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Generalised single valued neutrosophic number and its application to neutrosophic linear programming

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Abstract . In this paper, the concept of single valued neutrosophic number (SVN-number) is presented in a generalized way. Using this notion, a crisp linear programming problem (LP-problem) is extended to a neutrosophic linear programming problem (NLP-problem). The coefficients of the objective function of a crisp LP-problem are considered as generalized single valued neutrosophic number (G_{SVN}-number). This modified form of LP-problem is here called an NLP-problem. An algorithm is developed to solve NLP-problem by simplex method. Finally, this simplex algorithm is applied to a real life problem. The problem is illustrated and solved numerically.

Keywords : Single valued neutrosophic number; Neutrosophic linear programming problem; Simplex method.

1 Introduction

Introduction of fuzzy set by Zadeh [10] and then intuitionistic fuzzy set by Atanassov [8] brought a golden opportunity to handle the uncertainty and vagueness in our daily life activities. The fuzzy sets are evaluated by the membership grade of an object only, whereas intuitionistic fuzzy set meets the membership and the non-membership grade of an object simultaneously. To deal with uncertainty more precisely, Smarandache [3,4] initiated the notion of neutrosophic set (NS), a generalised version of classical set, fuzzy set, intuitionistic fuzzy set etc. In the neutrosophic logic, each proposition is estimated by a triplet $viz.$, truth grade, indeterminacy grade and falsity grade. The indeterministic part of uncertain data, introduced in NS theory, plays an important role to make a proper decision which is not possible by intuitionistic fuzzy set theory. Since indeterminacy always appears in our routine activities, the NS theory can analyse the various situations smoothly. But it is too difficult to apply the NS theory in real life scenario for it's initial character as pointed out by Smarandache. So to apply in real spectrum, Wang et al. [6] brought the concept of single valued neutrosophic set (SVN-set). Ranking of fuzzy number and intuitionistic fuzzy number is an interesting subject needed in decision making, optimization, even in developing of various mathematical structures. From time to time, several ranking methods [2,5,9,13-15] have been adopted by researchers. Naturally, the ranking of neutrosophic number also was come into consideration from beginning of NS theory. Deli and Subas [7] considered a ranking way of neutrosophic numbers and have used it to a decision making problems. Abdel-Baset [11,12] solved group decision making problems based on TOPSIS technique by use of neutrosophic number. To estimate and solve the NLP-problem in different direction, some respective attempts [1,16] by researchers are seen.

This paper introduces the structure of SVN-number in a different way to opt the notion of generalized single valued trapezoidal neutrosophic number (G_{SVTN}-number), generalized single valued triangular neutrosophic number (G_{SVTrN}-number) and develops an algorithm to solve NLP-problem by simplex method. The proposed simplex algorithm is applied to a real life problem. The problem is illustrated and solved numerically.

The organisation of this paper is as follows. Section 2 deals some preliminary definitions. The concept of G_{SVN}-number, G_{SVTN}-number, G_{SVTrN}-number and their respective parametric form are presented in Section 3. The concept of NLP-problem and it's solution procedure are proposed in Section 4 and Section 5, respectively. In Section 6, the simplex method is illustrated by suitable examples. Finally, the present work is summarised in Section 7.

2 Preliminaries

Some basic definitions are provided to bring the main thought of this paper here.

2.1 Definition [18]

A continuous t- norm $*$ and t- conorm $\diamond$ are two continuous binary operations assigning $[0,1]\times[0,1]\to[0,1]$ and obey the under stated principles :
(i) $*$ and $\diamond$ are both commutative and associative.
(ii) $x*1=1*x=x$ and $x\diamond 0=0\diamond x=x,\ \forall x\in[0,1]$.
(iii) $x*y\le p*q$ and $x\diamond y\le p\diamond q$ if $x\le p,\ y\le q$ with $x,y,p,q\in[0,1]$.
$x*y=xy, x*y=\min\{x,y\}, x*y=\max\{x+y-1,0\}$ are most useful t-norms and
$x\diamond y=x+y-xy, x\diamond y=\max\{x,y\}, x\diamond y=\min\{x+y,1\}$ are most useful t-conorms.

2.2 Definition [3]

An NS Q on an initial universe X is presented by three characterisations namely true value T_Q, indeterminant value I_Q and false value F_Q so that $T_Q, I_Q, F_Q : X \to]^-0,1^+[$. Thus Q can be designed as : $\{<u,(T_Q(u),I_Q(u),F_Q(u))>: u\in X\}$ with $^-0\le\sup T_Q(u)+\sup I_Q(u)+\sup F_Q(u)\le 3^+$. Here $1^+=1+\delta$, where 1 is standard part and δ is non-standard part. Similarly $^-0=0-\delta$. The non-standard set $]^-0,1^+[$ is basically practiced in philosophical ground and because of the difficulty to adopt it in real field, the standard subset of $]^-0,1^+[$ i.e., [0,1] is applicable in real neutrosophic environment.

2.3 Definition [6]

An SVN-set Q over a universe X is a set $Q=\{<x,T_Q(x),I_Q(x),F_Q(x)>: x\in X$ and $T_Q(x),I_Q(x),F_Q(x)\in[0,1]\}$ with $0\le\sup T_Q(x)+\sup I_Q(x)+\sup F_Q(x)\le 3$.

2.4 Definition [7]

Let $a_i,b_i,c_i,d_i\in\mathbf{R}$ (the set of all real numbers) with $a_i\le b_i\le c_i\le d_i\ (i=1,2,3)$ and $w_{\tilde p},u_{\tilde p},y_{\tilde p}\in[0,1]\subset\mathbf{R}$. Then an SVN-number $\tilde p=\langle([a_1,b_1,c_1,d_1];w_{\tilde p}),([a_2,b_2,c_2,d_2];u_{\tilde p}),([a_3,b_3,c_3,d_3];y_{\tilde p})\rangle$ is a special SVN-set on $\mathbf{R}$ whose true value, indeterminant value, false value are respectively defined by the mappings $T_{\tilde p}:\mathbf{R}\to[0,w_{\tilde p}]$, $I_{\tilde p}:\mathbf{R}\to[u_{\tilde p},1]$, $F_{\tilde p}:\mathbf{R}\to[y_{\tilde p},1]$ and they are given as :

$$T_{\tilde p}(x)=\begin{cases} g_T^l(x), & a_1\le x\le b_1,\\ w_{\tilde p}, & b_1\le x\le c_1,\\ g_T^r(x), & c_1\le x\le d_1,\\ 0, & \text{otherwise.}\end{cases}\quad I_{\tilde p}(x)=\begin{cases} g_I^l(x), & a_2\le x\le b_2,\\ u_{\tilde p}, & b_2\le x\le c_2,\\ g_I^r(x), & c_2\le x\le d_2,\\ 1, & \text{otherwise.}\end{cases}\quad F_{\tilde p}(x)=\begin{cases} g_F^l(x), & a_3\le x\le b_3,\\ y_{\tilde p}, & b_3\le x\le c_3,\\ g_F^r(x), & c_3\le x\le d_3,\\ 1, & \text{otherwise.}\end{cases}$$

The functions $g_T^l:[a_1,b_1]\to[0,w_{\tilde p}], g_I^r:[c_2,d_2]\to[u_{\tilde p},1], g_F^r:[c_3,d_3]\to[y_{\tilde p},1]$ are continuous and non-decreasing functions satisfying : $g_T^l(a_1)=0, g_T^l(b_1)=w_{\tilde p}, g_I^r(c_2)=u_{\tilde p}, g_I^r(d_2)=1, g_F^r(c_3)=y_{\tilde p}, g_F^r(d_3)=1$.

The functions $g_T^r : [c_1, d_1] \to [0, w_{\tilde{p}}], g_I^l : [a_2, b_2] \to [u_{\tilde{p}}, 1], g_F^l : [a_3, b_3] \to [y_{\tilde{p}}, 1]$ are continuous and non-increasing functions satisfying : $g_T^r(c_1) = w_{\tilde{p}}, g_T^r(d_1) = 0, g_I^l(a_2) = 1, g_I^l(b_2) = u_{\tilde{p}}, g_F^l(a_3) = 1, g_F^l(b_3) = y_{\tilde{p}}$.

2.4.1 Definition [7]

If $[a_1, b_1, c_1, d_1] = [a_2, b_2, c_2, d_2] = [a_3, b_3, c_3, d_3]$, then the SVN-number $\tilde{p}$ is reduced to a single valued trapezoidal neutrosophic number as : $\tilde{p} = \langle([a_1, b_1, c_1, d_1]; w_{\tilde{p}}, u_{\tilde{p}}, y_{\tilde{p}})\rangle$.
$\tilde{p} = \langle([a_1, b_1, d_1]; w_{\tilde{p}}, u_{\tilde{p}}, y_{\tilde{p}})\rangle$ is called a single valued triangular neutrosophic number if $b_1 = c_1$.

2.5 Definition [17]

The (α, β, γ)-cut of an NS P is denoted by $P_{(\alpha,\beta,\gamma)}$ and is defined as : $P_{(\alpha,\beta,\gamma)} = \{x \in X : T_P(x) \geq \alpha, I_P(x) \leq \beta, F_P(x) \leq \gamma\}$ with $\alpha, \beta, \gamma \in [0, 1]$ and $0 \leq \alpha + \beta + \gamma \leq 3$. Clearly, it is a crisp subset X.

2.6 Definition [14]

In parametric form, a fuzzy number P is a pair (P_L, P_R) of functions $P_L(r), P_R(r), r \in [0, 1]$ satisfying the followings.
(i) Both are bounded functions.
(ii) P_L is monotone increasing left continuous and P_R is monotone decreasing right continuous function.
(iii) $P_L(r) \leq P_R(r), 0 \leq r \leq 1$.
A trapezoidal fuzzy number is put as $P = (x_0, y_0, \delta, \zeta)$ where $[x_0, y_0]$ is interval defuzzifier and $\delta(> 0), \zeta(> 0)$ are respectively called left fuzziness, right fuzziness. $(x_0 - \delta, y_0 + \zeta)$ is the support of P and it's membership function is :

$$P(x) = \begin{cases} \frac{1}{\delta}(x - x_0 + \delta), & x_0 - \delta \leq x \leq x_0, \\ 1, & x \in [x_0, y_0], \\ \frac{1}{\zeta}(y_0 - x + \zeta), & y_0 \leq x \leq y_0 + \zeta, \\ 0, & \text{otherwise.} \end{cases}$$

In parametric form $P_L(r) = x_0 - \delta + \delta r,\ P_R(r) = y_0 + \zeta - \zeta r$.
For arbitrary trapezoidal fuzzy numbers $P = (P_L, P_R), Q = (Q_L, Q_R)$ and scalar $k > 0$, the addition and scalar multiplication are $P + Q, kQ$ and they are defined by :
$(P + Q)_L(r) = P_L(r) + Q_L(r),\ \ (P + Q)_R(r) = P_R(r) + Q_R(r)$ and
$(kQ)_L(r) = kQ_L(r),\ \ (kQ)_R(r) = kQ_R(r)$.

3 Generalised single valued neutrosophic number

Here, the structure of G_{SVN}-number, G_{SVTN}-number and G_{SVTrN}-number have been presented.

3.1 Definition

• The support of three components of an SVN-set Q over X are given by a triplet $(S_{Q_T}, S_{Q_I}, S_{Q_F})$ where $S_{Q_T} = \{u \in X | T_Q(u) > 0\}, S_{Q_I} = \{u \in X | I_Q(u) < 1\}, S_{Q_F} = \{u \in X | F_Q(u) < 1\}$.
• The height of the components of Q are given by a triplet $(H_{Q_T}, H_{Q_I}, H_{Q_F})$ where $H_{Q_T} = \max\{T_Q(u) | u \in X\}, H_{Q_I} = \max\{I_Q(u) | u \in X\}, H_{Q_F} = \max\{F_Q(u) | u \in X\}$.

3.1.1 *Example*

Define an SVN-set Q on $\{0, 1, \cdots, 10\} \subset \mathbf{Z}$ (the set of integers) as : $\{< u, (\frac{u}{1+u}, 1-\frac{1}{2^u}, \frac{1}{1+u}) > | 0 \le u \le 10\}$. Then $S_{Q_T} = \{1, \cdots, 10\}$, $S_{Q_I} = \{0, \cdots, 10\}$, $S_{Q_F} = \{1, \cdots, 10\}$ and $H_{Q_T} = 0.909$ at $u = 10$, $H_{Q_I} = 0.999$ at $u = 10$, $H_{Q_F} = 1$ at $u = 0$.

3.2 Definition

A G_{SVN}-number $\tilde{p} = \langle([a_1, b_1, \sigma_1, \eta_1]; w_{\tilde{p}}), ([a_2, b_2, \sigma_2, \eta_2]; u_{\tilde{p}}), ([a_3, b_3, \sigma_3, \eta_3]; y_{\tilde{p}})\rangle$ is a special SVN-set on $\mathbf{R}$ where $\sigma_i(> 0), \eta_i(> 0)$ are respectively called left spreads, right spreads and $[a_i, b_i]$ are the modal intervals of truth, indeterminacy and falsity functions for $i = 1, 2, 3$ respectively in $\tilde{p}$ and $w_{\tilde{p}}, u_{\tilde{p}}, y_{\tilde{p}} \in [0, 1] \subset \mathbf{R}$. The truth, indeterminacy and falsity functions are defined as follows :

$$T_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_1} w_{\tilde{p}}(x - a_1 + \sigma_1), & a_1 - \sigma_1 \le x \le a_1, \\ w_{\tilde{p}}, & x \in [a_1, b_1], \\ \frac{1}{\eta_1} w_{\tilde{p}}(b_1 - x + \eta_1), & b_1 \le x \le b_1 + \eta_1, \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_2}(a_2 - x + u_{\tilde{p}}(x - a_2 + \sigma_2)), & a_2 - \sigma_2 \le x \le a_2, \\ u_{\tilde{p}}, & x \in [a_2, b_2], \\ \frac{1}{\eta_2}(x - b_2 + u_{\tilde{p}}(b_2 - x + \eta_2)), & b_2 \le x \le b_2 + \eta_2, \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_3}(a_3 - x + y_{\tilde{p}}(x - a_3 + \sigma_3)), & a_3 - \sigma_3 \le x \le a_3, \\ y_{\tilde{p}}, & x \in [a_3, b_3], \\ \frac{1}{\eta_3}(x - b_3 + y_{\tilde{p}}(b_3 - x + \eta_3)), & b_3 \le x \le b_3 + \eta_3, \\ 1, & \text{otherwise.} \end{cases}$$

In parametric form, a G_{SVN}-number $\tilde{p}$ consists of three pairs $(T^l_{\tilde{p}}, T^u_{\tilde{p}}), (I^l_{\tilde{p}}, I^u_{\tilde{p}}), (F^l_{\tilde{p}}, F^u_{\tilde{p}})$ of functions $T^l_{\tilde{p}}(r), T^u_{\tilde{p}}(r)$, $I^l_{\tilde{p}}(r), I^u_{\tilde{p}}(r), F^l_{\tilde{p}}(r), F^u_{\tilde{p}}(r), r \in [0, 1]$ satisfying the followings.

(i) $T^l_{\tilde{p}}, I^u_{\tilde{p}}, F^u_{\tilde{a}}$ are bounded monotone increasing continuous function.

(ii) $T^u_{\tilde{p}}, I^l_{\tilde{p}}, F^l_{\tilde{a}}$ are bounded monotone decreasing continuous function.

(iii) $T^l_{\tilde{p}}(r) \le T^u_{\tilde{p}}(r), I^l_{\tilde{p}}(r) \ge I^u_{\tilde{p}}(r), F^l_{\tilde{p}}(r) \ge F^u_{\tilde{p}}(r), r \in [0, 1]$.

3.2.1 Definition

• The support of the components of a G_{SVN}-number $\tilde{p}$ are given by a triplet $(S_{P_T}, S_{P_I}, S_{P_F})$ where $S_{P_T} = \{x \in \mathbf{R} | T_{\tilde{p}}(x) > 0\}, S_{P_I} = \{x \in \mathbf{R} | I_{\tilde{p}}(x) < 1\}, S_{P_F} = \{x \in \mathbf{R} | F_{\tilde{p}}(x) < 1\}$.

• The height of the components of $\tilde{p}$ are given by a triplet $(H_{P_T}, H_{P_I}, H_{P_F})$ where $H\tilde{p}_T = w_{\tilde{p}}, H\tilde{p}_I = 1 - u_{\tilde{p}}, H\tilde{p}_F = 1 - y_{\tilde{p}}$.

• The boundaries of the truth function of $\tilde{p}$ are : $LB_{\tilde{p}_T} = (a_1 - \sigma_1, a_1)$ and $RB_{\tilde{p}_T} = (b_1, b_1 + \eta_1)$. $LB_{\tilde{p}_T}$ and $RB_{\tilde{p}_T}$ are respectively called left boundary and right boundary for truth function of $\tilde{p}$. Similarly, $LB_{\tilde{p}_I} = (a_2 - \sigma_2, a_2)$, $RB_{\tilde{p}_I} = (b_2, b_2 + \eta_2)$ and $LB_{\tilde{p}_F} = (a_3 - \sigma_3, a_3)$, $RB_{\tilde{p}_F} = (b_3, b_3 + \eta_3)$.

• The core for the truth function of $\tilde{p}$ is a set of points at which it's height is measured. Similarly, the core for other two components are defined.

3.2.2 *Example*

Consider a G_{SVN}-number $\tilde{p}$ on **R** whose three components are as follows :

$$T_{\tilde{p}}(x)=\begin{cases}\frac{0.6(x-11)}{4}, & x\in[11,15]\\ 0.6, & x\in[15,25]\\ \frac{0.6(36-x)}{11}, & x\in[25,36]\\ 0, & \text{otherwise.}\end{cases}\quad I_{\tilde{p}}(x)=\begin{cases}\frac{4.4-0.1x}{4}, & x\in[4,8]\\ 0.9, & x\in[8,13]\\ \frac{0.1x+5}{7}, & x\in[13,20]\\ 1, & \text{otherwise.}\end{cases}\quad F_{\tilde{p}}(x)=\begin{cases}\frac{26-x}{3}, & x\in[23,26]\\ 0, & x\in[26,30]\\ \frac{x-30}{8}, & x\in[30,38]\\ 1, & \text{otherwise.}\end{cases}$$

Then $S_{P_T}=(11,36),\ S_{P_I}=(4,20)$ and $S_{P_F}=(23,38)$.
For that $\tilde{p}$, $H\tilde{p}_T=0.6, H\tilde{p}_I=0.1, H\tilde{p}_F=1$. Here,
$LB_{\tilde{p}_T}=(11,15), RB_{\tilde{p}_T}=(25,36);\ \ LB_{\tilde{p}_I}=(4,8), RB_{\tilde{p}_I}=(13,20);\ LB_{\tilde{p}_F}=(23,26), RB_{\tilde{p}_F}=(30,38)$.
The core of truth, indeterminacy and falsity function are $[15,25]$, $[8,13]$, $[26,30]$ respectively.

3.3 Definition

Let us assume two G_{SVN}-numbers $\tilde{p}$ and $\tilde{q}$ as follows :
$\tilde{p}=\langle([a_1,a'_1,\sigma_1,\eta_1];w_{\tilde{p}}),([a_2,a'_2,\sigma_2,\eta_2];u_{\tilde{p}}),([a_3,a'_3,\sigma_3,\eta_3];y_{\tilde{p}})\rangle$,
$\tilde{q}=\langle([b_1,b'_1,\xi_1,\delta_1];w_{\tilde{q}}),([b_2,b'_2,\xi_2,\delta_2];u_{\tilde{q}}),([b_3,b'_3,\xi_3,\delta_3];y_{\tilde{q}})\rangle$.
Then for any real number x,

(i) Image of $\tilde{p}$:
$-\tilde{p}=\langle([-a'_1,-a_1,\eta_1,\sigma_1];w_{\tilde{p}}),([-a'_2,-a_2,\eta_2,\sigma_2];u_{\tilde{p}}),([-a'_3,-a_3,\eta_3,\sigma_3];y_{\tilde{p}})\rangle$.

(ii) Addition :
$\tilde{p}+\tilde{q}=\langle([a_1+b_1,a'_1+b'_1,\sigma_1+\xi_1,\eta_1+\delta_1];w_{\tilde{p}}*w_{\tilde{q}}),([a_2+b_2,a'_2+b'_2,\sigma_2+\xi_2,\eta_2+\delta_2];u_{\tilde{p}}\diamond u_{\tilde{q}}),$
$([a_3+b_3,a'_3+b'_3,\sigma_3+\xi_3,\eta_3+\delta_3];y_{\tilde{p}}\diamond y_{\tilde{q}})\rangle$.

(iii) Scalar multiplication :
$x\tilde{p}=\langle([xa_1,xa'_1,x\sigma_1,x\eta_1];w_{\tilde{p}}),([xa_2,xa'_2,x\sigma_2,x\eta_2];u_{\tilde{p}}),([xa_3,xa'_3,x\sigma_3,x\eta_3];y_{\tilde{p}})\rangle$
for $x>0$.
$x\tilde{p}=\langle([xa'_1,xa_1,-x\eta_1,-x\sigma_1];w_{\tilde{p}}),([xa'_2,xa_2,-x\eta_2,-x\sigma_2];u_{\tilde{p}}),([xa'_3,xa_3,-x\eta_3,-x\sigma_3];y_{\tilde{p}})\rangle$
for $x<0$.

3.4 Corollary

Let $\tilde{p}=\langle([a_1,b_1,\sigma_1,\eta_1];w_{\tilde{p}}),([a_2,b_2,\sigma_2,\eta_2];u_{\tilde{p}}),([a_3,b_3,\sigma_3,\eta_3];y_{\tilde{p}})\rangle$ be an G_{SVN}-number.
1. Any α-cut set of the G_{SVN}-number $\tilde{p}$ for truth function is denoted by $\tilde{p}_\alpha$ and is given by a closed interval as :

$$\tilde{p}_\alpha=[L_{\tilde{p}}(\alpha),R_{\tilde{p}}(\alpha)]=[a_1-\sigma_1+\frac{\sigma_1\alpha}{w_{\tilde{p}}},b_1+\eta_1-\frac{\eta_1\alpha}{w_{\tilde{p}}}],\quad \text{for } \alpha\in[0,w_{\tilde{p}}].$$

The value of $\tilde{p}$ corresponding α-cut set is denoted by $V_T(\tilde{p})$ and is calculated as :

$$\begin{aligned}V_T(\tilde{p}) &= \int_0^{w_{\tilde{p}}}[(a_1-\sigma_1+\frac{\sigma_1\alpha}{w_{\tilde{p}}})+(b_1+\eta_1-\frac{\eta_1\alpha}{w_{\tilde{p}}})]\alpha\,d\alpha\\ &= \int_0^{w_{\tilde{p}}}[a_1+b_1+\eta_1-\sigma_1-\frac{(\eta_1-\sigma_1)\alpha}{w_{\tilde{p}}}]\alpha\,d\alpha\\ &= \frac{1}{6}(3a_1+3b_1-\sigma_1+\eta_1)w_{\tilde{p}}^2.\end{aligned}$$

2. Any β- cut set of the G_{SVN}-number $\tilde{p}$ for indeterminacy membership function is denoted by $\tilde{p}^{\beta}$ and is given by a closed interval as :

$$\begin{aligned}\tilde{p}^{\beta} &= [L'_{\tilde{p}}(\beta), R'_{\tilde{p}}(\beta)] \\ &= [\frac{(u_{\tilde{p}}-\beta)\sigma_2+(1-u_{\tilde{p}})a_2}{1-u_{\tilde{p}}}, \frac{(\beta-u_{\tilde{p}})\eta_2+(1-u_{\tilde{p}})b_2}{1-u_{\tilde{p}}}], \quad \text{for } \beta \in [u_{\tilde{p}}, 1].\end{aligned}$$

The value of $\tilde{p}$ corresponding β- cut set is denoted by $V_I(\tilde{p})$ and is calculated as :

$$\begin{aligned}V_I(\tilde{p}) &= \int_{u_{\tilde{p}}}^{1}[\frac{(u_{\tilde{p}}-\beta)\sigma_2+(1-u_{\tilde{p}})a_2}{1-u_{\tilde{p}}}+\frac{(\beta-u_{\tilde{p}})\eta_2+(1-u_{\tilde{p}})b_2}{1-u_{\tilde{p}}}](1-\beta)\,d\beta \\ &= \int_{u_{\tilde{p}}}^{1}[a_2+b_2-\sigma_2+\eta_2+\frac{(\sigma_2-\eta_2)(1-\beta)}{1-u_{\tilde{p}}}](1-\beta)\,d\beta \\ &= \frac{1}{6}(3a_2+3b_2-\sigma_2+\eta_2)(1-u_{\tilde{p}})^2.\end{aligned}$$

3. Any γ-cut set of the G_{SVN}-number $\tilde{p}$ for falsity membership function is denoted by ${}^{\gamma}\tilde{p}$ and is given by a closed interval as :

$$\begin{aligned}{}^{\gamma}\tilde{p} &= [L''_{\tilde{p}}(\gamma), R''_{\tilde{p}}(\gamma)] \\ &= [\frac{(u_{\tilde{p}}-\gamma)\sigma_3+(1-y_{\tilde{p}})a_3}{1-y_{\tilde{p}}}, \frac{(\gamma-y_{\tilde{p}})\eta_3+(1-y_{\tilde{p}})b_3}{1-y_{\tilde{p}}}], \quad \text{for } \gamma \in [y_{\tilde{p}}, 1].\end{aligned}$$

The value of $\tilde{p}$ corresponding γ-cut set is denoted by $V_F(\tilde{p})$ and is calculated as :

$$\begin{aligned}V_F(\tilde{p}) &= \int_{y_{\tilde{p}}}^{1}[\frac{(u_{\tilde{p}}-\gamma)\sigma_3+(1-y_{\tilde{p}})a_3}{1-y_{\tilde{p}}}+\frac{(\gamma-y_{\tilde{p}})\eta_3+(1-y_{\tilde{p}})b_3}{1-y_{\tilde{p}}}](1-\gamma)\,d\gamma \\ &= \int_{y_{\tilde{p}}}^{1}[a_3+b_3-\sigma_3+\eta_3+\frac{(\sigma_3-\eta_3)(1-\gamma)}{1-y_{\tilde{p}}}](1-\gamma)\,d\gamma \\ &= \frac{1}{6}(3a_3+3b_3-\sigma_3+\eta_3)(1-y_{\tilde{p}})^2.\end{aligned}$$

3.5 Definition

For $\kappa \in [0,1]$, the κ-weighted value of an G_{SVN}-number $\tilde{b}$ is denoted by $V_{\kappa}(\tilde{b})$ and is defined as :
$V_{\kappa}(\tilde{b}) = \kappa^n V_T(\tilde{b}) + (1-\kappa^n)V_I(\tilde{b}) + (1-\kappa^n)V_F(\tilde{b}),\ n$ being any natural number.

Thus, the κ - weighted value for the G_{SVN}- number $\tilde{p}$ defined in Corollary 3.4 is :

$$\begin{aligned}V_{\kappa}(\tilde{p}) &= \frac{1}{6}[(3a_1+3b_1-\sigma_1+\eta_1)\kappa^n w_{\tilde{p}}^2+(3a_2+3b_2-\sigma_2+\eta_2)(1-\kappa^n)(1-u_{\tilde{p}})^2 \\ &\quad +(3a_3+3b_3-\sigma_3+\eta_3)(1-\kappa^n)(1-y_{\tilde{p}})^2].\end{aligned}$$

3.5.1 Property of κ - weighted value function

The κ- weighted value $V_{\kappa}(\tilde{p})$ and $V_{\kappa}(\tilde{q})$ of two G_{SVN}-numbers $\tilde{p}, \tilde{q}$ respectively obey the followings.
(i) $V_{\kappa}(\tilde{p} \pm \tilde{q}) \leq V_{\kappa}(\tilde{p}) + V_{\kappa}(\tilde{q}),\ V_{\kappa}(\tilde{p}+\tilde{q}) \geq V_{\kappa}(\tilde{p}) \sim V_{\kappa}(\tilde{q})$.
(ii) $V_{\kappa}(\tilde{p}-\tilde{p}) = V_{\kappa}(\tilde{0}),\ V_{\kappa}(\mu\tilde{p}) = \mu V_{\kappa}(\tilde{p}))$ for μ being any real number.

(iii) $V_\kappa(\tilde{p})$ is monotone increasing or decreasing or constant according as $V_T(\tilde{p}) > V_I(\tilde{p}) + V_F(\tilde{p})$ or $V_T(\tilde{p}) < V_I(\tilde{p}) + V_F(\tilde{p})$ or $V_T(\tilde{p}) = V_I(\tilde{p}) + V_F(\tilde{p})$ respectively.

Proof. We shall here prove (vi) only. Others can be easily verified by taking any two G_{SVN}-numbers. Here,

$$\begin{aligned} V_\kappa(\tilde{p}) &= \kappa^n V_T(\tilde{p}) + (1-\kappa^n)(V_I(\tilde{p}) + V_F(\tilde{p})) \\ \frac{dV_\kappa(\tilde{p})}{d\kappa} &= n\kappa^{n-1}[V_T(\tilde{p}) - (V_I(\tilde{p}) + V_F(\tilde{p}))] \end{aligned}$$

As $\kappa \in [0,1]$, so $\frac{dV_\kappa(\tilde{p})}{d\kappa} >, <, = 0$ for $[V_T(\tilde{p}) - (V_I(\tilde{p}) + V_F(\tilde{p}))] >, <, = 0$ respectively. This clears the fact.

3.6 Definition

Let $G_{SVN}(\mathbf{R})$ be the set of all G_{SVN}-numbers defined over $\mathbf{R}$. For $\kappa \in [0,1]$, a mapping $\Re_\kappa : G_{SVN}(\mathbf{R}) \longrightarrow \mathbf{R}$ is called a ranking function and it is defined as : $\Re_\kappa(\tilde{a}) = V_\kappa(\tilde{a})$ for $\tilde{a} \in G_{SVN}(\mathbf{R})$.

For $\tilde{a}, \tilde{b} \in G_{SVN}(\mathbf{R})$, their ranking is defined as :

$$\tilde{a} >_{\Re_\kappa} \tilde{b} \text{ iff } \Re_\kappa(\tilde{a}) > \Re_\kappa(\tilde{b}),\ \tilde{a} <_{\Re_\kappa} \tilde{b} \text{ iff } \Re_\kappa(\tilde{a}) < \Re_\kappa(\tilde{b}),\ \tilde{a} =_{\Re_\kappa} \tilde{b} \text{ iff } \Re_\kappa(\tilde{a}) = \Re_\kappa(\tilde{b}).$$

3.7 Definition

An G_{SVN}-number $\tilde{p}$ is called a G_{SVTN}-number if three modal intervals in $\tilde{p}$ are equal. Thus $\tilde{p} = \langle([a_0, b_0, \sigma_1, \eta_1]; w_{\tilde{p}}), ([a_0, b_0, \sigma_2, \eta_2]; u_{\tilde{p}}), ([a_0, b_0, \sigma_3, \eta_3]; y_{\tilde{p}})\rangle$ is an G_{SVTN}-number whose truth, indeterminacy and falsity functions are as follows :

$$T_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_1} w_{\tilde{p}}(x - a_0 + \sigma_1), & a_0 - \sigma_1 \le x \le a_0, \\ w_{\tilde{p}}, & x \in [a_0, b_0], \\ \frac{1}{\eta_1} w_{\tilde{p}}(b_0 - x + \eta_1), & b_0 \le x \le b_0 + \eta_1, \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_2}(a_0 - x + u_{\tilde{p}}(x - a_0 + \sigma_2)), & a_0 - \sigma_2 \le x \le a_0, \\ u_{\tilde{p}}, & x \in [a_0, b_0], \\ \frac{1}{\eta_2}(x - b_0 + u_{\tilde{p}}(b_0 - x + \eta_2)), & b_0 \le x \le b_0 + \eta_2, \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_3}(a_0 - x + y_{\tilde{p}}(x - a_0 + \sigma_3)), & a_0 - \sigma_3 \le x \le a_0, \\ y_{\tilde{p}}, & x \in [a_0, b_0], \\ \frac{1}{\eta_3}(x - b_0 + y_{\tilde{p}}(b_0 - x + \eta_3)), & b_0 \le x \le b_0 + \eta_3, \\ 1, & \text{otherwise.} \end{cases}$$

In parametric form for $r \in [0,1]$:

$$\begin{aligned} T^l_{\tilde{p}}(r) &= a_0 - \sigma_1 + \frac{\sigma_1 r}{w_{\tilde{p}}},\ T^u_{\tilde{p}}(r) = b_0 + \eta_1 - \frac{\eta_1 r}{w_{\tilde{p}}}; \\ I^l_{\tilde{p}}(r) &= \frac{(1-u_{\tilde{p}})a_0 + (u_{\tilde{p}} - r)\sigma_2}{1-u_{\tilde{p}}},\ I^u_{\tilde{p}}(r) = \frac{(1-u_{\tilde{p}})b_0 + (r - u_{\tilde{p}})\eta_2}{1-u_{\tilde{p}}}; \\ F^l_{\tilde{p}}(r) &= \frac{(1-y_{\tilde{p}})a_0 + (y_{\tilde{p}} - r)\sigma_3}{1-y_{\tilde{p}}},\ F^u_{\tilde{p}}(r) = \frac{(1-y_{\tilde{p}})b_0 + (r - y_{\tilde{p}})\eta_3}{1-y_{\tilde{p}}}. \end{aligned}$$

3.8 Definition

A G_{SVTN}-number $\tilde{p}$ is called a G_{SVTrN}-number if the modal interval in $\tilde{p}$ is reduced to a modal point. Thus $\tilde{p} = \langle([a_0, \sigma_1, \eta_1]; w_{\tilde{p}}), ([a_0, \sigma_2, \eta_2]; u_{\tilde{p}}), ([a_0, \sigma_3, \eta_3]; y_{\tilde{p}})\rangle$ is a G_{SVTrN}-number whose truth, indeterminacy and falsity functions are as follows :

$$T_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_1} w_{\tilde{p}}(x - a_0 + \sigma_1), & a_0 - \sigma_1 \leq x \leq a_0, \\ w_{\tilde{p}}, & x = a_0, \\ \frac{1}{\eta_1} w_{\tilde{p}}(a_0 - x + \eta_1), & a_0 \leq x \leq a_0 + \eta_1, \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_2}(a_0 - x + u_{\tilde{p}}(x - a_0 + \sigma_2)), & a_0 - \sigma_2 \leq x \leq a_0, \\ u_{\tilde{p}}, & x = a_0, \\ \frac{1}{\eta_2}(x - a_0 + u_{\tilde{p}}(a_0 - x + \eta_2)), & a_0 \leq x \leq a_0 + \eta_2, \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{p}}(x) = \begin{cases} \frac{1}{\sigma_3}(a_0 - x + y_{\tilde{p}}(x - a_0 + \sigma_3)), & a_0 - \sigma_3 \leq x \leq a_0, \\ y_{\tilde{p}}, & x = a_0, \\ \frac{1}{\eta_3}(x - a_0 + y_{\tilde{p}}(a_0 - x + \eta_3)), & a_0 \leq x \leq a_0 + \eta_3, \\ 1, & \text{otherwise.} \end{cases}$$

3.8.1 Definition

Let $\tilde{a}$ and $\tilde{b}$ be two G_{SVTrN}-numbers as follows :

$\tilde{a} = \langle([a, \sigma_1, \eta_1]; w_{\tilde{a}}), ([a, \sigma_2, \eta_2]; u_{\tilde{a}}), ([a, \sigma_3, \eta_3]; y_{\tilde{a}})\rangle$,

$\tilde{b} = \langle([b, \xi_1, \delta_1]; w_{\tilde{b}}), ([b, \xi_2, \delta_2]; u_{\tilde{b}}), ([b, \xi_3, \delta_3]; y_{\tilde{b}})\rangle$.

Then for any real number x,

(i) Image of $\tilde{a}$:

$-\tilde{a} = \langle([-a, \eta_1, \sigma_1]; w_{\tilde{a}}), ([-a, \eta_2, \sigma_2]; u_{\tilde{a}}), ([-a, \eta_3, \sigma_3]; y_{\tilde{a}})\rangle$.

(ii) Addition :

$\tilde{a} + \tilde{b} = \langle([a + b, \sigma_1 + \xi_1, \eta_1 + \delta_1]; w_{\tilde{a}} * w_{\tilde{b}}), ([a + b, \sigma_2 + \xi_2, \eta_2 + \delta_2]; u_{\tilde{a}} \diamond u_{\tilde{b}}),$
$([a + b, \sigma_3 + \xi_3, \eta_3 + \delta_3]; y_{\tilde{a}} \diamond y_{\tilde{b}})\rangle$.

(iii) Scalar multiplication :

$x\tilde{a} = \langle([xa, x\sigma_1, x\eta_1]; w_{\tilde{a}}), ([xa, x\sigma_2, x\eta_2]; u_{\tilde{a}}), ([xa, x\sigma_3, x\eta_3]; y_{\tilde{a}})\rangle$ for $x > 0$.

$x\tilde{a} = \langle([xa, -x\eta_1, -x\sigma_1]; w_{\tilde{a}}), ([xa, -x\eta_2, -x\sigma_2]; u_{\tilde{a}}), ([xa, -x\eta_3, -x\sigma_3]; y_{\tilde{a}})\rangle$ for $x < 0$.

(iv) The κ - weighted value $V_\kappa(\tilde{a})$ of $\tilde{a}$ is given as :

$$V_\kappa(\tilde{a}) = \frac{1}{6}[(6a - \sigma_1 + \eta_1)\kappa^n w_{\tilde{a}}^2 + \{(6a - \sigma_2 + \eta_2)(1 - u_{\tilde{a}})^2 + (6a - \sigma_3 + \eta_3)(1 - y_{\tilde{a}})^2\}(1 - \kappa^n)].$$

3.8.2 *Remark*

Definition 2.4.1 shows that the supports (i.e. the bases of trapeziums (triangles)) for truth, indeterminacy and falsity function are all same. Then the value of truth, indeterminacy and falsity function (i.e., the area of individual trapezium (triangle)) differs in respect to their corresponding height only. But by Definition 3.7, we consider different supports (i.e. bases of trapeziums (triangles) formed) for truth, indeterminacy and falsity functions. Thus we can allow the supports and heights together to differ the value of truth, indeterminacy and

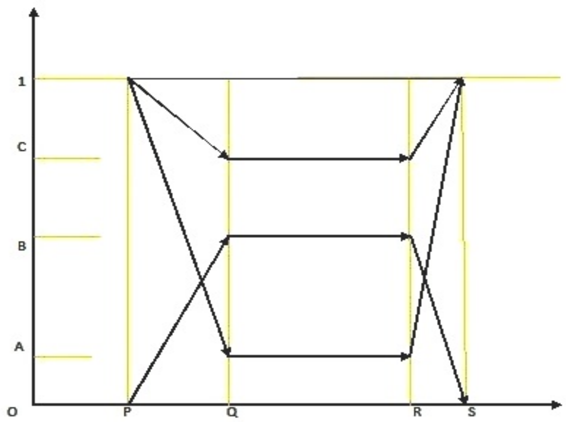

Figure – 1

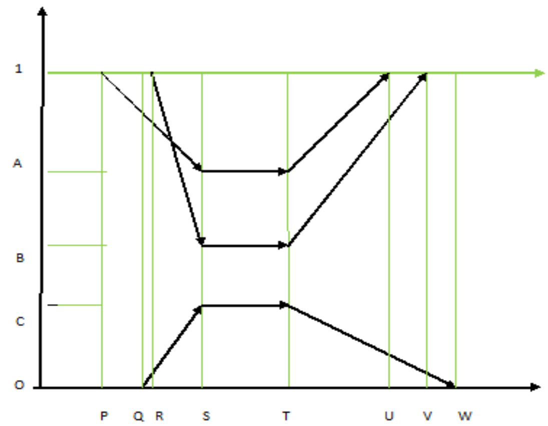

Figure - 2

falsity functions in the present study. Briefly, Definition 2.4.1 is a particular case of Definition 3.7. Hence decision maker has a scope of flexibility to choose and compare different G_{SVN}-numbers in their study. The facts are shown by the graphical Figure 1 and 2. Figure 1 and Figure 2 represent Definition 2.4.1 and Definition 3.7 respectively.

3.9 Definition

1. The zero G_{SVTN}-number is denoted by $\tilde{0}$ and is defined as :
   $\tilde{0} = \langle([0,0,0,0];1),([0,0,0,0];0),([0,0,0,0];0)\rangle$.
2. The zero G_{SVTrN}-number is denoted by $\tilde{0}$ and is defined as :
   $\tilde{0} = \langle([0,0,0];1),([0,0,0];0),([0,0,0];0)\rangle$.

4 Neutrosophic Linear Programming Problem

Before to discuss the main result, we shall remember the crisp concept of an LP-problem. The standard form of an LP-problem is :

$$\text{Max } z \;=\; cx \quad \text{such that } Ax = b,\; x \geq 0$$

where $c = (c_1, c_2, \cdots, c_n), b = (b_1, b_2, \cdots, b_n)^t$ and $A = [a_{ij}]_{m\times n}$.

In this problem, all the parameters are crisp. we shall now define NLP-problem.

4.1 Definition

An LP-problem having some parameters as G_{SVN}-number is called an NLP-problem. Considering the coefficient of the variables in the objective function in an LP-problem in term of G_{SVN}-numbers, an NLP-problem is designed as follows :

$$\begin{aligned} \text{Max } \tilde{z} \;&=_{\Re_\kappa}\; \tilde{c}x \\ \text{such that} \quad Mx \;&=\; b;\; x \geq 0 \end{aligned} \tag{4.1}$$

where $b \in \mathbf{R}^m, x \in \mathbf{R}^n, M \in \mathbf{R}^{m\times n}, \tilde{c}^t \in (G_{SVN}(\mathbf{R}))^n$ and $\Re_\kappa$ is a ranking function.

4.2 Definition

1. $x \in \mathbf{R}^n$ is a feasible solution to equation (4.1) if x satisfies the constraints of that.

2. A feasible solution x^* is an optimal solution if for all solutions x to (4.1), $\tilde{c}x^* \geq_{\Re_\kappa} \tilde{c}x$.

3. For the NLP-problem (4.1), suppose rank(M, b) = rank$(M) = m$. M is partitioned as $[B, N]$ where B is a non-singular $m \times m$ matrix i.e., rank$(B) = m$. A feasible solution $x = (x_B, x_N)^t$ to (4.1) obtained by setting $x_B = B^{-1}b, x_N = 0$ is called a neutrosophic basic feasible solution (N_{BFS}). Here B and N are respectively called basis and non basis matrix. x_B is called a basic variable and x_N is called a non-basic variable.

4. In an N_{BFS} if all components of $x_B > 0$, then x is non-degenerate N_{BFS} and if at least one component of $x_B = 0$, then x is degenerate N_{BFS}.

5 Simplex Method for NLP-problem

The NLP-problem (4.1) can be put as follows :

$$\begin{aligned} &\text{Max } \tilde{z} =_{\Re_\kappa} \tilde{c}_B x_B + \tilde{c}_N x_N \\ \text{such that} \quad &B x_B + N x_N = b; \quad x_B, x_N \geq 0 \end{aligned}$$

where the characters B, N, x_B and x_N are already stated. Then we have,

$$\begin{aligned} & x_B + B^{-1}N x_N = B^{-1}b && (5.1) \\ \Rightarrow\ & \tilde{c}_B x_B + \tilde{c}_B B^{-1} N x_N =_{\Re_\kappa} \tilde{c}_B B^{-1} b \\ \Rightarrow\ & \tilde{z} - \tilde{c}_N x_N + \tilde{c}_B B^{-1} N x_N =_{\Re_\kappa} \tilde{c}_B B^{-1} b \\ \Rightarrow\ & \tilde{z} + (\tilde{c}_B B^{-1} N - \tilde{c}_N) x_N =_{\Re_\kappa} \tilde{c}_B B^{-1} b. && (5.2) \end{aligned}$$

For an N_{BFS}, treating $x_N = 0$, we have $x_B = B^{-1}b$ and $\tilde{z} =_{\Re_\kappa} \tilde{c}_B B^{-1}b$ from (5.1) and (5.2), respectively. We can rewrite the NLP-problem as given in Table 1.

Table 1 : Tabular form of an NLP-problem.

	$\tilde{c}_j$	$\tilde{c}_B$	$\tilde{c}_N$	
	$\tilde{z}$	x_B	x_N	R.H.S
x_B	0	1	$B^{-1}N$	$B^{-1}b$
$\tilde{z}$	1	0	$\tilde{c}_B B^{-1} N - \tilde{c}_N$	$\tilde{c}_B B^{-1} b$

We can get all required initial information to proceed with the simplex method from Table 1. The neutrosophic cost row in the Table 1 is $\tilde{\lambda}_j =_{\Re_\kappa} (\tilde{c}_B B^{-1} a_j - c_j)_{a_j \notin B}$ giving $\tilde{\lambda}_j =_{\Re_\kappa} (\tilde{z}_j - \tilde{c}_j)$ for non-basic variables. The optimality arises if $\tilde{\lambda}_j \geq_{\Re_\kappa} \tilde{0}, \forall a_j \notin B$. If $\tilde{\lambda}_l <_{\Re_\kappa} \tilde{0}$ for any $a_l \notin B$, we need to replace x_{B_i} by x_l. We then compute $y_l = B^{-1}a_l$. If $y_l \leq 0$, then x_l can be increased indefinitely and so the problem admits unbounded optimal solution. But if y_l has at least one positive component, then one of the current basic variables blocks that increase, which drops to zero.

5.1 Theorem

In every column a_j of M, if $\tilde{z}_j - \tilde{c}_j \geq_{\Re_\kappa} \tilde{0}$ holds for an N_{BFS} x_B of the NLP-problem (4.1) then it is an optimal solution to that.

Proof. Let $M = [a_{ij}]_{m\times n} = [a_1, a_2, \cdots, a_n]$ where each $a_l = (a_{1l}, a_{2l}, \cdots, a_{ml})^t$ is m component column vector. Suppose $B = [\eta_1, \eta_2, \cdots, \eta_m]$ is the basis matrix and $\tilde{z}_B =_{\Re_\kappa} \tilde{c}_B x_B =_{\Re_\kappa} \sum_{i=1}^{m} \tilde{c}_{B_i} x_{B_i}$, where $\tilde{c}_{B_i}$ is the price corresponding to the basic variable x_{B_i}. Then any column a_l of M may be put as a linear combination of the vectors $\eta_1, \eta_2, \cdots, \eta_m$ of B. Let

$$a_l = y_{1l}\eta_1 + y_{2l}\eta_2 + \cdots + y_{ml}\eta_m = \sum_{i=1}^{m} y_{il}\eta_i = By_l \qquad \Rightarrow \; y_l = B^{-1}a_l.$$

where $y_l = (y_{1l}, y_{2l}, \cdots, y_{ml})^t$ being m component scalars represents a_l, the l-th vector of M. Assume that $\tilde{z}_l =_{\Re_\kappa} \tilde{c}_B y_l =_{\Re_\kappa} \sum_{i=1}^{m} \tilde{c}_{B_i} y_{il}$.

Let $x = [x_1, x_2, \cdots, x_n]^t$ be any other feasible solution of the NLP-problem (4.1) and $\tilde{z}$ be the corresponding objective function. Then,

$$Bx_B = b = Mx \qquad \Rightarrow \; x_B = B^{-1}(Mx) = (B^{-1}M)x = yx$$

where $B^{-1}M = y = [y_{ij}]_{m\times n} = [y_1, y_2, \cdots, y_n]$ with y_l defined as above. Thus,

$$\begin{pmatrix} x_{B_1} \\ x_{B_2} \\ \vdots \\ x_{B_m} \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Equating i-th component from both sides, we have $x_{B_i} = \sum_{j=1}^{n} y_{ij}x_j$. Now,

$$\begin{aligned} &\tilde{z}_j - \tilde{c}_j \geq_{\Re_\kappa} \tilde{0} \Rightarrow (\tilde{z}_j - \tilde{c}_j)x_j \geq_{\Re_\kappa} \tilde{0} \;[\text{ as } x_j > 0\,] \Rightarrow \sum_{j=1}^{n}(\tilde{z}_j - \tilde{c}_j)x_j \geq_{\Re_\kappa} \tilde{0} \\ \Rightarrow \; & \sum_{j=1}^{n} \tilde{z}_j x_j - \sum_{j=1}^{n} \tilde{c}_j x_j \geq_{\Re_\kappa} \tilde{0} \Rightarrow \sum_{j=1}^{n} x_j(\tilde{c}_B y_j) - \tilde{z} \geq_{\Re_\kappa} \tilde{0} \\ \Rightarrow \; & \sum_{j=1}^{n} x_j(\sum_{i=1}^{m} \tilde{c}_{B_i} y_{ij}) - \tilde{z} \geq_{\Re_\kappa} \tilde{0} \Rightarrow \sum_{i=1}^{m} \tilde{c}_{B_i}(\sum_{j=1}^{n} y_{ij}x_j) - \tilde{z} \geq_{\Re_\kappa} \tilde{0} \\ \Rightarrow \; & \sum_{i=1}^{m} \tilde{c}_{B_i}\tilde{x}_{B_i} - \tilde{z} \geq_{\Re_\kappa} \tilde{0} \Rightarrow \tilde{z}_B - \tilde{z} \geq_{\Re_\kappa} \tilde{0}. \end{aligned}$$

Thus $\tilde{z}_B$ is the maximum value of the objective function. This optimality criterion holds for all non-basic vectors of M. If a_l be in the basis matrix B, say $a_l = \eta_l$, then

$a_l = \eta_l = 0.\eta_1 + 0.\eta_2 + \cdots + 0.\eta_{l-1} + 1.\eta_l + 0.\eta_{l+1} + \cdots + 0.\eta_m$

i.e., y_l is a unit vector e_l with l-th component unity.

Since $a_l = \eta_l$, we have $\tilde{c}_l = \tilde{c}_{B_l}$ and so

$\tilde{z}_l - \tilde{c}_l =_{\Re_\kappa} (\tilde{c}_B y_l - \tilde{c}_l) =_{\Re_\kappa} (\tilde{c}_B e_l - \tilde{c}_l) =_{\Re_\kappa} (\tilde{c}_{B_l} - \tilde{c}_{B_l}) =_{\Re_\kappa} \tilde{0}.$

Thus as a whole $\tilde{z}_j - \tilde{c}_j \geq_{\Re_\kappa} \tilde{0}$ is the necessary condition for optimality.

5.2 Theorem

A non-degenerate N_{BFS} $x_B = B^{-1}b, x_N = 0$ is optimal to NLP-problem (4.1) iff $\tilde{z}_j - \tilde{c}_j \geq_{\Re_\kappa} \tilde{0}, \forall 1 \leq j \leq n$.

Proof. Suppose $x^* = (x_B^t, x_N^t)^t$ be an N_{BFS} to (4.1) where $x_B = B^{-1}b, x_N = 0$. If $\tilde{z}^*$ be the objective function corresponding to x^*, then $\tilde{z}^* =_{\Re_\kappa} \tilde{c}_B x_B =_{\Re_\kappa} \tilde{c}_B B^{-1}b$. Let $x = [x_1, x_2, \cdots, x_n]^t$ be another feasible solution of NLP-problem (4.1) and $\tilde{z}$ be the corresponding objective function. Then,

$$\tilde{z} =_{\Re_\kappa} \tilde{c}_B x_B + \tilde{c}_N x_N =_{\Re_\kappa} \tilde{c}_B B^{-1}b - \sum_{a_j \notin B} (\tilde{c}_B B^{-1} a_j - \tilde{c}_j) x_j =_{\Re_\kappa} \tilde{z}^* - \sum_{a_j \notin B} (\tilde{z}_j - \tilde{c}_j) x_j$$

This shows that the solution is optimal iff $\tilde{z}_j - \tilde{c}_j \geq_{\Re_\kappa} \tilde{0}$ for all $1 \leq j \leq n$.

5.3 Theorem

For any N_{BFS} to NLP-problem (4.1), if there is some column not in basis such that $\tilde{z}_l - \tilde{c}_l <_{\Re_\kappa} \tilde{0}$ and $y_{il} \leq 0$, $i = 1, 2, \cdots, m$, then (4.1) admits an unbounded solution.

Proof. Let x_B be a basic solution to the NLP-problem (4.1). Re-writing the constraints,

$$\begin{aligned}
& Bx_B + Nx_N = b \\
\Rightarrow\ & x_B + B^{-1}Nx_N = B^{-1}b \\
\Rightarrow\ & x_B + B^{-1}\sum_j (a_j x_j) = B^{-1}b,\ \ a_j s \text{ are the columns of } N \\
\Rightarrow\ & x_B + \sum_j (B^{-1}a_j x_j) = B^{-1}b \\
\Rightarrow\ & x_B + \sum_j (y_j x_j) = y_0,\ \text{ where } a_j = By_j, a_j \notin B \\
\Rightarrow\ & x_{B_i} + \sum_j (y_{ij} x_j) = y_{i0}, 1 \leq i \leq m, 1 \leq j \leq n \\
\Rightarrow\ & x_{B_i} = y_{i0} - \sum_j (y_{ij} x_j), 1 \leq i \leq m, 1 \leq j \leq n.
\end{aligned}$$

If x_l enters into the basis, then $x_l > 0$ and $x_j = 0$ for $j \neq B_i \cup l$. Since $y_{il} \leq 0,\ 1 \leq i \leq m$ hence $y_{i0} - y_{il}x_l \geq 0$. So, the basic solution remains feasible and for that, the objective function is :

$$\begin{aligned}
\tilde{z}^* &=_{\Re_\kappa} \tilde{c}_B x_B + \tilde{c}_N x_N =_{\Re_\kappa} \sum_{i=1}^m \tilde{c}_{B_i}(y_{i0} - y_{il}x_l) + \tilde{c}_l x_l =_{\Re_\kappa} \sum_{i=1}^m \tilde{c}_{B_i} y_{i0} - (\sum_{i=1}^m \tilde{c}_{B_i} y_{il} - \tilde{c}_l) x_l \\
&=_{\Re_\kappa} \tilde{c}_B y_0 - (\tilde{c}_B y_l - \tilde{c}_l) x_l =_{\Re_\kappa} \tilde{z} - (\tilde{z}_l - \tilde{c}_l) x_l.
\end{aligned}$$

It shows that $\tilde{z}^* >_{\Re_\kappa} \tilde{z}$, as $\tilde{z}_l - \tilde{c}_l <_{\Re_\kappa} \tilde{0}$ and this completes the fact.

5.4 Simplex algorithm for solving NLP-problem

To solve any NLP-problem by simplex method, the existence of an initial basic feasible solution is always assumed. This solution will be optimised through some iterations. The required steps are as follows :

Step 1. Check whether the objective function of the given NLP-problem is to be maximized or minimized. If it is to be minimized, then it is converted into a maximization problem by using the result $Min(\tilde{z}) = -Max(-\tilde{z})$.

Step 2. Convert all the inequations of the constraints ($\leq$ type) into equations by introducing slack variables. Put the costs of the respective variables equal to $\tilde{0}$.

Step 3. Obtain an N_{BFS} to the problem in the form $x_B = B^{-1}b = y_0$ and $x_N = 0$. The corresponding objective function is $\tilde{z} =_{\Re_\kappa} \tilde{c}_B B^{-1} b =_{\Re_\kappa} \tilde{c}_B y_0$.

Step 4. For each basic variable, put $\tilde{\lambda}_B =_{\Re_\kappa} \tilde{z}_B - \tilde{c}_B =_{\Re_\kappa} \tilde{0}$. For each non-basic variable, calculate $\tilde{\lambda}_j =_{\Re_\kappa} \tilde{z}_j - \tilde{c}_j =_{\Re_\kappa} \tilde{c}_B B^{-1} a_j - \tilde{c}_j$ in the current iteration. If all $\tilde{z}_j - \tilde{c}_j \geq_{\Re_\kappa} \tilde{0}$, then the present solution is optimal.

Step 5. If for some non-basic variables, $\tilde{\lambda}_j =_{\Re_\kappa} \tilde{z}_j - \tilde{c}_j <_{\Re_\kappa} \tilde{0}$ then find out $\tilde{\lambda}_l = \min\{\tilde{\lambda}_j\}$. If $y_{il} < 0$ for all $i = 1, \cdots, m$, then the given problem will have unbounded solution and stop the iteration. Otherwise to determine the index of the variable x_{B_r} that is to be removed from the current basis, compute

$$\frac{y_{r0}}{y_{rl}} = \min\{\frac{y_{i0}}{y_{il}} : y_{il} > 0, 1 \leq i \leq m\}.$$

Step 6. Update y_{i0} by replacing $y_{i0} - \frac{y_{r0}}{y_{rl}} y_{il}$ for $i \neq r$ and y_{r0} by $\frac{y_{r0}}{y_{rl}}$.

Step 7. Construct new basis and repeat the Step 4, Step 5 until the optimality is reached.

Step 8. Find the optimal solution and hence the optimal value of objective function.

6 Numerical Example

The NLP-problems with both G_{SVTN}-number and G_{SVTrN}-number are solved by the use of proposed algorithm. For simplicity, we define the κ-weighted value function for $n = 1$ in rest of the paper.

6.1 *Example*

Two friends F_1 and F_2 wish to invest in a raising share market. They choose two particular shares S_1 and S_2 of two multinational companies. They also decide to purchase equal unit of two shares individually. The maximum investment of F_1 is Rs. 4000 and that of F_2 is Rs. 7000. The price per unit of S_1 and S_2 are Re. 1 and Rs. 3, respectively when F_1 purchases. These are Rs. 2 and Rs. 5 at the time of purchasing of share by F_2. The current value of share S_1 and S_2 per unit is Rs. $\tilde{c}_1$ and Rs. $\tilde{c}_2$ (given in G_{SVN}-numbers), respectively. Now if they sell their shares, formulate an NLP-problem to maximize their returns.

The problem can be summarised as follows :

Table 2

Friends $\Downarrow$	Shares : S_1	S_2	Purchasing capacity $\Downarrow$
F_1	Re. 1	Rs. 3	Rs. 4000
F_2	Rs. 2	Rs. 5	Rs. 7000
Price per unit $\Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	

Let they individually purchase x_1 units of share S_1 and x_2 units of share S_2. The problem is formulated as :

$$\begin{aligned} & \text{Max } \tilde{z} =_{\Re_\kappa} \tilde{c}_1 x_1 + \tilde{c}_2 x_2 \\ \text{such that} \quad & x_1 + 3x_2 \leq 4000 \\ & 2x_1 + 5x_2 \leq 7000; \quad x_1, x_2 \geq 0 \end{aligned}$$

It is an NLP-problem where $\tilde{c}_1 = \langle([5, 8, 1, 3]; 0.2), ([5, 8, 3, 4]; 0.3), ([5, 8, 2, 1]; 0.4)\rangle$ and $\tilde{c}_2 = \langle([3, 7, 2, 4]; 0.3), ([3, 7, 1, 3]; 0.5), ([3, 7, 2, 5]; 0.6)\rangle$ are two G_{SVTN}-numbers with a pre-assigned $\kappa = 0.45$.

Rewriting the given constraints by introducing slack variables :

$$
\begin{aligned}
&x_1 + 3x_2 + x_3 = 4000\\
&2x_1 + 5x_2 + x_4 = 7000\\
&x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

We take the t-norm and s-norm as $p * q = \min\{p, q\}$ and $p \diamond q = \max\{p, q\}$, respectively. The first feasible simplex table is as follows :

Table 3 : First iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_3	1	3	1	0	4000
x_4	2	5	0	1	7000$\rightarrow$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(1)} \uparrow$	$\tilde{c}_2^{(1)}$	$\tilde{c}_3^{(1)}$	$\tilde{c}_4^{(1)}$	

Here $\tilde{c}_1^{(1)} = -\tilde{c}_1 = \langle([-8, -5, 3, 1]; 0.2), ([-8, -5, 4, 3]; 0.3), ([-8, -5, 1, 2]; 0.4)\rangle$,
$\tilde{c}_2^{(1)} = -\tilde{c}_2 = \langle([-7, -3, 4, 2]; 0.3), ([-7, -3, 3, 1]; 0.5), ([-7, -3, 5, 2]; 0.6)\rangle$
and $V_\kappa(\tilde{c}_3^{(1)}) = V_\kappa(\tilde{c}_4^{(1)}) = V_\kappa(\tilde{0})$.
Then $V_\kappa(\tilde{c}_1^{(1)}) = \frac{1}{6}(31.64\kappa - 33.28)$ and $V_\kappa(\tilde{c}_2^{(1)}) = \frac{1}{6}(10.4\kappa - 13.28)$ by Definition 3.5.
Clearly $V_\kappa(\tilde{c}_1^{(1)}) < 0,\ V_\kappa(\tilde{c}_2^{(1)}) < 0$ and $V_\kappa(\tilde{c}_1^{(1)}) - V_\kappa(\tilde{c}_2^{(1)}) < 0$ for $\kappa = 0.45$.
Then $\tilde{c}_1^{(1)} <_{\Re_\kappa} \tilde{c}_2^{(1)}$. So x_1 enters in the basis and as $\min\{4000/1, 7000/2\} = 3500$, the leaving variable is x_4. The revised table is :

Table 4 : Second iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_3	0	1/2	1	-1/2	500
x_1	1	5/2	0	1/2	3500
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(2)}$	$\tilde{c}_2^{(2)}$	$\tilde{c}_3^{(2)}$	$\tilde{c}_4^{(2)}$	$3500\tilde{c}_1$

where $V_\kappa(\tilde{c}_1^{(2)}) = V_\kappa(\tilde{c}_3^{(2)}) = V_\kappa(\tilde{0})$ and

$$
\begin{aligned}
\tilde{c}_2^{(2)} &= \frac{5}{2}\tilde{c}_1 - \tilde{c}_2\\
&= 2.5\langle([5, 8, 1, 3]; 0.2), ([5, 8, 3, 4]; 0.3), ([5, 8, 2, 1]; 0.4)\rangle\\
&\quad -\langle([3, 7, 2, 4]; 0.3), ([3, 7, 1, 3]; 0.5), ([3, 7, 2, 5]; 0.6)\rangle\\
&= \langle([5.5, 17, 6.5, 9.5]; 0.2), ([5.5, 17, 10.5, 11]; 0.5), ([5.5, 17, 10, 4.5]; 0.6)\rangle.\\
\tilde{c}_4^{(2)} &= \frac{1}{2}\tilde{c}_1 = \langle([2.5, 4, 0.5, 1.5]; 0.2), ([2.5, 4, 1.5, 2]; 0.3), ([2.5, 4, 1, 0.5]; 0.4)\rangle.
\end{aligned}
$$

Then $V_\kappa(\tilde{c}_2^{(2)}) = \frac{1}{6}(26.92 - 24.1\kappa)$ and $V_\kappa(\tilde{c}_4^{(2)}) = \frac{1}{6}(16.64 - 15.82\kappa)$ by Definition 3.5.
Clearly $V_\kappa(\tilde{c}_2^{(2)}) > 0$ and $V_\kappa(\tilde{c}_4^{(2)}) > 0$ for $\kappa = 0.45$.

Hence the optimality arises and Max $\tilde{z} =_{\Re_\kappa} 3500\tilde{c}_1$, which, using κ - weighted function, becomes Rs. 11107 approximately. Then corresponding return of F_1 and F_2 becomes Rs. 7607 and of Rs. 4107 respectively.

6.1.1 *Example*

Consider the NLP-problem defined in Example 6.1 with a pre-assigned $\kappa = 0.96$.

The initial simplex table (Table 5) is same as Table 3.

Table 5 : First iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_3	1	3	1	0	4000$\rightarrow$
x_4	2	5	0	1	7000
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(1)}$	$\tilde{c}_2^{(1)} \uparrow$	$\tilde{c}_3^{(1)}$	$\tilde{c}_4^{(1)}$	

Here $V_\kappa(\tilde{c}_3^{(1)}) = V_\kappa(\tilde{c}_4^{(1)}) = V_\kappa(\tilde{0})$ and $V_\kappa(\tilde{c}_1^{(1)}) < 0$, $V_\kappa(\tilde{c}_2^{(1)}) < 0$ with $V_\kappa(\tilde{c}_1^{(1)}) - V_\kappa(\tilde{c}_2^{(1)}) > 0$ for $\kappa = 0.96$. Then $\tilde{c}_1^{(1)} >_{\Re_\kappa} \tilde{c}_2^{(1)}$. So x_2 enters in the basis and as $\min\{\frac{4000}{3}, \frac{7000}{5}\} = \frac{4000}{3}$, the leaving variable is x_3. The revised table is :

Table 6 : Second iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_2	1/3	1	1/3	0	4000/3
x_4	1/3	0	-5/3	1	1000/3 $\rightarrow$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(2)} \uparrow$	$\tilde{c}_2^{(2)}$	$\tilde{c}_3^{(2)}$	$\tilde{c}_4^{(2)}$	$\frac{4000}{3}\tilde{c}_2$

where $V_\kappa(\tilde{c}_2^{(2)}) = V_\kappa(\tilde{c}_4^{(2)}) = V_\kappa(\tilde{0})$ and

$$\begin{aligned}
\tilde{c}_1^{(2)} &= \frac{1}{3}\tilde{c}_2 - \tilde{c}_1 = \langle([-7, -8/3, 11/3, 7/3]; 0.2), ([-7, -8/3, 13/3, 4]; 0.5), ([-7, -8/3, 5/3, 11/3]; 0.6)\rangle, \\
\tilde{c}_3^{(2)} &= \frac{1}{3}\tilde{c}_2 = \langle([1, 7/3, 2/3, 4/3]; 0.3), ([1, 7/3, 1/3, 1]; 0.5), ([1, 7/3, 2/3, 5/3]; 0.6)\rangle.
\end{aligned}$$

Then $V_\kappa(\tilde{c}_1^{(2)}) = \frac{1}{18}(31.32\kappa - 34.96)$ and $V_\kappa(\tilde{c}_3^{(2)}) = \frac{1}{18}(13.28 - 10.4\kappa)$.

Clearly $V_\kappa(\tilde{c}_1^{(2)}) < 0$ and $V_\kappa(\tilde{c}_3^{(2)}) > 0$ for $\kappa = 0.96$. So x_1 enters in the basis and as $\min\{\frac{4000/3}{1/3}, \frac{1000/3}{1/3}\} = 1000$, the leaving variable is x_4. The revised table is :

Table 7 : Third iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_2	0	1	2	-1	1000 $\rightarrow$
x_1	1	0	-5	3	1000
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(3)}$	$\tilde{c}_2^{(3)}$	$\tilde{c}_3^{(3)} \uparrow$	$\tilde{c}_4^{(3)}$	$1000(\tilde{c}_1 + \tilde{c}_2)$

where $V_\kappa(\tilde{c}_1^{(3)}) = V_\kappa(\tilde{c}_2^{(3)}) = V_\kappa(\tilde{0})$ and

$$\begin{aligned}\tilde{c}_3^{(3)} &= -5\tilde{c}_1 + 2\tilde{c}_2 = \langle([-34,-11,19,13];0.2),([-34,-11,22,21];0.5),([-34,-11,9,20];0.6)\rangle,\\ \tilde{c}_4^{(3)} &= 3\tilde{c}_1 - \tilde{c}_2 = \langle([8,21,7,11];0.2),([8,21,12,13];0.5),([8,21,11,5];0.6)\rangle.\end{aligned}$$

Then $V_\kappa(\tilde{c}_3^{(3)}) = \frac{1}{6}(48.2\kappa - 53.84) < 0$ and $V_\kappa(\tilde{c}_4^{(3)}) = \frac{1}{6}(34.96 - 31.32\kappa) > 0$ for $\kappa = 0.96$. So x_3 enters in the basis and the leaving variable is x_2. The revised table is :

Table 8 : Fourth iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_3	0	1/2	1	-1/2	500
x_1	1	5/2	0	1/2	3500
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(4)}$	$\tilde{c}_2^{(4)}$	$\tilde{c}_3^{(4)}$	$\tilde{c}_4^{(4)}$	$3500\tilde{c}_1$

where $V_\kappa(\tilde{c}_1^{(4)}) = V_\kappa(\tilde{c}_3^{(4)}) = V_\kappa(\tilde{0})$ and $\tilde{c}_2^{(4)} = \frac{5}{2}\tilde{c}_1 - \tilde{c}_2$ and $\tilde{c}_4^{(4)} = \frac{1}{2}\tilde{c}_1$. Then $V_\kappa(\tilde{c}_2^{(4)}) = \frac{1}{6}(26.92 - 24.1\kappa) > 0$ and $V_\kappa(\tilde{c}_4^{(4)}) = \frac{1}{6}(16.64 - 15.82\kappa) > 0$ for $\kappa = 0.96$.

Hence the optimality arises and the optimal solution is $x_1 = 3500, x_2 = 0$.

6.1.2 *Remark*

From Example 6.1 and Example 6.1.1, it is seen that the final simplex tables in both cases are same. So, if the optimality exists for an NLP-problem, the optimal solutions are always unique whatever the value of κ assigned. Depending upon the chosen κ, the number of iteration to reach at optimality stage may vary but it does not affect the optimal solutions. However, the character κ plays an important role to assign the optimal value of the objective function in a problem. The fact is shown in Table 9. So, the value of κ is an important factor in any such NLP-problem. Since the share market depends on so many factors, we claim κ as the degree of political turmoil of the country in the present problem.

6.1.3 Sensitivity analysis in post optimality stage

We shall analyse the results of the problem in Example 6.1 for different values of κ in post optimality stage, shown by the Table 9.

Table 9 : Sensitivity analysis

κ	0	0.1	0.2	0.3	0.4
x_1	3500	3500	3500	3500	3500
x_2	0	0	0	0	0
$V_\kappa(\tilde{z})$	19413.33	17567.67	15722	13876.33	12030.67

κ	0.5	0.6	0.7	0.8	0.9	1
x_1	3500	3500	3500	3500	3500	3500
x_2	0	0	0	0	0	0
$V_\kappa(\tilde{z})$	10185	8339.33	6493.67	4648	2802.33	956.67

6.2 *Example*

$$\begin{aligned} \text{Max } \tilde{z} \;=_{\Re_\kappa}\; & \tilde{c}_1 x_1 + \tilde{c}_2 x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \leq 4 \\ & 5x_1 + 4x_2 \leq 15 \\ & x_1, x_2 \geq 0 \end{aligned}$$

is an NLP-problem where $\tilde{c}_1 = \langle([8,1,3];0.6),([8,3,4];0.2),([8,2,1];0.5)\rangle$ and $\tilde{c}_2 = \langle([6,2,6];0.7),([6,4,3];0.4),([6,3,5];0.3)\rangle$ are two G_{SVTrN}-numbers with a pre-assigned $\kappa = 0.9$.

Rewriting the given constraints by introducing slack variables :

$$\begin{aligned} & 2x_1 + 3x_2 + x_3 = 4 \\ & 5x_1 + 4x_2 + x_4 = 15 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

The t-norm and s-norm are $p * q = \max\{p+q-1, 0\}$ and $p \diamond q = \min\{p+q, 1\}$, respectively. The first feasible simplex table is as follows :

Table 10 : First iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_3	2	3	1	0	$4 \rightarrow$
x_4	5	4	0	1	15
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(1)}$	$\tilde{c}_2^{(1)} \uparrow$	$\tilde{c}_3^{(1)}$	$\tilde{c}_4^{(1)}$	

Here $\tilde{c}_1^{(1)} = -\tilde{c}_1 = \langle([-8,3,1];0.6),([-8,4,3];0.2),([-8,1,2];0.5)\rangle$,
$\tilde{c}_2^{(1)} = -\tilde{c}_2 = \langle([-6,6,2];0.7),([-6,3,4];0.4),([-6,5,3];0.3)\rangle$
and $V_\kappa(\tilde{c}_3^{(1)}) = V_\kappa(\tilde{c}_4^{(1)}) = V_\kappa(\tilde{0})$.
Then $V_\kappa(\tilde{c}_1^{(1)}) = \frac{1}{6}(25.11\kappa - 43.11)$ and $V_\kappa(\tilde{c}_2^{(1)}) = \frac{1}{6}(11.62\kappa - 31.22)$ by Definition 3.8.1.
Clearly $V_\kappa(-\tilde{c}_1) < 0$, $V_\kappa(-\tilde{c}_2) < 0$ and $V_\kappa(-\tilde{c}_1) - V_\kappa(-\tilde{c}_2) > 0$ for $\kappa = 0.9$.

So x_2 enters in the basis and as $\min\{4/3, 15/4\} = 4/3$, the leaving variable is x_3. The revised table is as :

Table 11 : Second iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_2	2/3	1	1/3	0	$4/3 \rightarrow$
x_4	7/3	0	-4/3	1	29/3
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(2)} \uparrow$	$\tilde{c}_2^{(2)}$	$\tilde{c}_3^{(2)}$	$\tilde{c}_4^{(2)}$	$\frac{4}{3}\tilde{c}_2$

where $V_\kappa(\tilde{c}_2^{(2)}) = V_\kappa(\tilde{c}_4^{(2)}) = V_\kappa(\tilde{0})$ and

$$\begin{aligned}
\tilde{c}_1^{(2)} &= \frac{2}{3}\tilde{c}_2 - \tilde{c}_1 = \langle([-4, 13/3, 5]; 0.3), ([-4, 20/3, 5]; 0.6), ([-4, 3, 16/3]; 0.8)\rangle, \\
\tilde{c}_3^{(2)} &= \frac{1}{3}\tilde{c}_2 = \langle([2, 2/3, 2]; 0.7), ([2, 4/3, 1]; 0.4), ([2, 1, 5/3]; 0.3)\rangle.
\end{aligned}$$

Then $V_\kappa(\tilde{c}_1^{(2)}) = \frac{1}{18}(8.62\kappa - 14.92)$ and $V_\kappa(\tilde{c}_3^{(2)}) = \frac{1}{18}(31.22 - 11.62\kappa)$ by Definition 3.8.1. Clearly, $V_\kappa(\tilde{c}_1^{(2)}) < 0$ but $V_\kappa(\tilde{c}_3^{(2)}) > 0$ for $\kappa = 0.9$. So x_1 enters in the basis and as $\min\{\frac{4/3}{2/3}, \frac{29/3}{7/3}\} = 2$, the leaving variable is x_2. The revised table is :

Table 12 : Third iteration

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$x_B \Downarrow$	x_1	x_2	x_3	x_4	R.H.S
x_1	1	3/2	1/2	0	2
x_4	0	-7/2	-5/2	1	5
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(3)}$	$\tilde{c}_2^{(3)}$	$\tilde{c}_3^{(3)}$	$\tilde{c}_4^{(3)}$	$2\tilde{c}_1$

where $V_\kappa(\tilde{c}_1^{(3)}) = V_\kappa(\tilde{c}_4^{(3)}) = V_\kappa(\tilde{0})$ and

$$\begin{aligned}
\tilde{c}_2^{(3)} &= \frac{3}{2}\tilde{c}_1 - \tilde{c}_2 = \langle([6, 7.5, 6.5]; 0.3), ([6, 7.5, 10]; 0.6), ([6, 8, 4.5]; 0.8)\rangle, \\
\tilde{c}_3^{(3)} &= \frac{1}{2}\tilde{c}_1 = \langle([4, 0.5, 1.5]; 0.6), ([4, 1.5, 2]; 0.2), ([4, 1, 0.5]; 0.5)\rangle.
\end{aligned}$$

Then $V_\kappa(\tilde{c}_2^{(3)}) = \frac{1}{6}(7.46 - 4.31\kappa)$ and $V_\kappa(\tilde{c}_3^{(3)}) = \frac{1}{6}(21.555 - 12.555\kappa)$ by Definition 3.8.1. Obviously, $V_\kappa(\tilde{c}_2^{(3)}) > 0$ and $V_\kappa(\tilde{c}_3^{(3)}) > 0$ for $\kappa = 0.9$. Hence the optimality arises. The optimal solution is $x_1 = 2, x_2 = 0$ and so Max $\tilde{z} =_{\Re_\kappa} 2\tilde{c}_1$.

7 Conclusion

In this paper, the crisp LP-problem has been generalised by considering the coefficients of the objective function as G_{SVN}-numbers. This generalised form of crisp LP-problem is called NLP-problem. Then a simplex algorithm has been proposed to solve such NLP-problems. Finally, the newly developed simplex algorithm has been applied to a real life problem. The concept has been illustrated by suitable examples using both G_{SVTN}-numbers and G_{SVTrN}-numbers. In future, the concept of a linear programming problem may be extended in more generalised way by considering some or all of the parameters as G_{SVN}-numbers.

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