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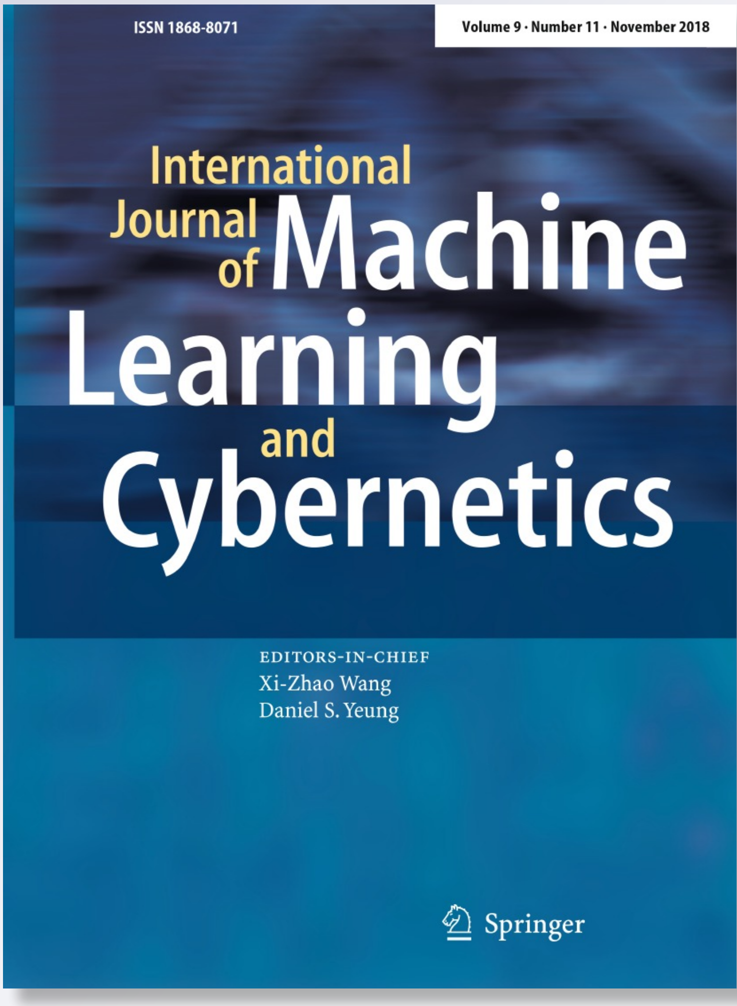
International Journal of Machine Learning and Cybernetics

ISSN 1868-8071
Volume 9
Number 11

Int. J. Mach. Learn. & Cyber. (2018)
9:1839-1855
DOI 10.1007/s13042-018-0860-3



Springer

International Journal of Machine Learning and Cybernetics (2018) 9:1839–1855
https://doi.org/10.1007/s13042-018-0860-3

ORIGINAL ARTICLE

Three-way *n*-valued neutrosophic concept lattice at different granulation

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Received: 16 April 2018 / Accepted: 28 July 2018 / Published online: 3 August 2018

Abstract

In recent year, the mathematics of three-way fuzzy concept lattice is introduced to characterize the attributes based on its acceptation, rejection and uncertain part. One of the suitable example is descriptive analysis of opinion of people in a democratic country. This became complex for the country like India where opinion (i.e. vote) of people to choose the particular leader is based on 29 independent states and their distinct issues. Adequate analysis of these type of 29-valued data based on its acceptation, rejection and uncertain part is major issue for the government and private agencies. To resolve this issue current paper introduces *n*-valued neutrosophic context and its graphical structure visualization for descriptive analysis. In the same time an another method is proposed to some of the similar three-way *n*-valued concepts. To zoom in and zoom out the *n*-valued neutrosophic context at user required information granules with an illustrative example.

Keywords Cognitive learning · Concept lattice · Formal concept analysis · *m*-polar fuzzy graph · *m*-polar fuzzy set · *n*-valued neutrosophic set · Three-way fuzzy concept lattice

1 Introduction

There are several data sets which contains *n*-valued logic [26, 31].[1] Handling these types of data sets is one of the crucial tasks for the researcher communities for knowledge discovery and representation tasks. One of the crucial problem arises with their precise mathematical representation. The reason is most of the mathematical functions provides their outputs beyond the binary values.[2] One of the suitable example is *n*th roots of any number includes *n*-values. Similarly, a function can contains maximum, minimum or saddle point for the given gradients. These types of multi-polar informations exists generally in human cognitive thought while taking multi-decision process [3, 39].[3] It may contain micro-expressions to represent the pleasure, sorrow, pain, fear, anger, surprise and many more. To deal with these types of multi-polar informations recently, the mathematical algebra of *m*-polar fuzzy sets [10] is introduced in applied lattice theory for knowledge processing tasks [26]. The graphical structure visualization of multi-polar fuzzy contexts [28] is considered as one of the generalization of fuzzy concept lattice [6, 21] beyond the unipolar space [5] to handle the heterogeneous context [4]. The most interesting part is its mathematical foundation laid down on the applied abstract algebra [12] and lattice theory [33]. In which, the set of objects (*X*) and their corresponding attributes (*Y*) used to represented by row and column of a matrix (*X*, *Y*, *R*) whereas entries represented by the corresponding relationship among them. Sometime the corresponding relationship among them may contains multi-polar valued which can be represented by *n*-valued context [26, 32]. One of the suitable example for *m*-polar fuzzy context is opinion of people in a democratic country like India depends on multi-polar informations [14, 28]. In this process, a problem arises when the condition of NOTA is used to determine the uncertainty beyond acceptance and rejection.[4] Dealing with this three-way attributes exists in *n*-dimensions (i.e. 29 states while considering the context of India) is a crucial problem for the researchers [17, 35, 37]. In this regard recently, Singh [22, 25] introduced the concept of three-way fuzzy concept lattice with its applications in various fields [26, 28].[5] However,

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1 https://en.wikipedia.org/wiki/Many_valued_logic.
2 https://en.wikipedia.org/wiki/Multivalued_function.
3 https://en.wikipedia.org/wiki/Polarity_international_relations.
4 https://en.wikipedia.org/wiki/None_of_the_above.
5 https://plato.stanford.edu/entries/logic-manyvalued/.

Table 1 Some basic advantages of using fuzzy, neutrosophic, m-polar or n-valued neutrosophic sets

	Fuzzy set	Neutrosophic set	m-polar fuzzy set	n-valued neutrosophic set
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Co-domain	Single-valued membership in [0, 1]	Three-valued membership in $[0, 1]^3$	m-valued membership in $[0, 1]^m$	n-valued in three-way space $[0, 1]^3$
Uncertainty	Yes	Yes	Yes	Yes
Truth-membership	Yes	Yes	Yes	Yes
False-membership	No	Yes	No	Yes
Indeterminacy	No	Yes	No	Yes
n-valued	No	No	Yes	Yes
Lattice	Yes	Yes	Yes	Yes
Partial order	Yes	Yes	Yes	Yes
Graph	Yes	Yes	Yes	Yes
Granulation	Yes	Yes	Yes	Yes

the current paper focuses on introducing the three-way fuzzy concept lattice for handling n-valued neutrosophic context at its navigation at user defined δ-granulation to refine some interesting or useful pattern.

Analysis of multi-polar data sets attracted several researchers recently for knowledge discovery and representation tasks [2, 23, 31]. One of the empirical analysis for these types of data sets is given in [28] that, the voting of people in a democratic country depends on multi-polar information. Afterward this study a problem is addressed that how to characterize them based on its acceptation, rejection and uncertain part [22]. This problem used to exists often in a democratic country where elections happens at each interval of time. It is a major issue for the democratic country like India where people of 29 states used to vote for the particular party based on their given parameters. This creates a problem with political as well as governing party to characterize this 29-valued data sets based on their acceptation, rejection and uncertain part. To deal with these types of issue mathematical algebra of neutrosophic set is developed [30] which is recently utilized for three-way fuzzy concept lattice representation [22] to navigate it at given threshold [25]. Some other researchers also focused on three-way decision space [36, 38] for analysis of uncertainty exists in the given decision attributes [15, 16]. The current paper focuses on precise analysis of n-valued data sets based on its acceptation, rejection and uncertain part. Table 1 shows that, the properties of n-valued neutrosophic set provides more generalize way to deal with these types of data sets when compared to any other approaches. It distinct the proposed methods when compared to any of the available approaches in three-way decision space as shown in Table 2. Some of the notable point which distinguish the proposed method from each of the available approaches are as follows:

1. The proposed method introduces a mathematical way to characterize the n-valued neutrosophic based on its n-valued truth, n-valued falsity and n-valued indeterminacy, independently.
2. The proposed method provides a granular based computing paradigm to find some δ-equal n-valued neutrosophic concepts.
3. The proposed method also provides a multiple ways to convert the given n-valued neutrosophic context into binary context at user required level of granulation.

The motivation of the current paper is to introduce a mathematical model for precise analysis of n-valued neutrosophic context with its graphical structure analytics for knowledge processing tasks as shown in Fig. 1. The objective is to extract some meaningful information from the given three-way n-valued neutrosophic data sets based on user required δ-granulation to solve the particular problem. To fulfil this objective current paper focuses on introducing three methods to deal with three-way n-valued neutrosophic contexts

Table 2 Some innovative researches on set theories discussed in Table 1

	Fuzzy set	Neutrosophic set	m-polar fuzzy set	n-valued neutrosophic set
Matrix	[6]	[8, 29, 30]	[14]	[2, 31]
Formal context	[4]	[24]	[32]	[1] *
Formal concept	[5]	[25]	[26]	*
Lattice	[12]	[22]	[28]	*
Interesting pattern	[11]	[20, 35, 37]	[10]	[7]
δ-granulation	[18, 34]	[24]	[17]	[7]
Applications	[21]	[13]	[3]	[7, 27]

* Research gap where less attention has been paid by researchers

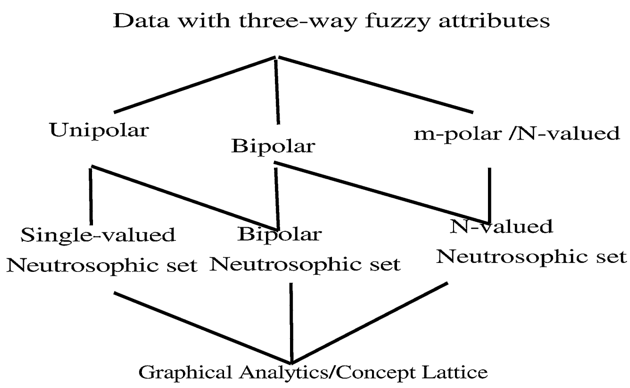

Fig. 1 Understanding the necessity of n-valued Neutrosophic concept lattice

with an illustrative example. The significant output of the proposed method is that it announces a first mathematical model for dealing with n-valued neutrosophic context and its navigation at user required granulation. It opens a door for knowledge processing researchers to utilize the mathematical paradigm of n-valued contexts in various fields. To solve the complexity of particular problem more precisely when compared to its numerical representations.

The rest of paper is organized as follows: Sect. 2 contains a brief background about FCA with three-way fuzzy settings. Section 3 demonstrate the steps of the proposed methods for extracting n-valued neutrosophic concepts with its illustration in Sect. 4. Section 5 contains discussions followed by conclusions and references.

2 Formal concept analysis with three-way fuzzy setting

Definition 1 (*Three-way fuzzy context*) [22, 29, 30] A three-polar fuzzy contexts can be written as $\mathbf{K} = (X, Y, \tilde{R})$ where X represents set of objects, Y represents set of three-polar attributes and $\tilde{R}$ represents three-polar relationship characterized by truth $(T_{\tilde{R}}(x,y))$, indeterminacy $(I_{\tilde{R}}(x,y))$ and falsity $(F_{\tilde{R}}(x,y))$ membership-value at which the object $x \in X$ has the attribute $y \in Y$ in three-polar fuzzy space $[0,1]^3$ where $(T_{\tilde{R}}(x,y))$,$(I_{\tilde{R}}(x,y))$ and $(F_{\tilde{R}}(x,y))$ are real standard or non-standard subsets of $]0^-, 1^+[$. It means $\tilde{R} = \{((x,y), T_{\tilde{R}}(x,y), I_{\tilde{R}}(x,y), F_{\tilde{R}}(x,y) : \forall x \in X, y \in Y\}$ where $0^- \leq T_{\tilde{R}}(x,y) + I_{\tilde{R}}(x,y) + F_{\tilde{R}}(x,y) \leq 3^+$. It should be noted that $0^- = 0 - \epsilon$ where 0 represents its standard part and ϵ represents its non-standard part. Similarly, $1^+ = 1 + \epsilon$ $(3^+ = 3 + \epsilon)$ where 1 (or 3) represents standard part and ϵ represents its non-standard part. In this case, the real standard format (0,1) or [0,1] can be also used to represent the relationship among objects and attributes set using neutrosophic set.

Table 3 A three-way fuzzy context for Example 1

	Corruption (y_1)
x_1	(0.6, 0.3, 0.1)
x_2	(0.3, 0.4, 0.3)
x_3	(0.8, 0.2, 0.0)

Example 1 Let us suppose, an expert wants to characterize the opinion of people in a democratic country based on their acceptation, rejection and uncertain part. In this case the expert can consider each of the political parties as a set of objects whereas the corresponding parameters as set of attributes. The corresponding three-way relationship to accept, reject or uncertain about the given political party based on chosen parameters can be written using the neutrosophic relations. The Table 3 represents a three-way fuzzy contexts where $\{x_1, x_2, x_3\}$ represents the political parties and y_1 represents attributes. The relation $\tilde{R}_{(x_1,y_1)} = (0.6, 0.3, 0.2)$ represents that 60% people voted the political party x_1. Due to its, stand against the corruption, 30% people uncertain about their stand on corruption whereas 10% people disagreed about their stand on the corruption. Similarly, other entries can be described. However a problem arises with expert while characterizing the opinion of people comes from 29-states of a democratic country like India (or USA) based on their the acceptation, rejection and uncertain part. To deal with this problem, a n-valued neutrosophic context representation is shown in the next definition.

Definition 2 (*Three-way neutrosophic graph*) [13] Let $G = (V, E)$ is a neutrosophic graph in which the vertices (V) can be characterized by a truth-membership function $T_V(v_i)$, a indeterminacy-membership function $I_V(v_i)$ and a falsity-membership function $F_V(v_i)$ where $\{(T_V(v_i), I_V(v_i), F_V(v_i)) \in [0,1]^3\}$ for all $v_i \in V$. Similarly the edges (E) can be defined as a neutrosophic set:
$\{(T_E(V \times V), I_E(V \times V), F_E(V \times V)) \in [0,1]^3\}$ for all $V \times V \in E$ such that:

$$T_E(v_i v_j) \leq min[T_E(v_i), T_E(v_j)],$$
$$I_E(v_i v_j) \geq max[I_E(v_i), I_E(v_j)],$$
$$F_E(v_i v_j) \geq max[F_E(v_i), F_E(v_j)].$$

The single-valued neutrosophic graph is complete iff:

$$T_E(v_i v_j) = min[T_E(v_i), T_E(v_j)],$$
$$I_E(v_i v_j) = max[I_E(v_i), I_E(v_j)],$$
$$F_E(v_i v_j) = max[F_E(v_i), F_E(v_j)].$$

Table 4 A three-way neutrosophic edges E for the Example 1

	Three-way decision edges (E)
v_1v_2	(0.3, 0.4, 0.3)
v_2v_3	(0.3, 0.4, 0.3)
v_3v_1	(0.6, 0.3, 0.2)

Table 5 A representation of Example 3 using three-way 29-valued context

	Corruption (y_1)
x_1	$\tilde{R}(x_1, y_1)((T_1, T_2, \dots, T_{29}), (I_1, I_2, \dots, I_{29}), (F_1, F_2, \dots, F_{29}))$
x_2	$\tilde{R}(x_2, y_1)((T_1, T_2, \dots, T_{29}), (I_1, I_2, \dots, I_{29}), (F_1, F_2, \dots, F_{29}))$
x_3	$\tilde{R}(x_3, y_1)((T_1, T_2, \dots, T_{29}), (I_1, I_2, \dots, I_{29}), (F_1, F_2, \dots, F_{29}))$

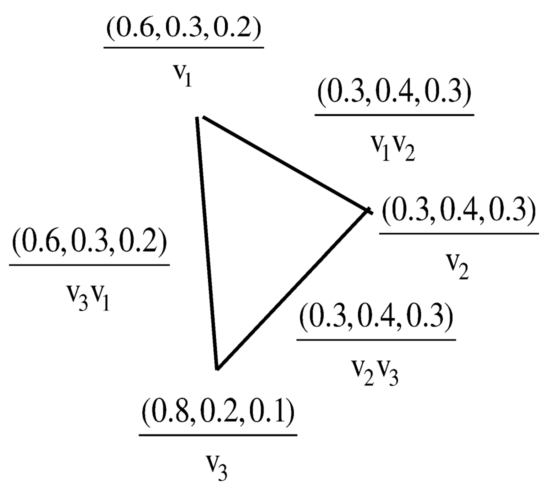

Fig. 2 A neutrosophic graph representation of context shown in Tables 3 and 4

It is noted that $\{(T_E(v_iv_j), I_E(v_iv_j), F_E(v_iv_j))\} = (0, 0, 0)$ $\forall (v_i, v_j) \in (V \times V \setminus E)$. Recently, one of the tool is developed for drawing the neutrosophic graph and its shortest path using Matlab [9]. Hence it is easier for any researchers to build the neutrosophic graph. This paper aimed at utilizing this property for handling the n-valued neutrosophic contexts for knowledge processing tasks.

Example 2 Let us suppose, an expert wants to visualize the opinion of people towards any given political parties $\{x_1, x_2, x_3\}$ shown in Table 3. In this case, the political parties can be considered as vertices $\{v_1, v_2, v_3\}$ of a neutrosophic graph. The corresponding relationship among their acceptance, rejection and uncertain part can be represented using the three-way neutrosophic set based edges $\{v_1v_2, v_2v_3, v_3v_1\}$ as shown in Table 4 based on neutrosophic relations. Fig. 2 represents the three-way neutrosophic graphs visualization of Table 3 and 4 in more descriptive way for further processing.

Definition 3 (*n-valued neutrosophic context*) [31, 32] An n-valued neutrosophic context provides a way to analyze the three-way fuzzy context based on n-types of truths membership-values $(T_1, T_2, \dots, T_n)$, n-types of indeterminacy membership-values $(I_1, I_2, \dots, I_n)$ and n-types of falsity membership-values $(F_1, F_2, \dots, F_n)$. It means the n-valued neutrosophic relation ($\tilde{R}$) on the set X and Y can be represented as follows:

$$\tilde{R} = \Big\{((x, y), \textstyle\sum_{i=1}^{n} T_{\tilde{R}_i}(x, y), \sum_{j=1}^{n} I_{\tilde{R}_j}(x, y), \sum_{k=1}^{n} F_{\tilde{R}_k}(x, y) :$$
$$\forall x \in X, y \in Y\} \text{ where } 0^- \leq \textstyle\sum_{i=1}^{n} T_{\tilde{R}_i}(x, y) + \sum_{j=1}^{n} I_{\tilde{R}_j}(x, y) +$$
$$\textstyle\sum_{k=1}^{n} F_{\tilde{R}_k}(x, y) \leq n^+ \text{ where } n = i + j + k.$$

Example 3 Now merge the Examples 1 and 2 as the expert wants to characterize the opinion of people from 29 states of a democratic country like India based on its acceptation, rejection and uncertain part about the political parties $\{x_1, x_2, x_3\}$ in case of their stand against corruption (y_1). This three-way context can be written using 29-truth membership-values, 29-indeterminacy membership-values and 29-falsity membership-values using the extensive properties of n-valued neutrosophic context as shown in Table 5. It can be considered as one of the first kind of representation in field of knowledge processing tasks when compared to three-way fuzzy context [22, 24] as well as m-polar fuzzy context [26, 28]. Furthermore, the central analysis of this paper is investigation of some interesting and useul knowledge from the given three-way n-valued neutrosophic context for knowledge processing tasks as given in further section.

Definition 4 (*n-valued Neutrosophic graph*) [7] Let us suppose, $G = (V, E)$ is a neutrosophic graph in which the vertices (V) can be characterized by n-valued truth-membership function $T_{1,\dots,n}(v_i)$, n-valued indeterminacy-membership function $I_{1,\dots,n}(v_i)$ and n-valued falsity-membership function $F_{1,\dots,n}(v_i)$ where $\{(T_{1,\dots,n}(v_i), I_{1,\dots,n}(v_i), F_{1,\dots,n}(v_i)) \in [0, 1]^n\}$ for all $v_i \in V$. Similarly, the edges (E) can be defined as n-valued neutrosophic set:

$\{(T_{1,\dots,n}(V \times V), I_{1,\dots,n}(V \times V), F_{1,\dots,n}(V \times V)) \in [0, 1]^3\}$ for all $V \times V \in E$ such that:

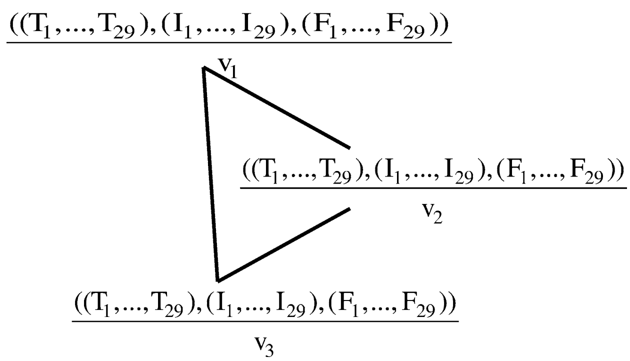

Fig. 3 A 29-valued neutrosophic graph to represent the cognition of 29 states of India

$$T_{1,\ldots,n}(v_iv_j) \le min\left[T_{1,\ldots,n}(v_i), T_{1,\ldots,n}(v_j)\right],$$
$$I_{1,\ldots,n}(v_iv_j) \ge max\left[I_{1,\ldots,n}(v_i), I_{1,\ldots,n}(v_j)\right],$$
$$F_{1,\ldots,n}(v_iv_j) \ge max\left[F_{1,\ldots,n}(v_i), F_{1,\ldots,n}(v_j)\right].$$

The n-valued neutrosophic graph is complete iff:

$$T_{1,\ldots,n}(v_iv_j) = min[T_{1,\ldots,n}(v_i), T_{1,\ldots,n}(v_j)],$$
$$I_{1,\ldots,n}(v_iv_j) = max[I_{1,\ldots,n}(v_i), I_{1,\ldots,n}(v_j)],$$
$$F_{1,\ldots,n}(v_iv_j) = max[F_{1,\ldots,n}(v_i), F_{1,\ldots,n}(v_j)].$$

It is noted that $\left\{(T_{1,\ldots,n}(v_iv_j), I_{1,\ldots,n}(v_iv_j), F_{1,\ldots,n}(v_iv_j))\right\} = (0, 0, 0)\ \forall (v_i, v_i) \in (V \times V \setminus E)$.

Example 4 Let us suppose, the expert wants to visualize the vote of people given from 29 states to each of the political parties. In this case, the opinion can be characterized based on its 29 acceptation, 29 rejection and 29 uncertain parts, resepectively for each political party. To represent these types of data sets the expert can utilize the three-way 29-valued context shown in Table 5 for descriptive analysis. The graphical structure visualization of given context can be resolved using the mathematical algebra of three-way n-valued graph where the the political parties can be considered as vertices of a defined three-way 29-valued neutrosophic graph $\left\{v_1, v_2, v_3\right\}$ and their corresponding three-way 29-valued relationship can be considered as edges (E). The compact representation of this context is shown in Fig. 3. In the next section, three methods are proposed to discover some useful information from the three-way n-valued neutrosophic context.

Definition 5 (*Three-way fuzzy concepts*) [24, 25] For any **L**-set $A \in L^X$ of objects an **L**-set $A^{\uparrow} \in L^Y$ of attributes can be defined using UP operator of Galois connection as follows:

$$A^{\uparrow}(y) = \wedge_{x\in X}(A(x) \rightarrow \tilde{R}(x, y)).$$

Similarly, for any **L**-set of $B \in L^Y$ of attributes an **L**-set $B^{\downarrow} \in L^X$ of objects set can be defined using down operator of Galois connection as given below :

$$B^{\downarrow}(x) = \wedge_{y\in Y}(B(y) \rightarrow \tilde{R}(x, y)).$$

$A^{\uparrow}(y)$ is interpreted as the **L**-set of attribute $y \in Y$ shared by all objects from A. Similarly, $B^{\downarrow}(x)$ is interpreted as the **L**-set of all objects $x \in X$ having the same attributes from B in common. This can be computed similarly for truth, indeterminacy and falsity membership values for the given neutrosophic set of attributes. The pair (A, B) is called as formal neutrosophic concept iff $B^{\downarrow} = (A, (T_A, I_A, F_A))$ and $A^{\uparrow} = (B, (T_B, I_B, F_B))$. The $\downarrow$ is applied on three-way fuzzy set of attributes as follows:

$$B^{\downarrow} = \left\{y_j, (T_B(y_j), I_B(y_j), F_B(y_j)) \in [0, 1]^3 : \forall y_j \in Y\right\}.$$

It provides the covering objects set as follows:

$$A = \left\{x_i, (T_A(x_i), I_A(x_i), F_A(x_i)) \in [0, 1]^3 : \forall x_i \in X\right\}.$$

The obtained object set have maximal membership with respect to integrating the information by the given attributes set. Now apply the $\uparrow$ on these constituted objects set which provide the three-way fuzzy attribute set with its membership value being maximal with respect to integrating the information from the constituted objects set. If the obtained pair make the three-way fuzzy concepts then any extra attribute (or object) cannot be discovered which will make the maximum membership value of the obtained set of attributes (objects). Similarly, it can be used for n-valued context as shown in [1, 7].

Figures 2 and 3 approves that any given n-valued neutrosophic context can be hierarchically analyzed using graph based lattice for cognitive concept learning as it is done in three-way [22, 25] and multi-polar fuzzy space [26, 28]. To accomplish this task, a method is proposed, in the next section for discovering all the n-valued neutrosophic concepts and their display in the concept lattice.

3 Proposed methods

In this section, three methods are proposed to extract some of the interesting information from the n-valued neutrosophic context. The first method provides a way to discover n-valued neutrosophic concepts and their concept lattice visualization whereas the second method provides a way to find some of the closest n-valued concepts at defined threshold. The third method given a multiple ways to zoom in and zoom out the given n-valued neutrosophic contexts at micro

Table 6 A three-way n-valued neutrosophic context

	y_1	…	y_m
x_1	$((T_{11},\dots,T_{1n}),(I_{11},\dots,I_{1n}),(F_{11},\dots,F_{1n})$	…	$((T_{11},\dots,T_{1n}),(I_{11},\dots,I_{1n}),(F_{11},\dots,F_{1n})$
.	.	…	
.	.	…	
.	.	…	
x_n	$((T_{n1},\dots,T_{nn}),(I_{n1},\dots,I_{nn}),(F_{n1},\dots,F_{nn})$	…	$((T_{n1},\dots,T_{nn}),(I_{n1},\dots,I_{nn}),(F_{n1},\dots,F_{nn})$

and macro level. Subsequently, the time complexity for each of the proposed method is given to solve the particular problem based on user or expert requirements.

3.1 A proposed method for generating the *n*-valued neutrosophic concepts

In this section, a method is proposed to generate the m-polar formal fuzzy concepts using the properties of m-polar fuzzy graph and component-wise Gödel residuated lattice. Let us suppose, an m-polar formal fuzzy context $\mathbf{F} = (X, Y, \tilde{R})$ where, $|X| = n$, $|Y| = k$ and, $\tilde{R}$ represents corresponding n-valued neutrosophic relationship among them. The n-valued formal concepts can be discovered as follows:

Step 1 Let us suppose a n-valued three-way fuzzy context $\mathbf{K} = (X, Y, \tilde{R_n})$ where, $|X| = k$, $|Y| = m$ and, $\tilde{R_n}$ represents a mapping for their corresponding n-valued neutrosophic relationship among them as shown in Table 6.

Step 2 Let us choose, any subset of attribute s_j with their acceptance value based on user requirement.

Step 3 The proposed method considers maximal acceptance of truth-membership value i.e. 1.0, minimum indeterminacy i.e. 0 and minimum falsity membership-value i.e. 0 in each component of given n-valued three-way fuzzy attributes as follows:

$B_{s_j} = ((1, 1, \dots, 1), (0, 0, \dots, 0), (0, 0, \dots, 0))$ i.e. $T_{B_n}(y_j) = (1, \dots, 1)_n$, $I_{B_n}(y_j) = (0, \dots, 0)n$, and $F_{B_n}(y_j) = (0, \dots, 0)_n$. It should be noted that, the user or expert can change the membership-values based on his/her requirements to discover the n-valued concepts.

Step 4 The maximal covering set of objects for the chosen subset of attributes (s_j) can be investigated using the Down operator (↓) i.e. $B_{s_j}^{\downarrow} = A_{s_i}$ of Galois connection.

Step 5 The membership value of the obtained objects set can be computed by considering each component of n-valued neutrosophic set as follows:

$$T_{A_{s_i}}(x_i) = min_{j \in T_{B_{s_j}}} \mu_T^{\tilde{R}}(x_i, y_j),$$

$$I_{A_{s_i}}(x_i) = max_{j \in I_{B_{s_j}}} \mu_I^{\tilde{R}}(x_i, y_j),$$

$$F_{A_{s_i}}(x_i) = max_{j \in F_{B_{s_j}}} \mu_F^{\tilde{R}}(x_i, y_j).$$

where $\tilde{R}$ is the corresponding three-way n-valued neutrosophic relationship among the object and attribute set.

Step 6 In the same time discover the covering attributes set i.e. B_{s_j} for the constituted objects which are closed with Galois connection (↑) i.e $A_{s_i}^{\uparrow}$ and their n-valued neutrosophic membership-values as follows:

$$T_{B_{s_j}}(y_j) = min_{j \in T_{A_{s_i}}} \mu_T^{\tilde{R}}(x_i, y_j),$$

$$I_{B_{s_j}}(y_j) = max_{j \in I_{A_{s_i}}} \mu_I^{\tilde{R}}(x_i, y_j),$$

$$F_{B_{s_j}}(y_j) = max_{j \in F_{A_{s_i}}} \mu_F^{\tilde{R}}(x_i, y_j).$$

Step 7 Similarly, apply the corresponding Galois connection (↓, ↑) to find the maximal closed set of objects and attributes.

Step 8 The obtained set of objects and attributes set (A_{s_i}, B_{s_j}) is considered as n-valued three-way fuzzy concepts iff any extra objects or attributes set cannot make it bigger.

Step 9 Similarly, other existing pattern in the given three-way n-valued neutrosophic context can be generated.

Step 10 Draw the n-valued three-way fuzzy concept lattice based on their subset. Table 7 summarizes the proposed algorithm established above.

Complexity Let, number of objects-$|X| = k$ and the number of attributes $|Y| = m$ in the given three-way fuzzy context. The proposed algorithm computes subset of attributes-(Y) which takes 2^m complexity to find the maximal pair of covering objects k for the n-valued in three-way component based on its truth, indeterminacy, and falsity-membership value. In this way the proposed method takes $O(2^m \cdot 3n \cdot k)$ time complexity. It can be observed that the proposed method provides a way to extract the three-way n-valued neutrosophic concepts. However to achieve this goal it takes more computational time which may become relevant in large context. To deal with this issue another method is proposed to decompose the context at user required δ -granulation.

Table 7 A proposed algorithm for discovery of n-valued neutrosophic concepts

Input: a n-valued three-way fuzzy context $\mathbf{K} = (X, Y, \tilde{R}_n)$
where $|X| = n, |Y| = m$ and $\tilde{R}_n$ is n-valued neutrosophic relation.
Output: the set of n-valued three-way fuzzy concepts.
$\{\{x_i, (T_{A_n}(x_i), I_{A_n}(x_i), F_{A_n}(x_i))\}, \{y_j, (T_{B_n}(y_j), I_{B_n}(y_j), F_{B_n}(y_j))\}\}$
where $i \leq k$ and $j \leq m$

1. Choose any subset s_j.
2. Decide the acceptance value for the chosen subset as follows:
   $T_{B_n}(y_j) = (1, \dots, 1)_n$, $I_{B_n}(y_j) = (0, \dots, 0)n$, and $F_{B_n}(y_j) = (0, \dots, 0)_n$
3. Discover the covering objects for the chosen attribute set :
   $\left\{y_j, (T_{B_{s_j}}(y_j), I_{B_{s_j}}(y_j), F_{B_{s_j}}(y_j))\right\}^{\downarrow} = \left\{x_i, (T_{A_{s_i}}(x_i), I_{A_{s_i}}(x_i), F_{A_{s_i}}(x_i))\right\}$
4. Compute the membership value for each n-valued objects set:
   $T_{A_{s_i}}(x_i) = min_{j \in T_{B_{s_j}}} \mu_T^{\tilde{R}}(x_i, y_j)$,
   $I_{A_{s_i}}(x_i) = max_{j \in I_{B_{s_j}}} \mu_I^{\tilde{R}}(x_i, y_j)$,
   $F_{A_{s_i}}(x_i) = max_{j \in F_{B_{s_j}}} \mu_F^{\tilde{R}}(x_i, y_j)$.
5. Discover the covering attributes for the obtained objects set:
   $\{x_i, (T_{A_n}(x_i), I_{A_n}(x_i), F_{A_n}(x_i))\}^{\uparrow}$
6. **if** $\left\{x_i, (T_{A_{s_i}}(x_i), I_{A_{s_i}}(x_i), F_{A_{s_i}}(x_i))\right\}^{\uparrow} = \left\{y_j, (T_{B_{s_j}}(y_j), I_{B_{s_j}}(y_j), F_{B_{s_j}}(y_j))\right\}$
7. Similarly, apply the corresponding Galois connection $(\downarrow, \uparrow)$.
8. The obtained closed pair of objects and attributes (A_{s_i}, B_{s_j}) is a concept.
9. Similarly, other concepts can be generated using the chosen subsets.
10. Draw the line diagram among generated concepts based on their subsets.

3.2 A method for extracting δ-equal *n*-valued neutrosophic concepts

In this section, a method is proposed for finding δ-equal three-way n-valued neutrosophic objects (or attributes) based on user defined set of attributes. The acceptance value for the chosen subset of attributes can be defined by user or expert requirement to solve the particular problem. In this way this method discovered all the objects (or attributes) having closed relationship with user defined set of attributes and its acceptance values. It means the chosen subset of attributes can be considered as information granules to finds some of the closest δ-equal concepts [20, 24]. In this way, the proposed method provides multiple ways to investigate the δ-equal pattern from the given n-valued contexts based on its objects and attributes without generating all the concepts. It is one of the significant advantages of the proposed method while considering the time constraints. To achieve this goal, the properties of granular computing is utilized to refine or coarse the obtained pattern based on user or expert requirement. The steps of the proposed method is as follows:

Step 1 Let us suppose, a three-way n-valued neutrosophic information granules $(C_1 = (A_1, B_1))$ to investigate some of the closest pattern.

Step 2 Enter a n-valued neutrosophic concept i.e. $(C_2 = (A_2, B_2))$ generated from the given data set.

Step 3 Compute the n-valued neutrosophic distance among them using their attributes (or object set) as follows:
$d(B_1, B_2) = \max(\sup|T_{B_1}(y) - T_{B_2}(y)|, \sup|I_{B_1}(y) - F_{B_2}(y)|, \sup|F_{B_1}(y) - F_{B_2}(y)|)$.

The distance will be computed for each component of n-valued neutrosophic set defined for the attributes. Similarly, it can be computed for the extent also.

Step 4 The distance of each attributes can be computed similarly using their summation: $\sum d(B_1, B_2)$.

Step 5 The average distance can be computed as follows: $AV(d) = \frac{\sum d(B_1, B_2)}{m}$ where m is total number of attributes in the given n-valued neutrosophic data sets.

Step 6 Decide the level of δ-granulation for extracting the similar information when compared to chosen information granules for the knowledge processing tasks.

Step 7 The chosen subset can be selected iff : $d(C_1, C_2) \leq 1 - \delta$.

Step 8 In similar way other closeness of other objects and attributes can be discovered.

Step 9 List out all the discovered set of objects based on their covering attributes from the given context.

Step 10 Derive the knowledge to solve the particular problem.

Table 8 A proposed algorithm for finding δ-similar n-valued concepts

Input: A three-way n-valued neutrosophic attributes (B_1)),
Output:δ-equal n-valued neutrosophic concepts $(C_2 = (A_2, B_2))$.
1. Decide any n-valued neutrosophic concept to find its similar concepts:
$(C_1 = (A_1, B_1))$
2. Enter any n-valued neutrosophic concepts :
$(C_2 = (A_2, B_2))$.
3. Compute its distance from chosen concept as follows:
$d(C_1, C_2) = \max(\sup\vert T_{C_1} - T_{C_2}\vert, \sup\vert I_{C_1} - I_{C_2}\vert, \sup\vert F_{C_1} - F_{C_2}\vert)$.
4. Sum the distance of each attributes in the intent i.e.:
$\sum d(C_1, C_2)$.
5. Compute the average distance $AV(d)$ as follows :
$AV(d) = \frac{\sum d(C_1,C_2)}{m}$ where m is number of attributes.
6. Define a δ-equality for selecting the concepts.
7. **if** $(d(C_1, C_2) \leq 1 - \delta)$
Select the concept.
else
remove the concept.
8. Similarly, other concepts can be chosen.
9. Represent the obtain concepts in their order.
10. Derive the knowledge from the obtained concepts.

Complexity The proposed algorithm shown in Table 8 provides a way to find the δ-equal n-valued concepts based on user required information granules. In this case the proposed method compares each component of n-valued concepts from the chosen information granules for n-valued truth, indeterminacy and falsity membership, function. In this case, it may take maximally $m \times n$ time for truth, indeterminacy and falsity membership values to compute the distance among them, respectively. Hence, the overall complexity of the proposed method cannot exceed the $O(m^3 \cdot n^3)$ computational time. In this way the proposed method reduces the complex fuzzy concepts and its computational time for knowledge processing tasks when compared to the proposed method in Table 6. To measure the interestingness the analysis derived from both of the methods are compared in this paper with demonstration.

3.3 A method for decomposition of *n*-valued neutrosophic contexts

It can be observed that, the above mentioned proposed methods generates exponential number of n-valued neutrosophic concepts. In this case it will be difficult to user or expert when they need some of the generalized or specialized concepts to derive the knowledge. To deal with this problem generally a method is applied to decomposition of given formal context at user required granulation for further processing. The reason mathematics of granular computing provides multiple ways to zoom in or zoom out the context for extracting precise information to solve the particular problem [19]. Toward this extension the properties of granular computing is applied in formal context [34] as well as neutrosophic context decomposition [25] for multi-decision process [17, 18]. Recently, the properties of granular computing is applied in three-way neutrosophic fuzzy context and its interval-valued also [25] to refine the neutrosophic context for knowledge processing tasks. In this paper the proposed method focuses on decomposing the n-valued neutrosophic context at user defined granulation for its n-valued truth, indeterminacy and false membership values, independently. i.e. $((\alpha_{T_1}, \dots, \alpha_{T_n}), (\beta_{I_1}, \dots, \beta_{I_n}), (\gamma_{F_1}, \dots, \gamma_{F_n}))$-cut.

Step 1 Let us suppose, a n-valued neutrosophic context $\mathbf{K} = (X, Y, \tilde{R}_n)$ where, $|X| = k, |Y| = m$ and, $\tilde{R}_n$ represents the n-valued neutrosophic relationship among them.

Step 2 The n-valued neutrosophic context can be decomposed based on user or expert requirements to solve the particular problem. The level of granulation can be decided based on its n-valued truth i.e. $(\alpha_{T_1}, \dots, \alpha_{T_n})$, indeterminacy i.e. $(\beta_{I_1}, \dots, \beta_{I_n})$, and falsity membership value i.e. $(\gamma_{F_1}, \dots, \gamma_{F_n})$.

Step 3 The n-valued context can be decomposed based on chosen multi-granulation as follows:

The truth membership value belongs to the same decomposed context iff: $\mathbf{K}^T_{(\alpha_{T_1},\dots,\alpha_{T_n})} = \{T_{\tilde{R}_n(x,y)} | T_{\tilde{R}_1(x,y)} \geq \alpha_{T_1} \wedge \cdots \wedge T_{\tilde{R}_n(x,y)} \geq \alpha_{T_n}\}$. The decomposition of context provides either 0 or 1 values. The obtained 1.0 values represents acceptance of truth membership-value for the chosen multi-granulation whereas 0 represents unacceptance.

Table 9 A proposed algorithm for $((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{I_1},\ldots,\beta_{I_n}),(\gamma_{F_1},\ldots,\gamma_{F_n})))$-cut of n-valued neutrosophic context

Input: A n-valued neutrosophic context $\mathbf{K}=(X,Y,\tilde{R}_n)$
where $|X|=k, |Y|=m$ and $(\tilde{R}_n=(T_{\tilde{R}_n(x,y)}, I_{\tilde{R}_n(x,y)}, F_{\tilde{R}_n(x,y)}))$.
Output: The set of decomposed context $\mathbf{K}_{((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{F_1},\beta_{F_n}),(\gamma_{F_1},\ldots,\gamma_{F_n})))}$.
1. Let us suppose, a n-valued neutrosophic context $\mathbf{K}=(X,Y,\tilde{R}_n)$.
2. Define the granulation for the n-valued truth, indeterminacy, and falsity membership as follows:
$((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{I_1},\ldots,\beta_{I_n}),(\gamma_{F_1},\ldots,\gamma_{F_n})))$.
3. Decomposed the given n-valued context as follows:
4. *if* $(T_{\tilde{R}_1(x,y)}\geq\alpha_{T_1}\wedge\ldots\wedge T_{\tilde{R}_n(x,y)}\geq\alpha_{T_n})$
then represent 1.0 at the place of truth membership value.
else 0.0 at the place of truth membership value.
5. *if* $(I_{\tilde{R}_1(x,y)}\leq\gamma_{I_1}\wedge\ldots, I_{\tilde{R}_n(x,y)}\leq\gamma_{I_1})$
then represent 0.0 at the place of indeterminacy membership value.
else 1.0 at the place of indeterminacy membership value.
6. *if* $(F_{\tilde{R}_1(x,y)}\leq\gamma_{F_1}\wedge\ldots\wedge F_{\tilde{R}_n(x,y)}\leq\gamma_{F_n})$
then represent 0.0 at the place of falsity membership value.
else 1.0 at the place of falsity membership value.
7. The decomposed context follows the equality:
$\mathbf{K}=\bigcup_{((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{I_1},\ldots,\beta_{I_n}),(\gamma_{F_1},\ldots,\gamma_{F_n}))}$.
8. Similarly, compute for all entries of n-valued neutrosophic context.
9. Represent the obtained decomposed context.
10. Derive the knowledge.

The indeterminacy membership value belongs to the same decomposed context iff: $\mathbf{K}^I_{(\beta_{I_1},\ldots,\beta_{I_n})}=\{I_{\tilde{R}_n(x,y)}|I_{\tilde{R}_1(x,y)}\leq\alpha_{I_1}\wedge\cdots\wedge I_{\tilde{R}_n(x,y)}\leq\beta_{I_n}\}$. In this case, 0 represent 0.0 acceptance of indeterminacy-value for the chosen multi-granulation where 1 as unacceptance.

The falsity membership value belongs to the same decomposed context iff: $\mathbf{K}^F_{(\gamma_{F_1},\ldots,\gamma_{F_n})}=\{F_{\tilde{R}_n(x,y)}|F_{\tilde{R}_1(x,y)}\leq\gamma_{F_1}\wedge\cdots\wedge F_{\tilde{R}_n(x,y)}\leq\gamma_{F_n}\}$. In this case, 0 represent 0.0 acceptance of indeterminacy-value for the chosen multi-granulation where 1 as unacceptance.

Step 4 The decomposed contexts $\mathbf{K}_{((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{I_1},\ldots,\beta_{I_n}),(\gamma_{F_1},\ldots,\gamma_{F_n})))}$ at user defined granulation follow an equality given below:

$\mathbf{K}=\bigcup_{((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{I_1},\ldots,\beta_{I_n}),(\gamma_{F_1},\ldots,\gamma_{F_n}))}$ where, α is a defined multi-granulation level for the n-valued neutrosophic truth membership-values, β is defined for n-valued indeterminacy membership-values, and γ is defined for n-valued falsity membership-values.

Step 5 The decomposed context also satisfies the following properties i. e. $\mathbf{K}_{\alpha_1,\beta_1,\gamma_1}\subseteq\mathbf{K}_{\alpha_2,\beta_2,\gamma_2}$ when $\alpha_1\leq\alpha_2$, $\beta_1\geq\beta_2$, $\gamma_1\geq\gamma_2$. It means the size of n-valued neutrosophic concept lattice can be zoom in and zoom out using the distinct granulation for the truth, indeterminacy and falsity membership value.

Step 7 Similarly, it can be computed for other each entries of n-valued neutrosophic context based on user required information granules.

Step 8 In last the decomposed context can be written in three-way decision space for further analysis of knowledge processing taks. The pseudo code for this proposed algorithm is shown in Table 8.

Complexity Table 9 shows the pseudo code for the decomposition of a given n-valued neutrosophic context having k number of objects and m number of attributes. The proposed method decomposed the context based on user defined granulation for the n-valued truth, indeterminacy, and falsity membership-values i.e. $((\alpha_{T_1},\ldots,\alpha_{T_n}),(\beta_{I_1},\ldots,\beta_{I_n}),(\gamma_{F_1},\ldots,\gamma_{F_n})))$-cut, independently. It may take $O(m^3\cdot n)$ or $O(k^3\cdot n)$ time complexity in case of considering intent or extent, respectively. In this way the proposed method reduces the computational time for processing the given n-valued context based on user or expert requirement to solve the particular problem.

4 Illustrations

In this section each of the proposed method is illustrated considering one of the n-valued neutrosophic context with their comparative study of the obtained results to validate the results.

Table 10 A three-way fuzzy context representation based on the vote given in State 1

Political parties	Corruption (y_1)	Criminals (y_2)	Un-educated (y_3)
x_1	(0.6, 0.3, 0.2)	(0.4, 0.3, 0.3)	(0.5, 0.2, 0.2)
x_2	(0.5, 0.3, 0.2)	(0.4, 0.3, 0.3)	(0.6, 0.2, 0.3)
x_3	(0.7, 0.2, 0.1)	(0.7, 0.1, 0.2)	(0.9, 0.1, 0.0)

Table 11 A three-way fuzzy context representation based on the vote given in State 2

Political parties	Corruption (y_1)	Criminals (y_2)	Un-educated (y_3)
x_1	(0.5, 0.3, 0.2)	(0.4, 0.3, 0.3)	(0.5, 0.2, 0.2)
x_2	(0.4, 0.2, 0.3)	(0.4, 0.3, 0.3)	(0.6, 0.2, 0.3)
x_3	(0.3, 0.4, 0.3)	(0.7, 0.1, 0.2)	(0.9, 0.1, 0.0)

4.1 Three-way *n*-valued neutrosophic concept lattice

Recently, several researchers have paid attention towards analysis of multi-polar informations [10, 26]. One of the suitable example is given in [28] to analyze the opinion of people in a democratic country based on some parameters. In this process, a problem arises while characterization of opinion based on their acceptation, rejection and uncertain part, independently [22]. This problem become more complex in case of handling the data from 29 states of a democratic country like India.[6] This analysis is indeed requirement for any political scientist or other experts to decide the win, loss or neutrality of a given party.[7] To deal with this problem, the current paper introduces a method based on three-way *n*-valued set in Sect. 3.1. In this section, the proposed method is demonstrated with an example as given below:

Example 5 Let us suppose, there are three political parties $\{x_1, x_2, x_3\}$ participating in a democratic country like India to win the particular election. The people from 29 states used to participate in the election. In this case people of 29 states used to vote these parties based on following parameters: y_1 = Corruption; y_2 = Criminals, y_3 = Un-education, y_4 = Control on Terrorism; y_5 = Un-employment; y_6 = Foreign policy; y_7 = Medical and health policy; y_8 = Education policy; y_9 = Economical policy; y_{10} = Defense policy;; or other issues also can be considered. The problem arises when an expert wants characterize the acceptation, rejection and uncertain part from opinion of people came from these 29-states. To demonstrate this issue the current paper considers three-potential attributes using the proposed method shown in Sect. 3.1. Table 10 represents the opinion of people from states 1 based on three-decision space. Similarly, Tables 11 and 12 represents the opinion of people from states 2 and 3. Table 13 represents the combined format of Tables 10, 11, 12 and 13 using the introduced three-way *n*-valued neutrosophic contexts.

[6] http://katehon.com/article/india-and-multipolarity.

[7] http://www.e-ir.info/2013/06/03/towards-a-multi-polar-international-system-which-prospects-for-global-peace/.

Table 12 A three-way fuzzy context representation based on the vote given in State 3

Political parties	Corruption (y_1)	Criminals (y_2)	Un-educated (y_3)
x_1	(0.5, 0.2, 0.3)	(0.4, 0.2, 0.3)	(0.4, 0.2, 0.2)
x_2	(0.3, 0.4, 0.3)	(0.4, 0.2, 0.4)	(0.3, 0.3, 0.4)
x_3	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.7, 0.1, 0.1)

Table 13 A composed representation of Tables 10, 11 and 12 using *n*-valued neutrosophic set

Political parties	Corruption (y_1)	Criminals (y_2)	Un-educated (y_3)
x_1	((0.6, 0.3, 0.2), (0.5, 0.3, 0.2), (0.5, 0.2, 0.3))	((0.4, 0.3, 0.3), (0.4, 0.3, 0.3), (0.4, 0.2, 0.3))	((0.5, 0.2, 0.2), (0.5, 0.2, 0.2), (0.4, 0.2, 0.2))
x_2	((0.3, 0.4, 0.3), (0.4, 0.2, 0.3), (0.3, 0.4, 0.3))	((0.4, 0.2, 0.4), (0.4, 0.3, 0.3), (0.4, 0.2, 0.4))	((0.3, 0.3, 0.4), (0.6, 0.2, 0.3), (0.3, 0.3, 0.4))
x_3	((0.8, 0.1, 0.1), (0.3, 0.4, 0.3), (0.8, 0.1, 0.1))	((0.9, 0.1, 0.4), (0.7, 0.1, 0.2), (0.9, 0.1, 0.0))	((0.7, 0.1, 0.1), (0.9, 0.1, 0.0), (0.7, 0.1, 0.1))

The three-valued neutrosophic relation $(x_1, y_1) = ((0.6, 0.3, 0.2)(0.5, 0.3, 0.2)(0.5, 0.2, 0.3))$ shown in Table 13 represents that 60% people of state 1 agreed that political party x_1 stands against corruption, 30% people are indeterminant for its stands on corruption whereas 20% people disagreed on its stands on corruption. 50% people of state 2 agreed that political party x_1 stand against corruption, 30% people are indeterminant for their stand on corruption whereas 20% people disagreed on their stand on corruption, 50% people of state 3 agreed that political party x_1 stands against corruption, 20% people are indeterminant for their stand on corruption whereas 30% people disagreed on their stand on corruption. Similarly, other three-way *n*-valued neutrosophic relation shown in Table 13 can be interpreted.

The goal is to find some of the interesting pattern from the given three way neutrosophic context which can be achieved using the proposed method in Sect. 3.1 as given below:

Step 1: In step 1 write the subset of n-valued neutrosophic attribute with their acceptance value to generate the concepts. In this paper maximal acceptance of truth-value, minimal indeterminacy-value, minimal falsity-value is considered for each component of n-valued neutrosophic attribute as given below:

$$1.\ \frac{(1.0, 1.0, 1.0), (0, 0, 0), (0, 0, 0)}{y_1} + \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_2} + \frac{((1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_3},$$

$$2.\ \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_1},$$

$$3.\ \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_2},$$

$$4.\ \frac{((1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_3},$$

$$5.\ \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_1} + \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_2},$$

$$6.\ \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_1} + \frac{((1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_3},$$

$$7.\ \frac{(1.0, 1, 1), (0, 0, 0), (0, 0, 0)}{y_2} + \frac{((1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_3},$$

$$8.\ \frac{(0, 0, 0), (0, 0, 0), (0, 0, 0)}{y_1} + \frac{(0, 0, 0), (0, 0, 0), (0, 0, 0)}{y_2} + \frac{((0, 0, 0), (0, 0, 0), (0, 0, 0)}{y_3},$$

Step 2: Consider the first subset to generate the concepts i.e.

$$1.\ \frac{(1.0, 1.0, 1.0), (0, 0, 0), (0, 0, 0)}{y_1} + \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_2} + \frac{((1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_3},$$

The concept can be generated using the operator (↓) as follows:

$$\left(\frac{(1.0, 1.0, 1.0), (0, 0, 0), (0, 0, 0)}{y_1} + \frac{(1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_2} + \frac{((1, 1, 1), (0, 0, 0), (0, 0, 0)}{y_3}\right)^{\downarrow}$$
$$= \frac{(1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)}{x_1} + \frac{(1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)}{x_2} + \frac{(1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)}{x_3}$$

Similarly, apply the ↑ on the obtained objects set as follows:

$$\left(\frac{(1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)}{x_1} + \frac{(1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)}{x_2} + \frac{(1.0, 1.0, 1.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)}{x_3}\right)^{\uparrow}$$
$$= \frac{(0.3, 0.4, 0.3), (0.5, 0.4, 0.3), (0.3, 0.4, 0.3)}{y_1} + \frac{(0.4, 0.3, 0.4), (0.4, 0.3, 0.3), (0.4, 0.2, 0.3)}{y_2} + \frac{(0.3, 0.3, 0.3), (0.5, 0.2, 0.3), (0.3, 0.3, 0.4)}{y_3}$$

In this way it provides following three-way n-valued neutrosophic concept:

1. The following concept can be generated using the subset number 1:

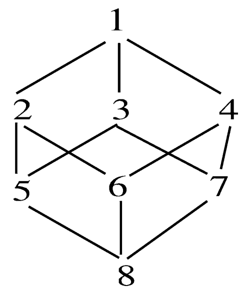

Fig. 4 Three-way n-valued neutrosophic concept lattice generated from Table 13

$$\text{Extent-}\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{x_1}+\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{x_2}+\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{x_3}$$

$$\text{Intent-}\frac{(0.3,0.4,0.3),(0.5,0.4,0.3),(0.3,0.4,0.3)}{y_1}+\frac{(0.4,0.3,0.4),(0.4,0.3,0.3),(0.4,0.2,0.3)}{y_2}+\frac{(0.3,0.3,0.3),(0.5,0.2,0.3),(0.3,0.3,0.4)}{y_3}$$

This concept represents that all the political parties are not acceptable simultaneously as per the opinion of people based on given parameters.

Step 3 Similarly, other concepts can be generated using the subsequent set of attributes as given below:

2. The following concept can be generated using the subset number 2:

$$\text{Extent-}\frac{(0.6,0.3,0.2),(0.5,0.3,0.2),(0.5,0.2,0.3)}{x_1}+\frac{(0.3,0.4,0.2),(0.4,0.2,0.3),(0.3,0.4,0.3)}{x_2}+\frac{(0.8,0.1,0.1),(0.3,0.4,0.3),(0.8,0.1,0.1)}{x_3}$$

$$\text{Intent-}\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_1}+\frac{(0.4,0.3,0.4),(0.4,0.3,0.3),(0.4,0.2,0.3)}{y_2}+\frac{(0.3,0.3,0.3),(0.5,0.2,0.3),(0.3,0.3,0.4)}{y_3}$$

This concept represents that the political party $\frac{(0.8,0.1,0.1),(0.3,0.4,0.3),(0.8,0.1,0.1)}{x_3}$ has maximal acceptance among people of given states based on their stand against corruption y_1.

3. The following concept can be generated using the subset number 3:

$$\text{Extent-}\frac{(0.4,0.3,0.3),(0.4,0.3,0.3),(0.4,0.2,0.3)}{x_1}+\frac{(0.4,0.2,0.4),(0.4,0.3,0.4),(0.4,0.2,0.4)}{x_2}+\frac{(0.9,0.1,0.4),(0.7,0.1,0.2),(0.9,0.1,0.0)}{x_3}$$

$$\text{Intent-}\frac{(0.3,0.4,0.3),(0.5,0.4,0.3),(0.3,0.4,0.3)}{y_1}+\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_2}+\frac{(0.3,0.3,0.3),(0.5,0.2,0.3),(0.3,0.3,0.4)}{y_3}$$

This concept represents that the political party $\frac{(0.9,0.1,0.4),(0.7,0.1,0.2),(0.9,0.1,0.0)}{x_3}$ has maximal acceptance among the people of given states based on their stand against criminals y_2.

4. The following concept can be generated using the subset number 4:

$$\text{Extent-}\frac{(0.5,0.2,0.2),(0.5,0.2,0.2),(0.4,0.2,0.2)}{x_1}+\frac{(0.3,0.3,0.4),(0.6,0.2,0.3),(0.3,0.3,0.4)}{x_2}+\frac{(0.7,0.1,0.1),(0.9,0.1,0.0),(0.7,0.1,0.0)}{x_3}$$

$$\text{Intent-}\frac{(0.3,0.4,0.3),(0.5,0.4,0.3),(0.3,0.4,0.3)}{y_1}+\frac{(0.4,0.3,0.4),(0.4,0.3,0.3),(0.4,0.2,0.3)}{y_2}+\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_3}$$

This shows that the political party $\frac{(0.7,0.1,0.1),(0.9,0.1,0.0),(0.7,0.1,0.0)}{x_3}$ has maximal acceptance among the people of given states based on their stand against un-educated leaders y_3.

5. The following concept can be generated using the subset number 5:

$$\text{Extent-}\frac{(0.4,0.3,0.3),(0.4,0.3,0.3),(0.4,0.2,0.3)}{x_1}+\frac{(0.3,0.4,0.4),(0.4,0.3,0.3),(0.3,0.4,0.4)}{x_2}+\frac{(0.8,0.1,0.1),(0.3,0.4,0.3),(0.8,0.1,0.1)}{x_3}$$

$$\text{Intent-}\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_1}+\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_2}+\frac{(0.3,0.3,0.3),(0.5,0.2,0.3),(0.3,0.3,0.4)}{y_3}$$

Table 14 Distance measurement of political party x_1 based on the proposed method shown in Table

	y_1	y_2	y_3	Total $AV(d)$
$\sup\|T_{x_1} - T_U\|$	0.4	0.6	0.5	
$\sup\|I_{x_1} - T_U\|$	0.5	0.5	0.6	
$\sup\|F_{x_1} - T_U\|$	0.5	0.6	0.6	
$AV(d)$	$(0.4 + 0.5 + 0.5)/3$	$(0.6 + 0.5 + 0.6)/3$	$(0.5 + 0.6 + 0.6)/3$	
	0.46	0.56	0.56	$(0.45 + 0.56 + 0.56)/3$
				$1.58/3 = 0.52$

Table 15 Distance measurement of political party x_2 based on the proposed method shown in Table

	y_1	y_2	y_3	Total $AV(d)$
$\sup\|T_{x_1} - T_U\|$	0.5	0.6	0.4	
$\sup\|I_{x_1} - T_U\|$	0.6	0.5	0.6	
$\sup\|F_{x_1} - T_U\|$	0.7	0.6	0.7	
$AV(d)$	$(0.5 + 0.6 + 0.7)/3$	$(0.6 + 0.5 + 0.6)/3$	$(0.4 + 0.6 + 0.7)/3$	
	0.56	0.56	0.56	$(0.56 + 0.56 + 0.56)/3$
				$1.68/3 = 0.56$

Table 16 Distance measurement of political party x_3 based on the proposed method shown in Sect. 3.1

	y_1	y_2	y_3	Total $AV(d)$
$\sup\|T_{x_1} - T_U\|$	0.3	0.3	0.1	
$\sup\|I_{x_1} - T_U\|$	0.3	0.2	0.2	
$\sup\|F_{x_1} - T_U\|$	0.3	0.1	0.3	
$AV(d)$	$(0.3 + 0.3 + 0.2)/3$	$(0.3 + 0.2 + 0.1)/3$	$(0.1 + 0.2 + 0.3)/3$	
	0.26	0.2	0.2	$(0.26 + 0.2 + 0.2)/3$
				0.22

Table 17 δ-similarity computation of each political parties based on given parameters

	Distance	δ-Similarity
x_1	0.52	0.48
x_2	0.56	0.44
x_3	0.22	0.78

Table 18 Selection of political parties based on user required granulation

δ-equal	Selected party
$0.78 \le \delta \le 1.0$	x_3
$0.56 \le \delta \le 0.77$	x_1
$0.0 \le \delta \le 0.55$	x_2

This shows that the political party $\frac{(0.8,0.1,0.1),(0.3,0.4,0.3),(0.8,0.1,0.1)}{x_3}$ has maximal acceptance among the people based on their stand against corruption y_1 and criminals (y_2).

6. The following concept can be generated using the subset number 6:

$$\text{Extent-}\frac{(0.5,0.3,0.2),(0.5,0.3,0.2),(0.4,0.2,0.3)}{x_1} + \frac{(0.3,0.4,0.4),(0.4,0.3,0.3),(0.3,0.4,0.4)}{x_2} + \frac{(0.7,0.1,0.1),(0.3,0.4,0.3),(0.7,0.1,0.1)}{x_3}$$

$$\text{Intent-}\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_1} + \frac{(0.4,0.3,0.4),(0.4,0.3,0.3),(0.4,0.2,0.3)}{y_2} + \frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_3}$$

This shows that the political party $\frac{(0.7,0.1,0.1),(0.3,0.4,0.3),(0.7,0.1,0.1)}{x_3}$ has maximal acceptance among the people based on their stand against corruption y_1 and un-educated leaders (y_3).

7. The following concept can be generated using the subset number 7:

Table 19 Some interesting level of granulation to decompose n-valued neutrosophic context

Level of granulation	Expert interpretation	Neutrosophic values
Level 1	Very Interesting	(0.7, 0.1, 0.1)
Level 2	Interesting	(0.5, 0.2, 0.2)
Level 3	Average Interesting	(0.4, 0.3, 0.2)
Level 4	Moderate Interesting	(0.3, 0.3, 0.3)
Level 5	Not Interesting	(0.2, 0.4, 0.3)

Table 20 A decomposition of context shown in Table 13 at (0.5, 0.2, 0.2)-granulation

Political parties	Corruption(y_1)	Criminals (y_2)	Un-educated(y_3)
x_1	(1, 1, 1)	(0, 1, 1)	(0, 0, 0)
x_2	(0, 1, 1)	(0, 1, 1)	((0, 1, 1)
x_3	(0, 1, 1)	(1, 0, 0)	(1, 0, 0)

Table 21 Comparison of proposed methods in this paper on various parameters

	Proposed method in Sect. 3.1	Proposed method in Sect. 3.1	Proposed method in Sect. 3.1
Objective	Super-sub hierarchy hierarchy	Similar concepts	Decomposition
Methodology	Subset based attributes	δ-equality	multi-granulation
Knowledge discovery	n-valued neutrosophic concepts	Closed n-valued concepts	Decomposed concepts at given granulation
n-valued pattern	Yes	No	No
Micro and macro level refinement	Yes	No	Yes
Graphical visualization	Yes	No	No
Time complexity	$O(2^m \cdot 3n \cdot k)$	$O(m^3 \cdot n^3)$	$O(m^3 \cdot n)$

$$\text{Extent-}\frac{(0.4,0.3,0.3),(0.4,0.3,0.3),(0.4,0.2,0.3)}{x_1} + \frac{(0.4,0.3,0.4),(0.4,0.3,0.3),(0.3,0.3,0.4)}{x_2} + \frac{(0.7,0.1,0.1),(0.7,0.1,0.2),(0.7,0.1,0.1)}{x_3}$$
$$\text{Intent-}\frac{(0.3,0.4,0.3),(0.5,0.4,0.3),(0.3,0.4,0.3)}{y_1} + \frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_2} + \frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_3}$$

This shows that the political party $\frac{(0.7,0.1,0.1),(0.7,0.1,0.2),(0.7,0.1,0.1)}{x_3}$ has maximal acceptance among the people based on their stand against criminals y_2 and un-educated leaders (y_3).

8. The following concept can be generated using the subset number 8:

$$\text{Extent-}\frac{(0.4,0.3,0.3),(0.4,0.3,0.3),(0.4,0.2,0.3)}{x_1} + \frac{(0.3,0.4,0.4),(0.4,0.3,0.3),(0.3,0.4,0.4)}{x_2} + \frac{(0.7,0.1,0.1),(0.3,0.4,0.3),(0.7,0.1,0.1)}{x_3}$$
$$\text{Intent-}\frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_1} + \frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_2} + \frac{(1.0,1.0,1.0),(0.0,0.0,0.0),(0.0,0.0,0.0)}{y_3}$$

This shows that the political party $\frac{(0.7,0.1,0.1),(0.3,0.4,0.3),(0.7,0.1,0.1)}{x_3}$ has maximal acceptance with regard to opinion of people in the given states. In this case it will be first preference to vote in the given election.

The above generated concepts are shown in Fig. 4 via properties of n-valued neutrosophic concept lattice. It shows

Table 22 Comparison of proposed methods with recently introduced related methods

	Neutrosophic concept lattice [22, 24]	m-polar concept lattice [26, 28]	Proposed method in this paper
Objective	Finding pattern in three-way data	Finding pattern in m-polar context	Finding pattern in n-valued neutrosophic data
Methodology	Subset based attributes	Subset based and Projection	δ-equality and, multi-granulation
Knowledge discovery	Three-way neutrosophic concepts	m-polar fuzzy concepts	n-valued neutrosophic concepts
Micro and macro level refinement	No	No	Yes
Graphical visualization	Yes	Yes	Yes
Lattice	Yes	Yes	Yes
Granulation decomposition	No	No	Yes
δ-equal similarity	No	No	Yes
Time complexity	$O(2^m \cdot 3n)$	$O(2^k \cdot m \cdot n)$ and $O(n \cdot m^2 + m \cdot k^2)$	$O(m^3 \cdot n^3)$ and $O(m^3 \cdot n)$

the generalized and specialized opinion of people about any given political party to decide their vote.

4.2 δ-equal three-way n-valued pattern

In the previous section, it is addressed that the number of concepts generated from the proposed method shown in Sect. 3.1 is exponential. It become irrelevant when an expert wants to analyze the win or loss of any political party based on his/her required subset of attributes and his/her. In this this the expert want to discover those parties which is maximally closed to the given subset of attributes. To deal with this problem, a method is proposed in Sect. 3.2 to find some of the closest three-way n-valued neutrosophic attributes (or objects set) at user required δ-granules. The objective is to extract some of the closest political party (i.e. objects) based on chosen set of attributes within a reasonable time period i.e. O($m^3 \times n^3$) when compared to subset based method [24]. To illustrate the proposed method the context shown in Table 13 is considered which represents the opinion of people towards three political party based on the given parameters. In general the user used to vote any political party which mandate is closed to the maximal acceptance, minimal indeterminacy, minimal falsity of his/her desired parameters i.e. (1, 0, 0). The goal of an expert is to classify the incongruous party and their closeness to maximal acceptance of people opinion. To achieve this goal the δ-equal distance among the given political party based on their parameters can be computed using the proposed method shown in Sect. 3.2. Tables 14, 15, 16 contains the δ-equal similarity measurement of each political party x_1, x_2, x_3 based on maximal acceptance of user defined parameters, respectively. Table 17 represents their ordering based on computed distance. Table 18 shows closeness of each political party based on user defined subset of attributes.

Table 18 shows that, the political party x_3 and its stand on the given parameter is maximal i.e. 78% when compared to user required parameters. In this case, people will may vote to this political party to win whereas the x_2 will be second. This information is similar to its n-valued neutrosophic concept lattice shown in Fig. 4. However, this method does not provide any graphical structure visualization except its δ-equal similarity analysis. In this way both of the proposed methods have their own advantages and disadvantages to solve the particular problem. It is based on user or expert to utilize any of them based on his/her requirement. None of these two approaches provides a general solution when an expert need a macro or micro level predictive analysis of three-way n-valued neutrosophic context to refine or coarse the pattern. To resolve this problem, another method is proposed in Sect. 3.3 which is illustrated in the next section.

4.3 Decomposition of three-way n-valued formal context using granular computing

It is observed that many times an expert need to analyze the given data sets based on micro and macro level to refine and coarser the hidden pattern. One of the most suitable reason is that the expert need a generalized as well as specialized pattern in hidden three-way n-valued neutrosophic context to solve the particular problem at his/her required multi-granulation [25]. To achieve this goal a method is proposed in Sect. 3.3 to decompose the given three-way n-valued context based on chosen granulation for the truth, indeterminacy, and falsity membership-values, independently. The level of granulation can be decided by user or expert requirement to zoom in zoom out the given three-way n-valued context as shown in Table 19. To illustrate the proposed method a three-way n-valued context is shown in Table 13. Table 20 shows the decomposition of given context based on granulation (0.5, 0.2, 0.2).

The decomposed context shown in Table 20 shows that the political party x_3 contains maximal acceptance (1, 0, 0) for attributes when compared to other parties. In this case the political party x_1 will be first preference for the people to vote. This derived analysis is similar to other methods illustrated in Sects. 4.1 and 4.2. In this way it can be observed that each of the proposed method provides multi-ways to describe the given n-valued neutrosophic context. Hence it based on user requirement to utilize any of them to solve the particular problem. In near future the author will focus on reducing the size of n-valued neutrosophic concept lattice and its applications in various fields for knowledge processing tasks.

5 Discussions

In the last decade several researchers paid data analytics using the mathematical properties of concept lattice theory [12]. In this regard, many extensions are proposed to deal with uncertainty in fuzzy attributes [21]. Towards this process recently mathematics of neutrosophic set [30] has been introduced [22] to characterize the acceptation, rejection and uncertain part in the fuzzy attributes [29]. This is an indeed requirement for descriptive analysis of indeterminacy exists in fuzzy attributes to quantize the information as for example mercury (Hg) is neither liquid nor solid at any normal temperature. Semiconductors are neither conductors not isolators. Handling these types of quantum superposition of states in the given three-way n-valued fuzzy context is a major issues for the researchers. To deal with this problem, one of the researcher tried to characterize them based on truth, indeterminacy, and falsity membership and its graphical structure visualization [22] to refine and coarse the pattern at user defined granulation [24, 25]. In this process a problem is addressed while handling the m-polar information [28] based on its acceptation, rejection and uncertain part. The second problem arises while computing some of the similar set of objects based on user defined subset of attributes. To traverse the three-way n-valued contexts at user defined granulation. To demonstrate these problems one of the suitable examples is given in this paper. Such that the m-polar opinion of people can be classified based on their maximal acceptance for truth, indeterminacy and falsity membership-values. To achieve this goal, the current paper introduces three different methods using the calculus of n-valued neutrosophic set, its δ-equal similarity as decomposition at user defined granulation in Sects. 3.1, 3.2, and 3.3, respectively.

Table 21 shows the comparison among each of the proposed methods in this paper based on several parameters. It represents that, each of the proposed methods includes their own utility. To solve the particular problem of three-way n-valued neutrosophic context. It is totally based on user or expert requirement to select the particular method based on his/her requirements. Table 22 provides a comparative study among each of the proposed method when compared to recently introduced three-way [22] as well as m-polar fuzzy concept lattice [26, 28]. The proposed methods in this paper are distinct from any of these approaches in following ways:

1. The proposed method provides a first mathematical tool to analyze the n-valued neutrosophic contexts more precisely based on its acceptation, rejection and uncertain part when compared to any of the available approaches.
2. The proposed method provides a method to investigate the closest three-way n-valued set of attributes based on user defined δ-equality.
3. The proposed method provides a multiple ways to zoom in and zoom out the three-way n-valued neutrosophic contexts at user defined granulation for truth, falsity and indeterminacy membership-values, respectively.
4. One of the suitable example is also given to demonstrate the proposed method for analyzing the opinion of people in a democratic country.

In this way, the proposed method will be helpful for many researchers working in the field of data sciences beyond the three-way as well as m-way decision space. Same time it will be more helpful for the government as well as private agencies for predictive analysis of opinion of people living in a democratic country precisely. In the near future, the author will focus on other applications of three-way n-valued neutrosophic contexts and its granular based refinement [20, 27] with an illustrative example.

6 Conclusions

This paper aimed at introducing a mathematical structure to characterize the n-valued contexts based its acceptation, rejection and uncertain part, respectively. To achieve this goal, the current paper introduced three methods based on calculus of n-valued neutrosophic set, δ-equality and granular computing. The first method provides a graphical structure compressed visualization of given three-way n-valued neutrosophic context in the concept lattice. The other two methods provides several ways to refine or coarse the pattern of given three-way n-valued neutrosophic context at user or expert require granulation with an illustrative example. The comparative study of each of the proposed method based on their advantages and disadvantages is shown in Table 21. Table 22 represents the comparison of the proposed method with recently available approahces in three-way [22] as well as multi-way [26] fuzzy concept lattice. In future, the author will focus on some other real life applications of three-way n-valued neutrosophic set and its fluctuation measurement.

Acknowledgements Author sincerely thanks the reviewers and Editor for their opinion to improve this paper.

Compliance with ethical standards

Conflict of interest Author declares that there is no conflict of interest.

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