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TODIM Method for Single-Valued Neutrosophic Multiple Attribute Decision Making

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Abstract: Recently, the TODIM has been used to solve multiple attribute decision making (MADM) problems. The single-valued neutrosophic sets (SVNSs) are useful tools to depict the uncertainty of the MADM. In this paper, we will extend the TODIM method to the MADM with the single-valued neutrosophic numbers (SVNNs). Firstly, the definition, comparison, and distance of SVNNs are briefly presented, and the steps of the classical TODIM method for MADM problems are introduced. Then, the extended classical TODIM method is proposed to deal with MADM problems with the SVNNs, and its significant characteristic is that it can fully consider the decision makers' bounded rationality which is a real action in decision making. Furthermore, we extend the proposed model to interval neutrosophic sets (INSs). Finally, a numerical example is proposed.

Keywords: multiple attribute decision making (MADM); single-valued neutrosophic numbers; interval neutrosophic numbers; TODIM method; prospect theory

1. Introduction

Multiple attribute decision making (MADM) is a hot research area of the decision theory domain, which has had wide applications in many fields, and attracted increasing attention [1,2]. Due to the fuzziness and uncertainty of the alternatives in different attributes, attribute values in decision making problems are not always represented as real numbers, and they can be described as fuzzy numbers in more suitable occasions, such as interval-valued numbers [3,4], triangular fuzzy variables [5–8], linguistic variables [9–13] or uncertain linguistic variables [14–21], intuitionistic fuzzy numbers (IFSs) [22–27] or interval-valued intuitionistic fuzzy numbers (IVIFSs) [28–31], and SVNSs [32] or INSs [33]. Since Fuzzy set (FS), which is a very useful tool to process fuzzy information, was firstly proposed by Zadeh [34], it has been regarded as an useful tool to solve MADM [35,36], fuzzy logic [37], and patterns recognition [38]. Atanassov [22] introduced IFSs with the membership degree and non-membership degree, which were extended to IVIFSs [28]. Smarandache [39,40] proposed a neutrosophic set (NS) with truth-membership function, indeterminacy-membership function, and falsity-membership function. Furthermore, the concepts of a SVNS [32] and an INS [33] were presented for actual applications. Ye [41] proposed a simplified neutrosophic set (SNS), including the SVNS and INS. Recently, SNSs (INSs, and SVNSs) have been utilized to solve many MADM problems [42–67].

In order to depict the increasing complexity in the actual world, the DMs' risk attitudes should be taken into consideration to deal with MADM [68–70]. Based on the prospect theory, Gomes and Lima [71] established TODIM (an acronym in Portuguese for Interactive Multi-Criteria Decision Making) method to solve the MADM problems with the DMs' psychological behaviors are considered. Some scholars have paid attention to depict the DMs' attitudinal characters in the MADM [72–74]. Also, some scholars proposed fuzzy TODIM models [75,76], intuitionistic fuzzy

TODIM models [77,78], the Pythagorean fuzzy TODIM approach [68], the multi-hesitant fuzzy linguistic TODIM approach [79,80], the interval type-2 fuzzy TODIM model [81], the intuitionistic linguistic TODIM method [82], and the 2-dimension uncertain linguistic TODIM method [83]. However, there is no scholar to investigate the TODIM model with SVNNS. Therefore, it is very necessary to pay abundant attention to this novel and worthy issue. The aim of this paper is to extend the TODIM idea to solve the MADM with the SVNNs, to fill up this vacancy. In Section 2, we give the basic concepts of SVNSs and the classical TODIM method for MADM problems. In Section 3, we propose the TODIM method for SVN MADM problems. In Section 4, we extend the proposed SVN TODIM method to INNs. In Section 5, an illustrative example is pointed out and some comparative analysis is conducted. We give a conclusion in Section 6.

2. Preliminaries

Some basic concepts and definitions of NSs and SVNSs are introduced.

2.1. NSs and SVNSs

Definition 1 [39,40]. *Let X be a space of points (objects) with a generic element in fix set X, denoted by x. NSs A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x) : X \to]^-0, 1^+[, I_A(x) : X \to]^-0, 1^+[$ and $F_A(x) : X \to]^-0, 1^+[$ and $0^- \le supT_A(x) + supI_A(x) + supF_A(x) \le 3^+$.*

The NSs was difficult to apply to real applications. Wang [32] develop the SNSs.

Definition 2 [32]. *Let X be a space of points (objects); a SVNSs A in X is characterized as the following:*

$$A = \{(x, T_A(x), I_A(x), F_A(x))|x \in X\} \tag{1}$$

where the truth-membership function $T_A(x)$, indeterminacy-membership $I_A(x)$ and falsity-membership function $F_A(x)$, $T_A(x) : X \to [0, 1], I_A(x) : X \to [0, 1]$ and $F_A(x) : X \to [0, 1]$, with the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For convenience, a SVNN can be expressed to be $A = (T_A, I_A, F_A)$, $T_A \in [0, 1], I_A \in [0, 1], F_A \in [0, 1]$, and $0 \le T_A + I_A + F_A \le 3$.

Definition 3 [50]. *Let $A = (T_A, I_A, F_A)$ be a SVNN, a score function $S(A)$ is defined:*

$$S(A) = \frac{(2 + T_A - I_A - F_A)}{3}, S(A) \in [0, 1]. \tag{2}$$

Definition 4 [50]. *Let $A = (T_A, I_A, F_A)$ be a SVNN, an accuracy function $H(A)$ of a SVNN is defined:*

$$H(A) = T_A - F_A, H(A) \in [-1, 1]. \tag{3}$$

to evaluate the degree of accuracy of the SVNN $A = (T_A, I_A, F_A)$, where $H(A) \in [-1, 1]$. The larger the value of $H(A)$ is, the higher the degree of accuracy of the SVNN A.

Zhang et al. [50] gave an order relation between two SVNNs, which is defined as follows:

Definition 5 [50]. *Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be two SVNNs, if $S(A) < S(B)$, then $A < B$; if $S(A) = S(B)$, then*

(1) *if $H(A) = H(B)$, then $A = B$;*
(2) *if $H(A) < H(B)$, then $A < B$.*

Definition 6 [32]. *Let A and B be two SVNNs, the basic operations of SVNNs are:*

(1) $A \oplus B = (T_A + T_B - T_A T_B, I_A I_B, F_A F_B)$;
(2) $A \otimes B = (T_A T_B, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B)$;
(3) $\lambda A = \left(1 - (1 - T_A)^{\lambda}, (I_A)^{\lambda}, (F_A)^{\lambda}\right), \lambda > 0$;
(4) $(A)^{\lambda} = \left((T_A)^{\lambda}, (I_A)^{\lambda}, 1 - (1 - F_A)^{\lambda}\right), \lambda > 0$.

Definition 7 [42]. *Let A and B be two SVNNs, then the normalized Hamming distance between A and B is:*

$$d(A, B) = \frac{1}{3}(|T_A - T_B| + |I_A - I_B| + |F_A - F_B|) \quad (4)$$

2.2. The TODIM Approach

The TODIM approach [71], developed to consider the DM's psychological behavior, can effectively solve the MADM problems. Based on the prospect theory, this approach depicts the dominance of each alternative over others by constructing a function of multi-attribute values [69].

Let $G = \{G_1, G_2, \cdots, G_n\}$ be the attributes, $w = (w_1, w_2, \cdots, w_n)$ be the weight of G_j, $0 \le w_j \le 1$, and $\sum_{j=1}^{n} w_j = 1$. $A = \{A_1, A_2, \cdots, A_m\}$ are alternatives. Let $A = (a_{ij})_{m \times n}$ be a decision matrix, where a_{ij} is given for the alternative A_i under the G_j, $i = 1, 2, \cdots, m$, and $j = 1, 2, \cdots, n$. We set $w_{jr} = w_j / w_r (j, r = 1, 2, \cdots, n)$ are relative weight of G_j to G_r, and $w_r = \max\{w_j | j = 1, 2, \cdots, n\}$, and $0 \le w_{jr} \le 1$.

Then the traditional TODIM model concludes the following computing steps:

Step 1. Normalizing $A = (a_{ij})_{m \times n}$ into $B = (b_{ij})_{m \times n}$.

Step 2. Computing the dominance degree of A_i over every alternative A_t under attribute G_j:

$$\delta(A_i, A_t) = \sum_{j=1}^{n} \phi_j(A_i, A_t),\ (i, t = 1, 2, \cdots, m) \quad (5)$$

where

$$\phi_j(A_i, A_t) = \begin{cases} \sqrt{w_{jr}(b_{ij} - b_{tj}) / \sum_{j=1}^{n} w_{jr}}, & if\ b_{ij} - b_{tj} > 0 \\ 0, & if\ b_{ij} - b_{tj} = 0 \\ -\frac{1}{\theta}\sqrt{\left(\sum_{j=1}^{n} w_{jr}\right)(b_{tj} - b_{ij}) / w_{jr}}, & if\ b_{ij} - b_{tj} < 0 \end{cases} \quad (6)$$

and the parameter θ shows the attenuation factor of the losses. If $b_{ij} - b_{tj} > 0$, then $\phi_j(A_i, A_t)$ represents a gain; if $b_{ij} - b_{tj} < 0$, then $\phi_j(A_i, A_t)$ signifies a loss.

Step 3. Deriving the overall dominance value of A_i by the Equation (7):

$$\phi(A_i) = \frac{\sum_{t=1}^{m} \delta(A_i, A_t) - \min_i\left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\}}{\max_i\left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\} - \min_i\left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\}},\ i = 1, 2, \cdots, m. \quad (7)$$

Step 4. Ranking all alternatives and selecting the most desirable alternative in accordance with $\phi(A_i)$. The alternative with minimum value is the worst. Inversely, the maximum value is the best one.

3. TODIM Method for SVN MADM Problems

Let $A = \{A_1, A_2, \cdots, A_m\}$ be alternatives, and $G = \{G_1, G_2, \cdots, G_n\}$ be attributes. Let $w = (w_1, w_2, \cdots, w_n)$ be the weight of attributes, where $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. Suppose that $R = (r_{ij})_{m\times n} = (T_{ij}, I_{ij}, F_{ij})_{m\times n}$ be a SVN matrix, where $\tilde{r}_{ij} = (T_{ij}, I_{ij}, F_{ij})$, which is an attribute value, given by an expert, for the alternative A_i under G_j, $T_{ij} \in [0,1], I_{ij} \in [0,1], F_{ij} \in [0,1]$, $0 \le T_{ij} + I_{ij} + F_{ij} \le 3$, $i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$.

To solve the MADM problem with single-valued neutrosophic information, we try to present a single-valued neutrosophic TODIM model based on the prospect theory and can depict the DMs' behaviors under risk.

Firstly, we calculate the relative weight of each attribute G_j as:

$$w_{jr} = w_j / w_r,\ j, r = 1, 2, \cdots, n. \tag{8}$$

where w_j is the weight of the attribute of G_j, $w_r = \max\{w_j | j = 1, 2, \cdots, n\}$, and $0 \le w_{jr} \le 1$.

Based on the Equation (8), we can derive the dominance degree of A_i over each alternative A_t with respect to the attribute G_j:

$$\phi_j(A_i, A_t) = \begin{cases} \sqrt{w_{jr} d(r_{ij}, r_{tj}) / \sum_{j=1}^{n} w_{jr}}, & if\ r_{ij} > r_{tj} \\ 0, & if\ r_{ij} = r_{tj} \\ -\frac{1}{\theta}\sqrt{\left(\sum_{j=1}^{n} w_{jr}\right) d(r_{ij}, r_{tj}) / w_{jr}}, & if\ r_{ij} < r_{tj} \end{cases} \tag{9}$$

$$d(r_{ij}, r_{tj}) = \frac{1}{3}\left(|T_{ij} - T_{tj}| + |I_{ij} - I_{tj}| + |F_{ij} - F_{tj}|\right). \tag{10}$$

where the parameter θ shows the attenuation factor of the losses, and $d(r_{ij}, r_{tj})$ is to measure the distances between the SVNNs r_{ij} and r_{tj} by Definition 7. If $r_{ij} > r_{tj}$, then $\phi_j(A_i, A_t)$ represents a gain; if $r_{ij} < r_{tj}$, then $\phi_j(A_i, A_t)$ signifies a loss.

For indicating functions $\phi_j(A_i, A_t)$ clearly, a dominance degree matrix $\phi_j = [\phi_j(A_i, A_t)]_{m\times m}$ under G_j is expressed as:

$$\phi_j = [\phi_j(A_i, A_t)]_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \left[\begin{array}{cccc} o & \phi_j(A_1, A_2) & \cdots & \phi_j(A_1, A_m) \\ \phi_j(A_2, A_1) & 0 & \cdots & \phi_j(A_2, A_m) \\ \vdots & \vdots & \cdots & \vdots \\ \phi_j(A_m, A_1) & \phi_j(A_m, A_2) & \cdots & 0 \end{array}\right] \end{array}, \ j = 1, 2, \cdots, n. \tag{11}$$

On the basis of Equation (11), the overall dominance degree $\delta(A_i, A_t)$ of the A_i over each A_t can be calculated:

$$\delta(A_i, A_t) = \sum_{j=1}^{n} \phi_j(A_i, A_t), \quad (i, t = 1, 2, \cdots, m). \tag{12}$$

Thus, the overall dominance degree matrix $\delta = [\delta(A_i, A_t)]_{m\times m}$ can be derived by Equation (12):

$$\delta = [\delta(A_i, A_t)]_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \left[\begin{array}{cccc} o & \delta(A_1, A_2) & \cdots & \delta(A_1, A_m) \\ \delta(A_2, A_1) & 0 & \cdots & \delta(A_2, A_m) \\ \vdots & \vdots & \cdots & \vdots \\ \delta(A_m, A_1) & \delta(A_m, A_2) & \cdots & 0 \end{array}\right] \end{array}. \tag{13}$$

Then, the overall value of each A_i can be calculated Equation (14):

$$\delta(A_i) = \frac{\sum_{t=1}^{m} \delta(A_i, A_t) - \min_i \left\{ \sum_{t=1}^{m} \delta(A_i, A_t) \right\}}{\max_i \left\{ \sum_{t=1}^{m} \delta(A_i, A_t) \right\} - \min_i \left\{ \sum_{t=1}^{m} \delta(A_i, A_t) \right\}}, \quad i = 1, 2, \cdots, m. \tag{14}$$

Also the greater the overall value $\delta(A_i)$, the better the alternative A_i.

In general, single-valued neutrosophic TODIM model includes the computing steps:

(Procedure one)

Step 1. Identifying the single-valued neutrosophic matrix $R = (r_{ij})_{m\times n} = (T_{ij}, I_{ij}, F_{ij})_{m\times n}$ in the MADM, where r_{ij} is a SVNN.

Step 2. Calculating the relative weight of G_j by using Equation (8).

Step 3. Calculating the dominance degree $\phi_j(A_i, A_t)$ of A_i over each alternative A_t under attribute G_j by Equation (9).

Step 4. Calculating the overall dominance degree $\delta(A_i, A_t)$ of A_i over each alternative A_t by using Equation (12).

Step 5. Deriving the overall value $\delta(A_i)$ of each alternative A_i using Equation (14).

Step 6. Determining the order of the alternatives in accordance with $\delta(A_i)(i = 1, 2, \cdots, m)$.

4. TODIM Method for Interval Neutrosophic MADM Problems

Furthermore, Wang et al. [33] defined INSs.

Definition 8 [33]**.** *Let X be a space of points (objects) with a generic element in fix set X, an INSs $\widetilde{A}$ in X is characterized as follows:*

$$\widetilde{A} = \left\{ \left(x, T_{\widetilde{A}}(x), I_{\widetilde{A}}(x), F_{\widetilde{A}}(x) \right) | x \in X \right\} \tag{15}$$

where truth-membership function $T_{\widetilde{A}}(x)$, indeterminacy-membership $I_{\widetilde{A}}(x)$ and falsity-membership function $F_{\widetilde{A}}(x)$ are interval values, $T_A(x) \subseteq [0,1], I_A(x) \subseteq [0,1]$ and $F_A(x) \subseteq [0,1]$, and $0 \le \sup\left(T_{\widetilde{A}}(x)\right) + \sup\left(I_{\widetilde{A}}(x)\right) + \sup\left(F_{\widetilde{A}}(x)\right) \le 3$.

An interval neutrosophic number (INN) can be expressed as $\widetilde{A} = \left(T_{\widetilde{A}}, I_{\widetilde{A}}, F_{\widetilde{A}}\right) = \left(\left[T_{\widetilde{A}}^L, T_{\widetilde{A}}^R\right], \left[I_{\widetilde{A}}^L, I_{\widetilde{A}}^R\right], \left[F_{\widetilde{A}}^L, F_{\widetilde{A}}^R\right] \right)$, where $\left[T_{\widetilde{A}}^L, T_{\widetilde{A}}^R\right] \subseteq [0,1], \left[I_{\widetilde{A}}^L, I_{\widetilde{A}}^R\right] \subseteq [0,1], \left[F_{\widetilde{A}}^L, F_{\widetilde{A}}^R\right] \subseteq [0,1]$, and $0 \le T_{\widetilde{A}}^R + I_{\widetilde{A}}^R + F_{\widetilde{A}}^R \le 3$.

Definition 9 [84]**.** *Let $\widetilde{A} = \left(\left[T_{\widetilde{A}}^L, T_{\widetilde{A}}^R\right], \left[I_{\widetilde{A}}^L, I_{\widetilde{A}}^R\right], \left[F_{\widetilde{A}}^L, F_{\widetilde{A}}^R\right] \right)$ be an INN, a score function S of an INN can be represented as follows:*

$$S\left(\widetilde{A}\right) = \frac{\left(2 + T_{\widetilde{A}}^L - I_{\widetilde{A}}^L - F_{\widetilde{A}}^L\right) + \left(2 + T_{\widetilde{A}}^R - I_{\widetilde{A}}^R - F_{\widetilde{A}}^R\right)}{6}, S\left(\widetilde{A}\right) \in [0,1]. \tag{16}$$

Definition 10 [84]**.** *Let $\widetilde{A} = \left(\left[T_{\widetilde{A}}^L, T_{\widetilde{A}}^R\right], \left[I_{\widetilde{A}}^L, I_{\widetilde{A}}^R\right], \left[F_{\widetilde{A}}^L, F_{\widetilde{A}}^R\right] \right)$ be an INN, an accuracy function $H\left(\widetilde{A}\right)$ is defined:*

$$H\left(\widetilde{A}\right) = \frac{\left(T_{\widetilde{A}}^L + T_{\widetilde{A}}^R\right) - \left(F_{\widetilde{A}}^L + F_{\widetilde{A}}^R\right)}{2}, H\left(\widetilde{A}\right) \in [-1,1]. \tag{17}$$

Tang [84] defined an order relation between two INNs.

Definition 11 [84]. *Let* $\widetilde{A} = \left(\left[T^L_{\widetilde{A}}, T^R_{\widetilde{A}}\right], \left[I^L_{\widetilde{A}}, I^R_{\widetilde{A}}\right], \left[F^L_{\widetilde{A}}, F^R_{\widetilde{A}}\right]\right)$ *and* $\widetilde{B} = \left(\left[T^L_{\widetilde{B}}, T^R_{\widetilde{B}}\right], \left[I^L_{\widetilde{B}}, I^R_{\widetilde{B}}\right], \left[F^L_{\widetilde{B}}, F^R_{\widetilde{B}}\right]\right)$ *be two INNs,* $S\left(\widetilde{A}\right) = \frac{\left(2+T^L_{\widetilde{A}}-I^L_{\widetilde{A}}-F^L_{\widetilde{A}}\right)+\left(2+T^R_{\widetilde{A}}-I^R_{\widetilde{A}}-F^R_{\widetilde{A}}\right)}{6}$ *and* $S\left(\widetilde{B}\right) = \frac{\left(2+T^L_{\widetilde{B}}-I^L_{\widetilde{B}}-F^L_{\widetilde{B}}\right)+\left(2+T^R_{\widetilde{B}}-I^R_{\widetilde{B}}-F^R_{\widetilde{B}}\right)}{6}$ *be the scores, and* $H\left(\widetilde{A}\right) = \frac{\left(T^L_{\widetilde{A}}+T^R_{\widetilde{A}}\right)-\left(F^L_{\widetilde{A}}+F^R_{\widetilde{A}}\right)}{2}$ *and* $H\left(\widetilde{B}\right) = \frac{\left(T^L_{\widetilde{B}}+T^R_{\widetilde{B}}\right)-\left(F^L_{\widetilde{B}}+F^R_{\widetilde{B}}\right)}{2}$ *be the accuracy function, then if* $S\left(\widetilde{A}\right) < S\left(\widetilde{B}\right)$, *then* $\widetilde{A} < \widetilde{B}$; *if* $S\left(\widetilde{A}\right) = S\left(\widetilde{B}\right)$, *then*

(1) *if* $H\left(\widetilde{A}\right) = H\left(\widetilde{B}\right)$, *then* $\widetilde{A} = \widetilde{B}$;

(2) *if* $H\left(\widetilde{A}\right) < H\left(\widetilde{B}\right)$, $\widetilde{A} < \widetilde{B}$.

Definition 12 [33,61]. *Let* $\widetilde{A}_1 = ([T^L_1, T^R_1], [I^L_1, I^R_1], [F^L_1, F^R_1])$ *and* $\widetilde{A}_2 = ([T^L_2, T^R_2], [I^L_2, I^R_2], [F^L_2, F^R_2])$ *be two INNs, and some basic operations on them are defined as follows:*

(1) $\widetilde{A}_1 \oplus \widetilde{A}_2 = ([T^L_1 + T^L_1 - T^L_1 T^L_1, T^R_1 + T^R_1 - T^R_1 T^R_1], [I^L_1 I^L_2, I^R_1 I^R_2], [F^L_1 F^L_2, F^R_1 F^R_2])$;

(2) $\widetilde{A}_1 \otimes \widetilde{A}_2 = \left(\begin{array}{c} [T^L_1 T^L_2, T^R_1 T^R_2], [I^L_1 + I^L_1 - I^L_1 I^L_1, I^R_1 + I^R_1 - I^R_1 I^R_1], \\ [F^L_1 + F^L_1 - F^L_1 F^L_1, F^R_1 + F^R_1 - F^R_1 F^R_1] \end{array}\right)$;

(3) $\lambda \widetilde{A}_1 = \left(\left[1 - (1 - T^L_1)^\lambda, 1 - (1 - T^R_1)^\lambda\right], \left[(I^L_1)^\lambda, (I^R_1)^\lambda\right], \left[(F^L_1)^\lambda, (F^R_1)^\lambda\right]\right), \lambda > 0$;

(4) $\left(\widetilde{A}_1\right)^\lambda = \left(\left[(T^L_1)^\lambda, (T^R_1)^\lambda\right], \left[(I^L_1)^\lambda, (I^R_1)^\lambda\right], \left[1 - (1 - F^L_1)^\lambda, 1 - (1 - F^R_1)^\lambda\right]\right), \lambda > 0.$

Definition 13 [84]. *Let* $\widetilde{A}_1 = ([T^L_1, T^R_1], [I^L_1, I^R_1], [F^L_1, F^R_1])$ *and* $\widetilde{A}_2 = ([T^L_2, T^R_2], [I^L_2, I^R_2], [F^L_2, F^R_2])$ *be two INNs, then the normalized Hamming distance between* $\widetilde{A}_1 = ([T^L_1, T^R_1], [I^L_1, I^R_1], [F^L_1, F^R_1])$ *and* $\widetilde{A}_2 = ([T^L_2, T^R_2], [I^L_2, I^R_2], [F^L_2, F^R_2])$ *is defined as follows:*

$$d\left(\widetilde{A}_1, \widetilde{A}_2\right) = \frac{1}{6}\left(\left|T^L_1 - T^L_2\right| + \left|T^R_1 - T^R_2\right| + \left|I^L_1 - I^L_2\right| + \left|I^R_1 - I^R_2\right| + \left|F^L_1 - F^L_2\right| + \left|F^R_1 - F^R_2\right|\right) \quad (18)$$

Let A, G *and* w *be presented as in Section 3. Suppose that* $\widetilde{R} = (\widetilde{r}_{ij})_{m \times n} = \left(\left[T^L_{ij}, T^R_{ij}\right], \left[I^L_{ij}, I^R_{ij}\right], \left[F^L_{ij}, F^R_{ij}\right]\right)_{m \times n}$ *is the interval neutrosophic decision matrix, where* $\left[T^L_{ij}, T^R_{ij}\right], \left[I^L_{ij}, I^R_{ij}\right], \left[F^L_{ij}, F^R_{ij}\right]$ *is truth-membership function, indeterminacy-membership function and falsity-membership function,* $\left[T^L_{ij}, T^R_{ij}\right] \subseteq [0,1]$, $\left[I^L_{ij}, I^R_{ij}\right] \subseteq [0,1]$, $\left[F^L_{ij}, F^R_{ij}\right] \subseteq [0,1]$, $0 \leq T^R_{ij} + I^R_{ij} + F^R_{ij} \leq 3$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$.

To cope with the MADM with INNs, we develop interval neutrosophic TODIM model.

Firstly, we calculate the relative weight of each attribute G_j as:

$$w_{jr} = w_j / w_r, j, r = 1, 2, \cdots, n \quad (19)$$

where w_j is the weight of the attribute of G_j, $w_r = \max\{w_j | j = 1, 2, \cdots, n\}$, and $0 \leq w_{jr} \leq 1$.

Based on the Equation (20), we can derive the dominance degree of A_i over each alternative A_t with respect to the attribute G_j:

$$\phi_j(A_i, A_t) = \begin{cases} \sqrt{w_{jr} d(\widetilde{r}_{ij}, \widetilde{r}_{tj}) / \sum\limits_{j=1}^{n} w_{jr}}, & if\ \widetilde{r}_{ij} > \widetilde{r}_{tj} \\ 0, & if\ \widetilde{r}_{ij} = \widetilde{r}_{tj} \\ -\frac{1}{\theta}\sqrt{\left(\sum\limits_{j=1}^{n} w_{jr}\right) d(\widetilde{r}_{ij}, \widetilde{r}_{tj}) / w_{jr}}, & if\ \widetilde{r}_{ij} < \widetilde{r}_{tj} \end{cases} \quad (20)$$

$$d(\widetilde{r}_{ij},\widetilde{r}_{tj}) = \frac{1}{6}\left(\left|T_{ij}^{L}-T_{tj}^{L}\right|+\left|T_{ij}^{R}-T_{tj}^{R}\right|+\left|I_{ij}^{L}-I_{tj}^{L}\right|+\left|I_{ij}^{R}-I_{tj}^{R}\right|+\left|F_{ij}^{L}-F_{tj}^{L}\right|+\left|F_{ij}^{R}-F_{tj}^{R}\right|\right). \quad (21)$$

where the parameter θ shows the attenuation factor of the losses, and $d(\widetilde{r}_{ij},\widetilde{r}_{tj})$ is to measure the distances between the INNs $\widetilde{r}_{ij}$ and $\widetilde{r}_{tj}$ by Definition 13. If $\widetilde{r}_{ij} > \widetilde{r}_{tj}$, then $\phi_j(A_i, A_t)$ represents a gain; if $\widetilde{r}_{ij} < \widetilde{r}_{tj}$, then $\phi_j(A_i, A_t)$ signifies a loss.

For indicating functions $\phi_j(A_i, A_t)$ clearly, a dominance degree matrix $\phi_j = [\phi_j(A_i, A_t)]_{m\times m}$ under G_j is expressed as:

$$\phi_j = [\phi_j(A_i, A_t)]_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \left[\begin{array}{cccc} o & \phi_j(A_1,A_2) & \cdots & \phi_j(A_1,A_m) \\ \phi_j(A_2,A_1) & 0 & \cdots & \phi_j(A_2,A_m) \\ \vdots & \vdots & \cdots & \vdots \\ \phi_j(A_m,A_1) & \phi_j(A_m,A_2) & \cdots & 0 \end{array}\right] \end{array}, j = 1,2,\cdots,n \quad (22)$$

On the basis of Equation (22), the overall dominance degree $\delta(A_i, A_t)$ of the A_i over each A_t can be calculated:

$$\delta(A_i, A_t) = \sum_{j=1}^{n} \phi_j(A_i, A_t), \quad (i, t = 1, 2, \cdots, m) \quad (23)$$

Thus, the overall dominance degree matrix $\delta = [\delta(A_i, A_t)]_{m\times m}$ can be derived by Equation (23):

$$\delta = [\delta(A_i, A_t)]_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \left[\begin{array}{cccc} o & \delta(A_1,A_2) & \cdots & \delta(A_1,A_m) \\ \delta(A_2,A_1) & 0 & \cdots & \delta(A_2,A_m) \\ \vdots & \vdots & \cdots & \vdots \\ \delta(A_m,A_1) & \delta(A_m,A_2) & \cdots & 0 \end{array}\right] \end{array} \quad (24)$$

Then, the overall value of each A_i can be calculated Equation (25):

$$\delta(A_i) = \frac{\sum_{t=1}^{m} \delta(A_i, A_t) - \min_i\left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\}}{\max_i\left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\} - \min_i\left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\}}, \ i = 1, 2, \cdots, m. \quad (25)$$

Also the greater the overall value $\delta(A_i)$, the better the alternative A_i.

In general, interval neutrosophic TODIM model includes the computing steps:

(Procedure two)

Step 1. Identifying the interval neutrosophic matrix $\widetilde{R} = (\widetilde{r}_{ij})_{m\times n} = \left(\left[T_{ij}^{L}, T_{ij}^{R}\right], \left[I_{ij}^{L}, I_{ij}^{R}\right], \left[F_{ij}^{L}, F_{ij}^{R}\right]\right)_{m\times n}$ in the MADM, where $\widetilde{r}_{ij}$ is an INN.

Step 2. Calculating the relative weight of G_j by using Equation (19).

Step 3. Calculating the dominance degree $\phi_j(A_i, A_t)$ of A_i over each alternative A_t under attribute G_j by Equation (20).

Step 4. Calculating the overall dominance degree $\delta(A_i, A_t)$ of A_i over each alternative A_t by using Equation (23).

Step 5. Deriving the overall value $\delta(A_i)$ of each alternative A_i using Equation (25).

Step 6. Determining the order of the alternatives in accordance with $\delta(A_i)(i = 1, 2, \cdots, m)$.

5. Numerical Example and Comparative Analysis

5.1. Numerical Example 1

In this part, a numerical example is given to show potential evaluation of emerging technology commercialization with SVNNs. Five possible emerging technology enterprises (ETEs) $A_i (i = 1, 2, 3, 4, 5)$ are to be evaluated and selected. Four attributes are selected to evaluate the five possible ETEs: ① G_1 is the employment creation; ② G_2 is the development of science and technology; ③ G_3 is the technical advancement; and ④ G_4 is the industrialization infrastructure. The five ETEs $A_i (i = 1, 2, 3, 4, 5)$ are to be evaluated by using the SVNNs under the above four attributes (whose weighting vector $\omega = (0.2, 0.1, 0.3, 0.4)^T$), as listed in the following matrix.

$$\widetilde{R} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{bmatrix} (0.5,0.8,0.1) & (0.6,0.3,0.3) & (0.3,0.6,0.1) & (0.5,0.7,0.2) \\ (0.7,0.2,0.1) & (0.7,0.2,0.2) & (0.7,0.2,0.4) & (0.8,0.2,0.1) \\ (0.6,0.7,0.2) & (0.5,0.7,0.3) & (0.5,0.3,0.1) & (0.6,0.3,0.2) \\ (0.8,0.1,0.3) & (0.6,0.3,0.4) & (0.3,0.4,0.2) & (0.5,0.6,0.1) \\ (0.6,0.4,0.4) & (0.4,0.8,0.1) & (0.7,0.6,0.1) & (0.5,0.8,0.2) \end{bmatrix} \end{array}$$

Then, we use **Procedure One** to select the best ETE.

Firstly, since $w_4 = \max\{w_1, w_2, w_3, w_4\}$, then G_4 is the reference attribute and the reference attribute's weight is $w_r = 0.4$. Then, we can calculate the relative weights of the attributes $G_j (j = 1, 2, 3, 4)$ as $w_{1r} = 0.50, w_{2r} = 0.25, w_{3r} = 0.75$ and $w_{4r} = 1.00$. Let $\theta = 2.5$, then the dominance degree matrix $\phi_j(A_i, A_t)(j = 1, 2, 3, 4)$ with respect to G_j can be calculated:

$$\phi_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.0000 & -0.4619 & -0.2828 & -0.5657 & -0.4619 \\ 0.2309 & 0.0000 & 0.2160 & 0.1633 & 0.2000 \\ 0.1414 & -0.4320 & 0.0000 & -0.4899 & -0.3651 \\ 0.2828 & -0.3266 & 0.2449 & 0.0000 & 0.2000 \\ 0.2309 & -0.4000 & 0.1826 & -0.4000 & 0.0000 \end{bmatrix} \end{array}$$

$$\phi_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.0000 & -0.4000 & 0.1291 & 0.0577 & 0.1732 \\ 0.1000 & 0.0000 & 0.1633 & 0.1155 & 0.1826 \\ -0.5164 & -0.6532 & 0.0000 & -0.5657 & -0.4619 \\ -0.2309 & -0.4619 & 0.1414 & 0.0000 & 0.1826 \\ -0.6928 & -0.7303 & 0.1155 & -0.7303 & 0.0000 \end{bmatrix} \end{array}$$

$$\phi_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.0000 & -0.4422 & -0.2981 & -0.2309 & -0.2667 \\ 0.3317 & 0.0000 & -0.3266 & 0.2828 & 0.2646 \\ 0.2236 & 0.2449 & 0.0000 & 0.2000 & 0.2236 \\ 0.1732 & -0.3771 & -0.2667 & 0.0000 & -0.3528 \\ 0.2000 & -0.3528 & -0.2981 & 0.2646 & 0.0000 \end{bmatrix} \end{array}$$

$$\phi_4 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.0000 & -0.3464 & -0.2582 & -0.1633 & 0.1155 \\ 0.3464 & 0.0000 & 0.2309 & 0.3055 & 0.3651 \\ 0.2582 & -0.2309 & 0.0000 & 0.2582 & 0.2828 \\ 0.1633 & -0.3055 & -0.2582 & 0.0000 & 0.2000 \\ -0.1155 & -0.3651 & -0.2828 & -0.2000 & 0.0000 \end{bmatrix} \end{array}$$

The overall dominance degree $\delta(A_i, A_t)$ of the candidate A_i over each candidate A_t can be derived by Equation (13):

$$\delta = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.0000 & -1.6505 & -0.7100 & -0.9022 & -0.4399 \\ 1.0090 & 0.0000 & 0.2836 & 0.8671 & 1.01234 \\ 0.1068 & -1.0712 & 0.0000 & -0.5974 & -0.3206 \\ 0.3884 & -1.4711 & -0.1386 & 0.0000 & 0.2298 \\ -0.3774 & -1.8482 & -0.2828 & -1.0657 & 0.0000 \end{bmatrix} \end{array}$$

Then, we get the overall value $\delta(A_i)(i = 1, 2, 3, 4, 5)$ by using Equation (14):

$$\delta(A_1) = 0.0000, \delta(A_2) = 1.0000, \delta(A_3) = 0.2648$$
$$\delta(A_4) = 0.3944, \delta(A_5) = 0.0187$$

Finally, we get order of ETEs by $\delta(A_i)(i = 1, 2, 3, 4, 5)$: $A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$, and thus the most desirable ETE is A_2.

5.2. Comparative Analysis 1

In what follows, we compare our proposed method with other existing methods including the SVNWA operator and SVNWG operator proposed by Sahin [85] as follows:

Definition 14 [85]. *Let* $A_j = (T_j, I_j, F_j)$ $(j = 1, 2, \cdots, n)$ *be a collection of SVNNs,* $w = (w_1, w_2, \cdots, w_n)^T$ *be the weight of* $A_j(j = 1, 2, \cdots, n)$*, and* $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$ *. Then*

$$\begin{aligned} r_i &= (T_i, I_i, F_i) \\ &= \mathrm{SVNWA}_w(r_{i1}, r_{i2}, \cdots, r_{in}) = \mathop{\oplus}_{j=1}^{n} (w_j r_{ij}) \\ &= \left(1 - \prod_{j=1}^{n} (1 - T_{ij})^{w_j}, \prod_{j=1}^{n} (I_{ij})^{w_j}, \prod_{j=1}^{n} (F_{ij})^{w_j}\right) \end{aligned} \tag{26}$$

$$\begin{aligned} r_i &= (T_i, I_i, F_i) \\ &= \mathrm{SVNWG}_\omega(r_{i1}, r_{i2}, \cdots, r_{in}) = \mathop{\otimes}_{j=1}^{n} (r_{ij})^{w_j} \\ &= \left(\prod_{j=1}^{n} (T_{ij})^{w_j}, 1 - \prod_{j=1}^{n} (1 - I_{ij})^{w_j}, 1 - \prod_{j=1}^{n} (1 - F_{ij})^{w_j}\right) \end{aligned} \tag{27}$$

By utilizing the $\widetilde{R}$, as well as the SVNWA and SVNWG operators, the aggregating values are derived in Table 1.

Table 1. The aggregating values of the emerging technology enterprises by the SVNWA (SVNWG) operators.

	SVNWA	SVNWG
A_1	(0.4591, 0.6307, 0.1473)	(0.4369, 0.6718, 0.1627)
A_2	(0.7449, 0.2000, 0.1625)	(0.7384, 0.2000, 0.2124)
A_3	(0.5627, 0.3868, 0.1692)	(0.5578, 0.4571, 0.1822)
A_4	(0.5497, 0.3464, 0.1762)	(0.4799, 0.4381, 0.2067)
A_5	(0.5822, 0.6389, 0.1741)	(0.5610, 0.6933, 0.2083)

According to the aggregating results in Table 1, the score functions are listed in Table 2.

Table 2. The score functions of the emerging technology enterprises.

	SVNWA	SVNWG
A_1	0.5604	0.5341
A_2	0.7942	0.7753
A_3	0.6689	0.6398
A_4	0.6757	0.6117
A_5	0.5898	0.5531

According to the score functions shown in Table 2, the order of the emerging technology enterprises are in Table 3.

Table 3. Order of the emerging technology enterprises.

	Order
SVNWA	$A_2 > A_4 > A_3 > A_5 > A_1$
SVNWG	$A_2 > A_3 > A_4 > A_5 > A_1$

From the above analysis, it can be seen that two operators have the same best emerging technology enterprise A_2 and two methods' ranking results are slightly different. However, the SVN TODIM approach can reasonably depict the DMs' psychological behaviors under risk, and thus, it may deal with the above issue effectively. This verifies the method we proposed is reasonable and effective in this paper.

5.3. Numerical Example 2

If the five possible emerging technology enterprises $A_i (i = 1, 2, 3, 4, 5)$ are to be evaluated by using the INNS under the above four attributes (whose weighting vector $\omega = (0.2, 0.1, 0.3, 0.4)^T$), as listed in the matrix $\widetilde{R}$, then:

$$\widetilde{R} = \begin{bmatrix} ([0.5,0.6],[0.8,0.9],[0.1,0.2]) & ([0.6,0.7],[0.3,0.4],[0.3,0.4]) & ([0.3,0.4],[0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.7,0.8],[0.1,0.2]) \\ ([0.7,0.9],[0.2,0.3],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2],[0.2,0.3]) & ([0.7,0.9],[0.2,0.3],[0.4,0.5]) & ([0.8,0.9],[0.2,0.3],[0.1,0.2]) \\ ([0.6,0.7],[0.7,0.8],[0.2,0.3]) & ([0.5,0.6],[0.7,0.8],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4],[0.1,0.2]) & ([0.6,0.7],[0.3,0.4],[0.2,0.3]) \\ ([0.8,0.9],[0.1,0.2],[0.3,0.4]) & ([0.6,0.7],[0.3,0.4],[0.4,0.5]) & ([0.3,0.4],[0.4,0.5],[0.2,0.3]) & ([0.5,0.6],[0.6,0.7],[0.1,0.2]) \\ ([0.6,0.7],[0.4,0.5],[0.4,0.5]) & ([0.4,0.5],[0.8,0.9],[0.1,0.2]) & ([0.7,0.8],[0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.8,0.9],[0.2,0.3]) \end{bmatrix}$$

Then, we use **Procedure Two** to select the best ETE.

Firstly, since $w_4 = \max\{w_1, w_2, w_3, w_4\}$, then G_4 is the reference attribute and the reference attribute's weight is $w_r = 0.4$. Then, we can calculate the relative weights of the attributes

$G_j (j = 1,2,3,4)$ as: $w_{1r} = 0.50, w_{2r} = 0.25, w_{3r} = 0.75$ and $w_{4r} = 1.00$. Let $\theta = 2.5$, then the dominance degree matrix $\phi_j(A_i, A_t)(j = 1,2,3,4)$ with respect to G_j can be calculated:

$$\phi_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left[\begin{array}{ccccc} 0.0000 & -0.4761 & -0.2828 & -0.5657 & -0.4619 \\ 0.2380 & 0.0000 & 0.2236 & 0.1528 & 0.2082 \\ 0.1414 & -0.4472 & 0.0000 & -0.4899 & -0.3651 \\ 0.2828 & -0.3055 & 0.2449 & 0.0000 & 0.2000 \\ 0.2309 & -0.4163 & 0.1826 & -0.4000 & 0.0000 \end{array}\right] \end{array}$$

$$\phi_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left[\begin{array}{ccccc} 0.0000 & -0.4619 & 0.1291 & 0.0577 & 0.1732 \\ 0.1155 & 0.0000 & 0.1732 & 0.1291 & 0.1915 \\ -0.5164 & -0.6928 & 0.0000 & -0.5657 & -0.4619 \\ -0.2309 & -0.5164 & 0.1414 & 0.0000 & 0.1826 \\ -0.6928 & -0.7659 & 0.1155 & -0.7303 & 0.0000 \end{array}\right] \end{array}$$

$$\phi_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left[\begin{array}{ccccc} 0.0000 & -0.4522 & -0.2981 & -0.2309 & -0.2667 \\ 0.3391 & 0.0000 & 0.2550 & 0.2915 & 0.2739 \\ 0.2236 & -0.3399 & 0.0000 & 0.2000 & 0.2236 \\ 0.1732 & -0.3887 & -0.2667 & 0.0000 & -0.3528 \\ 0.2000 & -0.3651 & -0.2981 & 0.2646 & 0.0000 \end{array}\right] \end{array}$$

$$\phi_4 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left[\begin{array}{ccccc} 0.0000 & -0.3266 & -0.2828 & -0.1155 & 0.1633 \\ 0.3266 & 0.0000 & 0.2309 & 0.3055 & 0.3651 \\ 0.2828 & -0.2309 & 0.0000 & 0.2582 & 0.2828 \\ 0.1155 & -0.3055 & -0.2582 & 0.0000 & 0.2000 \\ -0.1633 & -0.3651 & -0.2828 & -0.2000 & 0.0000 \end{array}\right] \end{array}$$

The overall dominance degree $\delta(A_i, A_t)$ of the candidate A_i over each candidate A_t can be derived by Equation (24):

$$\delta = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left[\begin{array}{ccccc} 0.0000 & -1.7168 & -0.7346 & -0.7506 & 0.0698 \\ 1.0192 & 0.0000 & 0.3727 & 0.3513 & 0.8305 \\ 0.1314 & -1.0310 & 0.0000 & -0.4726 & 0.0445 \\ 0.3406 & -1.5161 & -0.1386 & 0.2000 & 0.0298 \\ -0.4252 & -1.9124 & -0.8654 & -0.6657 & 0.0000 \end{array}\right] \end{array}$$

Then, we get the overall value $\delta(A_i)(i = 1,2,3,4,5)$ by using Equation (25):

$$\delta(A_1) = 0.1143, \delta(A_2) = 1.0000, \delta(A_3) = 0.3944$$
$$\delta(A_4) = 0.4322, \delta(A_5) = 0.0000$$

Finally, we get order of ETEs by $\delta(A_i)(i = 1,2,3,4,5)$: $A_2 \succ A_4 \succ A_3 \succ A_1 \succ A_5$, and thus the most desirable ETE is A_2.

5.4. Comparative Analysis 2

In what follows, we compare our proposed method with other existing methods including the INWA operator and INWG operator proposed by Zhang et al. [50] as follows:

Definition 15 [50]. *Let* $\widetilde{A}_j = \left(\left[T_j^L, T_j^R\right], \left[I_j^L, I_j^R\right], \left[F_j^L, F_j^R\right]\right)(j = 1, 2, \cdots, n)$ *be a collection of INNs,* $w = (w_1, w_2, \cdots, w_n)^T$ *be the weight of* $A_j(j = 1, 2, \cdots, n)$*, and* $w_j > 0$*,* $\sum_{j=1}^{n} w_j = 1$ *. Then*

$$\begin{aligned}
\widetilde{r}_i &= \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right) \\
&= \mathrm{INWA}_w(\widetilde{r}_{i1}, \widetilde{r}_{i2}, \cdots, \widetilde{r}_{in}) = \bigoplus_{j=1}^{n} (w_j \widetilde{r}_{ij}) \\
&= \left(\begin{array}{l} \left[1 - \prod_{j=1}^{n}\left(1 - T_{ij}^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - T_{ij}^R\right)^{w_j}\right], \\ \left[\prod_{j=1}^{n}\left(I_{ij}^L\right)^{w_j}, \prod_{j=1}^{n}\left(I_{ij}^R\right)^{w_j}\right], \left[\prod_{j=1}^{n}\left(F_{ij}^L\right)^{w_j}, \prod_{j=1}^{n}\left(F_{ij}^R\right)^{w_j}\right] \end{array} \right)
\end{aligned} \tag{28}$$

$$\begin{aligned}
\widetilde{r}_i &= \left(\left[T_i^L, T_i^R\right], \left[I_i^L, I_i^R\right], \left[F_i^L, F_i^R\right]\right) \\
&= \mathrm{INWG}_w(\widetilde{r}_{i1}, \widetilde{r}_{i2}, \cdots, \widetilde{r}_{in}) = \bigotimes_{j=1}^{n} (\widetilde{r}_{ij})^{w_j} \\
&= \left(\begin{array}{l} \left[\prod_{j=1}^{n}\left(T_{ij}^L\right)^{w_j}, \prod_{j=1}^{n}\left(T_{ij}^R\right)^{w_j}\right], \left[1 - \prod_{j=1}^{n}\left(1 - I_{ij}^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - I_{ij}^R\right)^{w_j}\right], \\ \left[1 - \prod_{j=1}^{n}\left(1 - F_{ij}^L\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - F_{ij}^R\right)^{w_j}\right] \end{array} \right)
\end{aligned} \tag{29}$$

By utilizing the decision matrix $\widetilde{R}$, and the INWA and INWG operators, the aggregating values are in Table 4.

Table 4. The aggregating values of the emerging technology enterprises by the INWA and INWG operators.

	INWA
A_1	([0.4591, 0.5611], [0.6307, 0.7342], [0.1116, 0.2144])
A_2	([0.7449, 0.8928], [0.1866, 0.2881], [0.1625, 0.2742])
A_3	([0.5627, 0.6634], [0.3868, 0.4925], [0.1692, 0.2734])
A_4	([0.5497, 0.6674], [0.3464, 0.4657], [0.1762, 0.2844])
A_5	([0.5822, 0.6863], [0.6389, 0.7421], [0.1741, 0.2825])
	INWG
A_1	([0.4369, 0.5395], [0.6718, 0.7805], [0.1223, 0.2227])
A_2	([0.7384, 0.8895], [0.1905, 0.2906], [0.2124, 0.3144])
A_3	([0.5578, 0.6581], [0.4571, 0.5685], [0.1822, 0.2825])
A_4	([0.4799, 0.5851], [0.4381, 0.5440], [0.2067, 0.3077])
A_5	([0.5610, 0.6624], [0.6933, 0.8082], [0.2083, 0.3097])

According to the aggregating values in Table 4, the score functions are in Table 5.

Table 5. The score functions of the emerging technology enterprises.

	INWA	INWG
A_1	0.5549	0.5298
A_2	0.7877	0.7700
A_3	0.6507	0.6209
A_4	0.6574	0.5948
A_5	0.5718	0.5340

According to the score functions shown in Table 5, the order of the emerging technology enterprises are in Table 6.

Table 6. Order of the emerging technology enterprises.

	Ordering
INWA	$A_2 > A_4 > A_3 > A_5 > A_1$
INWG	$A_2 > A_3 > A_4 > A_5 > A_1$

From the above analysis, it can be seen that two operators have the same best emerging technology enterprise A_2 and two methods' ranking results are slightly different. However, the interval neutrosophic TODIM approach can reasonably depict the DMs' psychological behaviors under risk, and thus, it may deal with the above issue effectively. This verifies the method we proposed is reasonable and effective.

6. Conclusions

In this paper, we will extend the TODIM method to the MADM with the single-valued neutrosophic numbers (SVNNs). Firstly, the definition, comparison and distance of SVNNs are briefly presented, and the steps of the classical TODIM method for MADM problems are introduced. Then, the extended classical TODIM method is proposed to deal with MADM problems with the SVNNs, and its significant characteristic is that it can fully consider the decision makers' bounded rationality which is a real action in decision making. Furthermore, we extend the proposed model to interval neutrosophic sets (INSs). Finally, a numerical example is proposed to verify the developed approach.

In the future, the application of the proposed models and methods of SVNSs and INSs needs to be explored in the decision making [86–99], risk analysis and many other uncertain and fuzzy environment [100–112].

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