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Neutrosophic Sociogram for Group Analysis

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Abstract. The sociogram is a technique of sociometry widely used in the field of sociology due to its simplicity and effectiveness. The purpose of this method is the graphical visualization of the relationships among the members of a social group. The sociogram has been extended to the fuzzy framework to include uncertainty in the so-called fuzzy sociograms. However, there could be indeterminate relationships among some members of the group, because they have not experience in performing some activities together, although potentially either future links or disagreements could be established among them. In this paper, we propose a neutrosophic sociogram, which allows representing the indeterminacies in the relationships among some members of a group. The advantage of neutrosophic sociograms over fuzzy sociograms is that the representation and calculation considering indeterminacy, allow us to achieve greater accuracy in the results, and a greater approach to the potentialities of the group in terms of the future bond among the members. A hypothetical example is proposed to illustrate the applicability of the method.

Keywords: Sociogram, neutrosophic sociogram, Neutrosociology, group analysis, sociometry analysis.

1 Introduction

The sociogram is a data analysis technique that focuses its attention on the way in which social relationships are established within any group, [1]. Jacob Levy Moreno, a Romanian psychiatrist, developed the technique in the mid-30s of the 20th century as a tool for exploratory and diagnostic purposes. Since its creation, sociometry appears as one of the most advanced and ordered strategies to describe and measure group dynamics, since it allows the quantitative study of interpersonal relationships in groups. The sociogram is an important example within sociometry.

In essence, the sociogram allows us to study the existing interpersonal preferences in a group of people. Currently it is widely used in various organizational settings, from small schools to large companies. It is also used in intelligence work in order to detect criminal networks. They can be briefly defined as graphics or tools used to determine the sociometry of a social space.

A social bond is a set of social relationships established between two or more individuals, which together, results in a group of social interaction, that is, when several members establish social bonds between them, forming a small social group. The social position is the specific place that every member occupies either in relation to the group of interaction or to the group in general.

This way, when applying a sociometric test or sociogram in a social group, the researcher may have knowledge of the way in which the group is socially related to each other, as well as the benefits and repercussions that this interaction has on each one of the members individually. This is very useful, since many times the degree of integration of an individual directly influences their performance. It is not groups dynamic but an easy-to-apply technique that can help us to better understand the world of relationships that is established in a social group.

Specifically, the sociogram starts from a questionnaire applied to the social group under investigation, where each member of the group specifies, in order of preference, with which other members they would like to carry out the activities asked in the questionnaire. This way, it starts with a matrix that is represented in the form of a graph, where the individual of the group preferred by the others and the isolated individual are determined.

In the classical sociogram, each member evaluates their preference through crisp values; however, some authors introduce the uncertainty that exists in these relationships, by using fuzzy graphs instead of crisp graphs with the so-called fuzzy sociogram, [2, 3]. Others make this type of graph even more complex with the definition of fuzzy graphs for polyfactor analysis, that is, fuzzy graphs that allow us studying more than one relationship between members of the social group. Some of these methods link this tool with classic cooperative game solutions such as Shapley value [2]. Sociograms can be applied in more than one moment to measure the change in relationships within the group.

It is not difficult to accept that the relationships between the members of the social groups may contain indeterminacies. Some members of the group may not know each other well, or may doubt on the behavior of the other in some activity. Therefore, in the classical sociogram and in the fuzzy sociogram it is not differentiated whether there is a mutual rejection between these individuals and therefore there is no possibility of a future relationship, or there is simply a potential bond that has not developed yet.

This fact has motivated the authors to propose a neutrosophic sociogram, where indeterminacy is included as part of the relationships between two individuals, because they are not well known, or there has been no possibility of creating a link between them or they have not determined the impossibility of such relationship.

Neutrosophy has served as the basis for sociology with the so-called Neutrosophic Sociology or Neutrosociology, which is defined as the study of sociology using neutrosophic scientific methods, [4, 5]. There are also neutrosophic graphs that allow us to measure concepts using graphs within the framework of Neutrosophy.

In this paper, neutrosophic sociograms are introduced, where the relationships among the members of a social group are graphically represented and quantitatively measured, including the indeterminacy of these relationships. Indeterminate relationships are considered as potential relationships in short, medium or long term, therefore it is a more accurate indicator than sociograms or fuzzy sociograms, since it guarantees a more precise measurement of group dynamics.

The paper is structured into the following sections: section 2 contains the main concepts related to sociograms and Neutrosophy. In section 3 the method proposed in this paper is introduced and a hypothetical example is used to illustrate how to apply it. The last section contains the conclusions.

2 Preliminaries

In this section, we summarize the main concepts of sociogram and Neutrosophy that will be used in this paper.

2.1 Sociogram

A sociogram is a graph that represents the relationship among the members of a social group. Firstly, the social group is identified. Then the investigator explains to the members the objective of the research. Next, the investigator designs a questionnaire for each member about the other members of the group he/she prefers to join in certain activities. E.g., in a group of students the teacher can ask every one of the members the following three questions [1]:

In order of preference, write the friends with whom
Q_1 : you want to join a quiz program.
Q_2 : you want to study in group.
Q_3 : you want to do volunteer activity.
Let us assume $S = \{s_1, s_2, \cdots, s_n\}$ denotes the set of interviewed. The results are represented in Table 1:

	Q_1	Q_2	Q_3
s_1	S_{11}	S_{12}	S_{13}
s_2	S_{21}	S_{22}	S_{23}
⋮	⋮	⋮	⋮
s_n	S_{n1}	S_{n2}	S_{n3}

Table 1: Generic table representing the relationship among the members of the social group.

The elements of Table 1 are the sets of members $S_{ij} \subset S$ $(i = 1,2, \dots, n)(j = 1,2,3)$ such that the member s_i has chosen for answering the j-th question (Q_1, Q_2, or Q_3).

The classical sociogram is formed from a square matrix where every member of S is represented in one row and one column, such that elements of the matrix contain one number from 1 to 3, which is used by every s_k to evaluate his/her preference for member s_l.

The results are depicted in a directed graph, where every node represents a member of the social group and the edges E_{kl} represent that k-th member of the group selected the l-th member. An example of sociogram is depicted in Figure 1.

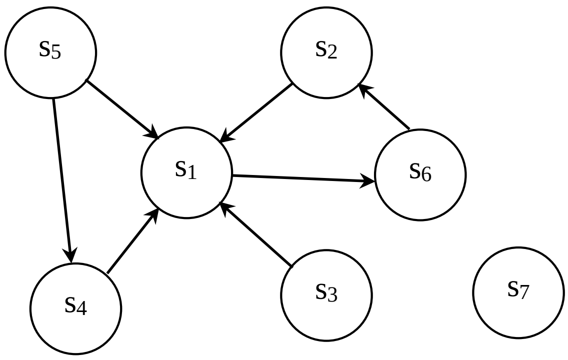

Figure 1: Example of sociogram of a group with seven members.

For example, in Figure 1 a social group of 7 members is investigated, where every node represents a member and every edge represents that one member prefers the other. Let us note in the example most of the members preferred s_1, while s_7 is isolated, he/she does not prefer anybody and nobody prefers him/her.

In the crisp sociogram, the graph is the final result, whereas in fuzzy sociogram the strength of every node (member) is measured with a function. f: $\{1, 2, \cdots, n\} \to [0, 1]$, where the closer is f(i) to 0 the more isolated member *i* is, thus it is an unpopular member possibly discriminated by the others, and the closer is f(i) to 1 the more linked member *i* is, then, *i* is a popular member or possibly the group's leader. This function can depend on fuzzy operators like t-norms or compensatory ones.

On the other hand, the preferred member can be selected using Shapley value [2]. Sometimes dendrograms are used to represent the sociogram [3].

2.2 Basic concepts on Neutrosophy

Definition 1: [6] Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \to]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of truthfulness, indetermination and falsehood of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

NS are useful only as a philosophical approach, so a *Single-Valued Neutrosophic Set* is defined to guarantee the applicability of Neutrosophy, see Definition 2.

Definition 2: ([6]) Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is an object of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A : X \to [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ denote the membership functions of truthfulness, indetermination and falsehood of x in A, respectively. For convenience, a *Single-Valued Neutrosophic Number* (SVNN)[7, 8] will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfies $0 \leq a + b + c \leq 3$.

Neutrosophic Logic (NL) extends fuzzy logic. As stated by Florentin Smarandache, its author, a proposition P is characterized by three components; see [9-12]:

$$NL(P) = (T,I,F) \quad (2)$$

Where component T is the degree of truthfulness, F is the degree of falsehood and I is the degree of indetermination. T, I and F belong to the interval [0, 1], and they are independent from each other.

A *neutrosophic number* is formed by the algebraic structure a+bI, where I = indetermination. Below we formally describe some important concepts.

Definition 3: ([13-18]) Let R be a ring. The *neutrosophic ring* $\langle R \cup I\rangle$ is also a ring, generated by R and I under the operation of R, where I is a neutrosophic element that satisfies the property $I^2 = I$. Given an integer n, then, n+I and nI are neutrosophic elements of $\langle R \cup I\rangle$ and in addition $0 \cdot I = 0$. Also, I^{-1}, the inverse of I is not defined.

E.g., a neutrosophic ring is $\langle \mathbb{Z} \cup I\rangle$ generated by $\mathbb{Z}$, which is the set of integers.

Some operation using I is $I + I + \ldots + I = nI$.

Definition 4: ([19, 20]) A *neutrosophic number* N is also defined as a number:

$$N = d + I \quad (3)$$

Where d is the *determined part* and I is the *indeterminate part* of N.

Example 1. N = 1+I, where 1 is the determined part and I is the indeterminate part, and for I = [0, 1] we have N = [1, 2].

Let $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ be two neutrosophic numbers, then some operations between them are:

1. $N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$ (Addition),

2. $N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$ (Difference),
3. $N_1 \times N_2 = a_1a_2 + (a_1b_2 + b_1a_2 + b_1b_2)I$ (Multiplication),
4. $\frac{N_1}{N_2} = \frac{a_1+b_1I}{a_2+b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1-a_1b_2}{a_2(a_2+b_2)}I$ (Division).

A *neutrosophic matrix* is a matrix whose components are elements of $\langle R \cup I \rangle$.

Thus, it is possible to generalize the operations between vectors and matrices on R to the ring $\langle R \cup I \rangle$. See Example 2.

Example 2. Given two matrices, $A = \begin{pmatrix} 3 & 9 \\ -1 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 8 & I \\ 1 & 3 & 2I \end{pmatrix}$, $AB = \begin{pmatrix} 12 & 51 & 21I \\ 6 & 13 & 13I \end{pmatrix}$.

A *neutrosophic graph* is a graph with at least one neutrosophic edge linking two nodes, that is to say, there is an edge with an indetermination on its two nodes connection, [6, 21-23], see Figure 2.

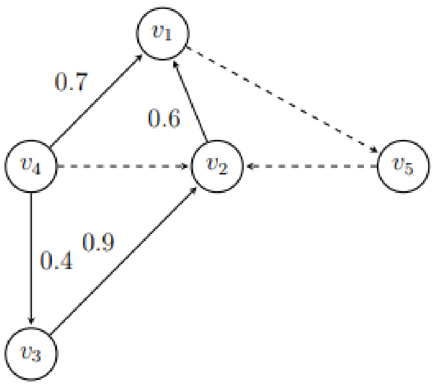

Figure 2: Example of neutrosophic graph. Source [6].

The de-neutrosophication process was introduced by Salmeron and Smarandache in [19], which converts a neutrosophic number in one numeric value. This process provides a range of numbers for centrality using as a base the maximum and minimum values of $I = [a_1, a_2] \subseteq [0, 1]$, based on Equation 4:

$$\lambda([a_1, a_2]) = \frac{a_1 + a_2}{2} \quad (4)$$

3 Neutrosophic sociogram

In this section, we introduce for the first time the concepts of neutrosophic sociograms. Firstly, the interviewers have to explain to the members of the social group the goal for applying the questionnaire and the type of possible answers required by the researchers[24, 25].

The new questionnaire is a variant of that summarized in Table 1. Now, we have $Q_1, Q_2, \ldots, Q_m$ the questions to be answered. Again, $S = \{s_1, s_2, \cdots, s_n\}$ denotes the set of interviewed.

The possible questions are the following:
In order of preference, write the friends with whom:
Q_1 : you want to join quiz program.
Q_2 : you want to study in group.
Q_3 : you want to do volunteer activity.
Apart, write the members of the group with whom:
Q_1 : you are not sure to join quiz program.
Q_2 : you are not sure to study in group.
Q_3 : you are not sure to do volunteer activity.

With this new method we maintain the elements of Table 1 like $S_{ij} \subset S$ $(i = 1,2, \ldots, n)(j = 1,2, \cdots, m)$ meaning the answers of s_i about his/her preferred members for doing activity asked in Q_j. Additionally, $O_{ij} \subset S$ $(i = 1,2, \ldots, n)(j = 1,2, \cdots, m)$ means the list of the members of the group which s_i is not sure to join in the activity asked in question Q_j, they satisfy $S_{ij} \cap O_{ij} = \emptyset$. Also, interviewer provides a weight to every question, which is denoted by $\Omega = \{\omega_1, \omega_2, \cdots, \omega_m\}$, where $\sum_{j=1}^{m} \omega_j = 1$ and $\omega_j \in [0, 1]$.

Then Table 1 converts into Table 2, where sets O_{ij} are included.

	Q_1	Q_2	$\cdots$	Q_m
s_1	$S_{11}; O_{11}$	$S_{12}; O_{12}$	$\cdots$	$S_{1m}; O_{1m}$
s_2	$S_{21}; O_{21}$	$S_{22}; O_{22}$	$\cdots$	$S_{2m}; O_{2m}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s_n	$S_{n1}; O_{n1}$	$S_{n2}; O_{n2}$	$\cdots$	$S_{nm}; O_{nm}$

Table 2: Generic table representing the relationship among the members of the social group for the neutrosophic sociogram.

According to Table 2, the interviewed has also the possibility to include those members of the group whom he/she is not sure to carry out the activity. We consider this indeterminate selected group is the potential extension of the links among the members of the group. The advantage is that we can influence those imprecise relationships to strength the group unity, instead of carrying out some external exercise, e.g. didactic activity in the group class, and later to apply another sociogram to study the dynamical changes in the social group.

Using Table 2 the *evaluation matrix* $R^j = \left(r^j_{kl}\right)$, where r^j_{kl} is the number of times (0 or 1) that s_k selects s_l in Q_j. When k = l we define $r^j_{kl} = 1$.

Thus, $F = \sum_{j=1}^{m} \omega_j R^j$, $F = (f_{kl})$ and if k = l we have $f_{kl} = 1$. f_{kl} means the degree of preference of s_l by s_k. If $f_{kl} = 1$ then s_k strongly prefers s_l and $f_{kl} = 0$ means s_k never prefers s_l.

The *fuzzy amicable degree* g_{kl} between s_k and s_l is calculated through formula 5:

$$\frac{2}{g_{kl}} = \frac{1}{f_{kl}} + \frac{1}{f_{lk}} \quad (5)$$

Where the arithmetic $1/_0 = \infty$ and $1/_\infty = 0$ is used.

Equivalently, $T^j = \left(t^j_{kl}\right)$, where t^j_{kl} is the number of times that s_k selects or hesitates about s_l in Q_j (0 or 1), $T = \sum_{j=1}^{m} \omega_j T^j$. When k = l we define $t^j_{kl} = 1$. Matrix T determines the preferences of s_l by s_k or the possibility that he/she would prefer him/her in the future. Therefore, the *neutrosophic amicable degree* u_{kl} between s_k and s_l is calculated with Equation 6:

$$\frac{2}{u_{kl}} = \frac{1}{t_{kl}} + \frac{1}{t_{lk}} \quad (6)$$

The fuzzy sociogram is represented with the elements of F, whereas the *neutrosophic sociogram* is a neutrosophic graph, such that the elements of the fuzzy sociogram are represented with continuous lines, and the other edges are represented with dashed lines. Every edge of the neutrosophic sociogram is associated with the fuzzy value g_{kl} and the other edges are associated with symbol I. Let us note that we are dealing with non-directed graphs.

The interval of indeterminacy is calculated as $I_{kl} = [g_{kl},\ u_{kl}]$. $\lambda(I_{kl})$ indicates a unique value for representing the amicable relationship between s_k and s_l, according to Equation 4, whereas $I_{kl} = u_{kl} - g_{kl}$ measures the degree of indeterminacy.

The leadership of the k-th member of the group is measured with the following index [2]:

$$\mu(k) = \frac{\sum_l g_{kl}}{\sum_k \sum_l g_{kl}} \quad (7)$$

Additionally, the potential leadership of the k-th member of the group is measured with the following index:

$$\theta(k) = \frac{\sum_l u_{kl}}{\sum_k \sum_l u_{kl}} \quad (8)$$

Below, we use an example for demonstrating how to use neutrosophic sociograms in a simulated case.

Example 3.

A teacher of a group of 10 elementary school students wants to investigate the relationships between the children and the potential links among the group members. To do this, he asks three questions to analyze preferences and possible future links among students. He also uses this study to determine current and potential leaders within the group and if there is any isolated student. That is why he decides to apply the neutrosophic sociogram.

The total questionnaire consists of the following pairs of questionnaires:

Write your friends with whom:

Q_1: you want to join a quiz program.

Q_2: you want to study in group.

Q_3: you want to do volunteer activity.

Apart, write the members of the group in with whom:

Q_1^I : you are not sure to join quiz program.

Q_2^I : you are not sure to study in group.

Q_3^I : you are not sure to do volunteer activity.

We denote by $S = \{s_1, s_2, \cdots, s_{10}\}$ the set of members of the group of class. Results are shown in Table 3.

	$Q_1; Q_1^I$	$Q_2; Q_2^I$	$Q_3; Q_3^I$
s_1	$s_3, s_6, s_8; s_4$	$s_2, s_6, s_{10}; s_4$	$s_3, s_6, s_9; s_4$
s_2	$s_3, s_4, s_5; s_7$	$s_1, s_4, s_9; s_7$	$s_1, s_3, s_4; s_7$
s_3	$s_1, s_2, s_9; s_6$	$s_5, s_8, s_9; s_2$	$s_2, s_6, s_7; s_{10}$
s_4	$s_2, s_5, s_6; s_9$	$s_3, s_6, s_{10}; s_9$	$s_3, s_5, s_6; s_2$
s_5	$s_3, s_7, s_{10}; s_9$	$s_4, s_7, s_8; s_1$	$s_4, s_8, s_{10}; s_1$
s_6	$s_1, s_7, s_8; s_9$	$s_1, s_7, s_8; s_9$	$s_1, s_2, s_9; s_3$
s_7	$s_4, s_5, s_9; s_2$	$s_2, s_4, s_9; s_5$	$s_6, s_8, s_9; s_1$
s_8	$s_5, s_7, s_9; s_3$	$s_4, s_6, s_9; s_7$	$s_1, s_2, s_5; s_3$
s_9	$s_1, s_8, s_{10}; s_2$	$s_2, s_4, s_5; s_1$	$s_1, s_4, s_5; s_2$
s_{10}	$s_2, s_5, s_7; s_9$	$s_2, s_3, s_5; s_4$	$s_2, s_4, s_5; s_8$

Table 3: Preferences and potential links between the members of the group.

In Table 3, for every child in the row, before the semicolon we have the children he/she prefers for performing the activity asked in questions Q_j. After the semicolon there are the children that the student is not sure about to perform activity asked in Q_j^I. Interestingly, student denoted by s_3 prefers to join quiz program with s_2, however he/she is not sure to study in group with s_2. This shows the capacity of the neutrosophic method to model more feelings of the members than its precedents do.

Here we assumed the three questions are equally important, thus, $\omega_j = \frac{1}{3}$ (j = 1,2,3).

Tables 4, 5, and 6 contains the evaluation matrices for Q_1, Q_2, and Q_3, respectively.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0	1	0	0	1	0	1	0	0
s_2	0	1	1	1	1	0	0	0	0	0
s_3	1	1	1	0	0	0	0	0	1	0
s_4	0	1	0	1	1	1	0	0	0	0
s_5	0	0	1	0	1	0	1	0	0	1
s_6	1	0	0	0	0	1	1	1	0	0
s_7	0	0	0	1	1	0	1	0	1	0
s_8	0	0	0	0	1	0	1	1	1	0
s_9	1	0	0	0	0	0	0	1	1	1
s_{10}	0	1	0	0	1	0	1	0	0	1

Table 4: Evaluation matrix R^1 for Q_1.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	1	0	0	0	1	0	0	0	1
s_2	1	1	0	1	0	0	0	0	1	0
s_3	0	0	1	0	1	0	0	1	1	0
s_4	0	0	1	1	0	1	0	0	0	1
s_5	0	0	0	1	1	0	1	1	0	0
s_6	1	0	0	0	0	1	1	1	0	0
s_7	0	1	0	1	0	0	1	0	1	0
s_8	0	0	0	1	0	1	0	1	1	0
s_9	0	1	0	1	1	0	0	0	1	0
s_{10}	0	1	1	0	1	0	0	0	0	1

Table 5: Evaluation matrix R^2 for Q_2

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0	1	0	0	1	0	0	1	0
s_2	1	1	1	1	0	0	0	0	0	0
s_3	0	1	1	0	0	1	1	0	0	0
s_4	0	0	1	1	1	1	0	0	0	0
s_5	0	0	0	1	1	0	0	1	0	1
s_6	1	1	0	0	0	1	0	0	1	0
s_7	0	0	0	0	0	1	1	1	0	1
s_8	1	1	0	0	1	0	0	1	0	0
s_9	1	0	0	1	1	0	0	0	1	0
s_{10}	0	1	0	1	1	0	0	0	0	1

Table 6: Evaluation matrix R^3 for Q_3.

Table 7 contains the result of F, the fuzzy matrix.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0.33	0.66	0	0	1	0	0.33	0.33	0.33
s_2	0.66	1	0.66	1	0.33	0	0	0	0.33	0
s_3	0.33	0.66	1	0	0.33	0.33	0.33	0.33	0.66	0
s_4	0	0.33	0.66	1	0.66	1	0	0	0	0.33
s_5	0	0	0.33	0.66	1	0	0.66	0.66	0	0.66
s_6	1	0.33	0	0	0	1	0.66	0.66	0.33	0
s_7	0	0.33	0	0.66	0.33	0.33	1	0.33	0.66	0.33
s_8	0.33	0.33	0	0.33	0.66	0.33	0.33	1	0.66	0
s_9	0.66	0.33	0	0.66	0.66	0	0	0.33	1	0.33
s_{10}	0	1	0.33	0.33	1	0	0.33	0	0	1

Table 7: Fuzzy matrix.

Table 8 summarizes the matrix G of fuzzy amicable degree.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0.44	0.44	0	0	1	0	0.33	0.44	0
s_2	0.44	1	0.66	0.5	0	0	0	0	0.33	0
s_3	0.44	0.66	1	0	0.33	0	0	0	0	0
s_4	0	0.5	0	1	0.66	0	0	0	0	0.33
s_5	0	0	0.33	0.66	1	0	0.44	0.66	0	0.8
s_6	1	0	0	0	0	1	0.44	0.44	0	0
s_7	0	0	0	0	0.44	0.44	1	0.33	0	0.33
s_8	0.33	0	0	0	0.66	0.44	0.33	1	0.44	0
s_9	0.44	0.33	0	0	0	0	0	0.44	1	0
s_{10}	0	0	0	0.33	0.8	0	0.33	0	0	1

Table 8: Matrix of fuzzy amicable degree.

Equivalently, we calculate matrices T^1, T^2, and T^3, which are summarized in Tables 9, 10, and 11, respectively.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0	1	1	0	1	0	1	0	0
s_2	0	1	1	1	1	0	1	0	0	0
s_3	1	1	1	0	0	1	0	0	1	0
s_4	0	1	0	1	1	1	0	0	1	0
s_5	0	0	1	0	1	0	1	0	1	1
s_6	1	0	0	0	0	1	1	1	1	0
s_7	0	1	0	1	1	0	1	0	1	0
s_8	0	0	1	0	1	0	1	1	1	0
s_9	1	1	0	0	0	0	0	1	1	1
s_{10}	0	1	0	0	1	0	1	0	1	1

Table 9: Evaluation matrix T^1 for Q_1.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	1	0	1	0	1	0	0	0	1
s_2	1	1	0	1	0	0	1	0	1	0
s_3	0	1	1	0	1	0	0	1	1	0
s_4	0	0	1	1	0	1	0	0	1	1
s_5	1	0	0	1	1	0	1	1	0	0
s_6	1	0	0	0	0	1	1	1	1	0
s_7	0	1	0	1	1	0	1	0	1	0
s_8	0	0	0	1	0	1	1	1	1	0
s_9	1	1	0	1	1	0	0	0	1	0
s_{10}	0	1	1	1	1	0	0	0	0	1

Table 10: Evaluation matrix $T^2$for Q_2.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0	1	1	0	1	0	0	1	0
s_2	1	1	1	1	0	0	1	0	0	0
s_3	0	1	1	0	0	1	1	0	0	1
s_4	0	1	1	1	1	1	0	0	0	0
s_5	1	0	0	1	1	0	0	1	0	1
s_6	1	1	1	0	0	1	0	0	1	0
s_7	1	0	0	0	0	1	1	1	0	1
s_8	1	1	1	0	1	0	0	1	0	0
s_9	1	1	0	1	1	0	0	0	1	0
s_{10}	0	1	0	1	1	0	0	1	0	1

Table 11: Evaluation matrix $T^3$for Q_3.

Table 12 contains the values of matrix T.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0.33	0.66	1	0	1	0	0.33	0.33	0.33
s_2	0.66	1	0.66	1	0.33	0	1	0	0.33	0
s_3	0.33	1	1	0	0.33	0.66	0.33	0.33	0.66	0.33
s_4	0	0.66	0.66	1	0.66	1	0	0	0.66	0.33
s_5	0.66	0	0.33	0.66	1	0	0.66	0.66	0.33	0.66
s_6	1	0.33	0.33	0	0	1	0.66	0.66	1	0
s_7	0.33	0.66	0	0.66	0.66	0.33	1	0.33	0.66	0.33
s_8	0.33	0.33	0.66	0.33	0.66	0.33	0.66	1	0.66	0
s_9	1	1	0	0.66	0.66	0	0	0.33	1	0.33
s_{10}	0	1	0.33	0.66	1	0	0.33	0.33	0.33	1

Table 12: Matrix T.

Table 13 summarizes the values of the amicable degrees in matrix $U = (u_{kl})$

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
s_1	1	0.44	0.44	0	0	1	0	0.33	0.5	0
s_2	0.44	1	0.8	0.8	0	0	0.8	0	0.5	0
s_3	0.44	0.8	1	0	0.33	0.44	0	0.44	0	0.33
s_4	0	0.8	0	1	0.66	0	0	0	0.66	0.44
s_5	0	0	0.33	0.66	1	0	0.66	0.66	0.44	0.8
s_6	1	0	0.44	0	0	1	0.44	0.44	0	0
s_7	0	0.8	0	0	0.66	0.44	1	0.44	0	0.33
s_8	0.33	0	0.44	0	0.66	0.44	0.44	1	0.44	0
s_9	0.5	0.5	0	0.66	0.44	0	0	0.44	1	0.33
s_{10}	0	0	0.33	0.44	0.8	0	0.33	0	0.33	1

Table 13: Matrix U.

According to Tables 8 and 13 we have that, for example, the relationship between the students s_2 and s_3 and its potentiality is $I_{23} = I_{32} = [0.66, 0.8]$, which means that currently the amicable degree between them is 0.66, however this degree can be potentially increased up to 0.8 in the future. Thus, the teacher should work to strengthen the relationship between these two students, instead of students s_1 and s_2 with $I_{12} = I_{21} = [0.44, 0.44]$, which seems to do not have the opportunity of changing and is weaker than the relationship between s_2 and s_3.

Calculating the leadership index with Equations 7 and 8, we have the following results:

$\mu =$
$(0.127217, 0.102159, 0.084811, 0.086739, 0.135698, 0.100231, 0.088666, 0.111796, 0.077101, 0.085582)$.

Whereas, $\theta =$
$(0.098068, 0.114461, 0.100117, 0.094262, 0.120609, 0.087822, 0.097190, 0.099532, 0.102459, 0.085480)$.

It is interpreted that student 5 is the leader according to $\mu(5) = 0.135698$ that is a maximum; however, potentially his/her leadership can slightly diminish because of $\theta(5) = 0.120609$.

Finally, we depict the neutrosophic sociogram of the example. See Figure 3.

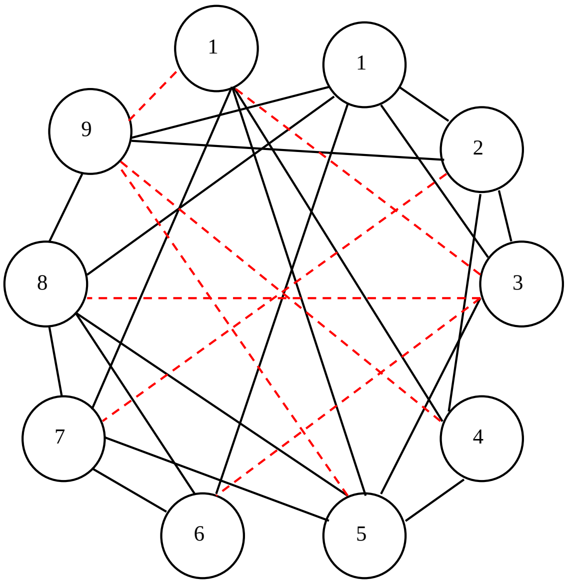

Figure 3: Neutrosophic sociogram of the example.

Let us note that the neutrosophic sociogram in Figure 3 shows in black continuous lines the relationships between the pair of students with fuzzy amicable degree bigger than 0, from Table 8. With dashed lines in red, we represent the edges with amicable degree bigger than 0 in matrix U and null value in matrix G, according to Table 13.

The results represented with continuous lines model the current preferences and the dashed lines represent the potential future links. For simplicity, we omitted in the graph the fuzzy amicable degrees values associated with the edges and the symbol I associated with the lines in red. Let us remark that this is a non-directed graph.

Conclusion

This paper introduces the neutrosophic sociograms. The crisp and fuzzy sociograms only take into account the preference relationships between individuals of the social group under investigation. However, it is possible that there are individuals in the group, especially if it is a large group, where some individuals do not know each other well and therefore are not designated as preferred ones. This type of relationship with lack of knowledge or lack of trust between two members can be consider an indeterminate relationship, where the future of the bond may be a preference relationship or a non-preference relationship, depending on group dynamics. Neutrosophic sociograms consider these indeterminacies, which are measured a possible future relationship. They are non-directed neutrosophic graphs. In this paper, we introduce a method to calculate the matrix of the graph, which is a neutrosophic matrix. The calculations include the weights or importance of each of the questions used to measure the preferred individuals to carry out the activities with. The advantage of defining a neutrosophic sociogram, instead of a crisp or fuzzy sociogram, is that it achieves greater accuracy in the representation of social relationships, and offers a better idea of what group dynamics is like and in which individuals the group cohesion can be strengthened. In future works we will study in depth the relationship of neutrosophic sociograms with Shapley value, taking into account that in the offsets [26, 27] there is an example of a solution for cooperative n-personal games using these neutrosophic sets [28].

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