NSS
Neutrosophic Sets and Systems {Special Issue: Neutrosophic Advancements And Their Impact on Research in Latin America}, Vol. 62, 2023
University of New Mexico

Neutrosophic Analysis of Supply Chain Resilience

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Abstract. The objective of this research is to extend the TOPSIS method by including single-valued neutrosophic sets to evaluate the vulnerabilities present in the supply chain of a footwear manufacturing company in the city of Ambato. To achieve this goal, principles of neutrosophic logic and the theory of single-valued neutrosophic sets were used. As a result of the application of this methodology, it was found that the most serious weaknesses identified were insufficient investment in technology and innovation, followed closely by the presence of obsolete and deficient communication systems. This study has provided solid evidence of the effectiveness and versatility of the method in a variety of scientific contexts and fields. The use of single-valued neutrosophic numbers as a resource for carrying out the analysis has confirmed the concrete applicability of neutrosophic set logic in practical situations.

Keywords: supply chain, decision making, neutrosophic set, vulnerability.

1 Introduction

The importance of decision making in everyday life and in the business environment is undeniable. In both contexts, decisions have a significant impact on outcomes and quality of life. However, it is often common to encounter situations where the desired objectives are in conflict or the available information is vague and uncertain [1]. This poses a crucial challenge in the decision-making process, as decisions based solely on accurate data may not be appropriate when it comes to dealing with the complexity and uncertainty of the real world.[2]

The inclusion of vague and indeterminate elements in decision-making procedures has become vital for finding effective solutions to complicated problems. Vagueness implies imprecision and ambiguity in accessible data, whereas indeterminacy relates to the uncertainty of information [3]. These elements are inherent in many situations, such as those related to the business environment, where conditions are constantly changing and information may be limited or imprecise.[4]

To address these challenges, researchers have developed approaches based on multi-criteria decision-making methods (MCDM) that enable decision-makers to account for vagueness and indeterminacy in their analysis. Such methods incorporate subjective and flexible evaluations that better capture the imprecise nature of the provided information. The incorporation of vague and uncertain elements into the decision-making process not only enhances adaptability to changing circumstances, but also mitigates suboptimal decisions influenced by oversimplified assumptions.[5]

To overcome the problem presented by uncertain and imprecise data, Zadeh introduced the fuzzy set (FS) theory in 1996. This theory enabled the representation of uncertainty by assigning degrees of membership to elements in a set. In decision making, this approach allowed decision makers to express their preferences and evaluate alternatives in terms of degrees of membership to decision criteria [6]. However, while fuzzy sets have been presented and applied to solve MCDM problems, there are still limitations in their ability to handle certain types of uncertainty in real-world scenarios.[7]

To address these limitations, neutrosophic set theory was formulated as an expansion of fuzzy set theory. Florentin Smarandache proposed this theory in 1995, introducing an innovative approach that enhanced the flexibility of representing uncertainty. Neutrosophic sets function independently with the ability for truth membership, indeterminacy membership, and falsity membership to take values within a nonstandard unit interval of]0-, 1+[.[8]

The integration of neutrosophic set theory into decision making methods has unlocked novel opportunities to address problems characterized by a high degree of vagueness, ambiguity, and uncertainty [9]. This enables decision-makers to more precisely model and manage uncertain information in complex situations. Moreover, neutrosophic set theory has demonstrated its immense value in multi-criteria decision-making, where the consideration of various factors and a more adaptable presentation of preferences are necessary.[10]–[12]

In the business context, decision making is of critical importance, as it can have a significant impact on the success and viability of a company. Every business decision, from investment in new projects to supply chain management, must be based on rigorous analysis and consider multiple criteria and alternatives. Moreover, in an ever-changing business environment, the ability to adapt and make informed decisions is essential.[13-18-20]

Supply chain resilience refers to its ability to withstand and recover from unexpected disruptions, such as natural disasters or interruptions in the flow of supplies. In this context, decision making plays a critical role in ensuring the continuity of operations and minimizing the impact of these unforeseen disruptions. Incorporating uncertainty and indeterminacy more fully enables decision-makers to evaluate supply chain management alternatives and risks more accurately. This identification process leads to development and implementation of robust and adaptive strategies to ensure supply chain continuity even under adverse conditions.[14-19-21-22]

The aim of this paper is to extend the TOPSIS method by applying single-valued neutrosophic sets to assess the vulnerabilities of the supply chain of a footwear company in the city of Ambato. The investigation begins with a detailed explanation of the basic concepts of single-valued neutrosophic sets and the underlying logic in Section 2, followed by a description of the TOPSIS method. In Section 3, this study presents a practical example of applying the aforementioned concepts to evaluate the resiliency of a supply chain within the chosen entity. The results are subsequently presented, followed by relevant conclusions derived from the study.

2 Method

First, some basic concepts of the neutrosophic theory and its relationship with the multicriteria method used are defined.

Definition 1. Let X be a space of points (objects) with generic elements in X denoted by x. A single-valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A (x), indeterminacy-membership function I_A (x), and falsity membership function F_A (x). Then, an SVNS A can be denoted by A = {x, $T_A(x)$, $I_A(x)$, $F_A(x)$ x ∈ X}, where T_A (x), I_A (x), F_A (x) ∈ [0,1] for each point x in X. Therefore, the sum of T_A (x), I_A (x) and F_A (x) satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.[15]

For convenience, a SVN number is denoted by A = ($a\ b\ c$), where a, b, c ∈ [0,1] and a + b + c ≤ 3

Definition 2. Let A_1= A_1= (a_1, b_1, c_1) and A_2 = (a_2, b_2, c_2) be two SVN numbers, then summation between A_1 y A_2 is defined as follows:

$$A1 + A2 = (a1 + a2 - a1a2, b1b2, c1c2) \quad (1)$$

Definition 3. Let A_1= (a_1, b_1, c_1) and A_2 = (a_2, b_2, c_2) be two SVN numbers, then multiplication between A_1 y A_2 is defined as follows:

$$A1 * A2 = (a1a2, b1 + b2 - b1b2, c1 + c2 - c1c2) \quad (2)$$

Definition 4. Let A = (a, b, c) be a SVN number and λ∈ℝ anarbitrary positive real number, then:

$$\lambda A = \left(1-(1-a)^{\lambda}, b^{\lambda}, c^{\lambda}\right), \lambda > 0 \quad (3)$$

Definition 5. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of n SVN numbers, where $A_j = (a_j, b_j, c_j)$ (j= 1, 2, …, n). The single value neutrosophic weighted average operator on them is defined by

$$\sum_{j=1}^{n} \lambda_j A_j = \left(1-\prod_{j=1}^{n}(1-a_j)^{\lambda_j}, \prod_{j=1}^{n} b_j^{\lambda_j}, \prod_{j=1}^{n} c_j^{\lambda_j}\right) \quad (4)$$

Where λ_j is the weight of A_j (j= 1, 2, …, n), $\lambda_j \in [0,1]$ and $\sum_{j=1}^{n} \lambda_j = 1$

Definition 6. Let $A^* = \{A_1^*, A_2^*, \ldots, A_n^*\}$ be a vector of n SVN numbers, such that $A_j^* = (a_j^*, b_j^*, c_j^*)$ (j= 1,2,…,n), and $B_i = \{B_{i1}, B_{i2}, \ldots, B_{im}\}$ (i= 1,2,…,m), (j= 1,2,…,n). Then the separation measure between B_i and A^* based on Euclidian distance is defined as follows:

$$s_i = \left(\frac{1}{3}\sum_{j=1}^{n}\left(|a_{ij}-a_j^*|\right)^2 + \left(|b_{ij}-b_j^*|\right)^2 + \left(|c_{ij}-c_j^*|\right)^2\right)^{\frac{1}{2}} \quad (5)$$

(i= 1, 2, …, m)

Next, a score function for ranking SVN numbers is proposed below:

Definition 7. Let A = (a, b, c) be a single valued neutrosophic number, a score function S of a single valued neutrosophic value, based on the truth-membership degree, indeterminacy-membership degree and falsity membership degree is defined by

$$S(A) = \frac{1+a-2b-c}{2} \quad (6)$$

where $S(A) \in [-1,1]$

The score function S is reduced the score function proposed by[16] if b = 0 and a + b ≤ 1.

The concept of a linguistic variable is very useful for solving decision making problems with complex content. The value of a linguistic variable is expressed as an element of its term set. Such linguistic values can be represented using single valued neutrosophic numbers.

In the method, there are k-decision makers, m-alternatives, and n-criteria. k-decision makers evaluate the importance of the m-alternatives under n-criteria and rank the performance of the n-criteria with respect to linguistic statements converted into single valued neutrosophic numbers. The importance weights based on single valued neutrosophic values of the linguistic terms is given as Table 1.

Table 1. Linguistic variable and SVNSs. Note: Source:[17]

Linguistic term	SVNSs
Very not influential / (VNI)	(0.9;0.1;0.1)
No influential / (NI)	(0.75;0.25;0.20)
Medium influential / (MI)	(0.50;0.5;0.50)
Influential / (I)	(0.35;0.75;0.80)
Very high influential / (VI)	(0.10;0.90;0.90)

2.1 The TOPSIS method for SVNS

Assuming that $A = \{\rho_1, \rho_2, \dots, \rho_m\}$ is a set of alternatives, and $G = \{\beta_1, \beta_2, \dots, \beta_n\}$ is a set of criteria, the following steps will be carried out:

Step 1: Determine the relative importance of the experts. For this purpose, the specialists evaluate according to the linguistic scale shown in Table 1, and the calculations are performed with their associated SVNN, let $A_t = (a_t, b_t, c_t)$ be the SVNS corresponding to the t-th decision-maker (t = 1, 2, ..., k). The weight is calculated by the following formula:

$$\delta_t = \frac{a_t + b_t\left(\frac{a_t}{a_t + c_t}\right)}{\sum_{t=1}^{k} a_t + b_t\left(\frac{a_t}{a_t + c_t}\right)} \quad (7)$$

$$\delta_t \geq 0 \; and \; \sum_{t=1}^{k} \delta_t = 1$$

Step 2: Construction of the neutrosophic decision matrix of aggregated single values. This matrix is defined by $D = \sum_{t=1}^{k} \lambda_t D^t$, where $d_{ij} = (u_{ij}, r_{ij}, v_{ij})$ and is used to aggregate all individual evaluations. d_{ij} is calculated as the aggregation of the evaluations given by each expert $(u_{ij}^t, r_{ij}^t, v_{ij}^t)$, using the weights λ_t of each one using Equation 4. In this way, a matrix $D = (d_{ij})_{ij}$, is obtained, where each d_{ij} is an SVNN (i = 1, 2, .., m; j = 1,2,…, n).

Step 3: Determination of the Weight of the Criteria. Suppose that the weight of each criterion is given by $W = (w_1, w_2, \dots, w_n)$, where w_j denotes the relative importance of the criterion $\lambda_t w_j^t = (a_j^t, b_j^t, c_j^t)$. S_i is the evaluation of the criterion λ_t by the t-th expert. Then Equation 4 is used to add w_j^t the weights λ_t.

Step 4: Construction of the neutrosophic decision matrix from the single-valued weighted average with respect to the criteria.

$$D^* = D * W, \quad (8)$$

$$where \; d_{ij} = (a_{ij}, b_{ij}, c_{ij})$$

Step 5: Calculation of the ideal positive and negative SVNN solutions. The criteria can be classified as cost type or benefit type. Let G_1 be the set of benefit-type criteria and G_2 be the cost-type criteria. The ideal alternatives will be defined as follows:

The positive ideal solution, corresponding to G_1.

$$\rho^+ = a_{\rho+w}(\beta_j), b_{\rho+w}(\beta_j), ac_{\rho+w}(\beta_j) \quad (9)$$

The negative ideal solution, corresponding to G_2.

$$\rho^- = (a_{\rho-w}(\beta_j), b_{\rho-w}(\beta_j), ac_{\rho-w}(\beta_j)) \quad (10)$$

Where:

$$a_{\rho+w}(\beta_j) = \begin{cases} max_i a_{\rho iw}(\beta_j), si \; j\epsilon G_1 \\ min_i a_{\rho iw}(\beta_j), si \; j\epsilon G_2, \end{cases} \qquad a_{\rho-w}(\beta_j) = \begin{cases} min_i a_{\rho iw}(\beta_j), si \; j\epsilon G_1 \\ max_i a_{\rho iw}(\beta_j), si \; j\epsilon G_2, \end{cases}$$

$$b_{\rho+w}(\beta_j) = \begin{cases} max_i b_{\rho iw}(\beta_j), si\ j\epsilon G_1 \\ min_i b_{\rho iw}(\beta_j), si\ j\epsilon G_2, \end{cases} \qquad b_{\rho-w}(\beta_j) = \begin{cases} min_i b_{\rho iw}(\beta_j), si\ j\epsilon G_1 \\ max_i b_{\rho iw}(\beta_j), si\ j\epsilon G_2, \end{cases}$$

$$c_{\rho+w}(\beta_j) = \begin{cases} max_i c_{\rho iw}(\beta_j), si\ j\epsilon G_1 \\ min_i c_{\rho iw}(\beta_j), si\ j\epsilon G_2, \end{cases} \qquad c_{\rho-w}(\beta_j) = \begin{cases} min_i c_{\rho iw}(\beta_j), si\ j\epsilon G_1 \\ max_i c_{\rho iw}(\beta_j), si\ j\epsilon G_2, \end{cases}$$

Step 6: Calculation of the distances to the positive and negative SVNN ideal solutions. The following equations are calculated using Equation 5:

$$d_i^+ = \left(\frac{1}{3}\sum_{j=1}^{n}\left\{\left(a_{ij}-a_j^+\right)^2+\left(b_{ij}-b_j^+\right)^2+\left(c_{ij}-c_j^+\right)^2\right\}\right)^{\frac{1}{2}} \qquad (11)$$

$$d_i^- = \left(\frac{1}{3}\sum_{j=1}^{n}\left\{\left(a_{ij}-a_j^-\right)^2+\left(b_{ij}-b_j^-\right)^2+\left(c_{ij}-c_j^-\right)^2\right\}\right)^{\frac{1}{2}} \qquad (12)$$

Step 7: Calculation of the Coefficient of Proximity (CP). The CP of each alternative is calculated with respect to the positive and negative ideal solutions.

$$\widetilde{\rho_J} = \frac{s^-}{s^+ + s^-} \qquad (13)$$

Where $0 \le \widetilde{\rho_J} \le 1$.

Step 8: Determination of the order of the alternatives. They are ordered according to the value of $\widetilde{\rho_J}$. The alternatives are ordered from highest to lowest, with the condition that $\widetilde{\rho_J} \rightarrow 1$ is the optimal solution.

3 Results

The analysis and literature review conducted by field specialists have identified several latent vulnerabilities that currently impact the integrity of the footwear company's supply chain. To evaluate these vulnerabilities, four criteria were generated through brainstorming and subsequently ratified by experts in the field.

The criteria selected for the development of data analysis imply:

1. Impact on Production: Evaluates how each vulnerability affects footwear production, from delays to complete shutdowns.
2. Mitigation capacity: Evaluates the real possibility of taking measures to mitigate each vulnerability.
3. Impact on Costs: Analyzes how each vulnerability affects operating costs and profitability.
4. Domino Effect: Evaluates how one vulnerability can cascade down the supply chain.

The analysis involved five experts in the field of study who are considered specialists in the matter. due to their extensive experience.

The criteria's weights were based on experts' evaluations, as presented in Table 1. Table 2 shows the vector of weights obtained in the study.

Table 2: Vector of weights of the analyzed criteria. Source: Own elaboration.

Criteria weights	SVNN
w_1	(0.87989;0.12011;0.11487)
w_2	(0.83428;0.16572;0.15849)
w_3	(0.82671;0.17329;0.15157)
w_4	(0.85573;0.14427;0.13195)

The experts assess detected vulnerabilities based on the criteria's impact, referencing the values in Table 1. The resultant data is translated into neutrosophic sets for future analyses. Table 3 displays the preliminary evaluations provided by each expert regarding the assessed criteria.

Table 3: Evaluation of decision alternatives with respect to the evaluation criteria. Source: own elaboration

Criterion 1: Impact on Production					
Vulnerabilities	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5
Supplier Dependence	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.75,0.25,0.2)

Criterion 1: Impact on Production					
Inefficient Transportation and Logistics	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.9,0.1,0.1)
Fluctuations in Raw Material Costs	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
Geographic location	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.5,0.5,0.5)
Government regulations	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
Raw material with very variable quality standards	(0.75,0.25,0.2)	(0.75,0.25,0.2)	(0.1,0.9,0.9)	(0.75,0.25,0.2)	(0.1,0.9,0.9)
Lack of investment in technology and innovation capacity in the supply chain.	(0.9,0.1,0.1)	(0.1,0.9,0.9)	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.1,0.9,0.9)
Poor and obsolete internal and external communication systems	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.5,0.5,0.5)
Criterion 2: Mitigation capacity					
Supplier Dependence	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.9,0.1,0.1)
Inefficient Transportation and Logistics	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)
Fluctuations in Raw Material Costs	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.9,0.1,0.1)
Geographic location	(0.75,0.25,0.2)	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.75,0.25,0.2)
Government regulations	(0.35,0.75,0.8)	(0.5,0.5,0.5)	(0.35,0.75,0.8)	(0.5,0.5,0.5)	(0.75,0.25,0.2)
Raw material with very variable quality standards	(0.5,0.5,0.5)	(0.35,0.75,0.8)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.75,0.25,0.2)
Lack of investment in technology and innovation capacity in the supply chain.	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.35,0.75,0.8)
Poor and obsolete internal and external communication systems	(0.9,0.1,0.1)	(0.35,0.75,0.8)	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.75,0.25,0.2)
Criterion 3: Impact on Costs					
Supplier Dependence	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)
Inefficient Transportation and Logistics	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)
Fluctuations in Raw Material Costs	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.5,0.5,0.5)
Geographic location	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)
Government regulations:	(0.5,0.5,0.5)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.5,0.5,0.5)	(0.35,0.75,0.8)
Raw material with very variable quality standards	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)
Lack of investment in technology and innovation capacity in the supply chain.	(0.5,0.5,0.5)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.35,0.75,0.8)
Poor and obsolete internal and external communication systems	(0.35,0.75,0.8)	(0.35,0.75,0.8)	(0.5,0.5,0.5)	(0.35,0.75,0.8)	(0.5,0.5,0.5)
Criterion 4: Domino Effect					
Supplier Dependence	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.75,0.25,0.2)	(0.75,0.25,0.2)
Inefficient Transportation and Logistics	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.75,0.25,0.2)	(0.75,0.25,0.2)
Fluctuations in Raw Material Costs	(0.9,0.1,0.1)	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.75,0.25,0.2)	(0.9,0.1,0.1)

Criterion 1: Impact on Production					
Geographic location	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
Government regulations	(0.75,0.25,0.2)	(0.5,0.5,0.5)	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.5,0.5,0.5)
Raw material with very variable quality standards	(0.75,0.25,0.2)	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
Lack of investment in technology and innovation capacity in the supply chain.	(0.75,0.25,0.2)	(0.75,0.25,0.2)	(0.1,0.9,0.9)	(0.75,0.25,0.2)	(0.1,0.9,0.9)
Poor and obsolete internal and external communication systems	(0.5,0.9,0.1)	(0.1,0.9,0.9)	(0.5,0.5,0.5)	(0.1,0.9,0.9)	(0.1,0.9,0.9)

The expert evaluations serve as the foundation for the method's operations performed to derive the decision matrix. Equation (8) is implemented to obtain the neutrosophic decision matrix of the single-valued weighted average with respect to the criteria. Table 4 shows the results obtained after applying the above procedure.

Table 4: Initial decision matrix. Source: own elaboration

Alternatives	Impact on Production	Mitigation capacity	Impact on Costs	Domino effect
Supplier Dependence	(0.573;0.427;0.383)	(0.62;0.38;0.35)	(0.266;0.81;0.842)	(0.573;0.427;0.383)
Inefficient Transportation and Logistics	(0.693;0.307;0.297)	(0.428;0.572;0.566)	(0.266;0.81;0.842)	(0.538;0.462;0.414)
Fluctuations in Raw Material Costs	(0.753;0.247;0.232)	(0.693;0.307;0.297)	(0.473;0.527;0.484)	(0.733;0.267;0.247)
Geographic location	(0.62;0.38;0.35)	(0.61;0.39;0.34)	(0.38;0.62;0.605)	(0.753;0.247;0.232)
Government regulations:	(0.714;0.286;0.269)	(0.442;0.582;0.568)	(0.316;0.725;0.733)	(0.62;0.38;0.35)
Raw material with very variable quality standards	(0.499;0.501;0.449)	(0.572;0.437;0.419)	(0.266;0.81;0.842)	(0.714;0.286;0.269)
Lack of investment in technology and innovation capacity in the supply chain.	(0.414;0.586;0.58)	(0.493;0.54;0.549)	(0.291;0.766;0.785)	(0.499;0.501;0.449)
Poor and obsolete internal and external communication systems	(0.631;0.369;0.36)	(0.676;0.329;0.306)	(0.316;0.725;0.733)	(0.247;0.829;0.58)

The results allow to obtain the ideal positive and negative values for each criterion. Subsequently, this allows to determine the ideal distances that are used to calculate the coefficient of proximity. Table 5 shows the distances to the positive and negative ideal values for each competence, according to the criteria, as well as the coefficients of proximity calculated.

Table 5: Distances to the positive and negative ideal values of each competence and coefficients of proximity

Alternatives	d+	d-	CP
Supplier Dependence	0.76	0.55	0.418
Inefficient Transportation and Logistics	0.82	0.57	0.41
Fluctuations in Raw Material Costs	0.72	0.45	0.386
Geographic location	0.72	0.43	0.375
Government regulations	0.79	0.52	0.398
Raw material with very variable quality standards	0.79	0.54	0.406
Lack of investment in technology and innovation capacity in the supply chain.	0.8	0.62	0.438
Poor and obsolete internal and external communication systems	0.78	0.69	0.468

The results obtained through the application of the method to identify the main vulnerabilities in the supply chain of the footwear company allowed several relevant conclusions to be determined. Firstly, it was observed that the dependence on certain suppliers, although significant, did not reach the level of maximum criticality compared to other alternatives evaluated. This finding suggests that while vendor lock-in represents a significant risk, there are other, even more concerning, vulnerabilities.

Inefficiency in transportation and logistics was highlighted as a relevant vulnerability. Although this vulnerability did not reach the category of the worst possible situation, the distances from the ideal values indicated that there was room for substantial improvements in this area. This underlines the need to optimize transportation and logistics processes in the supply chain.

On the other hand, the fluctuation in raw material costs presented a CP of 0.386, positioning itself as one of the least critical vulnerabilities according to the results obtained. Despite their relevance, other alternatives evaluated generated greater concern. This suggests that while it is important to manage and anticipate fluctuations in raw material costs, other vulnerabilities require more immediate attention.

Geographic location was identified as a significant vulnerability. The presence of vulnerability necessitates the consideration of strategies to reduce risks, particularly concerning the distance between the firm and its clients and suppliers. Government regulations have yielded a CP score of 0.398, positioning it at an intermediate level of criticality. These findings suggest that companies need to be ready to adjust to potential regulatory modifications, underscoring the significance of possessing regulatory flexibility in the supply chain.

The lack of investment in technology and innovation in the supply chain was positioned as one of the most critical vulnerabilities. This highlights the urgency of addressing the lack of investment in technology and innovation in the supply chain, as it represents a significant risk for the company. Likewise, poor and outdated communication systems emerged as the most critical vulnerability. This result emphasizes the essential priority of improving communication systems, both internal and external, to ensure supply chain efficiency and resilience.

4 Discussion

Neutrosophy, a philosophical and logical approach that addresses uncertainty and imprecision in decision making, has demonstrated its capability to produce results linked to the evaluation of vulnerabilities in the supply chain. Recognizing the complex nature of the system and enabling a more precise depiction of uncertainty (by considering three logical values instead of the conventional true or false) produces a better representation of the inherent indeterminacy present in supply chain vulnerabilities.

Furthermore, Neutrosophy serves as an effective tool in evaluating the significance of each vulnerability. Since supply chain vulnerabilities can vary in terms of their impact and probability, neutrosophy offers a framework for expressing the associated uncertainty. This is particularly crucial when allocating limited resources to address the most critical vulnerabilities.

Furthermore, the use of neutrosophic sets has promoted communication and consensus among various stakeholders. By acknowledging that evaluations cannot be reduced to simply true or false terms, but may present different levels of accuracy, inaccuracy, and uncertainty, a more comprehensive and subtle discourse about vulnerabilities and their consequences has emerged. Consequently, this development has permitted more knowledgeable judgments and the creation of more efficient tactics to reinforce the supply chain and guarantee the continuity of business operations.

Conclusion

In the field of business science, a wide range of processes constantly take place, leading to complex decisions influenced by multiple factors. Mathematical methods, specifically multicriteria problem-solving approaches, have proven to be immensely valuable in many scenarios.

The use of neutrosophy, as a tool for incorporating uncertainties inherent in complex decision-making processes in the business world, is essential in this dynamic context. In this study, the TOPSIS method was incorporated with neutrosophic logic to assess the supply chain vulnerabilities of a footwear manufacturing company located in Ambato, to improve its overall performance. As a result, it was discovered that the most critical vulnerabilities are the insufficient investment in technology and innovation, closely followed by outdated and deficient communication systems. This highlights the urgent need to allocate more resources and focus towards enhancing innovation and technology within the supply chain, as well as modernizing communication systems.

This study clearly demonstrates the effectiveness and versatility of the method in various scientific environments and fields. Using single-valued neutrosophic numbers to perform the analysis has confirmed the practical applicability of neutrosophic set logic. To broaden the scope of study and its relevance to real-life issues, it is recommended to explore and adopt other multi-criteria methods related to the multiple dimensions of neutrosophic logic.

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Received: October 20, 2023. **Accepted**: December 14, 2023