

# Multi-objective Cylindrical Skin Plate Design Optimization based on Neutrosophic Optimization Technique

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## Abstract

In this chapter, we develop a Neutrosophic Optimization (NSO) approach for optimizing the thickness and sag of skin plate of vertical lift gate with multi- objective subject to a specified constraint. In this optimum design formulation, the objective function is the thickness and sag of the skin plate of vertical lift gate; the design variables are the thickness and sag of skin plate of vertical lift gate; the constraint are the stress and deflection in member. A classical vertical lift gate optimization example is presented here in to demonstrate the efficiency of this technique. The test problem includes skin plate of vertical lift gate subjected to hydraulic load condition. This multi-objective structural optimization model is solved by fuzzy, intuitionistic fuzzy and neutrosophic multi-objective optimization technique. Numerical example is given to illustrate our NSO approach. The result shows that the NSO approach is very efficient in finding the best discovered optimal solutions.

## Keywords

Neutrosophic Set, Single Valued Neutrosophic Set, Neutrosophic Optimization, Cylindrical Skin Plate Design.

## 1 Introduction

Structural optimization is an important notion in civil engineering. Traditionally structural optimization is a well-known concept and in many situations, it is treated as single objective form, where the objective is known the weight function. The extension of this is the optimization where one or more constraints are simultaneously satisfied next to the minimization of the weight function. This does not always hold good in real world problems where multiple and conflicting objectives frequently exist. In this consequence, a methodology known as multi-objective structural optimization (MOSO) is introduced. In structural engineering design problems, the input data and parameters are often fuzzy/imprecise with nonlinear characteristics that necessitate the development of fuzzy optimum structural design method. Fuzzy set (FS) theory has long been introduced to handle inexact and imprecise data by Zadeh [2]. Later on Bellman and Zadeh [4] used the fuzzy set theory to the decision making problem. The fuzzy set theory also found application in structural design. Several researchers like Wang et al. [8] first applied  $\alpha$ -cut method to structural designs where the nonlinear problems were solved with various design levels  $\alpha$ , and then a sequence of solutions were obtained by setting different level-cut value of  $\alpha$ . Rao [3] applied the same  $\alpha$ -cut method to design a four-bar mechanism for function generating problem. Structural optimization with fuzzy parameters was developed by Yeh et al. [9] Xu [10] used two-phase method for fuzzy optimization of structures. Shih et al. [5] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources. Shih et al [6] developed an alternative  $\alpha$ -level-cuts method for optimum structural design with fuzzy resources. Dey et al. [11] used generalized fuzzy number in context of a structural design. Dey et al. [13] developed parameterized t-norm based fuzzy optimization method for optimum structural design. Also, Dey et.al [16] Optimized shape design of structural model with imprecise coefficient by parametric geometric programming. In such extension, Atanassov [1] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory and is characterized by a membership function, a non-membership function and a hesitancy function. In fuzzy sets, the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. A transportation model was solved by Jana et al [15] using multi-objective intuitionistic fuzzy linear programming. Dey et al. [12] solved two bar truss nonlinear problem by using intuitionistic fuzzy optimization problem. Dey et al. [14] used intuitionistic fuzzy optimization technique for multi objective optimum structural design. Intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and

inconsistent information. In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership which are independent. Neutrosophic theory was introduced by Smarandache [7]. The motivation of the present study is to give computational algorithm for solving multi-objective structural problem by single valued neutrosophic optimization approach. Neutrosophic optimization technique is very rare in application to structural optimization. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership function in such optimization process. The results are compared numerically both in fuzzy optimization technique, intuitionistic fuzzy optimization technique and neutrosophic optimization technique. From our numerical result, it is clear that neutrosophic optimization technique provides better results than fuzzy optimization and intuitionistic fuzzy optimization.

## 2 Multi-objective Structural Model

In the design problem of the structure i.e. lightest thickness of the structure and minimum sag that satisfies all stress and deflection constraints in members of the structure. In vertical lift gate structural system, the basic parameters (including allowable stress ,deflection etc.) are known and the optimization's target is that identify the optimal thickness and sag so that the structure is of the smallest total weight with minimum stress and deflection in a given load conditions .

The multi-objective structural model can be expressed as:

$$\begin{aligned}
 & \text{Minimize } G && (1) \\
 & \text{minimize } S \\
 & \text{subject to } \sigma \leq [\sigma] \\
 & \delta \leq [\delta] \\
 & G^{\min} \leq G \leq G^{\max} \\
 & S^{\min} \leq S \leq S^{\max}
 \end{aligned}$$

where  $G$  and  $S$  are the design variables for the structural design,  $\delta$  is the deflection of the vertical lift gate of skin plate due to hydraulic load.  $\sigma$  is the stress constraint and  $[\sigma]$ ,  $[\delta]$  are allowable stress of the vertical lift gate of skin plate under various conditions.  $G^{\min}$  and  $S^{\min}$ ,  $G^{\max}$  and  $S^{\max}$  are the lower and upper bounds of design variables respectively.

### 3 Mathematical Preliminaries

#### 3.1 Fuzzy Set

Let  $X$  be a fixed set. A fuzzy set:  $A$  set of  $X$  is an object having the form  $\tilde{A} = \{(x, T_A(x)) : x \in X\}$  where the function  $T_A : X \rightarrow [0,1]$  defined the truth membership of the element  $x \in X$  to the set  $A$ .

#### 3.2. Intuitionistic Fuzzy Set

Let a set  $X$  be fixed. An intuitionistic fuzzy set or IFS  $\tilde{A}^i$  in  $X$  is an object of the form  $\tilde{A}^i = \{ \langle X, T_A(x), F_A(x) \rangle \mid x \in X \}$  where  $T_A : X \rightarrow [0,1]$  and  $F_A : X \rightarrow [0,1]$  define the truth membership and falsity membership respectively, for every element of  $x \in X$ ,  $0 \leq T_A(x) + F_A(x) \leq 1$ .

#### 3.3. Neutrosophic Set

Let a set  $X$  be a space of points (objects) and  $x \in X$ . A neutrosophic set  $\tilde{A}^n$  in  $X$  is defined by a truth membership function  $T_A(x)$ , an indeterminacy-membership function  $I_A(x)$  and a falsity membership function  $F_A(x)$  and having the form  $\tilde{A}^n = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \}$ .  $T_A(x), I_A(x)$  and  $F_A(x)$  are real standard or non-standard subsets of  $]0^-, 1^+[$ . That is

$$T_A(x) : X \rightarrow ]0^-, 1^+[$$

$$I_A(x) : X \rightarrow ]0^-, 1^+[$$

$$F_A(x) : X \rightarrow ]0^-, 1^+[$$

There is no restriction on the sum of  $T_A(x), I_A(x)$  and  $F_A(x)$  so  $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ . [17-22]

#### 3.4. Single Valued Neutrosophic Set

Let a set  $X$  be the universe of discourse. A single valued neutrosophic set  $\tilde{A}^n$  over  $X$  is an object having the form  $\tilde{A}^n = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \}$  where:

$T_A : X \rightarrow [0,1], I_A : X \rightarrow [0,1], F_A : X \rightarrow [0,1]$  with  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$  for all  $x \in X$ .

#### 3.5. Complement of Neutrosophic Set

Complement of a single valued neutrosophic set  $A$  is denoted by  $c(A)$  and is defined by:

$$\begin{aligned} T_{c(A)}(x) &= F_A(x) \\ I_{c(A)}(x) &= 1 - F_A(x) \\ F_{c(A)}(x) &= T_A(x). \end{aligned}$$

### 3.6. Union of Neutrosophic Sets

The union of two single valued neutrosophic sets  $A$  and  $B$  is a single valued neutrosophic set  $C$ , written as  $C = A \cup B$ , whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$$\begin{aligned} T_{c(A)}(x) &= \max(T_A(x), T_B(x)) \\ I_{c(A)}(x) &= \max(I_A(x), I_B(x)) \\ F_{c(A)}(x) &= \min(F_A(x), F_B(x)) \text{ for all } x \in X \end{aligned}$$

### 3.7. Intersection of Neutrosophic Sets

The intersection of two single valued neutrosophic sets  $A$  and  $B$  is a single valued neutrosophic set  $C$ , written as  $C = A \cap B$ , whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$$\begin{aligned} T_{c(A)}(x) &= \min(T_A(x), T_B(x)) \\ I_{c(A)}(x) &= \min(I_A(x), I_B(x)) \\ F_{c(A)}(x) &= \max(F_A(x), F_B(x)) \text{ for all } x \in X. \end{aligned}$$

## 4 Mathematical Analysis

### 4.1. Neutrosophic Optimization Technique to Solve Minimization Type Multi-Objective Non-linear Programming Problem

A nonlinear multi-objective optimization of the problem is of the form

$$\text{Minimize } \{f_1(x), f_2(x), \dots, f_p(x)\} \quad (2)$$

Now the decision set  $\tilde{D}^n$ , a conjunction of Neutrosophic objectives and constraints is defined  $\tilde{D}^n = \left( \bigcap_{k=1}^p \tilde{G}_k^n \right) \cap \left( \bigcap_{j=1}^q \tilde{C}_j^n \right) = \{(x, T_{\tilde{D}^n}(x)) I_{\tilde{D}^n}(x), F_{\tilde{D}^n}(x)\}$

$$\text{Here } T_{\tilde{D}^n}(x) = \min \left\{ \begin{array}{l} T_{\tilde{G}_1^n}(x), T_{\tilde{G}_2^n}(x), T_{\tilde{G}_3^n}(x), \dots, T_{\tilde{G}_p^n}(x); \\ T_{\tilde{C}_1^n}(x), T_{\tilde{C}_2^n}(x), T_{\tilde{C}_3^n}(x), \dots, T_{\tilde{C}_q^n}(x) \end{array} \right\} \text{ for all } x \in X$$

$$I_{\tilde{D}^n}(x) = \min \left\{ \begin{array}{l} I_{\tilde{G}_1^n}(x), I_{\tilde{G}_2^n}(x), I_{\tilde{G}_3^n}(x), \dots, I_{\tilde{G}_p^n}(x); \\ I_{\tilde{C}_1^n}(x), I_{\tilde{C}_2^n}(x), I_{\tilde{C}_3^n}(x), \dots, I_{\tilde{C}_q^n}(x) \end{array} \right\} \text{ for all } x \in X$$

$$F_{\tilde{D}^n}(x) = \min \left\{ \begin{array}{l} F_{\tilde{G}_1^n}(x), F_{\tilde{G}_2^n}(x), F_{\tilde{G}_3^n}(x), \dots, F_{\tilde{G}_p^n}(x); \\ F_{\tilde{C}_1^n}(x), F_{\tilde{C}_2^n}(x), F_{\tilde{C}_3^n}(x), \dots, F_{\tilde{C}_q^n}(x) \end{array} \right\} \text{ for all } x \in X$$

where  $T_{\tilde{D}^n}(x), I_{\tilde{D}^n}(x), F_{\tilde{D}^n}(x)$  are truth-membership function, indeterminacy membership function, falsity membership function of neutrosophic decision set respectively. Now using the neutrosophic optimization, problem (2) is transformed to the non-linear programming problem as

$$\begin{aligned} & \text{Max } \alpha && (3) \\ & \text{Max } \gamma \\ & \text{Min } \beta \\ & \text{such that } T_{\tilde{G}_k^n}(x) \geq \alpha; T_{\tilde{C}_j^n}(x) \geq \alpha; I_{\tilde{G}_k^n}(x) \geq \gamma; I_{\tilde{C}_j^n}(x) \geq \gamma; \\ & F_{\tilde{G}_k^n}(x) \leq \beta; F_{\tilde{C}_j^n}(x) \leq \beta; \alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma; \\ & \alpha, \beta, \gamma \in [0, 1] \end{aligned}$$

Now this non-linear programming problem (3) can be easily solved by an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (1) by neutrosophic optimization approach.

#### 4.1.1 Computational Algorithm

**Step-1:** Solve the MONLP problem (2) as a single objective non-linear problem  $p$  times for each problem by taking one of the objectives at a time and ignoring the others. These solutions are known as ideal solutions. Let  $x^k$  be the respective optimal solution for the  $k^{\text{th}}$  different objective and evaluate each objective value for all these  $k^{\text{th}}$  optimal solution.

**Step-2:** From the result of step-1, determine the corresponding values for every objective for each derived solution, pay-off matrix can be formulated as follows

$$\begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \dots & f_p(x^1) \\ f_1(x^2) & f_2^*(x^2) & \dots & f_p(x^2) \\ \dots & \dots & \dots & \dots \\ f_1(x^p) & f_2(x^p) & \dots & f_p^*(x^p) \end{bmatrix}$$

**Step-3:** For each objective  $f_k(x)$  find lower bound  $L_k^\mu$  and the upper bound  $U_k^\mu$

$$U_k^T = \max \{ f_k(x^{r*}) \} \text{ and}$$

$$L_k^T = \min \{ f_k(x^{r*}) \} \text{ where } r = 1, 2, \dots, k$$

for truth membership of objectives.

**Step-4:** We represent upper and lower bounds for indeterminacy and falsity membership of objectives as follows:

for  $k=1,2,\dots,p$

$$U_k^F = U_k^T \quad \text{and} \quad L_k^F = L_k^T + t(U_k^T - L_k^T);$$

$$L_k^I = L_k^T \quad \text{and} \quad U_k^I = L_k^T + s(U_k^T - L_k^T)$$

Here  $t, s$  are predetermined real numbers in  $(0,1)$

**Step-5:** Define truth membership, indeterminacy membership and falsity membership functions as follows:

for  $k=1,2,\dots,p$

$$T_k(f_k(x)) = \begin{cases} 1 & \text{if } f_k(x) \leq L_k^T \\ \frac{U_k^T - f_k(x)}{U_k^T - L_k^T} & \text{if } L_k^T \leq f_k(x) \leq U_k^T \\ 0 & \text{if } f_k(x) \geq U_k^T \end{cases}$$

$$I_k(f_k(x)) = \begin{cases} 1 & \text{if } f_k(x) \leq L_k^I \\ \frac{U_k^I - f_k(x)}{U_k^I - L_k^I} & \text{if } L_k^I \leq f_k(x) \leq U_k^I \\ 0 & \text{if } f_k(x) \geq U_k^I \end{cases}$$

$$F_k(f_k(x)) = \begin{cases} 1 & \text{if } f_k(x) \leq L_k^F \\ \frac{U_k^F - f_k(x)}{U_k^F - L_k^F} & \text{if } L_k^F \leq f_k(x) \leq U_k^F \\ 0 & \text{if } f_k(x) \geq U_k^F \end{cases}$$

**Step-6:** Now neutrosophic optimization method for MONLP problem gives a equivalent nonlinear programming problem as:

$$\text{Maximize } (\alpha - \beta + \gamma) \tag{4}$$

$$\text{such that } T_k(f_k(x)) \geq \alpha; I_k(f_k(x)) \geq \gamma; F_k(f_k(x)) \leq \beta;$$

$$\alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma; \alpha, \beta, \gamma \in [0,1];$$

$$g_j(x) \leq b_j \quad x \geq 0, \quad k=1,2,\dots,p; \quad j=1,2,\dots,q$$

This is reduced to equivalent nonlinear programming problem as

$$\text{Maximize } (\alpha - \beta + \gamma)$$

$$\text{such that } f_k(x) + (U_k^T - L_k^T) \cdot \alpha \leq U_k^T;$$

$$f_k(x) + (U_k^I - L_k^I) \cdot \gamma \leq U_k^I;$$

$$f_k(x) + (U_k^F - L_k^F) \cdot \beta \leq L_k^F;$$

for  $k=1,2,\dots,p$

$$\alpha + \beta + \gamma \leq 3;$$

$$\alpha \geq \beta; \alpha \geq \gamma; \alpha, \beta, \gamma \in [0,1];$$

$$g_j(x) \leq b_j \quad x \geq 0. \tag{5}$$

## 5 Solution of Multi-Objective Structural Optimization Problem (MOSOP) by Neutrosophic Optimization Technique

To solve the MOSOP (1), step 1 of 4.1.1 is used. After that according to step to pay off matrix is formulated.

$$\begin{matrix} G & S \\ G^1 & \begin{bmatrix} G^1 & S^1 \\ S^2 & S^2 \end{bmatrix} \end{matrix}$$

According to step-2 the bound of weight objective  $U_G^T, L_G^T; U_G^I, L_G^I$  and  $U_G^F, L_G^F$  for truth, indeterminacy and falsity membership function respectively. Then:

$L_G^T \leq G \leq U_G^T; L_G^F \leq G \leq U_G^F; L_G^I \leq G \leq U_G^I$ . Similarly, the bound of deflection objective are  $U_S^T, L_S^T; U_S^F, L_S^F$  and  $U_S^I, L_S^I$  are respectively for truth, indeterminacy and falsity membership function.

Then:

$$L_S^T \leq S \leq U_S^T; L_S^F \leq S \leq U_S^F; L_S^I \leq S \leq U_S^I,$$

where  $U_G^F = U_G^T; L_G^F = L_G^T + \varepsilon_G; L_G^I = L_G^T, U_G^I = L_G^T + \varepsilon_G$

and  $U_S^F = U_S^T; L_S^F = L_S^T + \xi_S; L_S^I = L_S^T, U_S^I = L_S^T + \xi_S$ ,

such that

$$0 < \varepsilon_G < (U_G^T - L_G^T) \text{ and } 0 < \xi_S < (U_S^T - L_S^T).$$

According to neutrosophic optimization technique considering truth, indeterminacy and falsity membership function for MOSOP (1), and crisp non-linear programming problem can be formulated as

$$\text{Maximize } (\alpha + \gamma - \beta) \quad (6)$$

Subject to

$$T_G \geq \alpha; T_S \geq \alpha; F_G \leq \beta; F_S \leq \beta;$$

$$I_G \geq \gamma; I_S \geq \gamma; \sigma \leq [\sigma]; \delta \leq [\delta];$$

$$\alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma;$$

$$\alpha, \beta, \gamma \in [0, 1], \quad G^{\min} \leq G \leq G^{\max} \quad S^{\min} \leq S \leq S^{\max}$$



Solving the above crisp model (6) by an appropriate mathematical programming algorithm we get optimal solution and hence objective functions i.e structural weight and deflection of the loaded joint will attain Pareto optimal solution.

## 6 Numerical Illustration

A cylindrical skin plate of vertical lift gate (Guha A.L et al [17]) in fig-2 has been considered. The weight of the skin plate is about 40% of the weight of the vertical lift gate, thus the minimum weight of the vertical lift gate can be achieved by using minimum thickness of a skin plate with same number of horizontal girders for the particular hydraulic load. It is proposed to replace stiffened flat skin plate by unstiffened cylindrical skin plate. The stress developed in skin plate and its distribution mainly depends on water head, skin plate thickness, and sag and position of Horizontal girders. Stress and deflection are expressed in terms of water head, skin plate thickness, and sag based on finite element analysis.

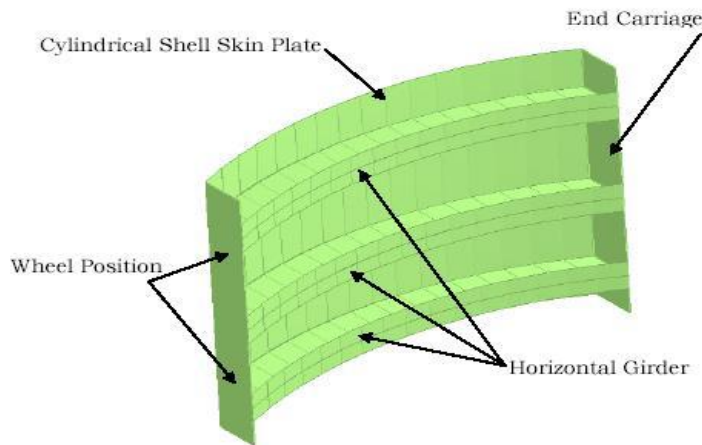


Fig. 1: Vertical lift gate with cylindrical shell type skin plate

The proposed expressions are furnished as stress  $\sigma(G, S, H) = K_1 G^{-n_1} S^{-n_2} H^{n_3}$  where,  $\sigma$  = stress in  $\text{Kg/cm}^2$ ;  $H$  = water Head in 'm'  $G$  = Thickness in 'mm'  $S$  = Sag in 'mm'  $K_1$  = Constant of variation and  $n_1$ ;  $n_2$  and  $n_3$  = constants depend on the properties of material Similarly, deflection:  $\delta(T, S, H) = K_2 T^{-n_4} S^{-n_5} H^{n_6}$   $\delta(G, S, H) = K_2 G^{-n_4} S^{-n_5} H^{n_6}$  where,  $K_2$  = constant of variation and  $n_4$ ;  $n_5$  and  $n_6$  = constants depend on the properties of material.

To minimize the weight of Vertical gate by simultaneous minimization of Thickness  $G$  and sag,  $S$  of skin plate subject to maximum allowable stress ( $\sigma_0$ ) and deflection ( $\delta_0$ ).

So, the model is

$$\text{Minimize } G \quad (7)$$

$$\text{Minimize } S$$

Subject to

$$\sigma(G, S, H) \equiv K_1 G^{-n_1} S^{-n_2} H^{n_3} \leq \sigma_0;$$

$$\delta(G, S, H) \equiv K_2 G^{-n_4} S^{-n_5} H^{n_6} \leq \delta_0$$

$$G, S > 0;$$

Input data of the problem is tabulated in Table. 1.

Table 1: Input data for crisp model (7)

constant of variation $K_1$	constant of variation $K_2$	constants depend on the properties of material	water head $H$ (m)	Maximum allowable stress $\sigma_0$ (Mpa)	Maximum allowable deflection of girder $\delta_0$ (Mpa)
$3.79 \times 10^{-3}$	$87.6 \times 10^{-5}$	$n_1 = 0.44; n_2 = 1.58;$ $n_3 = 1.0$ $n_4 = 0.729; n_5 = 0.895;$ $n_6 = 1.0$	25	137.5	5.5

Solution: According to step 2 of 4.1.1, pay-off matrix is formulated as follows:

$$\begin{matrix} & G & S \\ G^1 & \left[ 0.59 \times 10^{-5} & 37.61824 \right] \\ S^2 & \left[ 3528.536 & 0.10256 \times 10^{-2} \right] \end{matrix}$$

Here,

$$U_G^F = U_G^T = 3528.536, L_G^F = L_G^T + \varepsilon_G = 0.59 \times 10^{-5} + \varepsilon_G; L_G^L = L_G^T = 0.59 \times 10^{-5},$$

$$U_G^L = L_G^T + \xi_G = 0.59 \times 10^{-5} + \xi_G$$

$$\text{such that } 0 < \varepsilon_G, \xi_G < (3528.536 - 0.59 \times 10^{-5});$$

$$U_S^F = U_S^T = 37.61824, L_S^F = L_S^T + \varepsilon_S = 0.10256 \times 10^{-2} + \varepsilon_S;$$

$$L_S^I = L_S^T = 0.10256 \times 10^{-2}, U_S^I = L_S^T + \xi_S = 0.10256 \times 10^{-2} + \xi_S$$

such that  $0 < \varepsilon_S, \xi_S < (37.61824 - 0.10256 \times 10^{-2})$

Here, truth, indeterminacy, and falsity membership function for objective functions are  $G$  and  $S$  are defined as follows

$$T_G = \begin{cases} 1 & \text{if } G \leq 0.59 \times 10^{-5} \\ \frac{3528.536 - G}{3528.536 - 0.59 \times 10^{-5}} & \text{if } 0.59 \times 10^{-5} \leq G \leq 3528.536 \\ 0 & \text{if } G \geq 3528.536 \end{cases}$$

$$F_G = \begin{cases} 0 & \text{if } G \leq 0.59 \times 10^{-5} + \varepsilon_G \\ \frac{G - (0.59 \times 10^{-5} + \varepsilon_G)}{3528.536 - 0.59 \times 10^{-5} - \varepsilon_G} & \text{if } 0.59 \times 10^{-5} + \varepsilon_G \leq G \leq 3528.536; \\ 1 & \text{if } G \geq 3528.536 \end{cases}$$

$$I_G = \begin{cases} 1 & \text{if } G \leq 0.59 \times 10^{-5} \\ \frac{(0.59 \times 10^{-5} + \xi_G) - G}{\xi_G} & \text{if } 0.59 \times 10^{-5} \leq G \leq 0.59 \times 10^{-5} + \xi_G \\ 0 & \text{if } G \geq 0.59 \times 10^{-5} + \xi_G \end{cases}$$

$$T_S = \begin{cases} 1 & \text{if } S \leq 0.10256 \times 10^{-2} \\ \frac{37.61824 - S}{37.61824 - 0.10256 \times 10^{-2}} & \text{if } 0.10256 \times 10^{-2} \leq S \leq 37.61824; \\ 0 & \text{if } S \geq 37.61824 \end{cases}$$

$$F_S = \begin{cases} 0 & \text{if } S \leq 0.10256 \times 10^{-2} + \varepsilon_S \\ \frac{S - (0.10256 \times 10^{-2} + \varepsilon_S)}{37.61824 - 0.10256 \times 10^{-2} - \varepsilon_S} & \text{if } 0.10256 \times 10^{-2} + \varepsilon_S \leq S \leq 37.61824; \\ 1 & \text{if } S \geq 37.61824 \end{cases}$$

$$I_S = \begin{cases} 1 & \text{if } S \leq 0.10256 \times 10^{-2} \\ \frac{(0.10256 \times 10^{-2} + \xi_S) - S}{\xi_S} & \text{if } 0.10256 \times 10^{-2} \leq S \leq 0.10256 \times 10^{-2} + \xi_S \\ 0 & \text{if } S \geq 0.10256 \times 10^{-2} + \xi_S \end{cases}$$

Now using neutrosophic optimization technique with truth, indeterminacy and falsity membership functions we get

$$\begin{aligned}
 & \text{Maximize } (\alpha + \gamma - \beta) \\
 & \text{subject to } G + (3528.536 - 0.59 \times 10^{-5})\alpha \leq 3528.536; \\
 & S + (37.61824 - 0.10256 \times 10^{-2})\alpha \leq 37.61824; \\
 & G - (1 - \beta)(0.59 \times 10^{-5} + \varepsilon_G) \leq 3528.536\beta; \\
 & S - (1 - \beta)(0.10256 \times 10^{-2} + \varepsilon_S) \leq 37.61824\beta; \\
 & G + \xi_G \gamma \leq (0.59 \times 10^{-5} + \xi_G); \\
 & S + \xi_S \gamma \leq (0.10256 \times 10^{-2} + \xi_S); \\
 & (3.79 \times 10^{-3} \times 25)G^{-0.44}S^{-1.58} \leq 137.5; \\
 & (87.6 \times 10^{-5} \times 25)G^{-0.729}S^{-0.895} \leq 5.5; \\
 & \alpha \geq \beta; \alpha \geq \gamma; \alpha + \beta + \gamma \leq 3; \alpha, \beta, \gamma \in [0, 1]
 \end{aligned}$$

Table 2: Comparison of Optimal solution of MOSOP (7) based on different method

Methods	$G$ (mm)	$T$ (mm)
Fuzzy multi-objective nonlinear programming (FMONLP)	52.88329	0.5648067
Intuitionistic fuzzy multi-objective nonlinear programming (IFMONLP) $\varepsilon_G = 1764.268, \varepsilon_S = 2.57033$	52.88329	0.5648065
Neutrosophic optimization (NSO) $\varepsilon_G = \xi_G = 1764.268, \varepsilon_S = \xi_S = 22.57033$	44.28802	0.5676034

Here we get best solutions for the different tolerance  $\xi_G, \xi_S$  for indeterminacy membership function of objective functions. From the table 2, it shows that NSO technique gives better Pareto optimal result in the perspective of Structural Optimization.

## 7 Conclusions

The main objective of this work is to illustrate how much neutrosophic optimization technique reduces thickness and sag of nonlinear vertical lift gate in comparison of fuzzy and intuitionistic fuzzy optimization technique. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; rather, they are independent with degree of indeterminacy. Here we have considered a non-linear skin plate of vertical lift gate problem. In this problem, we find out minimum thickness of the structure as well as minimum sag of cylindrical skin plate. The comparisons of results obtained for the undertaken problem clearly show the superiority of neutrosophic optimization over fuzzy optimization and intuitionistic fuzzy optimization. The results of this study may lead to the development of effective neutrosophic technique for solving other model of nonlinear programming problem in different fields.

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