Neutrosophic Sets and Systems, Vol. 54, 2023
University of New Mexico

On Some Algorithms for Solving Different Types of Symbolic 2-Plithogenic Algebraic Equations

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Abstract:

The main goal of this paper is to study three different types of algebraic symbolic 2-plithogenic equations. The symbolic 2-plithogenic linear Diophantine equations, symbolic 2-plithogenic quadratic equations, and linear system of symbolic 2-plithgenic equations will be discussed and handled, where algorithms to solve the previous types will be presented and proved by transforming them to classical algebraic systems of equations.

Keywords: symbolic 2-plithogenic Diophantine equation, symbolic 2-plithogenic quadratic equation, linear system, symbolic 2-plithogenic field

Introduction and preliminaries

The process of extending classical algebraic structures by using logical symbols and elements can be considered as a novel approach to generalize algebraic structures, where many algebraic structures were generalized by using neutrosophic elements, fuzzy elements, and refined neutrosophic elements [1-15].

Smarandache has defined the concept of Symbolic 2-plithogenic sets and structures [16-20] as new generalizations of classical structures. Also, he has presented many open research problems [20].

In [21], Smarandache ideas was discussed in a special case of n=2, where the symbolic 2-plithogenic rings were defined and studied with many elementary interesting substructures and properties.

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:
$2-SP_R=\{a_0+a_1P_1+a_2P_2;\ a_i\in R, {P_j}^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\}$.
Smarandache has defined algebraic operations on $2-SP_R$ as follows:
Addition:
$[a_0+a_1P_1+a_2P_2]+[b_0+b_1P_1+b_2P_2]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2$.
Multiplication:
$[a_0+a_1P_1+a_2P_2].[b_0+b_1P_1+b_2P_2]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_1b_0{P_1}^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2{P_2}^2+a_1b_1P_1P_1=a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2$.
It is clear that $(2-SP_R)$ is a ring.
If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field [22].
Also, the following open problems were asked in [22]:

Problem (3):

If F is a field then $2-SP_F$ is called a 2-plithogenic symbolic field. Now, can we find a strong algorithm that explains how can we solve the previous equations by turning it into the classical equations.

Problem (4):

If Z is the ring of integers ring then $2-SP_Z$ is called a 2-plithogenic symbolic ring of integers. Can we find a strong algorithm that explains how can we solve the previous equations by turning it into the classical Diophantine equations.
In this paper, we solve the previous two open problems by suggesting effective algorithms that help us to transform symbolic 2-plithogenic equations to classical algebraic equations.

Main Results

Definition.

Let $2-SP_Z=\{a+bP_1+cP_2;\ a,b,c\in Z\}$ be the symbolic 2-plithogenic ring of integers, the Diophantine equation with two variables is defined as follows:

$AX+BY=C; A=a_0+a_1P_1+a_2P_2, B=b_0+b_1P_1+b_2P_2, C=c_0+c_1P_1+c_2P_2,$

$X=x_0+x_1P_1+x_2P_2, Y=y_0+y_1P_1+y_2P_2, a_i,b_i,c_i,x_i,y_i\in 2-SP_Z.$

The following theorem describes an algorithm to solve the symbolic 2-plithogenic linear Diophantine equation with two variables.

Theorem.

Let $AX+BY=C$ be the symbolic 2-plithogenic linear Diophantine equation with two variables, it is solvable if and only if the following linear Diophantine equations are solvable.

$$\begin{cases} a_0x_0+b_0y_0=c_0 \\ (a_0+a_1)(x_0+x_1)+(b_0+b_1)(y_0+y_1)=c_0+c_1 \\ (a_0+a_1+a_2)(x_0+x_1+x_2)+(b_0+b_1+b_2)(y_0+y_1+y_2)=c_0+c_1+c_2 \end{cases}$$

Proof.

The equation $AX+BY=C$ equivalents:

$$\begin{aligned} a_0x_0+b_0y_0&+(a_0x_1+a_1x_0+a_1x_1+b_0y_1+b_1y_0+b_1y_1)P_1 \\ &+(a_0x_2+a_2x_0+a_2x_2+a_1x_2+a_2x_1+b_0y_2+b_2y_0+b_2y_2+b_1y_2+b_2y_1)P_2 \\ &=c_0+c_1P_1+c_2P_2 \end{aligned}$$

$$\begin{cases} a_0x_0+b_0y_0=c_0 \ldots (1) \\ a_0x_1+a_1x_0+a_1x_1+b_0y_1+b_1y_0+b_1y_1=c_1 \ldots (2) \\ a_0x_2+a_2x_0+a_2x_2+a_1x_2+a_2x_1+b_0y_2+b_2y_0+b_2y_2+b_1y_2+b_2y_1=c_2 \ldots (3) \end{cases}$$

We add (1) to (2), and (1) to (2) to (3), we get:

$$\begin{cases} a_0x_0+b_0y_0=c_0 \\ (a_0+a_1)(x_0+x_1)+(b_0+b_1)(y_0+y_1)=c_0+c_1 \\ (a_0+a_1+a_2)(x_0+x_1+x_2)+(b_0+b_1+b_2)(y_0+y_1+y_2)=c_0+c_1+c_2 \end{cases}$$

And the proof is complete.

The description of the algorithm.

To solve $AX+BY=C$ in $2-SP_Z$, we must follow these steps.

Step1.

We compute $gcd(a_0,b_0), gcd(a_0+a_1,b_0+b_1), gcd(a_0+a_1+a_2,b_0+b_1+b_2)$.

If $gcd(a_0,b_0)/c_0$, $gcd(a_0+a_1,b_0+b_1)/c_0+c_1$, $gcd(a_0+a_1+a_2,b_0+b_1+b_2)/c_0+c_1+c_2$, then it is solvable.

Step2.

We solve the equivalent system and get the values of $x_i, y_i; 0 \leq i \leq 2$.

Example.

Consider the following symbolic 2-plithogenic linear Diophantine equation:

$(2 + P_1 + P_2)X + (3 + 2P_1 - P_2)Y = 8 + 5P_1 + 7P_2$.

$gcd(a_0, b_0) = gcd(2,3) = 1/8$.

$gcd(a_0 + a_1, b_0 + b_1) = gcd(3,5) = 1/c_0 + c_1 = 13$.

$gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = gcd(4,4) = 4/c_0 + c_1 + c_2 = 20$.

So that, the equation is solvable.

The equivalent system of linear Diophantine equations is:

$$\begin{cases} 2x_0 + 3y_0 = 8 \ldots (1) \\ 3(x_0 + x_1) + 5(y_0 + y_1) = 13 \ldots (2) \\ 4(x_0 + x_1 + x_2) + 4(y_0 + y_1 + y_2) = 20 \ldots (3) \end{cases}$$

The equation (1) has a solution $(x_0 = 1, y_0 = 2)$.

The equation (2) has a solution $(x_0 + x_1 = 1, y_0 + y_1 = 2)$, there for $(x_1 = 0, y_1 = 0)$.

The equation (3) has a solution $(x_0 + x_1 + x_2 = 2, y_0 + y_1 + y_2 = 3)$, there for $(x_2 = 1, y_2 = 1)$.

This implies a solution $X = 1 + P_2, Y = 2 + P_2$.

Example.

Consider the following:

$(3 + P_1 + 5P_2)X + (6 - 2P_1 + 10P_2)Y = 5 + P_1 + P_2$.

$gcd(a_0, b_0) = gcd(3,6) = 3 \ddagger 5$, there for it is not solvable.

2-symbolic plithogenic Quadratic equation.

Let $2 - SP_F$ be a symbolic 2-plithogenic field, the formula

$AX^2 + BY^2 + C = 0; A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2,$

$C = c_0 + c_1P_1 + c_2P_2, X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2, a_i, b_i, c_i, x_i, y_i \in 2 - SP_F$.

Is called the symbolic 2-plithogenic quadratic equation.

Theorem.

Let $AX^2 + BY^2 + C = 0$ be a symbolic 2-plithogenic quadratic equation over $2 - SP_F$, then it is solvable if and only if the following system is solvable:

$$\begin{cases} a_0{x_0}^2 + b_0{y_0}^2 + c_0 = 0 \ldots (1) \\ (a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(y_0 + y_1)^2 + (c_0 + c_1) = 0 \ldots (2) \\ (a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2)^2 + (c_0 + c_1 + c_2) = 0 \ldots (3) \end{cases}$$

Proof.

We have $X^2 = {x_0}^2 + P_1[(x_0 + x_1)^2 - {x_0}^2] + P_2[(x_0 + x_1 + x_2)^2 - (x_0 + x_1)^2]$, see [ ].

So that:

$$AX^2 = a_0{x_0}^2 + P_1[a_0(x_0 + x_1)^2 - a_0{x_0}^2 + a_1(x_0 + x_1)^2 - a_1{x_0}^2 + a_1{x_0}^2]$$
$$+ P_2[a_0(x_0 + x_1 + x_2)^2 - a_0(x_0 + x_1)^2 + a_1(x_0 + x_1 + x_2)^2 - a_1(x_0 + x_1)^2$$
$$+ a_2(x_0 + x_1 + x_2)^2 - a_2(x_0 + x_1)^2 + a_2{x_0}^2 + a_2(x_0 + x_1)^2 - a_2{x_0}^2]$$

$$AX^2 = a_0{x_0}^2 + P_1[(a_0 + a_1)(x_0 + x_1)^2 - a_0{x_0}^2]$$
$$+ P_2[(a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 - (a_0 + a_1)(x_0 + x_1)^2]$$

There for, the equation $AX^2 + BY^2 + C = 0$ is equivalent to:

$$\begin{cases} a_0{x_0}^2 + b_0{y_0}^2 + c_0 = 0 \ldots (1) \\ (a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(y_0 + y_1)^2 + (c_0 + c_1) = 0 \ldots (2) \\ (a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2)^2 + (c_0 + c_1 + c_2) = 0 \ldots (3) \end{cases}$$

The description of algorithm.

To solve $AX^2 + BY^2 + C = 0$ in $2 - SP_F$, follow these steps:

Step1.

Solve the equivalent classical system of quadratic equations. If $(1), (2)$, and (3) are solvable in the field F,then the symbolic 2-plithogenic quadratic equation is solvable.

Step2.

Discuss all possible cases of x_0, x_1, x_2.

Remark.

If $AX^2 + BY^2 + C = 0$ is solvable in $2 - SP_F$, then it has at most 8 solutions.

Example.

Consider the following:

$(1 + P_1 + P_2)X^2 + (3 - P_1)X - 4 - 12P_2 = 0$

We have:

$$\begin{cases} a_0 = 1, a_1 = 1, a_2 = 1 \\ b_0 = 3, b_1 = -1, b_2 = 0 \\ c_0 = -4, c_1 = 0, c_2 = -12 \end{cases}$$

The equivalent system is:

$$\begin{cases} {x_0}^2 + 3x_0 - 4 = 0 \ldots (1) \\ 2(x_0 + x_1)^2 + 2(x_0 + x_1) - 4 = 0 \ldots (2) \\ 3(x_0 + x_1 + x_2)^2 + 2(x_0 + x_1 + x_2) - 16 = 0 \ldots (3) \end{cases}$$

The solutions of (1): $x_0 = 1, x_0 = -4$.

The solutions of (2): $x_0 + x_1 = 1, x_0 + x_1 = -2$.

The solutions of (3): $x_0 + x_1 + x_2 = 2, x_0 + x_1 + x_2 = -\frac{8}{3}$.

Case1.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 2$, then $x_1 = 0, x_2 = 1$, and $X = 1 + P_2$.

Case2.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = 0, x_2 = -\frac{11}{3}$, and $X = 1 - \frac{11}{3}P_2$.

Case3.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 2$, then $x_1 = -3, x_2 = 4$, and $X = 1 - 3P_1 + 4P_2$.

Case4.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = -3, x_2 = -\frac{2}{3}$, and $X = 1 - 3P_1 - \frac{2}{3}P_2$.

Case5.

If $x_0 = -4, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 2$, then $x_1 = 5, x_2 = 1$, and $X = -4 + 5P_1 + P_2$.

Case6.

If $x_0 = -4, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = 5, x_2 = -\frac{11}{3}$, and $X = -4 + 5P_1 - \frac{11}{3}P_2$.

Case7.

If $x_0 = -4, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 2$, then $x_1 = 2, x_2 = 4$, and $X = -4 + 2P_1 + 4P_2$.

Case8.

If $x_0 = -4, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = 2, x_2 = -\frac{2}{3}$, and $X = -4 + 2P_1 - \frac{2}{3}P_2$.

So that, the solutions of the original symbolic 2-plithogenic quadratic equation are:

$$X \in \left\{-4 + 2P_1 - \frac{2}{3}P_2, -4 + 2P_1 + 4P_2, -4 + 5P_1 - \frac{11}{3}P_2, -4 + 5P_1 + P_2, 1 - 3P_1 - \frac{2}{3}P_2, 1 - 3P_1 + 4P_2, 1 - \frac{11}{3}P_2, 1 + P_2\right\}$$

Example.

Consider the following:

$(2 + 3P_1 - P_2)X^2 + (4 + P_1 + P_2)X - 6 - 4P_1 = 0$

We have:

$$\begin{cases} a_0 = 2, a_1 = 3, a_2 = -1 \\ b_0 = 4, b_1 = 1, b_2 = 1 \\ c_0 = -6, c_1 = -4, c_2 = 0 \end{cases}$$

The equivalent system is:

$$\begin{cases} 2{x_0}^2 + 4x_0 - 6 = 0 \dots (1) \\ 5(x_0 + x_1)^2 + 5(x_0 + x_1) - 10 = 0 \dots (2) \\ 4(x_0 + x_1 + x_2)^2 + 6(x_0 + x_1 + x_2) - 10 = 0 \dots (3) \end{cases}$$

The solutions of (1): $x_0 = 1, x_0 = -3$.

The solutions of (2): $x_0 + x_1 = 1, x_0 + x_1 = -2$.

The solutions of (3): $x_0 + x_1 + x_2 = 1, x_0 + x_1 + x_2 = -\frac{5}{2}$.

Case1.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 1$, then $x_1 = x_2 = 0$, and $X = 1$.

Case2.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = 0, x_2 = -\frac{7}{2}$, and $X = 1 - \frac{5}{2}P_2$.

Case3.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 1$, then $x_1 = -3, x_2 = 3$, and $X = 1 - 3P_1 + 3P_2$.

Case4.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = -3, x_2 = -\frac{1}{2}$, and $X = 1 - 3P_1 - \frac{1}{2}P_2$.

Case5.

If $x_0 = -3, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 1$, then $x_1 = 4, x_2 = 0$, and $X = -3 + 4P_1$.

Case6.

If $x_0 = -3, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = 4, x_2 = -\frac{7}{2}$, and $X = -3 + 4P_1 - \frac{7}{2}P_2$.

Case7.

If $x_0 = -3, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 1$, then $x_1 = 1, x_2 = 3$, and $X = -3 + P_1 + 3P_2$.

Case8.

If $x_0 = -3, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = 1, x_2 = -\frac{1}{2}$, and $X = -3 + P_1 - \frac{1}{2}P_2$.

So that, the solutions of the original symbolic 2-plithogenic quadratic equation are:

$$X \in \left\{1, 1 - \frac{5}{2}P_2, 1 - 3P_1 + 3P_2, 1 - 3P_1 - \frac{1}{2}P_2, -3 + 4P_1, -3 + 4P_1 - \frac{7}{2}P_2, -3 + P_1 + 3P_2, -3 + P_1 - \frac{1}{2}P_2\right\}$$

2-plithogenic Linear equations.

We begin the simplest case, a symbolic 2-plithogenic linear equation with one variable $A.X = B$.

This equation is solvable uniquely if and only if A is invertible and $X = A^{-1}B$.

According to [31], $A^{-1} = {a_0}^{-1} + P_1[(a_0 + a_1)^{-1} - {a_0}^{-1}] + P_2[(a_0 + a_1 + a_2)^{-1} - (a_0 + a_1)^{-1}]$.

Example.

Consider the equation $(2 + P_1 + P_2)X = 3 - P_1$ over $2 - SP_R$.

$a_0 = 2, {a_0}^{-1} = \frac{1}{2}, a_0 + a_1 = 3, (a_0 + a_1)^{-1} = \frac{1}{3}, a_0 + a_1 + a_2 = 4, (a_0 + a_1 + a_2)^{-1} = \frac{1}{4}$, thus:

$A^{-1} = \frac{1}{2} - \frac{1}{6}P_1 - \frac{1}{12}P_2$, there for:

$$X = \left(\frac{1}{2} - \frac{1}{6}P_1 - \frac{1}{12}P_2\right)(3 - P_1) = \frac{3}{2} - \frac{1}{2}P_1 - \frac{1}{2}P_1 + \frac{1}{6}P_1 - \frac{1}{4}P_2 + \frac{1}{12}P_2 = \frac{3}{2} - \frac{5}{6}P_1 - \frac{1}{6}P_2$$

The general case is about a linear system of n symbolic 2-plithogenic equations $A_i.X_i = B_i; 1 \le i \le n$.

To solve a system like that, we must transform it to an equivalent classical system. We present the following algorithm.

To solve the symbolic 2-plithogenic linear system:

$$\begin{cases} A_{11}.X_1 + A_{12}.X_2 + \cdots + A_{1n}.X_n = B_{1n} \\ A_{21}.X_1 + A_{22}.X_2 + \cdots + A_{2n}.X_n = B_{2n} \\ \vdots \\ A_{n1}.X_1 + A_{n2}.X_2 + \cdots + A_{nn}.X_n = B_{nn} \end{cases}$$

Where: $A_{ij} = a_{ij}^{(0)} + a_{ij}^{(1)}P_1 + a_{ij}^{(2)}P_2, X_i = X_i^{(0)} + X_i^{(1)}P_1 + X_i^{(2)}P_2, B_{ij} = b_{ij}^{(0)} + b_{ij}^{(1)}P_1 + b_{ij}^{(2)}P_2 \in 2 - SP_F$.

Follow these steps:

Step1.

Find the classical equivalent system as follows:

$$\begin{cases} \sum_{i,j=1}^{n} a_{ij}^{(0)} X_i^{(0)} = \sum_{i,j=1}^{n} b_{ij}^{(0)} \\ \sum_{i,j=1}^{n} \left(a_{ij}^{(0)} + a_{ij}^{(1)}\right)\left(X_i^{(0)} + X_i^{(1)}\right) = \sum_{i,j=1}^{n} \left(b_{ij}^{(0)} + b_{ij}^{(1)}\right) \\ \sum_{i,j=1}^{n} \left(a_{ij}^{(0)} + a_{ij}^{(1)} + a_{ij}^{(2)}\right)\left(X_i^{(0)} + X_i^{(1)} + X_i^{(2)}\right) = \sum_{i,j=1}^{n} \left(b_{ij}^{(0)} + b_{ij}^{(1)} + b_{ij}^{(2)}\right) \end{cases}$$

step2.

Solve each system and remark that:

The first system gives the values of $X_i^{(0)}; 1 \le i \le n$.

The second one gives the values of $X_i^{(0)} + X_i^{(1)}; 1 \le i \le n$.

The third one gives values of $X_i^{(0)} + X_i^{(1)} + X_i^{(2)}; 1 \le i \le n$.

Step3.

If each system is solvable, then the original 2-plithogenic system is solvable, and if the number of solutions of every classical system is k, then the number of solutions for the 2-plithogenic system is k^3.

Example.

Consider the following symbolic 2-plithogenic system of three linear equations with three variables:

$$\begin{cases} (1+P_2)X_1 + (3-P_1)X_2 + (1+P_1-P_2)X_3 = 5 \\ P_2X_1 + P_1X_2 + (P_1-P_2)X_3 = 2P_1 + 2P_2 \\ (1+P_1-P_2)X_1 + (4+3P_1-P_2)X_2 + (5+2P_2)X_3 = 11 + 4P_2 \end{cases}$$

the equivalent classical systems are:

$$\begin{cases} X_1^{(0)} + 3X_2^{(0)} + X_3^{(0)} = 5 \\ 0X_1^{(0)} + 0X_2^{(0)} + 0X_3^{(0)} = 0 \quad \ldots \text{system(1)} \\ 2X_1^{(0)} + 4X_2^{(0)} + 5X_3^{(0)} = 11 \end{cases}$$

$$\begin{cases} \left(X_1^{(0)} + X_1^{(1)}\right) + 2\left(X_2^{(0)} + X_2^{(1)}\right) + 2\left(X_3^{(0)} + X_3^{(1)}\right) = 5 \\ 0\left(X_1^{(0)} + X_1^{(1)}\right) + \left(X_2^{(0)} + X_2^{(1)}\right) + \left(X_3^{(0)} + X_3^{(1)}\right) = 2 \quad \ldots \text{system(2)} \\ 3\left(X_1^{(0)} + X_1^{(1)}\right) + 7\left(X_2^{(0)} + X_2^{(1)}\right) + 5\left(X_3^{(0)} + X_3^{(1)}\right) = 15 \end{cases}$$

$$\begin{cases} 2\left(X_1^{(0)} + X_1^{(1)} + +X_1^{(2)}\right) + 2\left(X_2^{(0)} + X_2^{(1)} + X_2^{(2)}\right) + \left(X_3^{(0)} + X_3^{(1)} + X_3^{(2)}\right) = 5 \\ \left(X_1^{(0)} + X_1^{(1)} + +X_1^{(2)}\right) + \left(X_2^{(0)} + X_2^{(1)} + X_2^{(2)}\right) + 2\left(X_3^{(0)} + X_3^{(1)} + X_3^{(2)}\right) = 4 \quad \ldots \text{system(3)} \\ 2\left(X_1^{(0)} + X_1^{(1)} + +X_1^{(2)}\right) + 6\left(X_2^{(0)} + X_2^{(1)} + X_2^{(2)}\right) + 7\left(X_3^{(0)} + X_3^{(1)} + X_3^{(2)}\right) = 15 \end{cases}$$

The system(1) has infinite solutions, thus the 2-plithogenic system has infinite solutions.

We will find some solutions to clarify the algorithm.

For example system(1) has a solution $X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 1$.

The system (2) has a solution $X_1^{(0)} + X_1^{(1)} = X_2^{(0)} + X_2^{(1)} = X_3^{(0)} + X_3^{(1)} = 1$, thus $X_1^{(1)} = X_2^{(1)} = X_3^{(1)} = 0$.

The system (3) has a solution $X_1^{(0)} + X_1^{(1)} + X_1^{(2)} = X_2^{(0)} + X_2^{(1)} + X_2^{(2)} = X_3^{(0)} + X_3^{(1)} + X_3^{(2)} = 1$, there for $X_1^{(2)} = X_2^{(2)} = X_3^{(2)} = 0$, and $X_1 = X_1^{(0)} + X_1^{(1)}P_1 + X_1^{(2)}P_2 = 1$, $X_2 = 1$,$X_3 = 1$ is a solution for the 2-plithogenic system.

Also, the system (1) has a solution $X_1^{(0)} = \frac{13}{2}, X_2^{(0)} = -\frac{1}{2}, X_3^{(0)} = 0$.

The system (2) has a solution $X_1^{(0)} + X_1^{(1)} = X_2^{(0)} + X_2^{(1)} = X_3^{(0)} + X_3^{(1)} = 1$.

The system (3) has a solution $X_1^{(0)} + X_1^{(1)} + X_1^{(2)} = X_2^{(0)} + X_2^{(1)} + X_2^{(2)} = X_3^{(0)} + X_3^{(1)} + X_3^{(2)} = 1$.

There for $X_1^{(1)} = 1 - \frac{13}{2} = -\frac{11}{2}, X_2^{(1)} = 1 + \frac{1}{2} = \frac{3}{2}, X_3^{(1)} = 1,\ X_1^{(2)} = X_2^{(2)} = X_3^{(2)} = 0$.

This implies that:

$X_1 = \frac{13}{2} - \frac{11}{2}P_1, X_2 = -\frac{1}{2} + \frac{3}{2}P_1, X_3 = P_1$ is a solution of the 2-plithogenic system.

Conclusion

In this paper, we have presented novel algorithms to solve many different types of 2-plithogenic algebraic equations (quadratic, linear, and linear Diophantine equations) by transforming them to classical systems of algebraic equations. Also, many examples were illustrated tp explain the validity of our work.

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Received: December 31, 2022. **Accepted:** March 30, 2023