A model for Smarandache's Anti-Geometry

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David Hilbert's *Foundations of Geometry* (1899) contain nineteen statements, labelled *axioms*, from which every theorem in Euclid's *Elements* can be derived by deductive inference, according to the classical rules of logic. The axioms use three property words —'point', 'straight' and 'plane'— and three relation words —'incident', 'between' and 'congruent'— for which no definition is given. These words have, of course, a so-called intuitive meaning in English (as the German equivalents actually used by Hilbert have in his language). But Hilbert believed they ought to be understood in whatever sense was compatible with the constraints prescribed by the axioms themselves.[1] To show that some of his axioms were not logical consequences of the others he unhesitatingly bestowed unorthodox meanings on the undefined terms. This enabled him to produce models that satisfied all the axioms but one, plus the negation of the excluded axiom.

The mathematician-philosopher Gottlob Frege showed little understanding for Hilbert's procedure. Frege thought that the undefined terms stood for properties and relations that Hilbert assumed to be well-known and that the axioms were intended as true statements about them. Hilbert disabused him: "I do not wish to presuppose anything as known; I see in my declaration in §1 the definition of the

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1 Although these constraints are very restrictive, the nineteen axioms admit two non-isomorphic models, viz., (i) the uncountable three-dimensional continuum that underlies Cartesian geometry and Newtonian analysis, and (ii) the countable set of points constructible with ruler and compasses from which Euclid built his figures. To suppress this ambiguity, Hilbert added the Axiom of Completeness (V.2) in Laugel's French translation (1900b); it subsequently was included in all German editions, beginning with the second (1903). With this addition, Hilbert's axiom system only admits isomorphic models.

concepts 'points', 'straights', 'planes', provided that one adds all the axioms in axiom groups I–V as expressing the defining characters" (Hilbert to Frege, 29.12.1899, in Frege 1967, p. 411). Frege had complained that Hilbert's concepts were equivocal, the predicate 'between' being applied to genuine geometrical points in §1 and to real number pairs in §9. Hilbert replied:

> Of course every theory is only a scaffolding or schema of concepts together with their necessary mutual relations, and the basic elements can be conceived in any way you wish. If I take for my points any system of things, for example, the system love, law, chimney-sweep, . . . and I just assume all my axioms as relations between these things, my theorems —for example, Pythagoras's— also hold of these things. In other words: every theory can always be applied to infinitely many systems of basic elements. One needs only to apply an invertible one–one transformation and to stipulate that the axioms for the transformed things are respectively the same. [. . .] This feature of theories can never be a shortcoming and is in any case inevitable.
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> (Hilbert to Frege, 29.12.1899; in Frege 1967, pp. 412–13)

Hilbert's reply has continued to sound artificial to those unwilling or unable to follow him in his leap to abstraction, because it is not possible to find a set of familiar relations among chimney-sweeps, laws and states of being in love which, when equated with Hilbert's relations of incidence, betweenness and congruence, would make his axioms to be true. But Hilbert's point can now be made crystal-clear thanks to Florian Smarandache's Anti-Geometry.[2]

Anti–Geometry rests on a system of nineteen axioms, each one of which is the negation of one of Hilbert's nineteen axioms.[3] Such wholesale negation brings

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2 My knowledge of this system is based on Sandy P. Chimienti and Mihaly Bencze's paper "Smarandache Anti-Geometry", as published in the Worldwide Web (http://www.gallup.unm.edu/~smarandache/prd-geo4.txt). I reproduce Smarandache's axioms from this paper, with mild stylistic corrections.

3 In the paper mentioned in ref. 2, Chimienti and Bencze say that "all Hilbert's 20 axioms of the Euclidean Geometry are denied in this vanguardist geometry". However, only the 19 axioms of 1899 are denied explicitly. Indeed, negation of Axiom V.2 is implicit, insofar as Smarandache's axioms of anti-geometry admit non-isomorphic models. For instance, if you change the date of the model proposed below from 31 December 1999 to 31 December 1899 you obtain at once a second model which is not isomorphic with mine (the total number of bank accounts in the United States was surely much less in 1899 than in 1999).

about a complete collapse of the constraints imposed by Hilbert's axioms on its conceivable models. The immediate consequence of this is that models of Anti-Geometry can be readily found in all walks of life.[4] On the other hand, and for the same reason, the truths concerning these models that can be obtained from Smarandache's axioms by deductive inference are somewhat uninteresting, to say the least.

I shall now state my interpretation of the undefined terms in Smarandache's (and Hilbert's) axioms and show, thereupon, that Smarandache's nineteen axioms come out true under this interpretation. Following Chimienti and Bencze (ref. 2), I say 'line' where Hilbert says 'straight' (*gerade*).[5] Points lying on one and the same line are said to be *collinear*; points or lines lying on one and the same plane are said to be *coplanar*. Two lines are said to *meet* or *intersect* each other if they have a point in common.

In my interpretation the geometrical terms employed in the axioms are made to stand for ordinary, non-geometric objects and relations, with which I assume the reader is familiar. As a matter of fact, Smarandache's system, despite its vaunted vanguardistic libertarianism, still imposes a few existential constraints on admissible models; for example, his Axiom III presupposes the existence of infinitely many of the objects called 'lines'. This has forced me to introduce three existence postulates which my model is required to comply with, at least one of which is plainly unnatural (EP3).

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I list below the meaning I bestow on Hilbert's property words:

(i) A *point* is the balance in a particular checking account, expressed in U.S. currency. (Points will be denoted by capital letters).

You can also extend the domain of my model, in direct contradiction with Hilbert's Axiom V.2, by adding all Swiss banks to the U.S. banks comprised in the extension of 'plane'.

4 I lighted on the model I shall present below while recovering from a long, delicious and calory-rich lunch with a poet, a psychiatrist and a philosopher, during which not a single word was said about geometry and I drank half a bottle of excellent Chilean merlot.

5 Was this deviant usage adopted because "some of our lines are curves", as Chimienti and Bencze note in their definition of 'angle' (following their Axiom IV.3)? That would bespeak a deep misunderstanding of Hilbertian axiomatics. I hope that my interpretation will make this clear. In it, *lines* are persons, and we might just as well have called them *straights*.

NOTE. Two points A and B may be distinct, because they are balances from different accounts, which may or may not belong to different persons, and yet be equal in amount, in which case we shall say that A *equals* B (symbolized A = B). If A and B are the same point, we say that A and B *are identical.* Of course, in current mathematical parlance "equal", signified by "=", means "identical", but, like Humpty Dumpty and David Hilbert, I feel free to use words any way I wish, provided that I explain their meaning clearly. I use the standard symbol < to express that a given balance is smaller than another.

(ii) A *line* is a person, who can be a human being or an angel. (Lines are denoted by lower case italics).

(iii) A *plane* is a U.S. bank, affiliated to the FDIC. (Planes are denoted by lower case Greek letters).

Here are the meanings I bestow on the relation words. All relations are supposed to hold at midnight E.S.T. of December 31, 1999.

1. Point A *lies on* line *a* if and only if person *a* owns the particular account that shows balance A. (For brevity's sake, I shall often say that *a* owns balance A when he or she owns the said account.)

2. Line *a lies on* plane α if and only if the person *a* has a checking account with bank α.

3. Point A *lies on* plane α if and only if the particular checking account that shows balance A is held with bank α.

4. Point B is *between* points A and C, if and only if balances A, B and C are the balances in three different accounts belonging to the same person *x*, and A = B < C.

Items 1–4 take care of *betweenness* and the three kinds of *incidence* we find in Hilbert and Smarandache. Hilbert's relation of *congruence* does not apply, however, to points, lines or planes, but to two sorts of figures constructed from points and lines, viz. *segments* and *angles.* I must therefore define these figures in terms of *my* points and lines.

DEF. I. If two balances A and B belong to the same person *x*, the collection formed by A, B and all balances Y belonging to *x* and such that A is less than Y and Y is less than B is called *the segment* AB.

NOTE. By our definition of "betweenness", the points belonging to segment AB but not identical with A or B do not lie between A and B. However, the Smarandache axioms are stated in such a way that none of them contradicts this surprising theorem.

DEF. II. If a balance O is owned in common by persons *h* and *k*, the set formed by *h*, *k* and O is called *the angle* (*h*,O,*k*) (symbolized ∠*h*O*k*).

NOTE 1. *h* and *k* could be the same person, in which case the qualification "in common" is trivial.

NOTE 2. If *h* and *k* are distinct persons, such that *h* besides O owns a balance P, not shared with *k*, and *k*, besides O, owns a balance Q, not shared with h, ∠*h*O*k* may be called "the angle POQ" and be symbolized by ∠POQ. In other words, the expression "∠POQ" has a referent if and only if there exist persons *h* and *k* who respectively own balance P and balance Q separately from one another, and share the balance O; otherwise, this expression has no referent.

DEF. III. Person *a acquired* balance A *partly from* person *b* if and only if a part of balance A was electronically transferred from funds owned by *b* to the account owned by *a* which shows balance A. Instead of "*a* acquired A partly from *b*" we write ☆(*a*,A,*b*)

I am now in a position to define Hilbert's two sorts of *congruence*.

5. Segment AB is congruent with segment CD if and only if there is a person *x* such that ☆(*h*,A,*x*) and ☆(*h*,B,*x*) and ☆(*k*,C,*x*) and ☆(*k*,D,*x*), where *h* denotes the owner of balances A and B, and *k* denotes the owner of balances C and D.

6. Angle (*h*,P,*k*) is congruent with angle (*f*,Q,*g*) if there is a person *x* such that ☆(*h*,P,*x*) and ☆(*k*,P,*x*) and ☆(*f*,Q,*x*) and ☆(*g*,Q,*x*).

We shall also need the following definitions:

DEF. IV. Two distinct lines *a* and *b* are said to be *parallel* if and only if persons *a* and *b* have accounts with the same bank α but do not own any balance in common.

DEF. V. Let A be a balance belonging to a person *h*. Any other balances owned by *h* can be divided into three classes: (i) those that are less than A, (ii) those that are greater than A, and (iii) those that are equal to A. Balances of class (i) and (ii) which are held by *h* in other accounts with the same bank where he has A will be said to lie, respectively, *on one* and on *the other side of* A (on *h*).

As I said, the fairly weak but nevertheless inescapable constraints implicit in some of Smarandache's axioms force me to adopt three existence postulates. The first of these is highly plausible; the second is, as far as I know, false in fact, but not implausible; while the third is quite unnatural, though not more so than the supposition, involved in Smarandache's Axiom III, that there are infinitely many distinct objects in any model of his system.

Existence postulates.

EP1. Mr. John Dee has four checking accounts, with balances of 5000, 5000, 5000 and 8000 dollars, respectively.

EP1 ensures the truth of Smarandache's Axiom II.3.

EP2. There are some checking accounts for whose balance two different banks are held responsible. I shall refer to such accounts as *two-bank* accounts.

EP2 is needed to ensure the truth of Smarandache's Axiom I.4; it is also presupposed by his Axiom I.6. We could be more specific and stipulate that checks drawn against such accounts will be cashed at the branches of either bank, that the banks share the maintenance costs and monthly service charges, etc. But all such details are irrelevant for the stated purpose..

EP3. There exist infinitely many supernatural persons who may secretly own bank accounts, usually in common.

EP3 is needed to take care of the last of the four situations contemplated in Smarandache's Axiom III (the Axiom of Parallels), which involves a point that is intersected by infinitely many lines. In our model, this amounts to a balance in current account that is owned in common by infinitely many persons. EP3 is certainly weird, but not more so than say, the postulation of points, lines and a plane at infinity in projective geometry. As in the latter case, we may regard talk of supernatural persons as a *façon de parler*. EP3 will perhaps sound less unlikely if the banks of our model are Swiss instead of American.

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I shall now show that —with one partial exception (I.7)—all of the axioms of Smarandache's Anti-Geometry hold in our model. As we shall see, the said exception is due to an inconsistency in Smarandache's axiom system.

Axiom I.1 Two distinct points A and B do not always completely determine a line.

Balance A and balance B need not belong to the same person.

Axiom I.2 There is at least one line *h* and at least two distinct points A and B of *h*, such that A and B do not completely determine the line *h*.

A and B are owned by *h* in common with a second person *k*.

Axiom I.3 Three points A, B, C, not on the same line, do not always completely determine a plane α.

Three balances belonging to different persons may pertain to accounts they have with different banks.

Axiom I.4 There is at least one plane α and at least three points A, B, C, which lie on α but not on the same line, such that A, B, C do not completely determine the plane α.

Three points A, B and C on plane α completely determine α if and only if any fourth point D, coplanar with A, B and C, also lies on α. However, according to EP2, the balances A, B and C may pertain to three two-bank accounts held, say, with bank α and bank β. In that case, D could belong to β and not to α.

Axiom I.5 Let two points A, B of a line *h* lie on a plane α. This does not entail that every point of *h* lies on α.

Obviously, a person *h* may hold accounts with other banks, besides α.

Axiom I.6 Let two planes α and β have a point A in common. This does not entail that α and β have another point B in common.

Balance A could be the balance in the one and only two-bank account for which banks α and β are jointly responsible (see EP2).

Axiom I.7 There exist lines on each one of which there lies only one point, or planes on each one of which there lie only two points, or a space which contains only three points.

Nothing in our model precludes the joint fulfilment of the first two disjuncts in this axiom, viz., “There exist lines on each one of which there lies only one point” (i.e. persons who own a single bank account) and “There exist planes on each one of which there lie only two points” (i.e. banks in which, at closing time on the last day of the twentieth century, only two checking accounts remained open). The third condition, however, cannot be fulfilled, for EP1 demands the existence of at least four points. However, EP1 was solely introduced to secure the truth of Axiom II.3, which actually requires the existence of four distinct points. Therefore Axiom II.3 cannot be satisfied in a model that satisfies the last disjunct of Axiom I.7. Thus, the Smarandache axioms of anti-geometry are inconsistent as stated. I propose to delete the last disjunct of I.7. By the way, ‘space’ is not a term used in Hilbert’s axioms. Indeed, since ‘space’ stands for the entire domain of application of Smarandache’s system it ought not to occur in it either.

Axiom II.1 Let A, B, and C be three collinear points, such that B lies between A and C. This does not entail that B lies also between C and A.

Obviously, if $A = B < C$, $C \neq B$. Thus, in fact, our model satisfies also the stronger axiom: “If B lies between A and C then B does not lie between C and A”.

Axiom II.2 Let A and C be two collinear points. Then, there does not always exist a point B lying between A and C, nor a point D such that C lies between A and D.

Obviously, if a given person owns A and C there is no reason why she or he should own a third checking account, let alone one with a balance that is either equal to A and less than C, or greater than both C and A.

Axiom II.3 There exist at least three collinear points such that one point lies between the other two, and another point lies also between the other two.

This is so, of course, if the line is Mr. John Dee (by EP1).

Axiom II.4 Four collinear points A, B, C, D cannot always be ordered so that (i) B lies between A and C and also between A and D, and (ii) C lies between A and D and also between B and D.

In fact, under our definition of betweenness four collinear points can *never* be ordered in this way. Condition (i) means than B equals A and is less than C and D; condition (ii) means that C equals A and B and is less than D. These two conditions are plainly incompatible.

Axiom II.5 Let A, B, and C be three non-collinear points, and *h* a line which lies on the same plane as points A,B, and C but does not pass through any of these points. Then, the line *h* may well pass through a point of egment AB, and yet not pass through a point of segment AC, nor through a point of segment BC

Suppose that *h* does not pass through A, B or C but passes nevertheless through a point of segment AB. This entails that person *h* owns in common with the owner of both A and B a checking account whose balance X is greater than A and less than B. Obviously, *h* need not own any balances in common with the owner of both B and C, nor with the owner of both A and C, let alone one that meets the requirements imposed by our definition of segment, viz., that the balance in question be greater than B and less than C, or greater than A and less than C.

III. ANTI-AXIOM OF PARALLELS

Let *h* be a line on a plane α and A a point on α but not on *h*. On plane α there can be drawn through point A either (i) no line, or (ii) only one line, or (iii) a finite number of lines, or (iv) an infinite number of lines which do(es) not intersect the line *h*. The line(s) is (are) called the parallel(s) to *h* through the given point A.[6]

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6 Two remarks are on order here: (i) Chimienti and Bencze label this axiom with the Roman number III, although the Hilbert axiom contradicted by it bears number IV. In Hilbert's book Axioms III (1–5) are the axioms of congruence. (ii) Chimienti and Bencze do not

Let A be the balance of a checking account with bank α and *h* a client of bank α who does not own that account. The account in question may belong to a person who shares another balance with *h* (case i), or to a person *b*, or to finitely many persons $c_1, \ldots, c_n$, none of whom shares a checking account with *h* (cases ii and iii). According to EP3, A may also be owned secretly by infinitely many supernatural persons who do not share an account with *h* (case iv). By DEF. IV, the lines comprised in cases (ii), (iii) and (iv) all meet the requirements for being parallel with *h*.

NOTE. InChimienti and Bencze's article (ref. 2), Axiom III includes the following supplementary condition, enclosed in parentheses: "(At least two of these situations should occur)", where 'these situations' are cases (i) through (iv). Since I do not understand what this condition means, I did not consider it in the preceding discussion. Anyway, the following is clear: No matter how you interpret the terms "point" and "line" and the predicates "coplanar" and "intersect", case (i) excludes cases (ii) and (iv). However, (i) implies (iii) and therefore can occur together with it, if by "finite number" you mean "any natural number" in Peano's sense, i.e. any integer equal to or greater than zero. In contemporary mathematical jargon, this would the usual meaning of the term in this context. By the same token, (ii) implies (iii), for "one" is a finite number. Finally, (iv) certainly implies (iii), for any infinite set includes a finite subset. In the light of this, the condition in parenthesis is obvious and trivial and few would think of mentioning it. Therefore, the fact that it is mentioned suggests to me that it is being given some other meaning, which eludes me.

Axiom IV.1 If A, B are two points on a line *h*, and A′ is a point on the same line or on another line *h*′, then, on a given side of A′ on line *h*′, we cannot always find a unique B so that the segment AB is congruent to the segment A'B'.

If balances A and B belong to person *h*, and A′ belongs to *h*′ (who may or may not be the same person as *h*), there is no reason at all why there should exist a unique balance B′ such that segments AB and A′B′ meet the condition of congruence, viz., that there exists a person *x* such that ☆(*h*,A,*x*) and ☆(*h*,B,*x*) and ☆(*h*′,A′,*x*) and ☆(*h*′,B′,*x*).

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explicitly require line *h* to lie on plane α; this is, however, a standard requirement of parallelism which I take to be understood.

NOTE. For the expression 'on a given side of A′', see DEF. V.

Axiom IV.2 If segment AB is congruent with segment A′B′ and also with segment A″B″, then segment A′B′ is not always congruent with segment A″B″.

Assume that (i) the owner of A and B got the monies in the respective accounts partly from a person *x* and partly from a person *y*; (ii) the owner of A′ and B′ got these monies partly from *x* but not from *y*; (iii) the owner of A″ and B″ got these monies partly from *y* but not from *x*. If these three conditions are met, Axiom IV.2 is satisfied.

Axiom IV.3 If AB and BC are two segments of the same line *h* which have no points in common besides the point B, and A′B′ and B′C′ are two segments of *h* or of another line *h′* which have no points in common besides B′, and segment AB is congruent with segment A′B′ and segment BC is congruent with B′C′, then it is not always the case that segment AC is congruent with segment A′C′.

Again, let B and B′ be acquired by *h* and *h′*, respectively, partly from *x* and partly from *y*; A and A′ from *x* but not from *y*; C and C′ from *y* but not from *x*. Then segment AB is congruent with segment A′B′; segment BC is congruent with segment B′C′, but segment AC is not congruent with segment A′C′.

Axiom IV.4. Let ∠*h*O*k* be an angle on plan α, and let *h′* be a line on plane β. Suppose that a definite side of *h′* on plane β is assigned and that a particular point O′ is distinguished on *h′*. Then there are on β either one, or more than one, or even no half-line *k′* issuing from the point O′ such that (i) ∠*h*O*k* is congruent with ∠*h′*O′*k′*, and (ii) the interior points of ∠*h′*O′*k′* lie upon one or both sides of *h′*.

This axiom is not easy to apply, for it contains the terms 'half-line', 'interior points (of an angle)' and 'side (of a line on a plane)' which have not been defined and are not used anywhere else in the axioms. I shall take the *half-line k issuing from* a point O to mean a person *k* who owns O and owns another bank balance less than

O in a different account with the same bank, but does not own a bank balance greater than O in a different account with the same bank. As for the other two expressions, since they are otherwise idle, we could simply ignore them. But if the readers do not like this expedient, they may equally well use the following one: Let ∠*a*P*b* be an angle, such that P is the balance held in common by *a* and *b* in their checking account with a particular branch of bank α; the *interior points* of ∠*a*P*b* are the cashiers of that particular branch. We say that the cashiers who are younger than *a*, lie *on one side* of *a* (on α), and that the cashiers who are older than *a*, lie *on the other side* of *a* (on α). The condition on interior points in axiom IV.4 will obviously be met if the line (i.e. bank client) *h*′ is so chosen that the branch of bank β where *h*′ holds the balance O′ in common with *k*′ has cashiers who are both younger and older than *h*′. Surely this requirement is not hard to meet, if β ranges freely over all banks in the U.S.

If the axiom is understood in this way, its meaning is clear enough. It is so weak that there is no difficulty in satisfying it. Take the arbitrarily assigned side of *h*′ to be *younger than*. It should be easy to find a bank β and a client *h*′ who owns a balance O′ in a branch of β, and is older than some cashiers of the branch and younger than others. Under this condition, there may or may not be a person *k*′ such that (i) *k*′ holds O′ in common with *h*′, (ii) *k*′ holds separately a balance less than O′ in another account with bank β (with my definitions this need not even be in the same branch of β), and (iii) there is a person *x* such that ☆(*h*,O,*x*) and ☆(*k*,O,*x*) and ☆(*h*′,O′,*x*) and ☆(*k*′,O′,*x*). Indeed, there may be several persons k_1, k_2, . . . , k_n, who simultaneously meet the conditions prescribed for *k*′.

Axiom IV.5 If ∠*h*O*k* is congruent with ∠*h*′O′*k*′ and also with ∠*h*″O″*k*″, then ∠*h*′O′*k*′ may not be congruent with ∠*h*″O″*k*″.

Let ∠*h*O*k* be congruent with ∠*h*′O′*k*′ because the vertices O and O′ —i.e. the shared balances— both stem partly from a donor *x* who contributes nothing to O″, while ∠*h*O*k* is congruent with ∠*h*″O″*k*″ because the vertices O and O″ stem partly from a debtor *y* who contributes nothing to O′.

Axiom IV.6 Let ABC and A′B′C′ be two triangles such that segment AB is congruent with segment A′B′, segment AC is congruent with segment A′C′, and ∠BAC is congruent with ∠B′A′C′. Then it is

> not always the case that ∠ABC is congruent with ∠A′B′C′ and that ∠ACB is congruent with ∠A′C′B′.

The triangle ABC is determined by three distinct balances A, B and C, such that A and B jointly belong to a person *c*, B and C jointly belong to a person *a* who is different from *c*, and C and A jointly belong to a person *b* who is different from both *a* and *c*. It follows that *a* and *b* are joint owners of C, *b* and *c* are joint owners of A, and *c* and *a* are joint owners of B. The axiom assumes:

(i) That segment AB is congruent with segment A′B′, i.e. that there is a person *x* such that ☆(*c*,A,*x*) and ☆(*c*,B,*x*) and ☆(*c*′,A′,*x*) and ☆(*c*′,B′,*x*);
(ii) That segment AC is congruent with segment A′C′, i.e. that there is a person *y* such that ☆(*b*,A,*y*) and ☆(*b*,C,*y*) and ☆(*b*′,A′,*y*) and ☆(*b*′,C′,*y*);
(iii) That ∠BAC is congruent with ∠B′A′C′, i.e. that there is a person *z* such that ☆(*c*,A,*z*) and ☆(*b*,A,*z*) and ☆(*c*′,A′,*z*) and ☆(*b*′,A′,*z*).

Obviously, conditions (i), (ii) and (iii) do not in any way imply that ∠ABC is congruent with ∠A′B′C′, i.e., that there is a person *v* such that ☆(*c*,B,*v*) and ☆(*a*,B,*v*) and ☆(*c*′,B′,*v*) and ☆(*a*′,B′,*v*), nor that ∠ACB is congruent with ∠A′C′B′, i.e. that there is a person *w* such that ☆(*b*,C,*w*) and ☆(*a*,C,*w*) and ☆(*b*′,C′,*w*) and ☆(*a*′,C′,*w*).

ANTI-AXIOM OF CONTINUITY (ANTI-ARCHIMEDEAN AXIOM)

> Let A, B be two points. Take the points A_1, A_2, A_3, A_4, so that A_1 lies between A and A_2, A_2 lies between A_1 and A_3, A_3 lies between A_2 and A_4, . . . , and the segments AA_1, A_1A_2, A_2A_3, A_3A_4, . . . are congruent to one another. Then, among this series of points, there does not always exist a certain point A_n such that B lies between A and A_n.

Let A and B be two checking account balances. Consider a series of *n* checking account balances A_1, A_2, . . . , A_n, such that all of them belong to the owner of A, and all except A_n amount to the same sum as A. Suppose that A_n is greater than A. Now, the condition denied in the apodosis, viz., that B lies between A and A_n, can hold if and only if B belongs to the owner of both A and A_n, and B is equal to A. Obviously this is not implied by the initial condition on B, viz., that B is a point, i.e. a checking account balance.

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There is a simple moral to be drawn from this exercise. Because Smarandache Anti-Geometry has removed the stringent constraints on *points*, *lines* and *planes* prescribed by the Hilbert axioms, it is child's play to find uninteresting applications for it, like the one proposed above. When first confronted with this model, Dr. Minh L. Perez wrote me that he had the impression that Smarandache's message was directed against axiomatization. Such an attack would be justified only if we take an equalitarian view of axiom systems. To my mind, equalitarianism in the matter of mathematical axiom systems—though favored by some early twentieth century philosophers—is like placing all games of wit and skill on an equal footing. The clever Indian who invented chess is said to have demanded 2^{64} corn grains minus 1 for his creation. Who would have the chutzpah to charge even a trillionth of that for tic-tac-toe? But Smarandache's Anti-Euclidean geometry does not derogate Hilbert's axiom system for Euclidean geometry. Indeed this system, as well as Hilbert's axiom system for the real number field (1900a), deserve *much more* —not *less*— attention and praise in view of the fact that one can also propose consistent yet vapid axiom systems.

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