Vector Similarity Measures For Simplified Neutrosophic hesitant fuzzy Set And Their Applications

Jun Ye[1], Tahir Mahmood[2], Qaisar Khan[1,*]
[2,1,*]Department of Mathematics and Statistic International Islamic University Islamabad
[1,*]qaisarkhan421@gmail.com,[2]tahirbakht@yahoo.com
[1]Department of Electrical and Information Engineering, Shoaxing University China
[1]yehjun@aliyun.com

May 1, 2016

Abstract

In this article we present three similarity measures between simplified neutrosophic hesitant fuzzy sets, which contain the concept of single valued neutrosophic hesitant fuzzy sets and interval valued neutrosophic hesitant fuzzy sets, based on the extension of Jaccard similarity measure, Dice similarity measure and Cosine similarity in the vector space. Then based on these three defined similarity measures we present a muliple attribute decision making method to solve the simplified neutrosophic hesitant fuzzy multiple criteria decision making problem, in which the evaluated values of the alternative with respect to the criteria is represented by simplified neutrosophic hesitant fuzzy elements. Further we applied the proposed similarity measures to pattern recognition. At the end a numerical examples are discussed to show the effectiveness of the proposed similarity measures.

Keywords: *Vector similarity measure, Jaccard similarity measure, Dice similarity measure, Cosine similarity measure, hesitant fuzzy sets, interval valued hesitant fuzzy sets, Single valued neutrosophic hesitant fuzzy sets, interval neutrosophic hesitant fuzzzy sets, multi-criteria decision making, pattern recognition.*

1 Introduction

The evidence provided by people to examine issues is frequently fuzzy and crisp, and the fuzzy set ($\widetilde{FS}$) has been verified to be very important in multi-criteria decision making. $\widetilde{FS}$ theory was first presented by L. A. Zadeh in 1965[46] seeing degrees of membership to progressively asses the membership of elements in a set. After the introduction of $\widetilde{FS}$, atanassov[1] presented the concept of intuitionistic fuzzy set ($\widetilde{IFS}$) by considering membership and non-membership degrees of an element to the set. Atanassov et al. extended $\widetilde{IFS}$ to interval valued intuitionistic fuzzy set ($\widetilde{IVIFS}$)[2]. After the introduction of $\widetilde{IFS}$ alot of similarity measures are proposed by many researchers as the extension of the similarity measures for $\widetilde{FS}$, as a similarity measure is an important tool for determining similarity between objects. Li and cheng[6] presented similarity measure for $\widetilde{IFS}$ which is the first to applied them to pattern recognition. S.K De et al. gave some application of $\widetilde{IFS}$ to medical daignosis[7]. Ye[12] presented cosine similarity measure for $\widetilde{IFS}$ and applied it to medical diagnosis and pattern recognition. Ye[13, 14] also presented cosine similarity measure and vector similarity measures for $\widetilde{IVIFS}$ and trapezoidal $\widetilde{IFN}$ and applied them to multiple criteria decision making problem.

Neutrosophic set($\widetilde{NS}$) was first presented by F. Samarandache[29, 30] which simplifies the concept of crisp set, $\widetilde{FS}$, $\widetilde{IFS}$, $\widetilde{IVIFS}$, and so on. In $\widetilde{NS}$ its indeterminacy is computed unambiguously and its truth-membership, indeterminacy-membership and falsity-membership are signified freely. $\widetilde{NS}$ simplifies all the above defined sets from philosophical point of view. Several similarity measures for $\widetilde{NS}$ has been defined by Broumi. S. et al. [31]. Majumdar et al.[26] presented similarity and entropy for $\widetilde{NS}$. $\widetilde{NS}$ is difficult to apply in real life and engineering problem. To overcome this difficulty wang et. al. [36, 37]. introduced the concept of single valued neutrosophic sets ($/\widetilde{SVNSs}$) and interval neutrosophic sets ($\widetilde{INSs}$) and provided basic operation and properties of $\widetilde{SVNSs}$ and $\widetilde{INSs}$. The $\widetilde{SVNSs}$ and $\widetilde{INSs}$ is a subclass of $\widetilde{NS}$. Sahin R. et al.[28] defined subsethood for $\widetilde{SVNSs}$. Zhang et al[42] also defined some operation for $\widetilde{INS}$. Ye[15, 16, 17, 18] defined correlation coefficient, improved correlation,entropy and similarity measure for $\widetilde{SVNS}$ and $\widetilde{INS}$ and gave their application in multiple attribute decision making problems ($\widetilde{MCDM}$). Broumi. S. et al. [32, 33] presented correlation and cosine similarity measure for $\widetilde{INS}$. Ye[19] presented the concept of simplified neutrosophic sets ($\widetilde{SNSs}$) which contained the concept of $\widetilde{SVNS}$ and $\widetilde{INS}$ and defined some basic operation for $\widetilde{SNSs}$ and applied them to multi-criteria decision making problem. Ye [20, 21] also defined vector similarity measures for $\widetilde{SNSs}$ and improved cosine similarity measures for $\widetilde{SNSs}$. Peng et al[11] find some drawbacks of the operation for $\widetilde{SNS}$ defined by Ye and presented new operations for $\widetilde{SNS}$.

Hesitant fuzzy set ($\widetilde{HFS}$) is another generalization of $\widetilde{FS}$ proposed by Torra and Narukawa[34, 35]. The advantege of $\widetilde{HFS}$ is that it authorizations the membership degree of an element to a given set with a limited different values, which can ascend in a group decision making problem. Recently a $\widetilde{HFS}$ has acknowledged more and more consideration since its presence. Xia and Xu [38, 39, 40, 41] presented hesitant fuzzy information, similarity measures for $\widetilde{HFS}$ and correlation for $\widetilde{HFS}$.Chen et al[4] presented some correlation coefficient for $\widetilde{HFS}$ and applied them to clustering analysis. Ye[22] presented vector similarity measures for $\widetilde{HFS}$. Chen et al.[3] further extended HFS to interval valued hesitant fuzzy set. ($\widetilde{IVHFS}$) Farhadinia. B [8, 9] defined information measures for $\widetilde{HFS}$, *and* $\widetilde{IVHFS}$. and distance measures for high order $\widetilde{HFS}$.

Dual hesitant fuzzy sets ($\widetilde{DHFS}$) was presented by Zhu[43, 44] which is the generalization of $\widetilde{HFS}$, $\widetilde{FS}$, $\widetilde{IFS}$ and fuzzy multi sets are special cases of $\widetilde{DHFS}$. Recently Su. Z[45] presented similarity measure for $\widetilde{DHFS}s$ and applied it to pattern recognition.Correlation of $\widetilde{DHFS}$ was presented by Ye[23] and applied them to $\widetilde{MCDM}$ under $\widetilde{DHF}$ informations. As mentioned above that hesitancy is the most common problem in decision making for which $\widetilde{HFS}$ is a suitable means by allowing sevrel possible values for an element to a set. However in $\widetilde{HFS}$ they consider only one truth-membership function and it cannot express this problem with a few different values assigned by truth-membership hesitant degree, indeterminacy-membership degree, and falsity-membership degrees due to doubts of decision makers.and also in $\widetilde{DHFS}$ they consider two functions that is membership and non-membership functions and can not consider indeterminacy-membership function. To overawed this problem and the decision maker can get more information Ye [24] presented the concept of single valued neutrosophic hesitant fuzzy sets ($\widetilde{SVNHF}$) and defined some basic operations, aggregation operators and applied it to $\widetilde{MCDM}$ under $\widetilde{SVNHF}$.environment. Lui P. [25] presented the concept of interval neutrosophic hesitant fuzzy set $\left(\widetilde{INHF}\right)$ and defined some basic operations, aggregation operators and applied them to multicriteria decision making problem.

Since vector similarity measures played an important role in decision making, so we present vector similarity measures for $\widetilde{SNHF}$ and then give its applications in $\widetilde{MCDM}$ and pattern recognition.

The rest of the article is organized as follows. In section 2 we defined some basic definitions related to our work. In section 3 we defined vector similarity measures for simplified neutrosophic hesitant fuzzy sets. In section 4 we apply the peoposed similarity measures to multi attribute decision making problem under simplified neutrosophic environment. in section 5 we apply the proposed similarity measures to pattern recognition. At the end we give some numerical examples to show the effectiveness of the proposed similarity measures, conclusion and refrences are given.

2 Preliminaries

In this section we define some basic definitions about hesitant fuzzy sets, interval valued hesitant fuzzy set, single valued neutrosophic hesitant fuzzy set, interval neutrosophic hesitant fuzzy sets and the vector similarity measures, such as Jaccard similarity measure, Dice similarity measure, Cosine similarity measure.

Definition 1 *[34, 35]Let $\bar{U}$ be a fixed set, a hesitant fuzzy set $\hat{M}$ on $\bar{U}$ is defined in terms of a function $f_{\hat{M}}(a)$ that when applied to $\bar{U}$ returns a finite subsets of* $[0,1]$. *Hesitant fuzzy set is mathematically represented as,*

$$\hat{M} = \{\langle a, f_{\hat{M}}(a) | a \in \bar{U}\rangle\},$$

Where $f_{\hat{M}}(a)$ is a set of some different values in $[0,1]$, denoting the possible membership degrees of the element $a \in \bar{U}$ to $\hat{M}$.

Definition 2 *[3]Let $\bar{U}$ be a fixed set, an interval valued hesitant fuzzy set $\hat{D}$ on $\bar{U}$ is defined in terms of a function $f_{\hat{M}}(a)$ that when applied to $\bar{U}$ returns a finite set of subintervals of* $[0,1]$. *An interval valued hesitant fuzzy set is mathematically represented as,*

$$\hat{D} = \{\langle a, f_{\hat{D}}(a) | a \in \bar{U}\rangle\},$$

Where $f_{\hat{D}}(a)$ is a set of some different values in $[0,1]$, denoting the possible membership degrees of the element $a \in \bar{U}$ to $\hat{D}$.

Definition 3 *[24]Let $\bar{U}$ be a non empty fixed set. A single valued neutrosophic hesitant fuzzy set $\hat{N}$ is the structure of the form:*

$$\hat{N} = \{\langle a, \breve{t}(a), \breve{\imath}(a), \breve{f}(a)\rangle\}$$

In which $\breve{t}(a), \breve{\imath}(a)$ and $\breve{f}(a)$ are three sets of some values in the real unit interval $[0,1]$, representing the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees and falsity-membership hesitant degree of the element $a \in \bar{U}$ to the set $\hat{N}$, respectively, with the condition that $0 \leq \varphi, \chi, \psi \leq 1$ and $0 \leq \varphi^+ + \chi^+ + \psi^+ \leq 3$, where $\varphi \in \breve{t}(a), \chi \in \breve{\imath}(a), \psi \in \breve{f}(a), \varphi^+ = \cup_{\varphi \in \breve{t}} \max\{\varphi\}, \chi^+ = \cup_{\chi \in \widetilde{i}} \max\{\chi\}, \psi^+ = \cup_{\psi \in \widetilde{f}} \max\{\psi\}$.

Definition 4 *[25]Let $\bar{U}$ be a non empty fixed set. A interval neutrosophic hesitant fuzzy set $\hat{H}$ is structure of the form*

$$\hat{H} = \{\langle a, \breve{t}(a), \breve{\imath}(a), \breve{f}(a)\rangle\}$$

In which $\breve{t}(a), \breve{\imath}(a)$ and $\breve{f}(a)$ are three sets of some interval values in the set of real unit interval $[0,1]$, representing the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees and falsity-membership hesitant degree of the element $a \in \bar{U}$ to the set $\hat{H}$, respectively, with the condition that $\varphi = [\varphi^l, \varphi^u] \in \breve{t}(a), \chi = [\chi^l, \chi^u] \in \breve{\imath}(a), \psi = [\psi^l, \psi^u] \in \breve{f}(a) \subseteq [0,1]$ and $0 \leq \sup \varphi^+ + \sup \chi^+ + \sup \psi^+ \leq 3, \varphi^+ = \cup_{\varphi \in \breve{t}} \max\{\varphi\}, \chi^+ = \cup_{\chi \in \widetilde{i}} \max\{\chi\}, \psi^+ = \cup_{\psi \in \widetilde{f}} \max\{\psi\}$.

2.1 Vector similarity measures

Let $A = (c_1, c_2, ..., c_m)$ and $B = (e_1, e_2, ..., e_m)$ be two vectors of length is m where all the coordinates are positive. The jaccard similarity measure of these two vectors [27] is given by

$$\tilde{J}(A,B) = \frac{A.B}{||A||_2^2 + ||B||_2^2 - A.B} = \frac{\sum_{k=1}^{m} c_k e_k}{\sum_{k=1}^{m} c_k^2 + \sum_{k=1}^{m} e_k^2 - \sum_{k=1}^{m} c_k e_k} \quad \text{(a)}$$

Where $A.B = \sum_{k=1}^{m} c_k e_k$ is the inner product of the vectors A and B and $||A||^2 = \sqrt{\sum_{k=1}^{m} c^2}$ and $||B||^2 = \sqrt{\sum_{k=1}^{m} e^2}$ are the Euclidean norms of A and B.

Then the Dice similarity measure [5] is defined as follows:

$$\tilde{D}(A,B) = \frac{2A.B}{||A||_2^2 + ||B||_2^2} = \frac{2\sum_{k=1}^{m} c_k e_k}{\sum_{k=1}^{m} c_k^2 + \sum_{k=1}^{m} e_k^2} \quad \text{(b)}$$

and the Cosine similarity measure[10] is defined as follows:

$$\tilde{C}(A,B) = \frac{A.B}{||A||_2.||B||_2} = \frac{\sum_{k=1}^{m} c_k e_k}{\sqrt{\sum_{k=1}^{m} c_k^2}.\sqrt{\sum_{k=1}^{m} e_k^2}} \quad \text{(c)}$$

The Cosine similarity measure is nothing just the cosine angle between two vectors.

the above formulas are same in the sence each take the value in the unit interval $[0, 1]$. Tha jaccard and dice and cosine similarity measures are respectively undefined if $c_k = e_k = 0,\ for\ k = 1, 2...m\ and\ \ c_k = 0\ or\ e_k = 0,\ k = 1, 2...m$. Now we assume that the cosine similarity measure equal to zero when $c_k = 0\ or\ e_k = 0,\ k = 1, 2...m$.

These vector similarity measure satisfy the following properties for the vectors $A\ and\ B$.

$$\begin{aligned}(1)\ J(A,B) &= J(B,A),\ D(A,B) = D(B,A),\ and\ C(A,B) = C(B,A)\\ (2)\ 0 &\le J(A,B),\ D(A,B), C(A,B) \le 1\\ (3)\ J(A,B) &= D(A,B) = C(A,B) = 1\ if\ A = B\end{aligned}$$

3 Vector Similarity Measure For Simplified Neutrosophic hesitant fuzzy Sets

If the simplified neutrosophic hesitant fuzzy set contains finite set of single points, that is single valued neutrosophic hesitant fuzzy set. Then the Jaccard, Dice and Cosine similarity measures are defined as follows:

Definition 5 *Let $\widetilde{A}$ and $\widetilde{B}$ be two $SV\widehat{NHFS}s$ on the universal set $\widehat{U} = \{u_1, u_2, ..., u_m\}$, respectively denoted by $\widetilde{A} = \{\langle u_j, \tau_{\widetilde{A}}(u_j), \iota_{\widetilde{A}}(u_j), \kappa_{\widetilde{A}}(u_j) | u_j \in \widehat{U}\}$ and $\widetilde{B} = \{\langle u_j, \tau_{\widetilde{B}}(u_j), \iota_{\widetilde{B}}(u_j), \kappa_{\widetilde{B}}(u_j) | u_j \in \widehat{U}\}$ for all $j = 1, 2, ..., m$. Then we define the jaccard,Dice and cosine similarity measures for $SV\widehat{NHFS}s$ $\widetilde{A}$ and $\widetilde{B}$ as follows:*

$$\widehat{J}_{SV\widehat{NHFS}}(\widetilde{A}, \widetilde{B}) = \frac{1}{m} \sum_{j=1}^{m} \frac{\sum_{i=1}^{le_j} \bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)}{\begin{array}{c} \sum_{i=1}^{le_j} \left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 \\ + \sum_{i=1}^{le_j} \left[\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 - \\ \left(\begin{array}{c} \sum_{i=1}^{le_j} \bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \\ \sum_{i=1}^{le_j} \bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) \end{array} \right) \end{array}} \tag{1}$$

$$\widehat{D}_{SV\widehat{NHFS}}(\widetilde{A}, \widetilde{B}) = \frac{1}{m} \sum_{j=1}^{m} \frac{2 \left(\begin{array}{c} \sum_{i=1}^{le_j} \bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) \\ + \sum_{i=1}^{le_j} \bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j) \end{array} \right)}{\begin{array}{c} \sum_{i=1}^{le_j} \left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 \\ + \sum_{i=1}^{le_j} \left[\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 \end{array}} \tag{2}$$

$$\widehat{C}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = \frac{1}{m}\sum_{j=1}^{m}\frac{\sum_{i=1}^{le_j}\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)}{\sqrt{\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2}+\sqrt{\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2}} \quad (3)$$

Here $le_j = \max(le(\bar{\tau}_{\widetilde{A}}(\tilde{u}_j),\bar{\iota}_{\widetilde{A}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{A}}(\tilde{u}_j)), le(\bar{\tau}_{\widetilde{B}}(\tilde{u}_j),\bar{\iota}_{\widetilde{B}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{B}}(\tilde{u}_j)))$ for all $\tilde{u}_j \in \widehat{U}$, where $le(\bar{\tau}_{\widetilde{A}}(\tilde{u}_j),\bar{\iota}_{\widetilde{A}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{A}}(\tilde{u}_j))$ and $le(\bar{\tau}_{\widetilde{B}}(\tilde{u}_j),\bar{\iota}_{\widetilde{B}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{B}}(\tilde{u}_j))$, respectively represents the number of values in $\bar{\tau}_{\widetilde{A}}(\tilde{u}_j),\bar{\iota}_{\widetilde{A}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{A}}(\tilde{u}_j)$ and $\bar{\tau}_{\widetilde{B}}(\tilde{u}_j),\bar{\iota}_{\widetilde{B}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{B}}(\tilde{u}_j)$. When the number of values in $\bar{\tau}_{\widetilde{A}}(\tilde{u}_j),\bar{\iota}_{\widetilde{A}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{A}}(\tilde{u}_j)$ and $\bar{\tau}_{\widetilde{B}}(\tilde{u}_j),\bar{\iota}_{\widetilde{B}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{B}}(\tilde{u}_j)$. are not equal that is $le(\bar{\tau}_{\widetilde{A}}(\tilde{u}_j),\bar{\iota}_{\widetilde{A}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{A}}(\tilde{u}_j)) \neq le(\bar{\tau}_{\widetilde{B}}(\tilde{u}_j),\bar{\iota}_{\widetilde{B}}(\tilde{u}_j),\bar{\kappa}_{\widetilde{B}}(\tilde{u}_j))$. Then the pessimestic add the minimum value while the optimistic add the maximum value. This depend on the decision makers that was successfully applied for hesitant fuzzy sets by [40].

The above defined three vector similarity measures satisfy the condition defined in Ye[22].

$(\underline{A_1})$ $\widehat{J}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = \widehat{J}_{SV\widehat{NHF}S}(\widetilde{B},\widetilde{A}), \widehat{D}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = \widehat{D}_{SV\widehat{NHF}S}(\widetilde{B},\widetilde{A}), \widehat{C}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = \widehat{C}_{SV\widehat{NHF}S}(\widetilde{B},\widetilde{A})$.

$(\underline{A_2})$ $0 \leq \widehat{J}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}), \widehat{D}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}), \widehat{C}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) \leq 1$.

$(\underline{A_3})$ $\widehat{J}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = \widehat{D}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = \widehat{C}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) =< 1,0,0 >$ $iff\ \widetilde{A} = \widetilde{B}$

Proof. $(A1)$ obviously it is true. ■

$(A2)$ obviously the property is true due to the inequality $\tilde{x}^2 + \tilde{y}^2 \geq 2\tilde{x}\tilde{y}$ for eq 1 and eq 2 and the cosine value for eq 3.

$(A3)$ When $\widetilde{A} = \widetilde{B}$ then $\bar{\tau}_{\widetilde{A}}(\tilde{u}_j) = \bar{\tau}_{\widetilde{B}}(\tilde{u}_j)$, $\bar{\iota}_{\widetilde{A}}(\tilde{u}_j) = \bar{\iota}_{\widetilde{B}}(\tilde{u}_j)$ and $\bar{\kappa}_{\widetilde{A}}(\tilde{u}_j) = \bar{\kappa}_{\widetilde{B}}(\tilde{u}_j)$ for each $\tilde{u}_j \in \widehat{U}$, $j = 1,2,...,m$. So there are

$\widehat{J}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = 1, \widehat{D}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B}) = 1$, and $\widehat{C}_{SV\widehat{NHFS}}(\widetilde{A},\widetilde{B}) = 1$.

In real life problem, the elements $\tilde{u}_j$ $(j = 1,2,...,m)$ in a universal set $\widehat{U} = \{\tilde{u}_1,\tilde{u}_2,...,\tilde{u}_j\}$ have different weights. Let $\widehat{\varpi} = (\varpi_1,\varpi_2,...,\varpi_j)^{\widehat{T}}$ be the weight vector of $\tilde{u}_j$ $(j = 1,2,...,m)$ with $\varpi_j \geq 0, j = 1,2,...,m,\ and\ \sum_{j=1}^{m}\varpi_j = 1$. Therefore we extend the above three similarity measures to weighted vector similarity measures for $SV\widehat{NHF}Ss$.

The weigthed Jaccard similarity measure for $SV\widehat{NHF}Ss$ $\widetilde{A}$ and $\widetilde{B}$ as follows:

$$\widehat{J}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_j\frac{\sum_{i=1}^{le_j}\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2\\+\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2-\\\left(\sum_{i=1}^{le_j}\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\end{array}} \tag{4}$$

The Dice similarity measure for $SV\widehat{NHF}Ss$ $\widetilde{A}$ and $\widetilde{B}$ as follows:

$$\widehat{D}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_j\frac{2\left(\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\\+\sum_{i=1}^{le_j}\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\end{array}\right)}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2\\+\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2\end{array}} \tag{5}$$

The Cosine similarity measure for $SV\widehat{NHF}Ss$ $\widetilde{A}$ and $\widetilde{B}$ as follows:

$$\widehat{C}_{SV\widehat{NHF}S}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_j\frac{\sum_{i=1}^{le_j}\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)}{\begin{array}{c}\left(\sqrt{\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2}\right)\\\left(\sqrt{\sum_{i=1}^{le_j}\left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2}\right)\end{array}} \tag{6}$$

If the $SNHFS$ contain a set of interval values instead of a set of single points. Then

$$\widehat{J}_{SV\widehat{NH}FS}(\widetilde{A},\widetilde{B})=\frac{1}{2m}\sum_{j=1}^{m}\frac{\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\\ +\sum_{i=1}^{le_j}\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\\ \sum_{i=1}^{le_j}\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\end{array}}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2-\\ \left(\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\\ +\sum_{i=1}^{le_j}\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\\ \sum_{i=1}^{le_j}\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\end{array}\right)\end{array}} \tag{7}$$

The dice similarity measure for $I\widehat{NHFS}s$

$$\widehat{D}_{SV\widehat{NHFS}}(\widetilde{A},\widetilde{B})=\frac{1}{4m}\sum_{j=1}^{m}\frac{2\left(\begin{array}{c}\left(\sum_{i=1}^{le_j}\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\\+\left(\sum_{i=1}^{le_j}\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\\+\left(\sum_{i=1}^{le_j}\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\end{array}\right)}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\kappa^{L}_{\widetilde{B}\sigma(i)}(u_j)\right]^2+\sum_{i=1}^{le_j}\left[\kappa^{U}_{\widetilde{B}\sigma(i)}(u_j)\right]^2\end{array}} \tag{8}$$

The cosine similarity measure for $INHFSs$.

$$\widehat{C}_{SV\widehat{NHFS}}(\widetilde{A},\widetilde{B})=\frac{1}{2m}\sum_{j=1}^{m}\frac{\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\\\sum_{i=1}^{le_j}\bar{\iota}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\end{array}}{\left(\sqrt{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\end{array}}\right)\left(\sqrt{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{L}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2\end{array}}\right)} \tag{9}$$

The weighted Jaccard similarity measures for $INHFS$

$$\widehat{WJ}_{SV\widehat{NH}FS}(\widetilde{A},\widetilde{B}) = \sum_{j=1}^{m} \varpi_j \frac{\begin{array}{c} \sum_{i=1}^{le_j} \bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \\ \sum_{i=1}^{le_j} \bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \\ \sum_{i=1}^{le_j} \bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j) \end{array}}{\begin{array}{c} \sum_{i=1}^{le_j} \left[\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \\ \sum_{i=1}^{le_j} \left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \\ \sum_{i=1}^{le_j} \left[\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \\ \sum_{i=1}^{le_j} \left[\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2 - \\ \left(\begin{array}{c} \sum_{i=1}^{le_j} \bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \\ \sum_{i=1}^{le_j} \bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j) \\ + \sum_{i=1}^{le_j} \bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j) \end{array} \right) \end{array}} \tag{10}$$

The weighted Dice similarity measure

$$\widehat{WD}_{SV\widehat{NH}FS}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_j\frac{2\left(\begin{array}{c}\left(\sum_{i=1}^{le_j}\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\\+\left(\sum_{i=1}^{le_j}\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\\+\left(\sum_{i=1}^{le_j}\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right)\end{array}\right)}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\kappa^L_{\widetilde{B}\sigma(i)}(u_j)\right]^2+\sum_{i=1}^{le_j}\left[\kappa^U_{\widetilde{B}\sigma(i)}(u_j)\right]^2\end{array}} \tag{11}$$

The weighted Cosine measure

$$\widehat{C}_{SV\widehat{NH}FS}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_j\frac{\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\\\sum_{i=1}^{le_j}\bar{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\end{array}}{\left(\sqrt{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\end{array}}\right)\left(\sqrt{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\bar{\iota}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^L_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^U_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2\end{array}}\right)} \tag{12}$$

4 Multi attribute decision making With single valued neutrosophic hesitant information

In this section we present a multi-criteria decision making method adapted from Ye[22] to utilize the vector similarity measures for $\widehat{SVNHFSs}$ and $\widehat{INHFSs}$ with $\widehat{SVNHF}$ and $\widehat{INHF}$ information respectively.

For a $\widehat{MCDM}$ problem with $\widehat{SVNHF}$ and $\widehat{INHF}$ information respectively, let $\breve{A} = \{\breve{A}_1, \breve{A}_2, ..., \breve{A}_n\}$ be a set of alternatives and $\breve{C} = \{\breve{C}_1, \breve{C}_2, ..., \breve{C}_m\}$ be a set of attributes. If for the alternatives $\breve{A}_j$ $(for\ j = 1, 2, ..., n)$ the decision makers provide several values under the criteria $\breve{C}_i$ $(for\ i = 1, 2, ..., m)$, then each value is considered as a $\widehat{SVNHF}$ element and $\widehat{INHF}$ element $(\bar{\tau}_{ji}, \bar{\iota}_{ji}, \bar{\kappa}_{ji})(\ for\ j = 1, 2, ..., n,\ i = 1, 2, ..., m)$.

Therefore we can stimulate a $\widehat{SVNHF}$ and $\widehat{INHF}$ decision matrix respectively $\tilde{D} = (\bar{\tau}_{ji}, \bar{\iota}_{ji}, \bar{\kappa}_{ji})_{n\times m}$, where $(\bar{\tau}_{ji}, \bar{\iota}_{ji}, \bar{\kappa}_{ji})(\ for\ j = 1, 2, ..., n,\ i = 1, 2, ..., m)$.is in the form of $\widehat{SVNHF}$ elements or $\widehat{INHF}$ elements.

For the selection of best alternatives in multiple-criteria decision making environments,the concept of ideal point has been used in the decision set. Although in real world the ideal point does not exists. The ideal point deliver a suitable theoretical hypothesis to evaluate alternatives. Therefore we define each value in each ideal $\widehat{SVNHF}$ or $\widehat{INHF}$ element $(\bar{\tau}^*_j, \bar{\iota}^*_j, \bar{\kappa}^*_j)$ for the ideal alternative $\breve{A}^* = \{\langle\breve{C}_i, (\bar{\tau}^*_j, \bar{\iota}^*_j, \bar{\kappa}^*_j)\rangle | \breve{C}_i \in \breve{C}\}$ as $\bar{\tau}^*_{j\sigma(k)} = 1, \bar{\iota}^*_{j\sigma(k)} = 0, \bar{\kappa}^*_{j\sigma(k)} = 0,\ or\ \bar{\tau}^*_{j\sigma(k)} = [1, 1], \bar{\iota}^*_{j\sigma(k)} = [0, 0], \bar{\kappa}^*_{j\sigma(k)} = [0, 0],\ \ for\ k = 1, 2, ..., le_i$, where le_i is the number of values or interval values in $(\bar{\tau}_{ji}, \bar{\iota}_{ji}, \bar{\kappa}_{ji})(\ for\ j = 1, 2, ..., n,\ i = 1, 2, ..., m)$.

For the different importance of each criteria the weighting vector of criteria is given as $\breve{\varpi} = (\varpi_1, \varpi_2, ..., \varpi_m)^T$, where $\varpi_i \geq 0, i = 1, 2, ..., m,\ and\ \ \sum_{i=1}^{m} \varpi_i = 1$.

Then we exploit the three weighted vector similarity measures for $\widehat{SVNHFSs}$ or $\widehat{INHFSs}$ for $\widehat{MCDM}$ under $\widehat{SVNHF}$ or $\widehat{INHF}$ information, which can be defined as follows:

Step 1. Calculate one of the three vector similarity measures between the alternative $\breve{A}_j$ $(for\ j = 1, 2, ..., n)$ and the ideal alternative $\breve{A}^*$ by using one of the three formulas:

$$\widehat{\varpi J}_{\widehat{SVNHF}}(\breve{A}_j, \breve{A}^*) = \sum_{j=1}^{m} \varpi_i \frac{\begin{array}{c}\sum_{i=1}^{le_j} \bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j) \\ + \sum_{i=1}^{le_j} \bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}}{\begin{array}{c}\sum_{i=1}^{le_j} \left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 \\ + \sum_{i=1}^{le_j} \left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2 \\ + \sum_{i=1}^{le_j} \left[\bar{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2 - \\ \left(\begin{array}{c}\sum_{i=1}^{le_j} \bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j) \\ + \sum_{i=1}^{le_j} \bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}\right)\end{array}} \tag{13}$$

$$\widehat{\varpi D}(\breve{A}_j, \breve{A}^*) = \sum_{j=1}^{m} \varpi_i \frac{2\left(\begin{array}{c}\sum_{i=1}^{le_j} \bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j) + \sum_{i=1}^{le_j} \bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j) \\ + \sum_{i=1}^{le_j} \bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}\right)}{\begin{array}{c}\sum_{i=1}^{le_j} \left[\bar{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 \\ + \sum_{i=1}^{le_j} \left[\bar{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2 \\ + \sum_{i=1}^{le_j} \left[\bar{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2 + \sum_{i=1}^{le_j} \left[\bar{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2\end{array}} \tag{14}$$

$$\widehat{\varpi C}(\breve{A}_j,\breve{A}^*)=\sum_{j=1}^{m}\varpi_i\frac{\sum_{i=1}^{le_j}\overline{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)}{\sqrt{\sum_{i=1}^{le_j}\left[\overline{\tau}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\iota}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\kappa}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2}\sqrt{\sum_{i=1}^{le_j}\left[\overline{\tau}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\iota}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\kappa}^*_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2}} \tag{15}$$

We use the following formulas under the$\widehat{SVNHF}$ information.

Under $\widehat{INHF}$ information, we use the following formulas:

$$\widehat{\varpi J}_{\widehat{INHFS}}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_i\frac{\begin{array}{c}\sum_{i=1}^{le_j}\overline{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\\+\sum_{i=1}^{le_j}\overline{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\\\sum_{i=1}^{le_j}\overline{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\kappa}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\overline{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\overline{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\overline{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\\\sum_{i=1}^{le_j}\left[\overline{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\overline{\kappa}^{*U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2-\\\left(\begin{array}{c}\sum_{i=1}^{le_j}\overline{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\\+\sum_{i=1}^{le_j}\overline{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\\\sum_{i=1}^{le_j}\overline{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\overline{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\overline{\kappa}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}\right)\end{array}} \tag{16}$$

$$\widehat{\varpi D}_{\widehat{INHFS}}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_i\frac{2\left(\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\\ \sum_{i=1}^{le_j}\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\\ +\sum_{i=1}^{le_j}\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}\right)}{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\iota}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{L}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{U}_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{*U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2\end{array}} \tag{17}$$

$$\widehat{\varpi C}_{\widehat{INHFS}}(\widetilde{A},\widetilde{B})=\sum_{j=1}^{m}\varpi_i\frac{\begin{array}{c}\sum_{i=1}^{le_j}\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\\ 2\sum_{i=1}^{le_j}\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\iota}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\\ +\sum_{i=1}^{le_j}\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)+\sum_{i=1}^{le_j}\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\bar{\kappa}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\end{array}}{\left[\sqrt{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\\ \sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2\\ +\sum_{i=1}^{le_j}\left[\bar{\kappa}^L_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^U_{\widetilde{A}\sigma(i)}(\tilde{u}_j)\right]^2\end{array}}\right]\left[\sqrt{\begin{array}{c}\sum_{i=1}^{le_j}\left[\bar{\tau}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\tau}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2\\ +\sum_{i=1}^{le_j}\left[\bar{\iota}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\iota}^{*U}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2\\ +\sum_{i=1}^{le_j}\left[\bar{\kappa}^{*L}_{\breve{A}^*\sigma(i)}(\tilde{u}_j)\right]^2+\sum_{i=1}^{le_j}\left[\bar{\kappa}^{*U}_{\widetilde{B}\sigma(i)}(\tilde{u}_j)\right]^2\end{array}}\right]} \tag{18}$$

Practical example

This example is adopted from Ye [22] is used as the demonstration of the effectiveness of of the proposed decision making method in real life problem.

For SVNH information

There is an investment company, which wants to invest some money in the best option. There is a panel with four possible alternatives to invest the money:

(1) A_1 is a car company (2) A_2 is a food company , (3) A_4 is a computers company, (4) A_4 is an arms company. The following three criterias on which the investment company must take decision are (1) C_1 is the risk, (2) C_2 is the growth, (3) C_3 is the environmental impact. The weight vector of the criteria is given as $(0.35, 0.25, 0.4)^T$ the four possible alternative

The decision matrix for attributes and alternative the data is represented by $\widehat{SVNHFN}$

$$\breve{D}=\left[\begin{array}{cccc} & \tilde{C}_1 & \tilde{C}_2 & \tilde{C}_3 \\ \tilde{A}_1 & \left\{\begin{array}{c}\{0.3,0.4\},\{0.2,0.3\},\\ \{0.5,0.6\}\end{array}\right\} & \left\{\begin{array}{c}\{0.4,0.5\},\{0.1,0.3\},\\ \{0.4,0.6\}\end{array}\right\} & \left\{\begin{array}{c}\{0.1,0.2\},\{0.3,0.4\},\\ \{0.6,0.7\}\end{array}\right\} \\ \tilde{A}_2 & \left\{\begin{array}{c}\{0.9,1\},\{0.05,0.07\},\\ \{0.01,0.02\}\end{array}\right\} & \left\{\begin{array}{c}\{0.8,0.9\},\{0.1,0.2\},\\ \{0.1,0.2\}\end{array}\right\} & \left\{\begin{array}{c}\{0.7,0.9\},\{0.1,0.2\},\\ \{0.01,0.02\}\end{array}\right\} \\ \tilde{A}_3 & \left\{\begin{array}{c}\{0.5,0.6\},\{0.2,0.3\},\\ \{0.1,0.2\}\end{array}\right\} & \left\{\begin{array}{c}\{0.5,0.7\},\{0.1,0.2\},\\ \{0.2,0.3\}\end{array}\right\} & \left\{\begin{array}{c}\{0.5,0.7\},\{0.1,0.2\},\\ \{0.2,0.3\}\end{array}\right\} \\ \tilde{A}_4 & \left\{\begin{array}{c}\{0.9,1\},\{0.1,0.2\},\\ \{0.15,0.25\}\end{array}\right\} & \left\{\begin{array}{c}\{0.8,0.9\},\{0.1,0.2\},\\ \{0.1,0.2\}\end{array}\right\} & \left\{\begin{array}{c}\{0.4,0.5\},\{0.2,0.4\},\\ \{0.3,0.5\}\end{array}\right\} \end{array}\right]$$

Table 1: The values obtained through the defined weighted vector similarity measures for $\widehat{SVNHFS}$

	$WJ_{SVNHF}(\tilde{A}^*,\hat{A}_k)$	$WD_{\widehat{SVNHF}}(\tilde{A}^*,\hat{A}_k)$	$WC_{\widehat{SVNHF}}(\tilde{A}^*,\hat{A}_k)$
$\tilde{A}_1$	0.2254	0.3384	0.4099
$\tilde{A}_2$	0.9420	0.9762	0.8752
$\tilde{A}_3$	0.68125	0.81120	0.8812
$\tilde{A}_4$	0.73040	0.8205	0.8395
Ranking order	$\tilde{A}_2 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_1$	$\tilde{A}_2 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_1$,	$\tilde{A}_3 \succ \tilde{A}_2 \succ \tilde{A}_4 \succ \tilde{A}_1$

The values obtained in table 1 shows that the Jaccard and Dice similarity measures have the same result that is $\widetilde{A}_2$ is the best alternative while the Cosine similarity measure shows that $\widetilde{A}_3$ is the best alternative. From table 1 we conclude that the Jaccard and Dice similarity measure are better to use in multi-criteria decision making.

For INS information

Now we consider the above example for $INHFSs$,

The decision matrix for attributes and alternative the data is represented by $I\widehat{NHF}N$

$$\breve{D}=\left[\begin{array}{cccc} & \tilde{C}_1 & \tilde{C}_2 & \tilde{C}_3 \\ \tilde{A}_1 & \left\{\begin{array}{c}\{[0.3,0.4],[0.4,0.5]\},\\ \{[0.2,0.3],[0.3,0.4]\},\\ \{[0.5,0.6],[0.6,0.7]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.4,0.5],[0.5,0.6]\},\\ \{[0.1,0.2],[0.3,0.4]\},\\ \{[0.4,0.5],[0.5,0.6]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.1,0.2],[0.2,0.3]\},\\ \{[0.3,0.4],[0.4,0.5]\},\\ \{[0.6,0.7],[0.7,0.8]\}\end{array}\right\} \\ \tilde{A}_2 & \left\{\begin{array}{c}\{[0.8,0.9],[0.9,1]\},\\ \{[0.01,0.02],[0.02,0.03]\},\\ \{[0.01,0.02],[0.02,0.03]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.8,0.9],[0.9,1]\},\\ \{[0.1,0.2],[0.2,0.3]\},\\ \{[0.1,0.2],[0.1,0.2]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.7,0.8],[0.9,1]\},\\ \{[0.05,0.15],[0.15,0.25]\},\\ \{[0.01,0.02],[0.05,0.09]\}\end{array}\right\} \\ \tilde{A}_3 & \left\{\begin{array}{c}\{[0.5,0.6],[0.6,0.7]\},\\ \{[0.2,0.3],[0.3,0.4]\},\\ \{[0.1,0.2],[0.2,0.3]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.4,0.5],[0.5,0.7]\},\\ \{[0.1,0.2],[0.2,0.3]\},\\ \{[0.2,0.3],[0.3,0.4]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.4,0.5],[0.5,0.7]\},\\ \{[0.1,0.2],[0.2,0.3]\},\\ \{[0.2,0.3],[0.3,0.4]\}\end{array}\right\} \\ \tilde{A}_4 & \left\{\begin{array}{c}\{[0.8,0.9],[0.9,1]\},\\ \{[0.1,0.2],[0.2,0.3]\},\\ \{[0.15,0.25],[0.25,0.35]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.7,0.8],[0.9,1]\},\\ \{[0.1,0.2],[0.2,0.3]\},\\ \{[0.1,0.2],[0.2,0.3]\}\end{array}\right\} & \left\{\begin{array}{c}\{[0.4,0.5],[0.5,0.6]\},\\ \{[0.2,0.3],[0.3,0.5]\},\\ \{[0.3,0.4],[0.4,0.5]\}\end{array}\right\} \end{array}\right]$$

Table 2: The values obtained throug using weighted vector similarity mea-

sures for $I\widehat{NHFN}$

	$WJ_{SVNHF}()$	$WD_{SVNHF}()$	$WJ_{SVNHF}()$
$\tilde{A}_1$	0.2825	0.4238	0.4357
$\tilde{A}_2$	0.9437	0.9708	0.9765
$\tilde{A}_3$	0.6100	0.7568	0.7297
$\tilde{A}_4$	0.7171	0.8197	0.8337
Ranking order	$\tilde{A}_2 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_1$	$\tilde{A}_2 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_1$,	$\tilde{A}_2 \succ \tilde{A}_4 \succ \tilde{A}_3 \succ \tilde{A}_1$

The values obtained from table 2 shows that the best alternatives is $\widetilde{A}_2$.

5 pattern recognition

In this section we appliad the above defined vector similarity measures to pattern recognition. The method use here is adapted from Ye[12].

Example 6 *We are given three sample pattern $\tilde{A}_1, \tilde{A}_2,$ and $\tilde{A}_3$ respectively which are represented by the following $SV\widehat{NHFS}s$ in the finite universe $\widetilde{U} = \{a_1, a_2, a_3\}$. We are also given an unkown pattern $\breve{Q}$. Our aim is to classify $\breve{Q}$ in one of the given pattern. The principle of recognizing pattern is that of the maximum degree of similarity between $SV\widehat{NHFS}s$ is discribed by*

$$N = \arg Max_{1 \le k \le 3} \left\{ \widetilde{C}_{SV\widehat{NHFS}}(\widetilde{A}_k, \tilde{Q}) \right\}$$

$$\begin{aligned}
\tilde{A}_1 &= \{\langle a_1, \{0.6, 0.7\}, \{0.2, 0.3\}, \{0.3, 0.4\}\rangle, \langle a_2, \{0.4, 0.5\}, \{0.3, 0.4\}, \{0.4, 0.5\}\rangle, \\
&\quad \langle a_3, \{0.3, 0.4\}, \{0.2, 0.4\}, \{0.5, 0.7\}\rangle\} \\
\tilde{A}_2 &= \{\langle a_1, \{0.4, 0.5\}, \{0.3, 0.5\}, \{0.5, 0.6\}\rangle, \langle a_2, \{0.6, 0.7\}, \{0.2, 0.3\}, \{0.3, 0.4\}\rangle, \\
&\quad \langle a_3, \{0.5, 0.6\}, \{0.2, 0.3\}, \{0.2, 0.4\}\rangle\} \\
\tilde{A}_3 &= \{\langle a_1, \{0.2, 0.3\}, \{0.3, 0.4\}, \{0.6, 0.7\}\rangle, \langle a_2, \{0.4, 0.5\}, \{0.3, 0.4\}, \{0.4, 0.5\}\rangle, \\
&\quad \langle a_3, \{0.6, 0.7\}, \{0.2, 0.3\}, \{0.3, 0.4\}\rangle\}
\end{aligned}$$

$$\begin{aligned}
\breve{Q} &= \{\langle a_1, \{0.1, 0.2\}, \{0.3, 0.4\}, \{0.7, 0.8\}\rangle, \langle a_2, \{0.2, 0.3\}, \{0.3, 0.4\}, \{0.6, 0.7\}\rangle, \\
&\quad \langle a_3, \{0.4, 0.5\}, \{0.3, 0.5\}, \{0.5, 0.6\}\rangle\}
\end{aligned}$$

$$(\tilde{A}_1, \breve{Q}) = 0.3501, (\tilde{A}_2, \breve{Q}) = 0.4317, (\mathbf{\tilde{A}_3}, \mathbf{\breve{Q}}) = \mathbf{0.4723}$$

One can observe that the pattern should be classified in $\widetilde{A}_3$ according to the pattern recognition principle. This result is the same obtained by Ye[12].

For I$\widehat{\textbf{NHFS}}$

Example 7 *For* $\widehat{\mathbf{INHFS}}$ *we use the same example defined above just the patterns are represented by* $\widehat{\mathbf{INHFS}}$*s.*

$$\begin{aligned}
\tilde{A}_1 &= \{\langle a_1, \{[0.4, 0.5], [0.6, 0.7]\}, \{[0.1, 0.2], [0.2, 0.3]\}, \{[0.2, 0.3], [0.3, 0.4]\}\rangle, \\
&\quad \langle a_2, \{[0.3, 0.4], [0.4, 0.5]\}, \{[0.3, 0.4], [0.3, 0.4]\}, \{[0.3, 0.4], [0.4, 0.5]\}\rangle, \\
&\quad \langle a_3, \{[0.2, 0.3], [0.3, 0.4]\}, \{[0.1, 0.3], [0.2, 0.4]\}, \{[0.4, 0.6], [0.5, 0.7]\}\rangle\} \\
\tilde{A}_2 &= \{\langle a_1, \{[0.3, 0.4], [0.4, 0.5]\}, \{[0.2, 0.3], [0.4, 0.5]\}, \{[0.3, 0.5], [0.4, 0.6]\}\rangle, \\
&\quad \langle a_2, \{[0.4, 0.6], [0.5, 0.7]\}, \{[0.2, 0.3], [0.3, 0.4]\}, \{[0.3, 0.4], [0.4, 0.5]\}\rangle, \\
&\quad \langle a_3, \{[0.4, 0.5], [0.6, 0.7]\}, \{[0.1, 0.2], [0.2, 0.3]\}, \{[0.1, 0.3], [0.2, 0.4]\}\rangle\} \\
\tilde{A}_3 &= \{\langle a_1, \{[0.1, 0.2], [0.2, 0.3]\}, \{[0.2, 0.3], [0.3, 0.4]\}, \{[0.5, 0.6], [0.6, 0.7]\}\rangle, \\
&\quad \langle a_2, \{[0.3, 0.4], [0.4, 0.5]\}, \{[0.2, 0.3], [0.3, 0.4]\}, \{[0.3, 0.4], [0.4, 0.5]\}\rangle, \\
&\quad \langle a_3, \{[0.5, 0.6], [0.6, 0.7]\}, \{[0.1, 0.2], [0.2, 0.3]\}, \{[0.2, 0.3], [0.3, 0.4]\}\rangle\}
\end{aligned}$$

$$\begin{aligned}
\breve{Q} &= \{\langle a_1, \{[0.1, 0.3], [0.2, 0.4]\}, \{[0.2, 0.3], [0.3, 0.4]\}, \{[0.6, 0.7], [0.7, 0.8]\}\rangle, \\
&\quad \langle a_2, \{[0.1, 0.2], [0.2, 0.3]\}, \{[0.2, 0.4], [0.4, 0.5]\}, \{[0.5, 0.7], [0.6, 0.8]\}\rangle, \\
&\quad \langle a_3, \{[0.2, 0.4], [0.4, 0.6]\}, \{[0.3, 0.4], [0.4, 0.5]\}, \{[0.4, 0.5], [0.5, 0.6]\}\rangle\}
\end{aligned}$$

$$(\tilde{A}_1, \breve{Q}) = 0.1954, (\tilde{A}_2, \breve{Q}) = 0.1960, (\tilde{\mathbf{A}}_3, \breve{\mathbf{Q}}) = \mathbf{0.2150}$$

One can observe that the pattern should be classified in $\widetilde{A}_3$ according to the pattern recognition principle. This result is the same obtained by Ye[12].

Discussion

As simplified neutrosophic hesitant fuzzy set is an important extension of the Fuzzy set($\widetilde{FS}$),Intuitionistic fuzzy set($\widetilde{IFS}$), Single valued neutrosophic set($\widetilde{SVNS}$), interval neutrosophic set ($\widetilde{INS}$),Hesitant fuzzy set($\widetilde{HFS}$) and Dual hesitant fuzzy set($\widetilde{DHFS}$), single valued neutrosophic hesitant fuzzyset ($\widetilde{SVNHFS}$), inteval neutrosophic hesitant fuzzy set ($\widetilde{INHFS}$). As mentioned above that hesitancy is the most common problem in decision making for which $\widetilde{HFS}$ is a suitable means by allowing sevrel possible values for an element to a set. However in $\widetilde{HFS}$ they consider only one truth-membership function and it cannot express this problem with a few different values assigned by truth-membership hesitant degree, indeterminacy-membership degree, and falsity-membership degrees due to doubts of decision makers.and also in $\widetilde{DHFS}$ they consider two functions that is membership and non-membership functions and can not consider indeterminacy-membership function. Therefore simplified neutrosophic sets is the more suitable for decision making and can handle incomplete, inconsistance and indeterminate information which occure in decision making. The other advantege of the simplified neutrosophic set is that it contain the concept of single valued neutrosophic hesitant fuzzy sets and as well as interval neutrosophic hesitant fuzzy sets. As comparing to other sets the simplified neutrosophic hesitant fuzzy set contain more information and the decision makers can get more information to take their decision.

Conclusion

As simplified neutrosophic hesitant fuzzy set ($\widehat{SNHFS}$) is a new extension which consists of the concept of Fuzzy set($\widetilde{FS}$),Intuitionistic fuzzy set($\widetilde{IFS}$), Single valued neutrosophic set($\widetilde{SVNS}$), interval neutrosophic set ($\widetilde{INS}$), Hesitant fuzzy set($\widetilde{HFS}$) and Dual hesitant fuzzy set($\widetilde{DHFS}$), single valued neutrosophic set ($\widetilde{SVNHFS}$), inteval neutrosophic hesitant fuzzy set ($\widetilde{INHFS}$). In this article we defined vector similarity measures for simplified neutrosophic hesitant fuzzy set and applied them to multi-criteria decision making and pattern recognition. We also observe that the Jaccard and Dice similarity measures are better to apply in multi-criteria decision making problem then the Cosine similarity measure. In future we shall apply the above distance measures to medical diagnosis.

References

[1] K. T. Atanassov, "Intuitionistic fuzzy sets." Fuzzy sets and Systems20.1 (1986): 87-96.

[2] K. T. Atanassov, and Georgi Gargov. "Interval valued intuitionistic fuzzy sets."Fuzzy sets and systems 31.3 (1989): 343-349.

[3] CHEN, Shu-wei, and Li-na CAI. "Interval-valued Hesitant Fuzzy Sets." Fuzzy Systems and Mathematics 6 (2013): 007.

[4] Chen, N., Xu, Z., & Xia, M. (2013). Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis. Applied Mathematical Modelling, 37(4), 2197-2211.

[5] Dice, L. R. (1945). Measures of the amount of ecologic association between species. Ecology, 26(3), 297-302.

[6] Dengfeng, Li, and Cheng Chuntian. "New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions." Pattern Recognition Letters23.1 (2002): 221-225.

[7] De, Supriya Kumar, Ranjit Biswas, and Akhil Ranjan Roy. "An application of intuitionistic fuzzy sets in medical diagnosis." Fuzzy sets and Systems 117.2 (2001): 209-213.

[8] Farhadinia, B. (2013). Information measures for hesitant fuzzy sets and interval-valued hesitant fuzzy sets. Information Sciences, 240, 129-144.

[9] Farhadinia, B. (2014). Distance and similarity measures for higher order hesitant fuzzy sets. Knowledge-Based Systems, 55, 43-48.

[10] Salton, G., & McGill, M. J. (1986). Introduction to modern information retrieval.

[11] . Peng, Juan-juan, Jian-qiang Wang, Jing Wang, Hong-yu Zhang, and Xiao-hong Chen. "Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems." International Journal of Systems Science ahead-of-print (2014): 1-17

[12] Ye, J. (2011). Cosine similarity measures for intuitionistic fuzzy sets and their applications. Mathematical and Computer Modelling, 53(1), 91-97.

[13] Ye, J. (2012). Multicriteria group decision-making method using vector similarity measures for trapezoidal intuitionistic fuzzy numbers. Group Decision and Negotiation, 21(4), 519-530.

[14] Ye, J. (2013). Interval-valued intuitionistic fuzzy cosine similarity measures for multiple attribute decision-making. International Journal of General Systems, 42(8), 883-891.

[15] Ye, J. (2013). Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42(4), 386-394.

[16] Ye, J. (2013). Another form of correlation coefficient between single valued neutrosophic sets and its multiple attribute decision-making method.Neutrosophic Sets and Systems, 1(1), 8-12.

[17] . Ye, J. (2014). Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. Journal of Intelligent and fuzzy Systems, 26(1), 165-172.

[18] Ye, J., & Zhang, Q. S. (2014). Single valued neutrosophic similarity measures for multiple attribute decision making. Neutrosophic Sets and Systems, 2, 48-54.

[19] Ye, J. (2014). A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology, 26(5), 2459-2466.

[20] Ye, J. (2014). Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. International Journal of Fuzzy Systems, 16(2), 204-215.

[21] Ye, J. (2015). Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artificial intelligence in medicine,63(3), 171-179.

[22] Jun, Ye. (2014). VECTOR SIMILARITY MEASURES OF HESITANT FUZZY SETS AND THEIR MULTIPLE ATTRIBUTE DECISION MAKING. Economic Computation & Economic Cybernetics Studies & Research, 48(4).

[23] Ye, Jun. "Correlation coefficient of dual hesitant fuzzy sets and its application to multiple attribute decision making." Applied Mathematical Modelling 38.2 (2014): 659-666.

[24] Ye, Jun. "Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment." Journal of Intelligent Systems 24.1 (2015): 23-36.

[25] Liu, Peide, and Lanlan Shi. "The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making." Neural Computing and Applications 26.2 (2015): 457-471.

[26] Majumdar, P., & Samanta, S. K. (2014). On similarity and entropy of neutrosophic sets. Journal of Intelligent and fuzzy Systems, 26(3), 1245-1252.

[27] Jaccard, P. (1901), Distribution de la flore alpine dans le Bassin des Drouces et dans quelques regions voisines; Bulletin de la Société Vaudoisedes Sciences Naturelles, volume 37, no. 140, 241–272;

[28] . Sahin, R., & Kucuk, A. (2014). Subsethood measure for single valued neutrosophic sets. Journal of Intelligent and Fuzzy Systems.

[29] Florentin Smarandache, Neutrosophy. Neutrosophic Probability, Set, and Logic, Amer. Res. Press, Rehoboth, USA, 105 p., 1998.

[30] Smarandache, Florentin. "A Unifying Field in Logics: Neutrosophic Logic."Philosophy (1999): 1-141.

[31] Broumi, Said, and Florentin Smarandache. "Several similarity measures of neutrosophic sets." Neutrosophic Sets and Systems 1.1 (2013): 54-62.

[32] Broumi, S., & Smarandache, F. (2013, December). Correlation coefficient of interval neutrosophic set. In Applied Mechanics and Materials (Vol. 436, pp. 511-517).

[33] Broumi, S., & Smarandache, F. (2014). Cosine similarity measure of interval valued neutrosophic sets. Neutrosophic Theory and Its Applications., 74

[34] . Torra, V., & Narukawa, Y. (2009, August). On hesitant fuzzy sets and decision. In Fuzzy Systems, 2009. FUZZ-IEEE 2009. IEEE International Conference on (pp. 1378-1382). IEEE.

[35] Torra, V. (2010). Hesitant fuzzy sets. International Journal of Intelligent Systems, 25(6), 529-539.

[36] Wang, Haibin, et al. "Single valued neutrosophic sets." Review (2010): 10

[37] Wang, Haibin, et al. "Interval neutrosophic sets." arXiv preprint math/0409113(2004)..

[38] Xia, Meimei, and Zeshui Xu. "Hesitant fuzzy information aggregation in decision making." International Journal of Approximate Reasoning 52.3 (2011): 395-407

[39] Xia, Meimei, Zeshui Xu, and Na Chen. "Some hesitant fuzzy aggregation operators with their application in group decision making." Group Decision and Negotiation 22.2 (2013): 259-279.

[40] Xu, Z., & Xia, M. (2011). Distance and similarity measures for hesitant fuzzy sets. Information Sciences, 181(11), 2128-2138.

[41] Xu, Z., & Xia, M. (2011). On distance and correlation measures of hesitant fuzzy information. International Journal of Intelligent Systems, 26(5), 410-425.

[42] Zhang, H. Y., Wang, J. Q., & Chen, X. H. (2014). Interval neutrosophic sets and their application in multicriteria decision making problems. The Scientific World Journal, 2014.

[43] Bin Zhu, Zeshui Xu, and Meimei Xia Dual Hesitant Fuzzy Sets Volume 2012 (2012), Journal of Applied Mathematics Article ID 879629, 13 pages

[44] Zhua, B., & Xub, Z. (2014). Some results for dual hesitant fuzzy sets. Journal of Intelligent & Fuzzy Systems, 26, 1657-1668.

[45] Su, Z., Xu, Z., Liu, H., & Liu, S. Distance and similarity measures for dual hesitant fuzzy sets and their applications in pattern recognitiontle. Journal of Intelligent and Fuzzy Systems.

[46] L. A. Zadeh, "Fuzzy sets." Information and control 8.3 (1965): 338-353