



The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment

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Abstract

Real-life decision-making problem has been demonstrated to cover the indeterminacy through single valued neutrosophic set. It is the extension of interval valued neutrosophic set. Most of the problems of real life involve some sort of uncertainty in it among which, one of the famous problem is finding a shortest path of the network. In this paper, a new score function is proposed for interval valued neutrosophic numbers and SPP is solved using interval valued neutrosophic numbers. Additionally, novel algorithms are proposed to find the neutrosophic shortest path by considering interval valued neutrosophic number, trapezoidal and triangular interval valued neutrosophic numbers for the length of the path in a network with illustrative example. Further, comparative analysis has been done for the proposed algorithm with the existing method with the shortcoming and advantage of the proposed method and it shows the effectiveness of the proposed algorithm.

Keywords Interval valued triangular neutrosophic number · Interval valued trapezoidal neutrosophic number · Ranking methods · Deneutrosophication · Neutrosophic shortest path problem · Network

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Introduction and literature of review

In this part, introduction to the objective of the paper is given by presenting basic concepts and procedure of the shortest path problem (SPP) and the literature of review have been collected to know the recent work related to the presented concept which shows the novelty of the presented work

Introduction

SPP is the ultimate and popular problem in the different areas also it is the heart of the network flows. In conventional problem, the distance between the nodes is considered to be certain and for the uncertain environment fuzzy numbers can be adopted to get an optimized result. Computing the minimum cost of the path from every vertex is called single source SPP. Especially in the process of finding shortest path, finding the path which has minimum number of bends is very important and will give the most optimized result. And the cost is the mapping of length and bends. The conventional SPP is to catch the minimum cost path from initial to end node and the cost is the addition of the costs of the curves on the path [1, 2, 4].

While applying in real time situations the vertices and the edges will be considered as follows. In transmission networks, telephone exchange, communication proficiency, satellites, work stations terminals and computers will be considered as the vertices and cables, wires and fiber optics will be treated as the arcs or paths and it is expected to meet transmission requirements at the minimum cost whereas in traffic control management the cost is due to only the paths with heavy traffic [8]. In the established network every path has a weight which will extend the flow in a recurrence fashion. The fusion of costs and weights proposes different ways of cost minimizing cycles. There may be cycles with negative cost which allow raise to perpetual instances and cost of minimum infinity and weight minimizing cycles which permits rise to a sink in such a way that it is inexpensive to consume a flow in an infinite cycle rather than transit to the station.

SPP plays an essential role in combinatorial optimization due to its elemental aspects and a broad range of applications. Investigating shortest paths is an essential thing in communication, computer networks, manufacturing systems and transportation. The weight of the path will represent the transportation timing from one end to other, i.e., the traveling time from the source to the destination. The efficiency of the transmission can be improved by speed up some of the routes to reduce the traveling time between some of the pairs of sources and terminals by minimizing the weights of the links. One needs some amount to reduce the traveling time by improving the road conditions for the faster traveling and the total cost supposed to be less to face the needs of the speedup [9].

In all the SPP, the source and terminal nodes should satisfy a set of conditions defined over a set of resources which associates to a quantity like the time, pickup of load by the vehicle or the duration of the break. The constraint of the resource will be given in the form of intervals which regulate the values that can be considered by the resources at each node on the path. SPP using complete graph can be encrypted as an assignment problem and is equivalent to an exceptional case of the assignment problem. Providing the shortest path is a necessary thing to the system of transport management, from a particular source node to the terminal node. The arc lengths are stimulated to represent time or cost of the transportation rather the geographical distances [10, 11].

The technique of using fuzzy numbers can be adopted for the environment with uncertainty. Crisp number is obtained from fuzzy number using defuzzification function and it is widely used in an optimization methods. SPP is not restricted to the geometric distance. Even though it is fixed, the traveling time within the cities may be represented by fuzzy variable. Since the weight of the arcs is uncertain in almost all the communication and transportation networks, it cannot be designed into crisp graphs. Dubois and Prade solved fuzzy shortest path problem for the first time. The most crucial combinatorial optimization problem is to find

the SP to the directed graph and its primary format unable to represent the situations where the value of the detached function should be found not only by the preference of each single arc [15–19].

Shortest path of the network can be found using neutrosophic set (NS) by considering edge weight as neutrosophic numbers (NNs) and that may be single and interval valued, and bipolar as well [21, 22]. Samarandache described about neutrosophic for the first time in the year 1995 and proposed an important mathematical mechanism called neutrosophic set theory to handle imprecise, uncertain and indeterminate problems which cannot be dealt by fuzzy and its various type. NS is obtained by three autonomous mapping such as truth (T), indeterminacy (I) and falsity (F) and takes values from $]0^-, 1^+[$. It is very difficult to utilize NS directly.

While getting uncertainty in the set of vertices and edge then fuzzy graph can be adopted for SPP, but if there is indeterminacy exist between the relation of nodes and vertices then neutrosophic will be the appropriate concept to deal the real life problems [23]. Since indeterminacy is also treated seriously, NSs can be able to handle uncertainty in a better way [35]. The model of the NS is an important mechanism to deal with real scientific and engineering as it is able to deal uncertain, inconsistent and also indeterminate information [36]. Route maintenance or supply with uncertainty is playing a primary role in intelligent transport systems.

Due to inadequate data, as the stochastic shortest path needs accurate probability distributions, it is unable give the optimized result. Due to accuracy, adoptability and rapport to a system, single valued neutrosophic graph (SVNG) gets more attention and produce optimized solution than other types of fuzzy sets. Application of probabilities in machine learning is done by the score function. These functions play an essential role to find the minimum cost path in SPP and minimum spanning tree (MST) to UIVNGs (undirected interval valued neutrosophic graphs). When the data are in the form of intervals then that can dealt effectively by considering interval valued neutrosophic setting [40, 41]. Many group decision making methods including hybrid methods have been proposed to solve decision making problems such as supplier selection, project selection under triangular and trapezoidal neutrosophic environment [55–64].

The rest of the paper is arranged as follows. In Sect. 1.2, literature of review has been collected. In Sect. 2, over view of interval valued neutrosophic set is given. In Sect. 3, novel algorithms are proposed to find the neutrosophic shortest path under interval valued neutrosophic environment and interval valued triangular and trapezoidal neutrosophic environments with the help of proposed score function. In Sect. 4, shortcoming of the existing methods, advantages of the proposed method and comparative analysis are presented for the proposed method with the existing method. In Sect. 5, conclusion of the presented work is given.

Literature of review

The authors of, Ahuja et al. [1] proposed a different model redistributive heap as a rapid algorithm to find SP of the network. Yang et al. [2] presented a graph-theoretic strategy of rectilinear paths on bends and lengths. Ibarra and Zheng [3] proved that the single-origin shortest path problem for permutation graphs can be determined by order of the logarithmic of n . Arsham [4] examined the robustness of the shortest path problem. Tzoreff [5] examined the disconnected SPP with group path lengths. Batagelj et al. [6] proposed generalized SPP.

Zhang and Lin [7] introduced the calculation of the reverse SPP. Vasantha and Samaranadache [8] proposed primary neutrosophic algebraic framework. Also their utilization to fuzzy and NEUTROSOPHIC models as well. Roditty and Zwick [9] acquired some results associated with effective forms of the SPP. Irnich and Desaulniers [10] proposed SPP with support force. Buckley and Jowers [11] introduced SPP using the concept of fuzzy logic. Wastlund [12] analyzed the relationship between random assignment and SPP problem on the complete graph. Turner [13] attained strongly polynomial algorithms for a collection of SPP on acyclic and normal digraphs. Deng et al. [14] proposed fuzzy Dijkstra algorithm for SPP for imprecise environment.

Biswas et al. [15] introduced an algorithm for deriving shortest path in intuitionistic fuzzy environment. Arnautovic et al. [16] obtained the complement of the ant colony development for the SPP using open MP and CUDA. Gabrel and Murat [17] presented different models, methods and principle for the stability of the SPP. Grigoryan and Harutyunyan [18] proposed SPP in the Knodel graph. Rostami et al. [19] proposed quadratic SPP. Randour et al. [20] presented algorithms to incorporate the approaches with various securities on the length allocation of the paths instead of decreasing its normal value. Broumi et al. [21] solved SPP under neutrosophic setting using Dijkstra algorithm. Broumi et al. [22] introduced SPP based on triangular fuzzy neutrosophic environment.

Broumi et al. [23] proposed assertive types of SVNGs and examination of properties with validation and examples. Nancy and Harish [24] proposed an improved score function and applied in decision making process. Sahin and Liu [25] maximized method of deviation for neutrosophic decision making problem with a support of incomplete weight. Broumi et al. [26] proposed the measurements for SPP using SV-triangular neutrosophic numbers. Broumi et al. [27] calculated MST in interval valued bipolar neutrosophic (IVBN) setting. Hu and Sotirov [28] proposed amenity of semi definite programming for the quadratic SPP and performed some arithmetic operations to solve the QSPP using branch and bound algorithm. Dragan and Leitert [29] solved SPP on

minimal peculiarity. Zhang et al. [30] proposed stable SPP with circulated uncertainty.

Broumi et al. [31] solved SPP using SVNG. Broumi et al. [32] solved SSP under bipolar neutrosophic environment. Peng and Dai [33] proposed interval-based algorithms based on neutrosophic environment for decision making process. Liu and You [34] proposed muirhead mean operators and employed them in decision making problem. Smarandache [35] solved SPP using trapezoidal neutrosophic knowledge. Wang et al. [36] applied SV-trapezoidal neutrosophic preference in decision making problem. Deli and Subas [37] proposed a ranking method of SVNNs and applied in decision making problem. Broumi et al. [38] proposed matrix algorithm for MST in undirected IVNG. Enayattabar et al. [39] applied Dijkstra algorithm to find the shortest path under IV Pythagorean fuzzy setting. Broumi et al. [40] proposed IVN soft graphs. Broumi et al. [41] proposed some notion with respect to neutrosophic set with triangular and trapezoidal concept and primary operations as well. Also done a contingent analysis with the existing concepts and proposed neutrosophic numbers.

Broumi et al. [42] proposed an innovative system and technique for the planning of telephone network using NG. Broumi et al. [43] proposed SPP under interval valued neutrosophic setting. Bolturk and Kahraman [44] presented a novel IVN AHP with cosine similarity measure. Wang et al. [45] proposed interval neutrosophic set and logic in detail. Biswas et al. [46] proposed distance measure using interval trapezoidal neutrosophic numbers. Deli [47] given detailed work on expansion and contraction on conventional neutrosophic soft set. Deli [48] solved a decision making problem using interval valued neutrosophic soft numbers.

Deli [49] proposed theory of npn-soft set and its application. Deli [50] proposed single valued trapezoidal neutrosophic operators and applied them in a decision making problem. Deli and Subas [51] proposed weighted geometric operators under single valued triangular neutrosophic numbers and applied in a decision making problem. Deli et al. [52] solved a decision making problem using neutrosophic soft sets. Basset et al. [53] proposed framework of hybrid neutrosophic group AND-TOPSIS for supplier selection. Chang et al. [54] experimented in detail about framework for the pattern of reuse necessary decision from theoretical perspective to practices.

Basset et al. [55] proposed a hybrid method of neutrosophic sets and method of DEMATEL to develop criteria for supplier selection. Basset et al. [56] proposed a structure based on VIKOR technique for e-government website evaluation. Basset et al. [57] Introduced a framework to evaluate cloud computing services. Basset et al. [58] proposed a hybrid method for project selection under neutrosophic environment. Basset et al. [59] proposed a new method for a neutrosophic linear programming problem.

Basset et al. [60] proposed an economic tool for risk quantification for supply chain. Basset et al. [61] proposed a framework for AHP-QFD to solve a supplier selection. Basset et al. [62] proposed neutrosophic AHP-Delphi group decision model under trapezoidal neutrosophic numbers. Basset et al. [63] solved a group decision making problem using neutrosophic analytic hierarchy process. Basset et al. [64] proposed a group decision making problem using triangular neutrosophic numbers. Kumar et al. [65] proposed an algorithm to solve neutrosophic shortest path problem under triangular and trapezoidal neutrosophic environment.

From this literature review, to the best of our knowledge, there is no contribution of research for SPP using interval neutrosophic numbers under triangular and trapezoidal environments. Additionally, this is the first study that SPP is solved by considering interval valued triangular and trapezoidal neutrosophic numbers for the length of the arc for a given network.

Overview on interval valued neutrosophic set

Here, a brief description of some basic concepts on NSs, SVNSSs, IVNSs and some existing ranking functions for IVNNs are given.

Definition 2.1 [35] NS is constructed by $N = \{ \langle x; T_N(x), I_N(x), F_N(x) \rangle, x \in X \}$, where X be an universal set of elements x and $T_N(x), I_N(x), F_N(x) : X \rightarrow]-0, 1^+[$ are the truth, indeterminacy and also falsity membership functions and satisfies the criterion,

$$-0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+ \tag{1}$$

Definition 2.2 [36] SVNNS is defined by $\dot{N} = \{ \langle x; T_N(x), I_N(x), F_N(x) \rangle, x \in X \}$ and for every

$$x \in X, \quad T_N(x), I_N(x), F_N(x) \in [0, 1], \tag{2}$$

and the sum of these three is less than or equal to 3.

Definition 2.3 [45] An interval valued NS is defined by $\dot{N} = \left\{ \langle x : \left[T_N^L(x), T_N^U(x) \right], \left[I_N^L(x), I_N^U(x) \right], \left[F_N^L(x), F_N^U(x) \right] \right\rangle, x \in X$, where $T_N(x) = \left[T_N^L(x), T_N^U(x) \right] \subseteq [0, 1]$,

$$\begin{aligned} I_N(x) &= \left[I_N^L(x), I_N^U(x) \right] \subseteq [0, 1], \\ F_N(x) &= \left[F_N^L(x), F_N^U(x) \right] \subseteq [0, 1] \text{ and} \end{aligned} \tag{3}$$

$$0 \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3. \tag{4}$$

Now we assume some mathematical operations on IVNNs (interval valued neutrosophic numbers).

Definition 2.4 [45] Let $\dot{N}_1 = \left\{ \langle x : \left[T_{N_1}^L, T_{N_1}^U \right], \left[I_{N_1}^L, I_{N_1}^U \right], \left[F_{N_1}^L, F_{N_1}^U \right] \rangle, x \in X \right\}$ and $\dot{N}_2 = \left\{ \langle x : \left[T_{N_2}^L, T_{N_2}^U \right], \left[I_{N_2}^L, I_{N_2}^U \right], \left[F_{N_2}^L, F_{N_2}^U \right] \rangle, x \in X \right\}$ be two IVNNs and $\delta > 0$ then we have the following operational laws.

$$\begin{aligned} \dot{N}_1 \oplus \dot{N}_2 &= \left\langle \left[T_{N_1}^L + T_{N_2}^L - T_{N_1}^L T_{N_2}^L, T_{N_1}^U + T_{N_2}^U - T_{N_1}^U T_{N_2}^U \right], \right. \\ &\quad \left. \left[I_{N_1}^L, I_{N_2}^L, I_{N_1}^U, I_{N_2}^U \right], \left[F_{N_1}^L, F_{N_2}^L, F_{N_1}^U, F_{N_2}^U \right] \right\rangle \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{N}_1 \otimes \dot{N}_2 &= \left\langle \left[T_{N_1}^L T_{N_2}^L, T_{N_1}^U T_{N_2}^U \right], \left[I_{N_1}^L + I_{N_2}^L - I_{N_1}^L I_{N_2}^L, I_{N_1}^U + I_{N_2}^U - I_{N_1}^U I_{N_2}^U \right], \right. \\ &\quad \left. \left[F_{N_1}^L + F_{N_2}^L - F_{N_1}^L F_{N_2}^L, F_{N_1}^U + F_{N_2}^U - F_{N_1}^U F_{N_2}^U \right] \right\rangle \end{aligned} \tag{6}$$

$$\begin{aligned} \delta \dot{N} &= \left\langle \left[1 - (1 - T_N^L)^\delta, 1 - (1 - T_N^U)^\delta \right], \right. \\ &\quad \left. \left[(T_N^L)^\delta, (T_N^U)^\delta \right], \left[(F_N^L)^\delta, (F_N^U)^\delta \right] \right\rangle \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{N}^\delta &= \left\langle \left[(T_N^L)^\delta, (T_N^U)^\delta \right], \left[1 - (1 - I_N^L)^\delta, 1 - (1 - I_N^U)^\delta \right], \right. \\ &\quad \left. \left[1 - (1 - F_N^L)^\delta, 1 - (1 - F_N^U)^\delta \right] \right\rangle. \end{aligned} \tag{8}$$

Deneutrosophication formulas for IVNNs: To compare two IVNNs \dot{N}_1 and \dot{N}_2 . We use the score function (SF) which represents a map from $[N(R)]$ into the real line. In the literature there are some deneutrosophication formulas, here paper, we focus on some of types [24, 25, 33, 34, 44] defined as follows:

$$\begin{aligned} S_{\text{Bolturk}}(\dot{N}_1) &= \left(\frac{(T_x^L + T_x^U)}{2} + \left(1 - \frac{(I_x^L + I_x^U)}{2} \right) \right) \\ &\quad * (I_x^U) - \left(\frac{(F_x^L + F_x^U)}{2} \right) * (1 - F_x^U) \end{aligned} \tag{9}$$

$$S_{\text{Ridvan}}(\dot{N}_1) = \left(\frac{1}{4} \right) \times (2 + T_x^L + T_x^U - 2I_x^L - 2I_x^U - F_x^L - F_x^U) \tag{10}$$

$$S_{\text{Peng}}(\dot{N}_1) = \left[\frac{2}{3} + \frac{(T_x^L + T_x^U)}{6} - \frac{(I_x^L + I_x^U)}{6} - \frac{(F_x^L + F_x^U)}{6} \right] \tag{11}$$

$$S_{\text{Liu}}(\dot{N}_1) = \left[2 + \frac{(T_x^L + T_x^U)}{2} - \frac{(I_x^L + I_x^U)}{2} - \frac{(F_x^L + F_x^U)}{2} \right] \tag{12}$$

$$S_{\text{Harish}}(\dot{N}_1) = \left(\frac{1}{8} \right) \times [4 + (T_x^L + T_x^U - F_x^L - F_x^U - 2I_x^L - 2I_x^U)(4 - T_x^L - T_x^U - F_x^L - F_x^U)]. \tag{13}$$

The ranking of \dot{N}_1 and \dot{N}_2 by SF is defined as follows:

- (i) $\dot{N}_1 < \dot{N}_2$ if $\mathbb{S}(\dot{N}_1) < \mathbb{S}(\dot{N}_2)$
- (ii) $\dot{N}_1 > \dot{N}_2$ if $\mathbb{S}(\dot{N}_1) > \mathbb{S}(\dot{N}_2)$
- (iii) $\dot{N}_1 = \dot{N}_2$ if $\mathbb{S}(\dot{N}_1) = \mathbb{S}(\dot{N}_2)$

Definition 2.5 [36] Let $R_N = \langle [R_T, R_I, R_M, R_E], (T_R, I_R, F_R) \rangle$ and $S_N = \langle [S_T, S_I, S_M, S_E], (T_S, I_S, F_S) \rangle$ be two trapezoidal neutrosophic numbers (TpNNs) and $\theta \geq 0$, then

$$R_N \oplus S_N = \langle [R_T + S_T, R_I + S_I, R_M + S_M, R_E + S_E], (T_R + T_S - T_R T_S, I_R I_S, F_R F_S) \rangle \tag{14}$$

$$R_N \otimes S_N = \langle [R_T \cdot S_T, R_I \cdot S_I, R_M \cdot S_M, R_E \cdot S_E], (T_R \cdot T_S, I_R + I_S - I_R \cdot I_S, F_R + F_S - F_R \cdot F_S) \rangle \tag{15}$$

$$\theta R_N = \langle [\theta R_T, \theta R_I, \theta R_M, \theta R_E], (1 - (1 - T_R)^\theta, (I_R)^\theta, (F_R)^\theta) \rangle. \tag{16}$$

Definition 2.6 [36] Let $R = [R_T, R_I, R_M, R_E]$ and $R_T \leq R_I \leq R_M \leq R_E$ then the centre of gravity (COG) in R is

$$\text{COG}(R) = \begin{cases} R & \text{if } R_T = R_I = R_M = R_E \\ \frac{1}{3} \left[R_T + R_I + R_M + R_E - \frac{R_E R_M - R_I R_T}{R_E + R_M - R_I - R_T} \right] & \text{otherwise} \end{cases} \tag{17}$$

Definition 2.7 [36] Let $S_N = \langle [S_T, S_I, S_M, S_E], (T_S, I_S, F_S) \rangle$ be a TpNN then the score, accuracy and certainty functions are as follows

$$\mathbb{S}(S_N) = \text{COG}(R) \times \frac{(2 + T_S - I_S - F_S)}{3} \tag{18}$$

$$a(S_N) = \text{COG}(R) \times (T_S - F_S) \tag{19}$$

$$C(S_N) = \text{COG}(R) \times (T_S). \tag{20}$$

Definition 2.8 [36] Let $R_N = \langle [R_T, R_I, R_P], (T_R, I_R, F_R) \rangle$ be a triangular neutrosophic number then the score and accuracy function are,

$$\mathbb{S}(R_N) = \frac{1}{12} [R_T + 2 \cdot R_T + R_P] \times [2 + T_R - I_R - F_R] \tag{21}$$

$$a(R_N) = \frac{1}{12} [R_T + 2 \cdot R_T + R_P] \times [2 + T_R - I_R + F_R]. \tag{22}$$

Definition 2.9 [46] Let N be a trapezoidal neutrosophic number in the set of real numbers with the truth, indeterminacy and falsity membership functions are defined by

$$T_N(x) = \begin{cases} \frac{(x-a)t_N}{b-a}, & a \leq x < b \\ t_N, & b \leq x \leq c \\ \frac{(d-x)t_N}{d-c}, & c < x \leq d \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

$$I_N(x) = \begin{cases} \frac{b-x+(x-a)i_N}{b-a}, & a \leq x < b \\ i_N, & b \leq x \leq c \\ \frac{x-c+(d-x)i_N}{d-c}, & c < x \leq d \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

$$F_N(x) = \begin{cases} \frac{b-x+(x-a)f_N}{b-a}, & a \leq x < b \\ f_N, & b \leq x \leq c \\ \frac{x-c+(d-x)f_N}{d-c}, & c < x \leq d \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

where $t_N = [t^L, t^U] \subset [0, 1]$, $i_N = [i^L, i^U] \subset [0, 1]$ and $f_N = [f^L, f^U] \subset [0, 1]$ are interval numbers. Then the number N can be denoted by $([a, b, c, d]; [t^L, t^U], [i^L, i^U], [f^L, f^U])$ and is called interval valued trapezoidal neutrosophic number.

- If $b = c$ in interval valued trapezoidal neutrosophic number then it becomes interval valued triangular neutrosophic number.

Proposed improved algorithm and score function

To find the length of the arc, the following algorithm and score function are proposed as follows.

Improved algorithm to solve SPP under interval valued neutrosophic number

- Step 1: Determine the source node (SN) arc length $l_1 = \langle [1, 1], [0, 0], [0, 0] \rangle$ and classify SN, node 1 by $[l_1 = \langle [1, 1], [0, 0], [0, 0] \rangle, -]$
- Step 2: Find the minimum of the length of n_1 with its acquaintance node using $l_i = \min\{l_i \oplus l_{ij}\}, j = 2, 3, \dots, r.$
- Step 3: If there is a minimum in the node and equating to the singular measure of i (i.e., $i = k$), then classify that node j as $[l_j, k].$
- Step 4: If the minimum value exists in the node matching to more values from i then it can be concluded that there are more IVN paths between SN (i) and DN (j) and select any value of $i.$
- Step 5: Classify the destination node (DN) (node r) by $[l_r, 1].$ Then the interval valued neutrosophic distance (IVND) among SN $l_r.$
- Step 6: Find the IVNSP between initial and terminal node according to $[l_r, 1]$ and check the label of n_1 and is denoted by $[l_a, d].$ Classify node a and so on. Rerun the process until get $n_1.$
- Step 7: By connecting all the nodes acquired by repeating the process in step 4, IVNSP can be found.

Note: If $\mathbb{S}(N_i) < \mathbb{S}(N_p)$ then the interval valued neutrosophic number (IVNN) is the minimum of $N_p,$ where $N_i, i = 1, 2, \dots, r$ is the set of IVNN and \mathbb{S} is the score function.

Proposed score function

The novel SF for finding the minimum cost path under interval valued neutrosophic shortest path (IVNSP) problem is provided as follows

$$\mathbb{S}_{Nagarajan}(\dot{N}_1) = \frac{1}{2} \left[(T_x^L + T_x^U) - (I_x^L \cdot I_x^U) + (I_x^U - 1)^2 + (F_x^U) \right]. \tag{26}$$

Numerical example:

For the edge 1–2: $\mathbb{S}_{Nagarajan}(\ddot{A}_1) = \frac{1}{2} [(0.1 + 0.2) - (0.2)(0.3) + (0.3 - 1)^2 + (0.5)] = 0.125$

For the edge 1–3: $\mathbb{S}_{Nagarajan}(\ddot{A}_1) = \frac{1}{2} [(0.2 + 0.4) - (0.3)(0.5) + (0.5 - 1)^2 + (0.2)] = 0.2.$

Similarly for other edges.

Note: Formulas used in the proposed algorithms.

Score function used in the proposed algorithm under IVN environment and COG for TFN are

$$\mathbb{S}(\theta) = \text{COG}(R) \times \frac{1}{2} \left[T^L + T^U - (I^L \cdot I^U) + (I^U - 1)^2 + F^U \right] \tag{27}$$

$$\text{COG for TFN is } \frac{1}{3} \left[R_T + 2R_M + R_E - \frac{R_M(R_E - R_I)}{(R_E - R_I)} \right]. \tag{28}$$

Computation of shortest path using IVNNs

Illustrate to the basic process of the improved algorithm, one simple example is shown.

Fig. 1 Interval-valued neutrosophic network

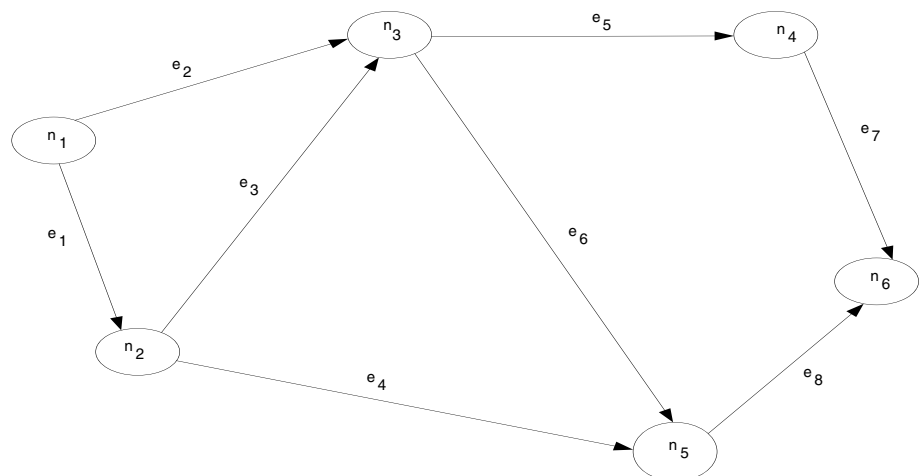


Table 1 The details of edges information in term of IVNNs

Edges	Interval valued neutrosophic distance	Edges	Interval valued neutrosophic distance
1–2 (e_1)	([0.1, 0.2], [0.2, 0.3], [0.4, 0.5])	3–4 (e_5)	([0.2, 0.3], [0.2, 0.5], [0.4, 0.5])
1–3 (e_2)	([0.2, 0.4], [0.3, 0.5], [0.1, 0.2])	3–5 (e_6)	([0.3, 0.6], [0.1, 0.2], [0.1, 0.4])
2–3 (e_3)	([0.3, 0.4], [0.1, 0.2], [0.3, 0.5])	4–6 (e_7)	([0.4, 0.6], [0.2, 0.4], [0.1, 0.3])
2–5 (e_4)	([0.1, 0.3], [0.3, 0.4], [0.2, 0.3])	5–6 (e_8)	([0.2, 0.3], [0.3, 0.4], [0.1, 0.5])

Illustrative example

This section is based on a numerical problem adapted from Broumi et al. [40] to show the potential application of the proposed algorithm and score function.

Consider a network Fig. 1 with six nodes and eight edges with SN, node 1 and DN, node 6. The interval valued neutrosophic distance is given in Table 1.

In this situation, we need to evaluate the shortest distance from SN, i.e., node 1 to DN, i.e., node 6.

Calculating the shortest path using proposed algorithm of interval valued neutrosophic path problem is given as follows.

Here $r = 6$, since there are totally 6 nodes.

Let, $l_1 = \langle [1, 1], [0, 0], [0, 0] \rangle$ and classify the SN $n_1 = \langle [1, 1], [0, 0], [0, 0], - \rangle$.

To find the value of l_j , $j = 2, 3, 4, 5, 6$.

Iteration no. 1:

Since n_2 has only n_1 as the predecessor, let $i = 1, j = 2$ in step 2.

To find l_2 :

$$l_2 = \min \{ l_1 \oplus l_{12} \}$$

$$= \min \{ \langle [1, 1], [0, 0], [0, 0] \rangle \oplus \langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle \}$$

$$= \langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle$$

Since, minimum occurs for $i = 1$, classify the node

$$n_2 = \langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5], 1 \rangle$$

Iteration no. 2:

Since n_3 has two predecessors n_1 and n_2 , let $i = 1, 2 \& j = 3$ in step 2.

To find l_3 :

$$l_3 = \min \{ l_1 \oplus l_{13}, l_2 \oplus l_{23} \}$$

$$= \min \{ \langle [1, 1], [0, 0], [0, 0] \rangle \oplus \langle [0.2, 0.4], [0.3, 0.5], [0.1, 0.2] \rangle, \langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle \oplus \langle [0.3, 0.4], [0.1, 0.2], [0.3, 0.5] \rangle \}$$

$$= \min \{ \langle [1 + 0.2 - 1(0.2), 1 + 0.4 - 1(0.4)], [0(0.3), 0(0.5)], [0(0.1), 0(0.2)] \rangle, \langle [0.1 + 0.3 - (0.1)(0.3), 0.2 + 0.4 - (0.2)(0.4)], [(0.2)(0.1), (0.3)(0.2)], [(0.4)(0.5), (0.5)(0.5)] \rangle \}$$

$$= \min \{ \langle [1, 1], [0, 0], [0, 0] \rangle, \langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25] \rangle \}$$

$$= \langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25] \rangle$$

Since the score function values are,

$$\mathbb{S}(\langle [1, 1], [0, 0], [0, 0] \rangle)$$

$$= \frac{1}{2} [(1 + 1) - (0 \times 0) + (0 - 1)^2 + 0] = 1.5$$

$$\mathbb{S}(\langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25] \rangle)$$

$$= \frac{1}{2} [(0.37 + 0.52) - (0.02 \times 0.06) + (0.06 - 1)^2 + 0.25]$$

$$= 0.9$$

and the minimum occurs for $i = 2$, then classify the node $n_3 = \langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25], 2 \rangle$.

Iteration no. 3:

Since n_4 has one predecessors n_3 , let $i = 3 \& j = 4$ in step 2.

To find the value of l_4 :

$$l_4 = \min \{ l_3 \oplus l_{34} \}$$

$$= \min \{ \langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25] \rangle \oplus \langle [0.2, 0.3], [0.2, 0.5], [0.4, 0.5] \rangle \}$$

$$= \langle [0.6, 0.67], [0.004, 0.018], [0.048, 0.125] \rangle$$

Since, minimum occurs for $i = 3$, hence classify the node $n_4 = \langle [0.6, 0.67], [0.004, 0.018], [0.048, 0.125], 3 \rangle$.

Iteration no. 4:

Since n_5 has two predecessors n_2 and n_3 , let $i = 2, 3 \& j = 5$ in step 2.

To find the value of l_5 :

$$l_5 = \min \{ l_2 \oplus l_{25}, l_3 \oplus l_{35} \}$$

$$= \min \{ \langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle \oplus \langle [0.1, 0.3], [0.3, 0.4], [0.2, 0.3] \rangle, \langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25] \rangle \oplus \langle [0.3, 0.6], [0.1, 0.2], [0.1, 0.4] \rangle \}$$

$$= \min \{ \langle [0.19, 0.47], [0.06, 0.12], [0.08, 0.15] \rangle, \langle [0.56, 0.81], [0.002, 0.012], [0.012, 0.1] \rangle \}$$

$$= \langle [0.19, 0.47], [0.06, 0.12], [0.08, 0.15] \rangle$$

Since the score function values are,

$$\mathbb{S}(\langle [0.19, 0.47], [0.06, 0.12], [0.08, 0.15] \rangle) = 0.75$$

$\mathbb{S}(\langle [0.56, 0.81], [0.002, 0.012], [0.012, 0.1] \rangle) = 1$
 and the minimum occurs for $i = 2$, hence classify the node
 $n_5 = \langle [0.19, 0.47], [0.06, 0.12], [0.08, 0.15] \rangle, 2$

Iteration no. 5:

Since n_6 has two predecessors n_4 and n_5 , let
 $i = 4, 5$ & $j = 6$ in step 2.

To find the value of l_6 :

$$l_6 = \min\{l_4 \oplus l_{46}, l_5 \oplus l_{56}\}$$

$$= \min\{\langle [0.6, 0.67], [0.004, 0.018], [0.048, 0.125] \rangle$$

$$\oplus \langle [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] \rangle,$$

$$\langle [0.19, 0.47], [0.06, 0.12], [0.08, 0.15] \rangle$$

$$\oplus \langle [0.2, 0.3], [0.3, 0.4], [0.1, 0.5] \rangle\}$$

$$= \min\{\langle [0.76, 0.87], [0.008, 0.0018], [0.0048, 0.0375] \rangle,$$

$$\langle [0.352, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle\}$$

$$= \langle [0.35, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle.$$

Since the score function values are,

$$\mathbb{S}(\langle [0.76, 0.87], [0.008, 0.0018], [0.0048, 0.0375] \rangle) = 1$$

$$\mathbb{S}(\langle [0.352, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle) = 0.82$$

and the minimum occurs for $i = 5$ hence classify
 $n_6 = \langle [0.35, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle, 5$.

Since n_6 is the DN of the given network, IVNSP between
 n_1 and n_6 is $\langle [0.35, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle$.

Now, IVNSP from n_1 and n_6 is obtained as follows.

Since, $n_6 = \langle [0.35, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle,$
 $5 \Rightarrow$ a person is coming from 5 $\rightarrow 6n_5 = \langle [0.19, 0.47], [0.06,$
 $0.12], [0.08, 0.15] \rangle, 2 \Rightarrow$ a person is coming from 2 $\rightarrow 5$
 $n_2 = \langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle, 1 \Rightarrow$ a person is coming
 from 1 $\rightarrow 2$.

By joining all the acquired nodes, interval valued neutrosophic shortest path from n_1 and n_6 is obtained.

Hence IVNSP of the given network is $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$.

The IVNS distance and IVNSP of all the nodes from SN node 1 in the below Table 2 and the classification of all the nodes are shown in Fig. 2.

The following table is formed using different deneutrosophic functions called score functions for all the possible edges and using proposed improved score function in the last column (Table 3).

According to the improved score function proposed in Sect. 3, the shortest path from node one to node six can be computed as follows (Table 4).

Therefore, the path $P : 1 \rightarrow 2 \rightarrow 5 \rightarrow 6$. is identified as the neutrosophic shortest path.

Algorithm: a new approach to find SPP using TplVNN and TIVNN

Consider a directed and noncyclic graph, where the length of the arcs is represented by IVNN. The introduced algorithm

Table 2 Interval valued neutrosophic shortest path

Node number (j)	l_j	IVNSP between j th and node 1
2	$\langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \rangle$	$1 \rightarrow 2$
3	$\langle [0.37, 0.52], [0.02, 0.06], [0.12, 0.25] \rangle$	$1 \rightarrow 2 \rightarrow 3$
4	$\langle [0.6, 0.67], [0.004, 0.018], [0.048, 0.125] \rangle$	$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$
5	$\langle [0.19, 0.47], [0.06, 0.12], [0.08, 0.15] \rangle$	$1 \rightarrow 2 \rightarrow 5$
6	$\langle [0.35, 0.63], [0.018, 0.048], [0.008, 0.075] \rangle$	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$

Fig. 2 Interval-valued neutrosophic shortest path

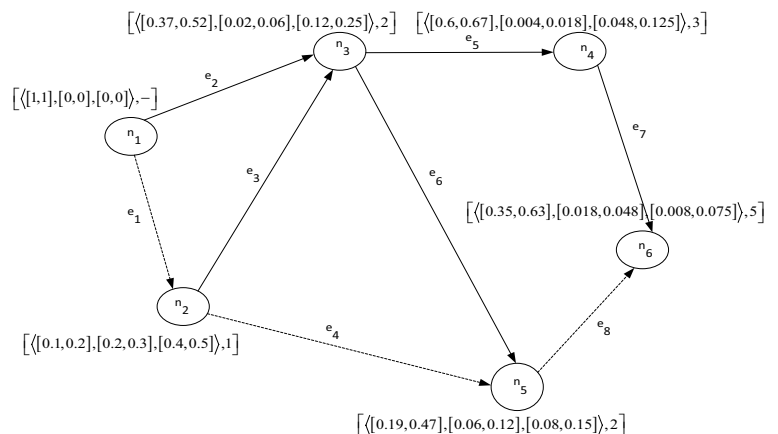


Table 3 Different deneutrosophication value of edge (i, j)

Edges	S_{Ridvan} [43]	$S_{Nagarajan}$
1–2	0.1	0.125
1–3	0.175	0.2
2–3	0.325	0.17
2–5	0.125	0.11
3–4	0.05	0.325
3–5	0.45	0.32
4–6	0.35	0.43
5–6	0.125	0.26

Table 4 Crisp path length for proposed algorithm

The proposed algorithm based $S_{Nagarajan}$	Crisp path length	Ranking
1 → 2 → 5 → 6	0.485	1
1 → 3 → 5 → 6	0.78	2
1 → 2 → 3 → 5 → 6	0.875	3
1 → 3 → 4 → 6	0.955	4
1 → 2 → 3 → 4 → 6	1.05	5

determines the shortest path from the initial node to the terminal node. The algorithm is described as follows.

- Step 1: Let n be the total number of paths from the initial node to terminal one. Find the score function of every arc length for the given network using Eqs. (18), (19) and (24), (25).
- Step 2: Find all the available paths $P_i, i = 1, 2, \dots, n$ and the corresponding path length. Also every n paths can be dealt as an arc which are represented by IVNN.
- Step 3: Find the sum of all score functions $S(\theta_i)$ of each available path.
- Step 4: The path which have minimum score value will represent an optimized interval valued shortest path by ranking in ascending order.

End

Note: TpIVNN-Trapezoidal interval valued neutrosophic number.

TIVNN-Triangular interval valued neutrosophic number.

Table 5 Trapezoidal interval valued neutrosophic distance

Edges	Trapezoidal interval valued neutrosophic distance	Edges	Trapezoidal interval valued neutrosophic distance
1–2 (e_1)	$\langle(1, 2, 3, 4); [0.1, 0.2], [0.2, 0.3], [0.4, 0.5]\rangle$	3–4 (e_5)	$\langle(2, 4, 8, 9); [0.2, 0.3], [0.2, 0.5], [0.4, 0.5]\rangle$
1–3 (e_2)	$\langle(2, 5, 7, 8); [0.2, 0.4], [0.3, 0.5], [0.1, 0.2]\rangle$	3–5 (e_6)	$\langle(3, 4, 5, 10); [0.3, 0.6], [0.1, 0.2], [0.1, 0.4]\rangle$
2–3 (e_3)	$\langle(3, 7, 8, 9); [0.3, 0.4], [0.1, 0.2], [0.3, 0.5]\rangle$	4–6 (e_7)	$\langle(7, 8, 9, 10); [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]\rangle$
2–5 (e_4)	$\langle(1, 5, 7, 9); [0.1, 0.3], [0.3, 0.4], [0.2, 0.3]\rangle$	5–6 (e_8)	$\langle(2, 4, 5, 7); [0.2, 0.3], [0.3, 0.4], [0.1, 0.5]\rangle$

Table 6 Available paths and its score value

Available path	$S(\theta_i)$	Ranking
$P_1 : 1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	4.18	1
$P_2 : 1 \rightarrow 3 \rightarrow 5 \rightarrow 6$	8.25	2
$P_4 : 1 \rightarrow 3 \rightarrow 4 \rightarrow 6$	12.43	3
$P_3 : 1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$	13.31	4
$P_5 : 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 6$	17.5	5

Illustrative example to find the shortest path using TpIVNN

For the validation of the proposed algorithm, a network is adopted from Broumi et al. [43] and Kumar et al. [65].

Consider a network with six nodes and eight edges. The TpIVN cost is given below (Tables 5, 6).

Applying steps 1–4 of the proposed algorithm, it is found that $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$ is IVNP with lowest cost 4.18 and the IVNP is $\langle(4, 11, 15, 20); [0.35, 0.608], [0.018, 0.048], [0.008, 0.075]\rangle$.

Illustrative example to find the shortest path using TIVNN

For the validation of the proposed algorithm, an example network is adopted from Broumi et al. [26, 35].

Consider a network with six nodes and eight edges. The TIVN cost is given below (Tables 7, 8).

Applying steps 1–4 of the proposed algorithm, it is found that $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$ is IVNP with lowest cost 4.18 and the IVNP is $\langle(4, 11, 15); [0.35, 0.61], [0.02, 0.05], [0.01, 0.08]\rangle$.

Comparative study of the proposed algorithm

In this section, a comparative study is carried out with the shortcomings and advantage of the proposed algorithm and it shows the effectiveness of the proposed algorithm

Shortcoming of the existing method

The compared existing method is unable to handle the interval-based information about the length of the arc and

Table 7 Triangular interval valued neutrosophic distance

Edges	Triangular interval valued neutrosophic distance	Edges	Triangular interval valued neutrosophic distance
1–2 (e_1)	$\langle(1, 2, 3); [0.1, 0.2], [0.2, 0.3], [0.4, 0.5]\rangle$	3–4 (e_5)	$\langle(2, 4, 8); [0.2, 0.3], [0.2, 0.5], [0.4, 0.5]\rangle$
1–3 (e_2)	$\langle(2, 5, 7); [0.2, 0.4], [0.3, 0.5], [0.1, 0.2]\rangle$	3–5 (e_6)	$\langle(3, 4, 5); [0.3, 0.6], [0.1, 0.2], [0.1, 0.4]\rangle$
2–3 (e_3)	$\langle(3, 7, 8); [0.3, 0.4], [0.1, 0.2], [0.3, 0.5]\rangle$	4–6 (e_7)	$\langle(7, 8, 9); [0.4, 0.6], [0.2, 0.4], [0.1, 0.3]\rangle$
2–5 (e_4)	$\langle(1, 5, 7); [0.1, 0.3], [0.3, 0.4], [0.2, 0.3]\rangle$	5–6 (e_8)	$\langle(2, 4, 5); [0.2, 0.3], [0.3, 0.4], [0.1, 0.5]\rangle$

Table 8 Available paths and its score value

Available path	$S(\theta_i)$	Ranking
$P_1 : 1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	4.9	1
$P_2 : 1 \rightarrow 3 \rightarrow 5 \rightarrow 6$	8.27	2
$P_4 : 1 \rightarrow 3 \rightarrow 4 \rightarrow 6$	11.1	3
$P_3 : 1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$	12.86	4
$P_5 : 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 6$	15.69	5

shortest path cannot be obtained for interval-based neutrosophic network.

Advantage of the proposed algorithm

If the length of the path is interval-based one then the shortest path of the given network can be obtained by interval valued neutrosophic numbers for an optimized path. Since triangular and trapezoidal numbers are widely used in many of the real world applications for their simplicity of computation, interval valued triangular and trapezoidal neutrosophic numbers have been used to find the neutrosophic shortest path. This is the advantage of the proposed algorithm.

Comparative study of algorithm

This section provides a comparative study of the proposed algorithm with the existing method of for neutrosophic shortest path problems.

A comparison of the results between existing and new techniques is shown in Table 9.

The result shows that the proposed algorithm provides sequence of visited nodes which shown to be similar with neutrosophic shortest path.

Table 9 Comparison of sequence of nodes using neutrosophic shortest path and our proposed algorithm

Algorithm of Broumi	Path	Crisp path length
S_{Ridvan} [43]	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	0.35
$S_{Nagarajan}$	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	0.485

The neutrosophic shortest path (abbr.NSP) remains the same namely $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$, but the crisp shortest path length (CSPL) differs namely $\langle[0.35, 0.60], [0.01, 0.04], [0.008, 0.075]\rangle$, respectively. From here we come to the conclusion that there exists no unique method for comparing neutrosophic numbers and different methods may satisfy different desirable criteria (Table 10).

Conclusion and future implication

The heart of the network community is nothing but the SPP. The objective of this problem is finding the minimum cost path among all other paths. This issue has been solved using many methods starts from conventional SPP with crisp weights. As many of the real world applications have uncertain vertices and edges fuzzy environment was useful to handle this problem. But still fuzzy setting cannot handle indeterminacy of the information, neutrosophic sets are found to be the best choice to handle this issue and has applied successfully. In this paper, neutrosophic SPP has been solved under interval valued neutrosophic, trapezoidal and triangular interval valued neutrosophic environments as it handles interval values. Also an improved score function and center of gravity has been proposed and applied to find the minimum cost of the path. Our proposed score function is without having the lower membership of falsity and which saves the time naturally. Further comparative analysis is done for Broumi’s algorithm with different

Table 10 Sequence of nodes with shortest path length

Possible path	Sequence of nodes	Neutrosophic shortest path length
Neutrosophic shortest path with interval valued neutrosophic numbers [43]	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	$\langle[0.35, 0.60], [0.01, 0.04], [0.008, 0.075]\rangle$
Proposed algorithm on $S_{Nagarajan}$	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	$\langle[0.35, 0.60], [0.01, 0.04], [0.008, 0.075]\rangle$

deneutrosophication function and proposed one. It is found that minimum cost is less compare than other existing method using proposed algorithms and score function. Also the proposed algorithm and improved score function have less computational complexity and saves the time. In future, the SPP would be extended to neutrosophic soft and rough set environments for interval-based path lengths. Also the proposed concept will be extended to complex neutrosophic environment.

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