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The Shapley weighting vector-based neutrosophic aggregation operator in DEMATEL method

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Abstract. Most of the existing aggregation operators do not consider the interactive characteristics in the group decision making. However, in the real situation, the input data are interdependent and correlative which is cannot be evaluated by solely additive measures. To deal with the shortcoming, we propose a Shapley weighting vector based single valued neutrosophic aggregation operator into DEMATEL method. The Shapley weighting vector is suitable for aggregating correlated neutrosophic information. The λ- fuzzy measure is employed to obtain a simple fuzzy measure on expert set. A DEMATEL algorithm with our proposed aggregation operator is developed to group decision making. The developed algorithm is applied to coastal erosion problem to illustrate the effectiveness of the proposed method.

1. Introduction

The aggregation is an essential step in any group decision making where all the decision makers' evaluation need to be fused into one collective evaluation before proceed with any other steps. The classical aggregation methods are usually based on either arithmetic or geometric mean methods. Countless extended aggregation operators have been developed based on these two traditional aggregation methods. For the past decades, many researches have been done to find the suitable aggregation operator for certain type of sets; in fuzzy sets (FS)s [3-5], intuitionistic fuzzy sets (IFS)s [6-9], interval-valued intuitionistic fuzzy sets (IVIFS)s [10-11], hesitant fuzzy sets (HFS)s [12-14] and etc.

The aggregation of neutrosophic information is very important as well. Ye [15] introduced the simplified neutrosophic weighted aggregation operators and applied them to multi-criteria group decision making (MCGDM) problem under simplified neutrosophic environment. Peng et al. [16] developed an aggregation operator within simplified neutrosophic sets (SNS)s environment. Zhang et al. [17] proposed a multi-criteria decision making (MCDM) method based on aggregation operators with interval neutrosophic sets (INS)s. Liu and Wang [18] investigated single-valued neutrosophic normalized weighted Bonferroni mean. Liu et al. [19] developed some combination of generalized aggregation operator and Hamacher operations to NSs. Tian et al. [20] developed simplified neutrosophic linguistic normalized weighted Bonferroni mean. Peng et al. [21] applied the power aggregation operator to aggregate the multi-valued neutrosophic sets (MVNS) and then solved an MCGDM problem.

Most existing neutrosophic aggregation operators only consider the situations where all the elements are independent. However, in the real application, the elements of the problem are mostly dependent and correlative to each other. The aforementioned aggregation operators are mostly based on solely additive measures. Thus, the existing additive measures cannot handle this situation. In what follows, Sugeno introduced the λ- fuzzy measure which can handle the correlative and interaction among elements [22].

Although some Choquet integral and Benferroni mean based aggregation operators considered interactive phenomenon in group decision making, the overall interaction among combination is not properly reflected. To overcome this shortage, the Shapley value based fuzzy measure is employed in this study. The Shapley value has been established as the main solution concepts of transferable utility games and it has been applied to problems of revenue sharing and cost allocations [23]. In this study the Shapley value is employed to obtain the allocation weights of experts, without ignoring the correlation between experts and also overall combinations of expert sets. The fuzzy measure on expert set is determined by a similarity based method. An aggregation operator called the shapley weighted based single valued neutrosophic aggregation (SSVNA) operator is proposed.

The rest of this paper is arranged as follows. Section 2 presents some basic theories of neutrosophic sets. Section 3 elaborates the proposed method. Section 4 illustrates the proposed methodology into coastal erosion multi-criteria decision problem and Section 5 provides some concluding remarks.

2. Preliminaries

In this section, some basic concepts of neutrosophic sets (NS)s are given. The definition 1 below shows the definition of NSs.

Definition 1 ([1]) Let X be a space of points (objects) with generic elements in X denoted by x. A neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity membership function $F_A(x)$. The function $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $\left]0^-,1^+\right[$. That is, $T_A(x)\to \left]0^-,1^+\right[$, $I_A(x)\to \left]0^-,1^+\right[$, and $F_A(x)\to \left]0^-,1^+\right[$. Thus there is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x)+\sup I_A(x)+\sup F_A(x)\le 3^+$.

Deneutrosophication is the process to obtain a real number from a neutrosophic number. The definition below is the definition of deneutrosophication.

Definition 2 ([24]) Deneutrosophication of SVNS $\tilde{N}$ can be defined as a process of mapping $\tilde{N}$ into a single crisp output $\psi^* \in X$, i.e., $f:\tilde{N}\to\psi^*$ for $\psi^*\in X$. If $\tilde{N}$ is discrete set then the vector of tetrads $\tilde{N}=\left\{\left(x \mid \left\langle T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\right\rangle\right) \mid x\in X\right\}$ is reduced to a single scalar quantity $\psi^*\in X$ by deneutrosophication. The obtained scalar quantity $\psi^*\in X$ best represents the aggregate distribution of three membership degrees of neutrosophic element $\left\langle T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\right\rangle$.

Therefore, the deneutrosophication can be obtained as follows:

$$\psi^* = 1-\sqrt{\left\{\left(1-T_{\tilde{N}}(x)\right)^2+\left(I_{\tilde{N}}(x)\right)^2+\left(F_{\tilde{N}}(x)\right)^2\right\}/3}. \tag{1}$$

3. Proposed Method

Based on MCGDM problem, there are n factors altogether, $C_1, C_2, ..., C_n$. Suppose that, experts $e_k=\{e_1,e_2,...,e_m\}$ are invited to give evaluation among factors. Let X^k and X^l be the direct-relation matrix given by expert *k* and *l* respectively and $k, l \in \{1, 2, ..., m\}$ .The SSVNA operator is proposed to aggregate all individual experts' preferences into one aggregated preference. The proposed SSVNA operator is employed before carrying out the DEMATEL procedure. The computational procedures are proposed as below.

Step 1: Identify goal and factors.
Identify the decision goal and find out the factors influencing the goal. At this point, a lot of literature reviews need to be done in order to collect the relevant information.

Step 2: Preference evaluation and the construction of individual direct-relation matrices.
A survey of questionnaire is distributed among experts to investigate the interrelationship between factors of coastal erosion. Each of decision makers need to evaluate the direct influence between any two factors by using linguistic variables according to Table 1.

Table 1. Linguistic variables and Single Valued Neutrosophic Numbers (SVNNs)[24].

Linguistic variable	**SVNNs**
No influence/ Not important (NI)	$\langle 0.1, 0.8, 0.9 \rangle$
Low influence/important (LI)	$\langle 0.35, 0.6, 0.7 \rangle$
Medium influence/important (MI)	$\langle 0.5, 0.4, 0.45 \rangle$
High influence/important (HI)	$\langle 0.8, 0.2, 0.15 \rangle$
Very high influence/important (VHI)	$\langle 0.9, 0.1, 0.1 \rangle$

The direct relation matrix created based on each decision makers' preferences is given by:

$$X^k, X^l = [x_{ij}]_{n \times n} \tag{2}$$

where X^k and X^l are the preference made by *k*-th and *l*-th decision maker for *i*-th factor in comparison with the *j*-th factor and *n* represents the number of factors. x_{ij} is the SVNN represented by three elements, $\langle T_x, I_x, F_x \rangle$.

Step 3: Obtain an aggregated direct-relation matrix.
In this step, the SSVNA operator is developed. The important step is to aggregate all the individual direct-relation matrices, $X^k = \{X^1, X^2, ..., X^m\}$ into one collective direct-relation matrix, $A = (a_{ij})_{n \times n}$ where $a_{ij} = \langle T_a, I_a, F_a \rangle$. The following sub-steps must be followed before getting the aggregated direct-relation matrix, *A*.

Step 3.1: Similarity degree and λ-fuzzy measure.
In order to calculate the Shapley value, the $\lambda-$fuzzy measure of any combination of expert sets should be determined. In what follows, a similarity based formula is developed to obtain the $\lambda-$fuzzy measure of each expert. The similarity degree between the direct-relation matrix X^k and X^l can be express as

$$S(X^k, X^l) = \frac{\sum_{j=1}^{n}\sum_{i=1}^{n}\left\{\min\left\{T_{X^k}(x_{ij}), T_{X^l}(x_{ij})\right\} + \min\left\{I_{X^k}(x_{ij}), I_{X^l}(x_{ij})\right\} + \min\left\{F_{X^k}(x_{ij}), F_{X^l}(x_{ij})\right\}\right\}}{\sum_{j=1}^{n}\sum_{i=1}^{n}\left\{\max\left\{T_{X^k}(x_{ij}), T_{X^l}(x_{ij})\right\} + \max\left\{I_{X^k}(x_{ij}), I_{X^l}(x_{ij})\right\} + \max\left\{F_{X^k}(x_{ij}), F_{X^l}(x_{ij})\right\}\right\}} \quad (3)$$

Moreover, the overall similarity measure is given as:

$$S\left(X^k\right) = \frac{1}{m}\sum_{l=1}^{m} S\left(X^k, X^l\right) \quad (4)$$

Step 3.2: Solve for λ value and λ-fuzzy measure of combination of experts.
Solve for λ value using Eq. (5) and the λ-fuzzy measure of any combination of expert set can be obtained by Eq. (6).

$$\prod_{e_k \in \{e_1, e_2, \ldots, e_m\}} \left(1 + \lambda \cdot g_\lambda(\{e_k\})\right) = 1 + \lambda \quad (5)$$

$$g_\lambda(V) = \frac{1}{\lambda}\left(\prod_{e_m \in V} \left(\lambda \cdot g_\lambda(\{e_k\}) + 1\right) - 1\right), \qquad V \subseteq \{e_1, e_2, \ldots, e_m\} \quad (6)$$

Step 3.3: Compute Shapley weight.
The Shapley weight vector can be computed as below:

$$Sh_k\left(\{e_1, e_2, \ldots, e_m\}, g_\lambda\right) = \sum_{e_k \in V \subset \{e_1, e_2, \ldots, e_m\}} \frac{(|V|-1)!\,(|m|-|V|)!}{(m)!}\left[g_\lambda(V) - g_\lambda(V \setminus \{e_k\})\right] \quad \forall e_k \in m \quad (7)$$

is the Shapley weight with respect to expert e_k.

Step 3.4: Aggregation using SSVNA operator.
The developed SSVNA operator is as follows:

$$A = SSVNA\left(a_{ij}\right)_{n\times n} = \sum_{i,j=1}^{n} Sh_j a_{ij} = \left(1 - \prod_{i,j=1}^{n}\left(1 - T_A\left(a_{ij}\right)\right)^{Sh_j}, \prod_{i,j=1}^{n}\left(I_A\left(a_{ij}\right)^{Sh_j}\right), \prod_{i,j=1}^{n}\left(F_A\left(a_{ij}\right)^{Sh_j}\right)\right) \quad (8)$$

Step 4: Normalized direct-relation matrix.
Following the standard DEMATEL procedure, the normalized direct-relation matrix can be obtained as the equation below:

$$D = A \times S, \quad (9)$$

where

$$S = \frac{1}{\max_{1\le i\le n} \sum_{j=1}^{n} a_{ij}}. \quad (10)$$

Step 5: Obtain the total relation matrix.
Total relation matrix can be obtained by the following formula:

$$T = D(I - D)^{-1}, \quad (11)$$

where I is the identity matrix.

Step 6: Obtain the R and C values.
The R and C values can be obtained by summing elements of rows and columns of total relation matrix T, respectively. The $(R+C)$ values indicate the importance of each factor.

4. Illustrative Example
In order to test out the proposed methodology, an example related to coastal erosion problem is applied in this study. Three experts, $e_k, k \in \{1,2,3\}$ are invited to give their preference on cause-effect relationship among factors of coastal erosion.

Step 1: Identify goal and factors.
The twelve factors are chosen based on literature reviews and consultation with experts; Hydrodynamic wave and current (C1), Imbalance sediment supply (C2), Storm surge (C3), Tidal range (C4), Global warming (C5), Bottom beach profile and shoreline instability (C6), Sea level rise (C7), Sand mining activities (C8), Coastal development (C9), Coastal protection (C10), Budgetary revenue (C11) and Coastal zone management and policy (C12)

Step 2: Preference evaluation and the construction of individual direct-relation matrices.
The evaluations made by each expert are transformed into individual direct-relation matrices. The direct-relation matrix of expert 1 is as below.

$$X^1 = \begin{pmatrix} \text{NI} & \text{VHI} & \text{HI} & \text{NI} & \text{LI} & \text{VHI} & \text{MI} & \text{MI} & \text{LI} & \text{VHI} & \text{VHI} & \text{VHI} \\ \text{LI} & \text{NI} & \text{LI} & \text{NI} & \text{HI} & \text{VHI} & \text{HI} & \text{LI} & \text{VHI} & \text{VHI} & \text{VHI} & \text{VHI} \\ \text{HI} & \text{HI} & \text{NI} & \text{NI} & \text{HI} & \text{HI} & \text{HI} & \text{HI} & \text{VHI} & \text{VHI} & \text{HI} & \text{VHI} \\ \text{LI} & \text{HI} & \text{VHI} & \text{NI} & \text{MI} & \text{VHI} & \text{HI} & \text{HI} & \text{VHI} & \text{HI} & \text{VHI} & \text{HI} \\ \text{HI} & \text{VHI} & \text{VHI} & \text{VHI} & \text{NI} & \text{HI} & \text{VHI} & \text{HI} & \text{VHI} & \text{VHI} & \text{VHI} & \text{VHI} \\ \text{MI} & \text{VHI} & \text{MI} & \text{MI} & \text{HI} & \text{NI} & \text{LI} & \text{VHI} & \text{MI} & \text{VHI} & \text{HI} & \text{HI} \\ \text{VHI} & \text{HI} & \text{VHI} & \text{MI} & \text{VHI} & \text{VHI} & \text{NI} & \text{HI} & \text{VHI} & \text{VHI} & \text{VHI} & \text{VHI} \\ \text{LI} & \text{VHI} & \text{MI} & \text{LI} & \text{HI} & \text{VHI} & \text{MI} & \text{NI} & \text{VHI} & \text{HI} & \text{VHI} & \text{VHI} \\ \text{VHI} & \text{VHI} & \text{VHI} & \text{LI} & \text{MI} & \text{VHI} & \text{VHI} & \text{HI} & \text{NI} & \text{VHI} & \text{VHI} & \text{VHI} \\ \text{VHI} & \text{VHI} & \text{MI} & \text{MI} & \text{MI} & \text{VHI} & \text{VHI} & \text{MI} & \text{HI} & \text{NI} & \text{VHI} & \text{MI} \\ \text{HI} & \text{VHI} & \text{MI} & \text{LI} & \text{MI} & \text{MI} & \text{VHI} & \text{HI} & \text{VHI} & \text{VHI} & \text{NI} & \text{VHI} \\ \text{MI} & \text{VHI} & \text{LI} & \text{NI} & \text{LI} & \text{HI} & \text{HI} & \text{VHI} & \text{VHI} & \text{VHI} & \text{HI} & \text{NI} \end{pmatrix}$$

All the individual direct-relation matrices need to be converted into SVNNs (see Table 1). This step is omitted due to large matrices and page limitation.

Step 3: Obtain an aggregated direct-relation matrix.
The following sub-steps are required to obtain the aggregated direct-relation matrix.

Step 3.1: Similarity degree and λ-fuzzy measure.
By applying Eqs. (3) and (4), the overall similarity measure of each experts' evaluation which is also λ-fuzzy measure are obtained as:

$g_\lambda(\{e_1\}) = S(X^1) = 0.73$, $g_\lambda(\{e_2\}) = S(X^2) = 0.74$ and $g_\lambda(\{e_3\}) = S(X^3) = 0.75$.

Step 3.2: Solve for λ value and λ-fuzzy measure of any combination of expert set.
By Eq. (5), the λ values are obtained, $\lambda = -3.0764$ and -0.9792. Since $\lambda \geq -1$ is only acceptable, therefore we take $\lambda = -0.9792$. Then, by computing Eq. (6), the λ-fuzzy measure of any combination of expert set are obtained as below:

$$g_\lambda(\{e_1, e_2\}) = 0.941,\ g_\lambda(\{e_1, e_3\}) = 0.944,\ g_\lambda(\{e_2, e_3\}) = 0.946,\ g_\lambda(\{e_1, e_2, e_3\}) = 1.$$

Step 3.3: Compute Shapley weight.
The Shapley weight with respect to experts can be computed using Eq. (7), obtained as below:

$Sh_1 = 0.220$, $Sh_2 = 0.224$ and $Sh_3 = 0.227$

Step 3.4: Aggregation using SSVNA operator.
The aggregation of all individual direct-relation matrices can be obtained by using SSVNA operator, Eq. (8).

Step 4: Normalized direct-relation matrix.
The aggregated direct-relation matrix is transformed into the normalized direct-relation matrix using Eqs. (9) and (10).

Step 5: Obtain the total relation matrix.
The total relation matrix can be obtained by following Eq. (11). Note that the aggregated direct-relation matrix, normalized direct-relation matrix, and total relation matrix are not shown here due to the huge matrices and page limitation.

Step 6: Obtain the R and C values.
By computing the DEMATEL procedures in step 4-6, the values of R and C can be obtained. The R and C values and the importance of coastal erosion factors, $(R+C)$ values are shown in the Table 2.

Table 2. The R and C values

	R	C	$(R+C)$
C1	5.176	5.418	10.594
C2	5.087	6.183	11.27
C3	5.872	4.478	10.35
C4	5.922	4.436	10.358
C5	6.413	4.121	10.534
C6	5.395	6.164	11.559
C7	6.184	5.444	11.628
C8	5.847	4.880	10.727
C9	5.765	5.973	11.738
C10	5.078	6.404	11.482
C11	4.772	6.415	11.187
C12	4.402	5.997	10.399

Based on Table 2, the importance of the twelve factors can be prioritized as C9 > C7>C6> C10> C2> C11 > C8 > C1> C5> C12 > C4>C3 based on $(R+C)$ values, where the coastal development is the most important factor with the value of 11.738, while storm surge is the least important criterion with the value of 10.35. Therefore, the coastal development factor which is the most important factor needs an extra attention in any coastal erosion mitigation plans.

5. Conclusion

This paper has proposed an aggregation operator, SSVNA operator with SVNNs. The SSVNA operator involves Shapley weighting vector, $\lambda -$fuzzy measure and similarity degrees in its development. Shapley weighting vector can reflect the interaction among combination of expert sets. The proposed methodology of SSVNA operator with DEMATEL method has been applied to figure out the cause-effect relationship among factors of coastal erosion. This proposed method successfully determined the importance of each factors towards coastal erosion. So, the policy maker can apply any appropriate mitigation approach to ensure the coastal erosion does not get worse in the future.

The procedure presented in this paper provides a relevant model to orderly identify the critical factors of coastal erosion. This proposed methodology can be also used in the field of supplier selection, manufacturing, organization management, information system and social science. Besides, it is applicable to all systems facing problems that require to segment indeterminate factors and further figure out the importance of factors by group decision in a neutrosophic circumstance. And this paper also contributes an effective way to continuously improve the coastal erosion problem as a whole in a comprehensive perspective.

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