I_{A_1U} \circ I_{A_2U})(x, x_2), (I_{A_1U} \circ I_{A_2U})(x, y_2))$**,**

Similarly, we can show that

$(F_{B_1L} \circ F_{B_2L})((x, x_2)(x, y_2)) \geq \max((F_{A_1L} \circ F_{A_2L})(x, x_2), (F_{A_1L} \circ F_{A_2L})(x, y_2))$**,**

$(F_{B_1U} \circ F_{B_2U})((x, x_2)(x, y_2)) \geq \max((F_{A_1U} \circ F_{A_2U})(x, x_2), (F_{A_1U} \circ F_{A_2U})(x, y_2))$**.**

Hence, $G_1 \circ G_2$ is not Strong Interval Valued Netrosophic Intuitionistic Fuzzy Graph, Which is a contradiction. This completes the proof.

Conclusion

In this Paper, Cartesian product, Composition and join of two SIVIFG and SIVNIFGs are discussed. Our future plan to extend our research to some other operations on Strong Interval Valued Neutrosophic Intuitionistic Fuzzy Graph.

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