



A Study of Algebraic Curves in Neutrosophic Real Ring $R(I)$ by Using the One-Dimensional Geometric AH-Isometry

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Abstract: The objective of this paper is to study and define some algebraic curves with neutrosophic variables in neutrosophic real field $R(I)$, where we study what are the relationships between classical algebraic curves and neutrosophic algebraic curves depending on the geometric isometry (AH-Isometry).

Keywords: Neutrosophic real ring $R(I)$, AH-isometry, Neutrosophic algebraic curves.

Introduction

Algebraic Geometry is one of the branches of algebra that deals with the study of geometric shapes through familiar algebraic concepts and theories [1]. There were several approaches to geometry, all of which are usually classified as algebraic geometry, at the end of the nineteenth century. Lazare Carnot (1753-1823) attributed to algebraic geometry which is about algebraic curves and their intersection with the sides of a triangle [2], but this concept developed a lot in the second half of the nineteenth century.

Neutrosophy is a new branch of philosophy concerns with the indeterminacy in all areas of life and science. It has become a useful tool in generalizing many classical systems such as equations [30], number theory [3], and linear spaces [4,5], and ring of matrices [19-31].

Recently, Abobala, and Hatip have presented the concept of one-dimensional AH-isometry to study the correspondence between neutrosophic plane $R(I)$ and the classical module $R \times R$.

In this work, we use the one-dimensional AH-isometry to turn the general case of algebraic curves in real ring $R(I)$ with one variable into two classical algebraic curves so we will go from $R(I)$ space into $R \times R$ space, we study the properties of our algebraic curves then we go back to $R(I)$ space using AH-isometry.

Neutrosophic Functions on $R(I)$.

Definition:

Let $R(I) = \{a + bI; a, b \in R\}$ where $I^2 = I$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [49]

$$\begin{aligned} T: R(I) &\rightarrow R \times R \\ T(a + bI) &= (a, a + b) \end{aligned}$$

Definition:

Let $f: R(I) \rightarrow R(I); f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.

Example:

$$\begin{aligned} \text{Take } f: R(I) \rightarrow R(I); f(X) &= X^2 + IX + 2I = (x + yI)^2 + I(x + yI) + 2I \\ &= x^2 + I(y^2 + 2xy + x + y + 2) \end{aligned}$$

Theorem:

Let $f: R(I) \rightarrow R(I)$ be a neutrosophic real function with one variable, $X = x + yI \in R(I)$ then f can be turned into two classical real functions.

Computing Powers in $R(I)$.

To compute such equation: $(a + bI)^{c+dI}; a, b, c, d \in R$ we need the one-dimensional isometry again:

$$T[(a + bI)^{c+dI}] = (a, a + b)^{(c, c+d)} = (a^c, (a + b)^{c+d}),$$

Which means

$$\begin{aligned} (a + bI)^{c+dI} &= T^{-1}(a^c, (a + b)^{c+d}), \\ &= a^c + I[(a + b)^{c+d} - a^c]. \end{aligned}$$

Theorem:

Let $R(I)$ be the neutrosophic field of reals, we have:

1. $\sin(a + bI) = \sin a + I[\sin(a + b) - \sin a]$
2. $\cos(a + bI) = \cos a + I[\cos(a + b) - \cos a]$
3. $e^{x+Iy} = e^x + I(e^{x+y} - e^x)$

Algebraic Curves In Neutrosophic Real Ring $R(I)$:**Definition : Neutrosophic Strophoide.**

Let $Y = y_1 + y_2I, X = x_1 + x_2I, A = a_1 + a_2I \in R(I), a_1, a_2, x_1, x_2, y_1, y_2 \in \mathbf{R}$, then we define a neutrosophic strophoide as follows:

$$Y^2 = X^2 \cdot \frac{A + X}{A - X} ; A > 0$$

This equation can be written as follows:

$$(y_1 + y_2I)^2 = (x_1 + x_2I)^2 \cdot \frac{(a_1 + a_2I) + (x_1 + x_2I)}{(a_1 + a_2I) - (x_1 + x_2I)} ; a_1 + a_2I > 0$$

Theorem :

Let $Y = y_1 + y_2I, X = x_1 + x_2I, A = a_1 + a_2I \in R(I)$, then if $A = a_1 + a_2I$ is invertible, the neutrosophic strophoide $(y_1 + y_2I)^2 = (x_1 + x_2I)^2 \cdot \frac{(a_1+a_2I)+(x_1+x_2I)}{(a_1+a_2I)-(x_1+x_2I)}$ is equivalent to the direct product of two classical strophoide.

Proof. Consider the equation $(y_1 + y_2I)^2 = (x_1 + x_2I)^2 \cdot \frac{(a_1+a_2I)+(x_1+x_2I)}{(a_1+a_2I)-(x_1+x_2I)}$

Now, we have:

$$y_1^2 + (y_2^2 + 2y_1y_2)I = [x_1^2 + (x_2^2 + 2x_1x_2)I] \cdot \frac{(a_1 + x_1) + (a_2 + x_2)I}{(a_1 - x_1) + (a_2 - x_2)I}$$

$$\begin{aligned} y_1^2 + (y_2^2 + 2y_1y_2)I &= [x_1^2 + (x_2^2 + 2x_1x_2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} \right. \\ &\quad \left. + \frac{(a_1 - x_1)(a_2 + x_2) - (a_1 + x_1)(a_2 - x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} I \right] \end{aligned}$$

by computing its direct image with AH-isometry, we get:

$$\begin{aligned} T(y_1^2 + (y_2^2 + 2y_1y_2)I) \\ = T(x_1^2 + (x_2^2 + 2x_1x_2)I)T\left(\frac{(a_1 + x_1)}{(a_1 - x_1)}\right) \\ + \frac{(a_1 - x_1)(a_2 + x_2) - (a_1 + x_1)(a_2 - x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} I \end{aligned}$$

$$\begin{aligned} (y_1^2, y_1^2 + y_2^2 + 2y_1y_2) \\ = (x_1^2, x_1^2 + x_2^2 + 2x_1x_2) \cdot \left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_1 + x_1)}{(a_1 - x_1)} \right) \\ + \frac{(a_1 - x_1)(a_2 + x_2) - (a_1 + x_1)(a_2 - x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} \end{aligned}$$

Then.

$$(y_1^2, (y_1 + y_2)^2) = (x_1^2, (x_1 + x_2)^2)$$

$$\cdot \left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_1 + x_1)[(a_1 + a_2) - (x_1 + x_2)] + (a_1 - x_1)(a_2 + x_2) - (a_1 + x_1)(a_2 - x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} \right)$$

$$(y_1^2, (y_1 + y_2)^2)$$

$$\begin{aligned} = (x_1^2, (x_1 \\ + x_2)^2) \cdot \left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_1 + x_1)[(a_1 + a_2) - (x_1 + x_2) - (a_2 - x_2)] + (a_1 - x_1)(a_2 + x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} \right) \end{aligned}$$

$$(y_1^2, (y_1 + y_2)^2) = (x_1^2, (x_1 + x_2)^2) \cdot \left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_1 + x_1)[(a_1 - x_1)] + (a_1 - x_1)(a_2 + x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} \right)$$

$$(y_1^2, (y_1 + y_2)^2) = (x_1^2, (x_1 + x_2)^2) \cdot \left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_1 - x_1)(a_1 + x_1 + a_2 + x_2)}{(a_1 + x_1)[(a_1 + a_2) - (x_1 + x_2)]} \right)$$

$$(y_1^2, (y_1 + y_2)^2) = (x_1^2, (x_1 + x_2)^2) \cdot \left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_1 + a_2) + (x_1 + x_2)}{(a_1 + a_2) - (x_1 + x_2)} \right)$$

$$(y_1^2, (y_1 + y_2)^2) = \left(x_1^2 \frac{(a_1 + x_1)}{(a_1 - x_1)}, (x_1 + x_2)^2 \frac{(a_1 + a_2) + (x_1 + x_2)}{(a_1 + a_2) - (x_1 + x_2)} \right)$$

So that we have:

$$\left\{ \begin{array}{l} \Gamma_1: y_1^2 = x_1^2 \frac{(a_1 + x_1)}{(a_1 - x_1)}; a_1 > 0 \\ \Gamma_2: (y_1 + y_2)^2 = (x_1 + x_2)^2 \frac{(a_1 + a_2) + (x_1 + x_2)}{(a_1 + a_2) - (x_1 + x_2)}; (a_1 + a_2) > 0 \end{array} \right.$$

Remark :

If $a_1 + a_2I$ is invertible, we can write the equation of neutrosophic strophoide as follows:

$$(y_1 + y_2I)^2 = (x_1 + x_2I)^2 \cdot \frac{(a_1 + a_2I) + (x_1 + x_2I)}{(a_1 + a_2I) - (x_1 + x_2I)}.$$

Now, we should discuss the cases of non-invertible of $a_1 + a_2I$.

$a_1 + a_2I$ is not invertible, then we have cases:

- 1- $a_1 = 0, a_1 + a_2 \neq 0$, this means that the neutrosophic strophoide will be equivalent to direct product of classical strophoide $(y_1 + y_2)^2 = (x_1 + x_2)^2 \frac{(a_1 + a_2) + (x_1 + x_2)}{(a_1 + a_2) - (x_1 + x_2)}; (a_1 + a_2) > 0$ with classical image two line $\begin{cases} y_1 = i x_1 \\ y_1 = -i x_1 \end{cases}$.
- 2- $a_1 \neq 0, a_1 + a_2 = 0$, this implies that the neutrosophic strophoide will be equivalent to direct product of classical strophoide $y_1^2 = x_1^2 \frac{(a_1 + x_1)}{(a_1 - x_1)}; a_1 > 0$ with classical image two line $\begin{cases} y_1 = i(x_1 + x_2) \\ y_1 = -i(x_1 + x_2) \end{cases}$.
- 3- If $a_1 = 0, a_1 + a_2 = 0$, this implies that the neutrosophic strophoide will be equivalent to direct product of classical image two line $\begin{cases} y_1 = i x_1 \\ y_1 = -i x_1 \end{cases}$ with classical image two line $\begin{cases} y_1 = i(x_1 + x_2) \\ y_1 = -i(x_1 + x_2) \end{cases}$.

Theorem:

Let Γ_1, Γ_2 are two classical strophoide, then the direct product of Γ_1, Γ_2 is equivalent to the neutrosophic strophoide Γ .

Proof.

Let Γ_1, Γ_2 are two classical strophoide, where:

$$\left\{ \begin{array}{l} \Gamma_1: y_1^2 = x_1^2 \cdot \frac{(a_1 + x_1)}{(a_1 - x_1)}; a_1 > 0 \\ \Gamma_2: y_2^2 = x_2^2 \cdot \frac{(a_2 + x_2)}{(a_2 - x_2)}; a_2 > 0 \end{array} \right.$$

Now, we take the inverse image of the AH-isometry, we have:

$$T^{-1}(y_1^2, y_2^2) = T^{-1}(x_1^2, x_2^2) \cdot T^{-1}\left(\frac{(a_1 + x_1)}{(a_1 - x_1)}, \frac{(a_2 + x_2)}{(a_2 - x_2)}\right)$$

$$y_1^2 + (y_2^2 - y_1^2)I = [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} + \left(\frac{(a_2 + x_2)}{(a_2 - x_2)} - \frac{(a_1 + x_1)}{(a_1 - x_1)} \right) I \right]$$

$$\begin{aligned} y_1^2 + (y_2^2 - y_1^2)I &= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} \right. \\ &\quad \left. + \left(\frac{(a_2 + x_2)(a_1 - x_1) - (a_2 - x_2)(a_1 + x_1)}{(a_1 - x_1)(a_2 - x_2)} \right) I \right] \end{aligned}$$

$$y_1^2 + (y_2^2 - y_1^2)I = [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} + \left(\frac{(a_2 + x_2)}{(a_2 - x_2)} - \frac{(a_1 + x_1)}{(a_1 - x_1)} \right) I \right]$$

$$\begin{aligned} y_1^2 + (y_2^2 - y_1^2)I &= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} \right. \\ &\quad \left. + \left(\frac{(a_2 + x_2)}{(a_2 - x_2)} - \frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 + x_1)}{(a_2 - x_2)} - \frac{(a_1 + x_1)}{(a_2 - x_2)} \right) I \right] \end{aligned}$$

$$\begin{aligned} y_1^2 + (y_2^2 - y_1^2)I &= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} \right. \\ &\quad \left. + \left(\left\{ \frac{(a_2 + x_2)}{(a_2 - x_2)} - \frac{(a_1 + x_1)}{(a_2 - x_2)} \right\} + \left\{ -\frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 + x_1)}{(a_2 - x_2)} \right\} \right) I \right] \end{aligned}$$

$$\begin{aligned} y_1^2 + (y_2^2 - y_1^2)I &= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} \right. \\ &\quad \left. + \left\{ \frac{(a_2 + x_2) - (a_1 + x_1)}{(a_2 - x_2)} - \frac{(a_1 + x_1)(a_2 - x_2) - (a_1 + x_1)(a_1 - x_1)}{(a_1 - x_1)(a_2 - x_2)} \right\} I \right] \end{aligned}$$

$$\begin{aligned}
& y_1^2 + (y_2^2 - y_1^2)I \\
&= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} \right. \\
&\quad \left. + \frac{(a_1 - x_1)[(a_2 + x_2) - (a_1 + x_1)] - (a_1 + x_1)(a_2 - x_2) - (a_1 + x_1)(a_1 - x_1)}{(a_1 - x_1)(a_2 - x_2)} I \right]
\end{aligned}$$

$$y_1^2 + (y_2^2 - y_1^2)I = [x_1^2 + (x_2^2 - x_1^2)I].$$

$$\begin{aligned}
& \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 - x_1)[(a_2 + x_2) - (a_1 + x_1)] - (a_1 + x_1)[(a_2 - x_2) - (a_1 - x_1)]}{(a_1 - x_1)(a_2 - x_2)} I \right] \frac{(a_1 + x_1)}{(a_1 - x_1)} \\
& \quad + \frac{(a_1 - x_1)[(a_2 + x_2) - (a_1 + x_1)] - (a_1 + x_1)[(a_2 - x_2) - (a_1 - x_1)]}{(a_1 - x_1)(a_2 - x_2)} I \\
&= \frac{(a_1 + x_1) + [(a_2 + x_2) - (a_1 + x_1)]I}{(a_1 - x_1) + [(a_2 - x_2) - (a_1 - x_1)]I}
\end{aligned}$$

$$\begin{aligned}
y_1^2 + (y_2^2 - y_1^2)I &= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{(a_1 + x_1) + [(a_2 + x_2) - (a_1 + x_1)]I}{(a_1 - x_1) + [(a_2 - x_2) - (a_1 - x_1)]I} \right] \\
y_1^2 + (y_2^2 - y_1^2)I &= [x_1^2 + (x_2^2 - x_1^2)I] \cdot \left[\frac{a_1 + (a_2 - a_1)I + [x_1 + (x_2 - x_1)I]}{a_1 + (a_2 - a_1)I - [x_1 + (x_2 - x_1)I]} \right] \dots (*)
\end{aligned}$$

We let $X = x_1 + (x_2 - x_1)I$, $Y = y_1 + (y_2 - y_1)I$, $A = a_1 + (a_2 - a_1)I$, then we can prove that:

$$Y^2 = y_1^2 + (y_2^2 - y_1^2)I, X^2 = x_1^2 + (x_2^2 - x_1^2)I$$

Then the equation (*) can be written as follows:

$$\Gamma: Y^2 = X^2 \cdot \frac{A + X}{A - X}; A > 0$$

This equation is a neutrosophic strophoide Γ .

Example:

Let the equation by a neutrosophic strophoide:

$$\Gamma: (y_1 + y_2I)^2 = (x_1 + x_2I)^2 \cdot \frac{(4 - 2I) + (x_1 + x_2I)}{(4 - 2I) - (x_1 + x_2I)}$$

Then, its equation be equivalent to direct product of two classical strophoide:

$$\left\{ \begin{array}{l} \Gamma_1: y_1^2 = x_1^2 \left(\frac{4 + x_1}{4 - x_1} \right) \\ \Gamma_2: (y_1 + y_2)^2 = (x_1 + x_2)^2 \left(\frac{2 + x_1}{2 - x_1} \right) \end{array} \right.$$

Example:

Let Γ_1, Γ_2 are two classical strophoide, where:

$$\begin{cases} \Gamma_1: y_1^2 = x_1^2 \left(\frac{1+x_1}{2} \right) \\ \Gamma_2: y_2^2 = x_2^2 \left(\frac{2+x_2}{2-x_2} \right) \end{cases}$$

Then by theorem 6.4 we have.

$$\Gamma: Y^2 = X^2 \cdot \frac{\left(\frac{1}{2} + \frac{3}{2}I\right) + X}{\left(\frac{1}{2} + \frac{3}{2}I\right) - X}$$

.Definition: Neutrosophic Cycloide.

Let $Y = y_1 + y_2I, X = x_1 + x_2I, R = r_1 + r_2I, t = t_1 + t_2I \in$

$\mathbf{R}(I), r_1, r_2, t_1, t_2, x_1, x_2, y_1, y_2 \in \mathbf{R}$, then we define a neutrosophic Cycloide as follows:

$$X = \mathbf{R}(1 - \sin t), Y = \mathbf{R}(1 - \cos t)$$

This equation can be written as follows:

$$x_1 + x_2I = (r_1 + r_2I)(1 - \sin(t_1 + t_2I)), y_1 + y_2I = (r_1 + r_2I)(1 - \cos(t_1 + t_2I))$$

Theorem:

Let $Y = y_1 + y_2I, X = x_1 + x_2I, R = r_1 + r_2I, t = t_1 + t_2I \in \mathbf{R}(I)$, then if $r_1 + r_2I$ is invertible, the neutrosophic Cycloide $X = \mathbf{R}(1 - \sin t), Y = \mathbf{R}(1 - \cos t)$ is equivalent to the direct product of two classical Cycloide.

Proof. Consider the equation $X = \mathbf{R}(1 - \sin t), Y = \mathbf{R}(1 - \cos t)$

Now, we have:

$$x_1 + x_2I = (r_1 + r_2I) \left(1 - \sin(t_1) - I(\sin(t_1 + t_2) - \sin(t_1)) \right)$$

$$y_1 + y_2I = (r_1 + r_2I) \left(1 - \cos(t_1) - I(\cos(t_1 + t_2) - \cos(t_1)) \right)$$

by computing its direct image with AH-isometry, we get:

$$T(x_1 + x_2I) = T(r_1 + r_2I).T(1 - \sin(t_1) - I[\sin(t_1 + t_2) - \sin(t_1)])$$

$$(x_1, x_1 + x_2) = (r_1, r_1 + r_2) \cdot (1 - \sin(t_1), 1 - \sin(t_1 + t_2))$$

$$(x_1, x_1 + x_2) = (r_1(1 - \sin(t_1)), (r_1 + r_2)(1 - \sin(t_1 + t_2)))$$

Then.

$$\begin{cases} x_1 = r_1(1 - \sin(t_1)) \\ x_1 + x_2 = (r_1 + r_2)(1 - \sin(t_1 + t_2)) \end{cases}$$

By a similar, we have.

$$\begin{cases} y_1 = r_1(1 - \cos(t_1)) \\ y_1 + y_2 = (r_1 + r_2)(1 - \cos(t_1 + t_2)) \end{cases}$$

So that we have:

$$\begin{cases} \Gamma_1: x_1 = r_1(1 - \sin(t_1)), y_1 = r_1(1 - \cos(t_1)) \\ \Gamma_2: x_1 + x_2 = (r_1 + r_2)(1 - \sin(t_1 + t_2)), y_1 + y_2 = (r_1 + r_2)(1 - \cos(t_1 + t_2)) \end{cases}$$

Remark:

If $r_1 + r_2I$ is invertible, we can write the equation of neutrosophic cycloide as follows:

$$X = R(1 - \sin t) , Y = R(1 - \cos t).$$

Now, we should discuss the cases of non-invertible of $r_1 + r_2I$.

The $r_1 + r_2I$ is not invertible, then we have two cases:

- 1- $r_1 = 0, r_1 + r_2 \neq 0$, this means that the neutrosophic cycloide will be equivalent to direct product of classical cycloide $x_1 + x_2 = (r_1 + r_2)(1 - \sin(t_1 + t_2)), y_1 + y_2 = (r_1 + r_2)(1 - \cos(t_1 + t_2))$ with the origin point $(0,0)$.
- 2- $r_1 \neq 0, r_1 + r_2 = 0$, this means that the neutrosophic cycloide will be equivalent to direct product of classical cycloide $x_1 = r_1(1 - \sin(t_1)), y_1 = r_1(1 - \cos(t_1))$ with the origin point $(0,0)$.
- 3- If $r_1 = 0, r_1 + r_2 = 0$, this implies that the neutrosophic cycloide will be equivalent to the origin point $(0,0)$.

Theorem:

Let Γ_1, Γ_2 are two classical cycloide, then the direct product of Γ_1, Γ_2 is equivalent to the neutrosophic cycloide Γ .

Proof.

Let Γ_1, Γ_2 are two classical cycloide, where:

$$\begin{cases} \Gamma_1: x_1 = r_1(1 - \sin t_1), y_1 = r_1(1 - \cos t_1) \\ \Gamma_2: x_2 = r_2(1 - \sin t_2), y_2 = r_2(1 - \cos t_2) \end{cases}$$

Now, we take the inverse image of the AH-isometry, we have:

$$T^{-1}(x_1, x_2) = T^{-1}(r_1, r_2).T^{-1}((1 - \sin t_1), (1 - \sin t_2))$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I]. [1 - \sin t_1 + ((1 - \sin t_2) - (1 - \sin t_1))I]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I]. [1 - \sin t_1 + (1 - \sin t_2 - 1 + \sin t_1)I]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I]. [1 - \sin t_1 - (\sin t_2 - \sin t_1)I]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I]. [1 - (\sin t_1 + (\sin t_2 - \sin t_1))I]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I]. [1 - (\sin t_1 + (\sin(t_2 - t_1 + t_1) - \sin t_1))I]$$

We let $X = x_1 + (x_2 - x_1)I, R = r_1 + (r_2 - r_1)I, t = t_1 + (t_2 - t_1)I$, then we can prove that:

$$\sin t = \sin[t_1 + (t_2 - t_1)I] = \sin t_1 + (\sin(t_2 - t_1 + t_1) - \sin t_1)I$$

Then, we have.

$$X = R. (1 - \sin t)$$

Now, by the same argument, we have.

$$Y = R. (1 - \cos t)$$

So.

$$\Gamma: \begin{cases} X = R. (1 - \sin t) \\ Y = R. (1 - \cos t) \end{cases}$$

This equation is a neutrosophic cycloid Γ .

Example:

Let the equation by a neutrosophic cycloide:

$$\begin{cases} x_1 + x_2I = (3 - 2I). [1 - \sin(t_1 + t_2I)] \\ y_1 + y_2I = (3 - 2I). [1 - \cos(t_1 + t_2I)] \end{cases}$$

Then, its equation be equivalent to direct product of two classical cycloide:

$$\begin{cases} \Gamma_1: x_1 = 3(1 - \sin t_1), y_1 = 3(1 - \cos t_1) \\ \Gamma_2: x_1 + x_2 = 1 - \sin(t_1 + t_2), y_1 + y_2 = 1 - \cos(t_1 + t_2) \end{cases}$$

Example:

Let Γ_1, Γ_2 are two classical cycloide, where:

$$\begin{cases} \Gamma_1: x_1 = 2(1 - \sin t_1), y_1 = 2(1 - \cos t_1) \\ \Gamma_2: x_2 = 5(1 - \sin t_2), y_2 = 5(1 - \cos t_2) \end{cases}$$

$$\Gamma: \begin{cases} X = (2 + 3I)(1 - \sin t) \\ Y = (2 + 3I)(1 - \cos t) \end{cases}$$

Definition: Neutrosophic Cardioide.

Let $\rho = \rho_1 + \rho_2 I, \theta = \theta_1 + \theta_2 I \in R(I), \rho_1, \rho_2, \theta_1, \theta_2 \in R$, then we define a neutrosophic Cardioide as follows:

$$\rho = (1 + \cos \theta)$$

This equation can be written as follows:

$$\rho_1 + \rho_2 I = (1 + \cos \theta_1) + [\cos(\theta_1 + \theta_2) - \cos \theta_1] I$$

Theorem:

Let $\rho = \rho_1 + \rho_2 I, \theta = \theta_1 + \theta_2 I \in R(I)$, then if $\theta_1 + \theta_2 I$ is invertible, the neutrosophic Cardioide

$\rho = (1 + \cos \theta)$ is equivalent to the direct product of two classical Cardioide.

Proof. Consider the equation $\rho = (1 + \cos \theta)$

Now, we have:

$$\rho_1 + \rho_2 I = (1 + \cos \theta_1) + [\cos(\theta_1 + \theta_2) - \cos \theta_1] I$$

by computing its direct image with AH-isometry, we get:

$$T(\rho_1 + \rho_2 I) = T((1 + \cos \theta_1) + [\cos(\theta_1 + \theta_2) - \cos \theta_1] I)$$

$$(\rho_1, \rho_1 + \rho_2) = (1 + \cos \theta_1, 1 + \cos(\theta_1 + \theta_2))$$

Then.

$$\begin{cases} \rho_1 = 1 + \cos \theta_1 \\ \rho_1 + \rho_2 = 1 + \cos(\theta_1 + \theta_2) \end{cases}$$

So that we have:

$$\begin{cases} \Gamma_1: \rho_1 = 1 + \cos\theta_1 \\ \Gamma_2: \rho_1 + \rho_2 = 1 + \cos(\theta_1 + \theta_2) \end{cases}$$

Remark:

If $\theta_1 + \theta_2 I$ is invertible, we can write the equation of neutrosophic Cardioide as follows:

$$\rho = (1 + \cos\theta).$$

Now, we should discuss the cases of non-invertible of $\theta_1 + \theta_2 I$.

The $\theta_1 + \theta_2 I$ is not invertible, then we have two cases:

- 1- $\theta_1 = 0, \theta_1 + \theta_2 \neq 0$, this means that the neutrosophic Cardioide will be equivalent to direct product of classical Cardioide $(\rho_1 + \rho_2) = 1 + \cos(\theta_1 + \theta_2)$ with the classical circle $\rho_1 = 2$.
- 2- $\theta_1 \neq 0, \theta_1 + \theta_2 = 0$, this means that the neutrosophic Cardioide will be equivalent to direct product of classical Cardioide $\rho_1 = 1 + \cos(\theta_1)$ with the classical circle $(\rho_1 + \rho_2) = 2$.
- 3- If $\theta_1 = 0, \theta_1 + \theta_2 = 0$ this means that the neutrosophic Cardioide will be equivalent to direct product of classical circle $(\rho_1 + \rho_2) = 2$ with the classical circle $\rho_1 = 2$.

Theorem:

Let Γ_1, Γ_2 are two classical Cardioide, then the direct product of Γ_1, Γ_2 is equivalent to the neutrosophic Cardioide Γ .

Proof.

Let Γ_1, Γ_2 are two classical Cardioide, where:

$$\begin{cases} \Gamma_1: \rho_1 = 1 + \cos\theta_1 \\ \Gamma_2: \rho_2 = 1 + \cos\theta_2 \end{cases}$$

Now, we take the inverse image of the AH-isometry, we have:

$$T^{-1}(\rho_1, \rho_2) = T^{-1}(1 + \cos\theta_1, 1 + \cos\theta_2)$$

$$\rho_1 + (\rho_2 - \rho_1)I = [1 + \cos\theta_1 + (1 + \cos\theta_2 - (1 + \cos\theta_1))I]$$

$$\rho_1 + (\rho_2 - \rho_1)I = [1 + \cos\theta_1 + (\cos\theta_2 - \cos\theta_1)I]$$

$$\rho_1 + (\rho_2 - \rho_1)I = 1 + [\cos\theta_1 + (\cos(\theta_1 + [\theta_2 - \theta_1]) - \cos\theta_1)I]$$

We let $\rho = \rho_1 + (\rho_2 - \rho_1)I, \theta = \theta_1 + (\theta_2 - \theta_1)I$, then we can prove that:

$$\cos\theta = \cos(\theta_1 + [\theta_2 - \theta_1]I) = \cos\theta_1 + (\cos(\theta_1 + [\theta_2 - \theta_1]) - \cos\theta_1)I$$

Then, we have.

$$\rho_1 + (\rho_2 - \rho_1)I = 1 + \cos(\theta_1 + [\theta_2 - \theta_1]I)$$

So.

$$\Gamma: \rho = 1 + \cos\theta$$

This equation is a neutrosophic Cardioide Γ .

Example:

Let the equation by a neutrosophic Cardioide:

$$\rho_1 + \rho_2 I = 1 + \cos\left(\frac{\pi}{3} + \frac{\pi}{4}I\right)$$

Then, its equation be equivalent to direct product of two classical Cardioide:

$$\left\{ \begin{array}{l} \Gamma_1: \rho_1 = 1 + \cos\left(\frac{\pi}{3}\right) \\ \Gamma_2: \rho_1 + \rho_2 = 1 + \cos\left(\frac{7\pi}{12}\right) \end{array} \right.$$

Example:

Let Γ_1, Γ_2 are two classical Cardioide, where:

$$\left\{ \begin{array}{l} \Gamma_1: \rho_1 = 1 + \cos\left(\frac{\pi}{4}\right) \\ \Gamma_2: \rho_2 = 1 + \cos\left(\frac{\pi}{2}\right) \end{array} \right.$$

$$\Gamma: \left\{ \rho = 1 + \cos\left(\frac{\pi}{4} + \frac{\pi}{4}I\right) \right.$$

Conclusions

In this paper we have studied some concepts of neutrosophic real analysis depending on the one-dimensional AH-isometry. We have provided a strict definition of some algebraic curves in neutrosophic real ring $\mathbf{R}(I)$, and we study the properties of this curves, and we

proved some theorems for this curves, also, we find relationships between a classical algebraic curves and neutrosophic algebraic curves.

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