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An effective container inventory model under bipolar neutrosophic environment

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Abstract

The fuzzy set and its application play a major role to most of the uncertainty situations of inventory management problem. At present, as an enlargement of fuzzy set, the notion of neutrosophic set is initiated to implement in inventory models for uncertain parameters. In this paper, the Trapezoidal Bipolar Neutrosophic Number (TrBNN) is enforced to the container inventory model. Because of the imbalanced flow of containers, the container management organization faces the major issue of scarcity of containers. One-way Free Use (OFU) of container and renting of containers are employed to restore the shortfall units. An algorithm is designed to make a decision on various conditions to compute the expected total cost. Also, this paper scrutinizes a condition that some fraction of deficit containers is one-way free used and the remaining are leased. Unpredictable parameters such as the fraction of received containers after used, the fraction of amendable containers from received units, and the fraction of one-way free used containers are presumed as TrBNN. In the view of reduce the total cost, a neutrosophic container inventory model is framed to obtain the optimal duration of inspection process and the optimal duration of leasing process. To flourish this study more effective, the proposed container inventory model is compared with the model by presuming Triangular Bipolar Neutrosophic Number (TBNN).
Keywords: Trapezoidal bipolar neutrosophic number, Triangular bipolar neutrosophic number, Container inventory, One-way free use, Lease.

1. Introduction

The major aspect of any business trading is to attract the customers from the competitors throughout the globe. So that, the import and export trading has been developing day by day in most of the countries. The import and export traders spend lots of cost for cargoes transportation and thus for minimizing their cost and transporting cargoes safely, they chosen the Reusable Containers (RCs). In order to transporting goods from one destination to another, the non-vessel operating common carrier, or the shipping companies provides the RCs to the consigners. This study examines some of the issues and the various cost of maintaining the reusable container faced by the Container Management Organization (CMO).

The objective of any CMO is to satisfy the consigner's requirements. Because of imbalanced business trading, shortage of container will occur. This research prescribes the CMO to implement the following strategies in order to avoid from scarcity of containers. One of the main strategies is one-way free use; the OFU containers only make one trip. To return the containers at OFU vendor's depot, they allow OFU of containers from their surplus area to slack area instead of repositioning under free of cost. OFU containers help business to minimize the costs. Since, the OFU vendor offers only limited number of units, all the deficit containers are unable to replace under OFU option. Thus, the proportion of shortfall units are one-way free used and the remaining are restored from another CMO as rent. There is no cost spent for OFU when compared to leasing or purchasing a container. That is, OFU of container does not incurred costs like repositioning cost, carrying charge as well as repairing charge but it incurred only the screening charge.

Since the proportion of received containers after used, the proportion of amendable containers from received RCs, and the fraction of OFU RCs are uncertain, the present model framed under bipolar neutrosophic arena to reach the most approximate solutions. A container inventory model with price sensitive demand is developed and the costs under various strategies are framed. Neutrosophication of the various proportions of the proposed model leads the CMO to attain most approximate ratio. In this study, these proportions are considered as single type linear TrBNN and then removal area technique of de-bipolarization is applied. The proposed study is compared with TrBNN and TBNN. The research works related to these topics are discussed as follows.

Buchanan and Abad [9] framed the single and N-periods study on inventory model for routine system of reusable transport containers. A notion of repositioning empty containers on container inventory model along with leasing option under (s,S) policy is analyzed by Yun et al. [38]. Further, in 2014, Kim and Glock [22] scrutinized the RFID and its usage to the management of reusable containers by presuming the proportion of returned units as stochastic. Glock and Kim [16] studied an inventory model of combined finished goods and RTIs under some safety measures. Hariga et al. [19] designed the reusable container inventory model with finished goods of single vendor single retailer along with the renting option of RTIs for delay returns.

Cobb [13] studied the container inventory control problem to attain the optimal inspection length, the optimal mending period and the optimal purchasing. This study presumed that the investigation procedure as well as the mending procedure done subsequently and analyzed about the early returned containers which are stored as safety stock. The studies [15, 20, 23, 24] examine the container inventory model under various strategies. Further, the works [17] and [25] analyzed the container inventory model with the notion of repositioning of empty container. Maity et al. [26] analyzed the EOQ model under cloudy fuzzy logic. Rajeswari et al. [30] examined the work of Cobb [13] and presented a container management model with customer charge sensitive demand by utilizing the ECR as well as the renting option instead of buying new containers under fuzzy arena. Recently, [31 and 32] studied the notion of prepayment strategy in fuzzy EOQ model under various situations.

The uncertainty situation leads the researchers of various fields to use the approach of the fuzzy set and its applications. First of all, the fuzzy set and its approach were initiated by Zadeh [39]. Heilpern [21] formulated the expected value along with the expected value interval of fuzzy number. Then Atanassov [2] elongated the fuzzy set as intuitionistic fuzzy set and some researchers classified the intuitionistic fuzzy set under various types and applied to various situations. Shaw and Row [33] established the trapezoidal intuitionistic fuzzy set along with the arithmetic operations then applied to reach the accurate result.

As an enlargement of the fuzzy set and intuitionistic fuzzy set, the neutrosophic set is established by Smarandache [34]. For easy understanding, Wang et al. [36] designed the neutrosophic set as the singled valued neutrosophic set. For effective results, [3, 4, 8, 18, 29, 37] utilizes various measures on neutrosophic numbers to decision making models under various environments. A new de-netrosophication method of pentagonal neutrosophic number using removal area method is established by Chakraborty et al. [10] and utilized this method to a minimal spanning tree. Further, [27, 28] developed the inventory models with various strategies under neutrosophic arena.

By the view of negative and positive part of the human decision-making techniques, Bosc and Pivert [5] established the bipolar fuzzy set. Subsequently, Abdullah et al. [1] designed the bipolar fuzzy soft set and utilized to decision theory. Followed by these works, the concept bipolar neutrosophic set is initiated by Deli et al. [14] and applied its notion to decision making problem. In 2016, Broumi et al. [6] proposed the approach of bipolar single valued neutrosophic under graph theory and [7] utilizes the application of bipolar neutrosophic set and solved the shortest path problem. For effective outcomes, [11, 12, 35] framed the various measures on different bipolar neutrosophic numbers to decision making models under various situations.

The objective of this research is to help the CMO in order to supply the RC to their consigner without scarcity under minimum cost. The novelty of this study is to utilizing the OFU and leasing option to avoid the revenue loss from deficit containers instead of repositioning the empty container or purchasing new container. To attain the outcomes pore effective, this model is proposed under bipolar neutrosophic environment. The main contribution of this paper is the OFU option, which helps the CMO reduce the total cost. The advantage of utilizing the one-way free use strategy is the CMO compensates for the shortfall containers without spending much cost.

The elementary definitions on fuzzy set, Intuitionistic fuzzy set, Neutrosophic set and bipolar neutrosophic set are provided in section 2. In section 3, the container inventory costs under various strategies say serviceable container cost, OFU cost, ECR cost and least cost are computed, and an algorithm is given to make decision on choosing these strategies. Then the container inventory model under bipolar neutrosophic environment is framed by presuming the proportions of return rate, repair rate, and the OFU rate as TrBNN and also the de-bipolrization of TrBNN and TBNN are provided. The numerical computation and sensitivity analysis is performed in section 4. In the last section, the outcomes of the study are deliberated.

2. Preliminaries

Some of the preliminaries related to this research are as follows.

***Fuzzy set* [39]:** A set of 2-tuples $\tilde{\xi} = (z', \mu_{\tilde{\xi}}(z')) : z' \in \mathcal{U}$ is said to be a a fuzzy set $\tilde{\xi}$ in $\mathcal{U}$ (Universe of discourse) where $\mu_{\tilde{\xi}}(z')$ represents the membership degree of z' such that $\mu_{\tilde{\xi}}(z') \in [0,1]$.

***Fuzzy number* [39]:** Let $\mathbb{R}$ be a real line then $\tilde{\xi} \subset \mathbb{R}$ is said to be a fuzzy number whose membership degree $\mu_{\tilde{\xi}}$ satisfies the given conditions

i. $\mu_{\tilde{\xi}}(z')$ is piecewise continuous in its domain.

ii. $\tilde{\xi}$ is normal, i.e., $\exists\, z'_0 \in \tilde{\xi}$ such that $\mu_{\tilde{\xi}}(z'_0) = 1$.

iii. $\tilde{\xi}$ is convex, i.e., $\mu_{\tilde{\xi}}(\epsilon z'_1 + (1-\epsilon) z'_2) \geq min\left(\mu_{\tilde{\xi}}(z'_1), \mu_{\tilde{\xi}}(z'_2)\right) \forall\, z'_1, z'_2$ in $\mathcal{U}$.

***Trapezoidal Fuzzy number*:[3]** Let $\tilde{\xi} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ be a trapezoidal fuzzy number then it has a membership degree

$$\mu_{\tilde{\xi}}(z') = \begin{cases} \frac{z' - \sigma_1}{\sigma_2 - \sigma_1}, & \sigma_1 \leq z' \leq \sigma_2 \\ 1, & \sigma_2 \leq z' \leq \sigma_3 \\ \frac{\sigma_4 - z'}{\sigma_4 - \sigma_3}, & \sigma_3 \leq z' \leq \sigma_4 \\ 0, & otherwise \end{cases}.$$

where, $\mu_{\tilde{\xi}}(z')$ satisfies the following conditions:

(i) $\mu_{\tilde{\xi}}(z')$ is a continuous mapping from $\mathbb{R}$ to [0,1],

(ii) $\mu_{\tilde{\xi}}(z') = 0$ for every $z' \in (-\infty, \sigma_1]$,

(iii) $\mu_{\tilde{\xi}}(z')$ is strictly increasing and continuous on $[\sigma_1, \sigma_2]$,

(iv) $\mu_{\tilde{\xi}}(z') = 0$ for every $z' \in [\sigma_2, \sigma_3]$,

(v) $\mu_{\tilde{\xi}}(z')$ is strictly decreasing and continuous on $[\sigma_3, \sigma_4]$,

(vi) $\mu_{\tilde{\xi}}(z') = 0$ for every $z' \in (\sigma_4, \infty,]$.

***Intuitionistic Fuzzy Set* [2]:** An intuitionistic fuzzy set $\tilde{\xi}$ in $\mathcal{U}$ (finite universe of discourse) is given as $\tilde{\xi} = \{\langle z', \varphi_{\tilde{\xi}}(z'), \psi_{\tilde{\xi}}(z')\rangle \,|z' \in \mathcal{U}\}$, where $\varphi_{\tilde{\xi}}: \mathcal{U} \to [0,1]$ and $\psi_{\tilde{\xi}}: \mathcal{U} \to [0,1]$ satisfying the condition $0 \le \varphi_{\tilde{\xi}}(z') + \psi_{\tilde{\xi}}(z') \le 1$ and $\varphi_{\tilde{\xi}}(z')$ denotes the membership degree of $z' \in \mathcal{U}$ in $\tilde{\xi}$ and $\psi_{\tilde{\xi}}(z')$ represents the non-membership function of $z' \in \mathcal{U}$ in $\tilde{\xi}$. Along with this, the hesitance degree of $z' \in \mathcal{U}$ in $\tilde{\xi}$ is denoted as $\varepsilon_{\tilde{\xi}}(z')$ and is given as $\varepsilon_{\tilde{\xi}}(z') = 1 - \varphi_{\tilde{\xi}}(z') - \psi_{\tilde{\xi}}(z')$. For convenience, the intuitionistic fuzzy number is considered as $\tilde{\xi} = \left(\varphi_{\tilde{\xi}}(z'), \psi_{\tilde{\xi}}(z')\right)$.

***Trapezoidal Intuitionistic Fuzzy Number* [33]:** A fuzzy number $\tilde{\mathcal{F}} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4), (\sigma_1^1, \sigma_2^1, \sigma_3^1, \sigma_4^1)$ is a trapezoidal intuitionistic fuzzy number where $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_1^1, \sigma_2^1, \sigma_3^1, \sigma_4^1$ are real and its membership degree $\mu_{\tilde{\mathcal{F}}}(g)$ and non-membership degree $\vartheta_{\tilde{\mathcal{F}}}(g)$ are given as below:

$$\mu_{\tilde{\mathcal{F}}}(g) = \begin{cases} \left(\frac{g-\sigma_1}{\sigma_2-\sigma_1}\right), & \sigma_1 \le g \le \sigma_2 \\ 1, & \sigma_2 \le g \le \sigma_3 \\ \left(\frac{\sigma_4-g}{\sigma_4-\sigma_3}\right), & \sigma_3 \le g \le \sigma_4 \\ 0, & otherwise \end{cases}$$

$$\text{and } \vartheta_{\tilde{\mathcal{F}}}(g) = \begin{cases} \left(\frac{\sigma_2^1-g}{\sigma_2^1-\sigma_1^1}\right), & \sigma_1^1 \le g \le \sigma_2^1 \\ 0, & \sigma_2^1 \le g \le \sigma_3^1 \\ \left(\frac{g-\sigma_3^1}{\sigma_4^1-\sigma_3^1}\right), & \sigma_3^1 \le g \le \sigma_4^1 \\ 1, & otherwise \end{cases}.$$

***Neutrosophic Set* [34]:** A neutrosophic set $\tilde{\mathcal{A}}_N$ in $\mathcal{U}$ (Universe of discourse) is catogorized as functions of a truth membership $T_{\tilde{\mathcal{A}}_N}(g)$, an indeterminacy membership $I_{\tilde{\mathcal{A}}_N}(g)$ and a falsity membership $F_{\tilde{\mathcal{A}}_N}(g)$. The functions $T_{\tilde{\mathcal{A}}_N}$, $I_{\tilde{\mathcal{A}}_N}$ and $F_{\tilde{\mathcal{A}}_N}$ are real standard or non-standard subsets of $]^{-}0, 1^{+}[$ i.e., $T_{\tilde{\mathcal{A}}_N}: \mathcal{U} \to]^{-}0, 1^{+}[$; $I_{\tilde{\mathcal{A}}_N}: \mathcal{U} \to]^{-}0, 1^{+}[$; $F_{\tilde{\mathcal{A}}_N}: \mathcal{U} \to]^{-}0, 1^{+}[$. $T_{\tilde{\mathcal{A}}_N}(g)$, $I_{\tilde{\mathcal{A}}_N}(g)$, and $F_{\tilde{\mathcal{A}}_N}(g)$ satisfy the relation $^{-}0 \le \sup T_{\tilde{\mathcal{A}}_N}(g) \le \sup I_{\tilde{\mathcal{A}}_N}(g) \le \sup F_{\tilde{\mathcal{A}}_N}(g) \le 3^{+}$, where $g \in \mathcal{U}$

***Fuzzy Neutrosophic Number* [3]:** Let $\tilde{\mathcal{A}}_N$ be a fuzzy neutrosophic number in the set of real numbers $\mathbb{R}$ then its validity membership degree, indeterminacy membership degree and negation membership degree are respectively written as below:

$$T_{\tilde{\mathcal{A}}_N}(g) = \left\langle \begin{matrix} T^L_{\tilde{\mathcal{A}}_N}(g) & \sigma_{11} \le g \le \sigma_{12} \\ 1 & \sigma_{12} \le g \le \sigma_{13} \\ T^U_{\tilde{\mathcal{A}}_N}(g) & \sigma_{13} \le g \le \sigma_{14} \\ 0 & otherwise \end{matrix} \right\rangle$$

$$I_{\tilde{\mathcal{A}}_N}(y) = \begin{cases} I^L_{\tilde{\mathcal{A}}_N}(g) & \sigma_{21} \leq g \leq \sigma_{22} \\ 0 & \sigma_{22} \leq g \leq \sigma_{23} \\ I^U_{\tilde{\mathcal{A}}_N}(g) & \sigma_{23} \leq g \leq \sigma_{24} \\ 1 & otherwise \end{cases}$$

$$F_{\tilde{\mathcal{A}}_N}(y) = \begin{cases} F^L_{\tilde{\mathcal{A}}_N}(g) & \sigma_{31} \leq g \leq \sigma_{32} \\ 0 & \sigma_{32} \leq g \leq \sigma_{33} \\ F^U_{\tilde{\mathcal{A}}_N}(g) & \sigma_{33} \leq g \leq \sigma_{34} \\ 1 & otherwise \end{cases}$$

where $0 \leq \sup T_{\tilde{\mathcal{A}}_N}(g) \leq \sup I_{\tilde{\mathcal{A}}_N}(g) \leq \sup F_{\tilde{\mathcal{A}}_N}(g) \leq 3$, $\forall\, g \in \mathcal{U}$ and for all $i = 1,2,3; j = 1,2,3,4$; $\sigma_{ij} \in$ R, such that $\sigma_{11} \leq \sigma_{12} \leq \sigma_{13} \leq \sigma_{14}$; $\sigma_{21} \leq \sigma_{22} \leq \sigma_{23} \leq \sigma_{24}$ and $\sigma_{31} \leq \sigma_{32} \leq \sigma_{33} \leq \sigma_{34}$. Here $T^L_{\tilde{\mathcal{A}}_N}(g), I^U_{\tilde{\mathcal{A}}_N}(g), F^U_{\tilde{\mathcal{A}}_N}(g) \in [0,1]$ are continuous monotonic increasing functions and $T^U_{\tilde{\mathcal{A}}_N}(g), I^L_{\tilde{\mathcal{A}}_N}(g), F^L_{\tilde{\mathcal{A}}_N}(g) \in [0,1]$ are continuous monotonic decreasing functions.

Single-valued Neutrosophic Set **[36]:** A single-valued neutrosophic set $\tilde{\mathcal{A}}_N$ in $\mathcal{U}$ (Universe of discourse) is catogorized as functions of a truth membership $T_{\tilde{\mathcal{A}}_N}(g)$, an indeterminacy membership $I_{\tilde{\mathcal{A}}_N}(g)$ and a falsity membership $F_{\tilde{\mathcal{A}}_N}(g)$and is given by

$\tilde{\mathcal{A}} = \{g, \langle T_{\tilde{\mathcal{A}}_N}(g),\, I_{\tilde{\mathcal{A}}_N}(g), F_{\tilde{\mathcal{A}}_N}(g)\rangle \mid g \in \mathcal{U}\}$.

Here $T_{\tilde{\mathcal{A}}_N}(g)$, $I_{\tilde{\mathcal{A}}_N}(g), F_{\tilde{\mathcal{A}}_N}(g) \in [0,1]$ and the relation $0 \leq \sup T_{\tilde{\mathcal{A}}_N}(g) \leq \sup I_{\tilde{\mathcal{A}}_N}(g) \leq \sup F_{\tilde{\mathcal{A}}_N}(g) \leq 3$ holds for all $g \in \mathcal{U}$.

Trapezoidal neutrosophic number **[3]:**

A trapezoidal fuzzy neutrosophic number (TrFNN) $\tilde{\mathcal{A}}_N$ in $\mathcal{U}$ (Universe of discourse) is defined as follows:

$$\tilde{\mathcal{A}}_N = \langle(\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}), (\sigma_{21}, \sigma_{22}, \sigma_{23}, \sigma_{24}), (\sigma_{31}, \sigma_{32}, \sigma_{33}, \sigma_{34})\rangle$$

where $\sigma_{11} \leq \sigma_{12} \leq \sigma_{13} \leq \sigma_{14}$; $\sigma_{21} \leq \sigma_{22} \leq \sigma_{23} \leq \sigma_{24}$ and $\sigma_{31} \leq \sigma_{32} \leq \sigma_{33} \leq \sigma_{34}$. Its truth member function is given as

$$T_{\tilde{\mathcal{A}}_N}(g) = \begin{cases} \frac{g-\sigma_{11}}{\sigma_{12}-\sigma_{11}} & \sigma_{11} \leq g \leq \sigma_{12} \\ 1 & \sigma_{12} \leq g \leq \sigma_{13} \\ \frac{\sigma_{14}-g}{\sigma_{14}-\sigma_{13}} & \sigma_{13} \leq g \leq \sigma_{14} \\ 0 & otherwise \end{cases}$$

Its indeterminacy membership function is written as

$$I_{\tilde{\mathcal{A}}_N}(g) = \begin{cases} \frac{\sigma_{22}-g}{\sigma_{22}-\sigma_{21}} & \sigma_{21} \leq g \leq \sigma_{22} \\ 0 & \sigma_{22} \leq g \leq \sigma_{23} \\ \frac{g-\sigma_{24}}{\sigma_{24}-\sigma_{23}} & \sigma_{23} \leq g \leq \sigma_{24} \\ 1 & otherwise \end{cases}$$

and its falsity membership function is written as

$$F_{\tilde{\mathcal{A}}_N}(g) = \begin{cases} \frac{\sigma_{32}-g}{\sigma_{32}-\sigma_{31}} & \sigma_{31} \leq g \leq \sigma_{32} \\ 0 & \sigma_{32} \leq g \leq \sigma_{33} \\ \frac{g-\sigma_{34}}{\sigma_{34}-\sigma_{33}} & \sigma_{33} \leq g \leq \sigma_{34} \\ 1 & otherwise \end{cases}$$

Bipolar Neutrosophic Set **[14]:** A set $\tilde{\mathcal{A}}_{Bi}$ in $\mathcal{U}$ (finite universe of discourse) is said to be the bipolar neutrosophic set if

$\tilde{\mathcal{A}}_{Bi} = \{\langle g, [T_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g), I_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g), F_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g), T_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g), I_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g), F_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g)]\rangle \mid g \in \mathcal{U}\}$, where $T_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g): \mathcal{U} \to [0,1]$, $T_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g): \mathcal{U} \to [-1,0]$ represents the truth membership function, $I_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g): \mathcal{U} \to [0,1]$, $I_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g): \mathcal{U} \to [-1,0]$ represents the indeterminacy membership function and $F_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g): \mathcal{U} \to [0,1]$, $F_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g): \mathcal{U} \to [-1,0]$ represents the falsity membership function.

Triangular bipolar neutrosophic number **[3]:**

A triangular bipolar neutrosophic number (TBNN) $\tilde{\mathcal{A}}_{Bi}$ in $\mathcal{U}$ (Universe of discourse) is defined as follows:

$$\tilde{\mathcal{A}}_{Bi} = \langle(\sigma_{11}, \sigma_{12}, \sigma_{13}), (\sigma_{21}, \sigma_{22}, \sigma_{23}), (\sigma_{31}, \sigma_{32}, \sigma_{33})\rangle$$

where $\sigma_{11} \le \sigma_{12} \le \sigma_{13}$; $\sigma_{21} \le \sigma_{22} \le \sigma_{23}$ and $\sigma_{31} \le \sigma_{32} \le \sigma_{33}$. Its validity membership degree, indeterminacy membership degree and the negation membership degree of TBNN are written as

$$T_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g) = \begin{cases} \frac{g-\sigma_{11}}{\sigma_{12}-\sigma_{11}} & \sigma_{11} \le g < \rho_2 \\ 1 & g = \sigma_{12} \\ \frac{\sigma_{13}-g}{\sigma_{13}-\sigma_{12}} & \sigma_{12} < g \le \rho_4 \\ 0 & otherwise \end{cases}, \quad T_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g) = \begin{cases} \frac{\sigma_{12}-g}{\sigma_{12}-\rho_1} & \sigma_{11} \le g < \rho_2 \\ -1 & g = \sigma_{12} \\ \frac{g-\sigma_{13}}{\sigma_{13}-\sigma_{12}} & \sigma_{12} < g \le \sigma_{13} \\ 0 & otherwise \end{cases};$$

$$I_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g) = \begin{cases} \frac{\sigma_{22}-g}{\sigma_{22}-\sigma_{21}} & \sigma_{21} \le g < \sigma_{22} \\ 0 & g = \sigma_{22} \\ \frac{g-\sigma_{23}}{\sigma_{23}-\sigma_{22}} & \sigma_{22} < g \le \sigma_{23} \\ 1 & otherwise \end{cases}, \quad I_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g) = \begin{cases} \frac{g-\sigma_{22}}{\sigma_{22}-\sigma_{21}} & \sigma_{21} \le g < \sigma_{22} \\ 0 & g = \sigma_{22} \\ \frac{\sigma_{23}-g}{\sigma_{23}-\sigma_{22}} & \sigma_{22} < g \le \sigma_{23} \\ -1 & otherwise \end{cases} \text{ and}$$

$$F_{\tilde{\mathcal{A}}_{Bi}}{}^{+}(g) = \begin{cases} \frac{\sigma_{32}-g}{\sigma_{32}-\sigma_{31}} & \sigma_{31} \le g < \sigma_{32} \\ 0 & g = \sigma_{32} \\ \frac{g-\sigma_{33}}{\sigma_{33}-\sigma_{32}} & \sigma_{32} < g \le \sigma_{33} \\ 1 & otherwise \end{cases}, \quad F_{\tilde{\mathcal{A}}_{Bi}}{}^{-}(g) = \begin{cases} \frac{g-\vartheta_2}{\vartheta_2-\sigma_{31}} & \sigma_{31} \le g < \sigma_{32} \\ 0 & g = \sigma_{32} \\ \frac{\sigma_{33}-g}{\sigma_{33}-\vartheta_3} & \sigma_{32} < g \le \sigma_{33} \\ -1 & otherwise \end{cases}$$

where $-3 \le T_{\tilde{\mathcal{A}}_{Bi}}(g) + I_{\tilde{\mathcal{A}}_{Bi}}(g) + F_{\tilde{\mathcal{A}}_{Bi}}(g) \le 3+$.

Trapezoidal bipolar neutrosophic number **[12]:** A trapezoidal bipolar neutrosophic number (TrBNN) $\tilde{\boldsymbol{\mathcal{A}}}_{\boldsymbol{Bi}}$ in $\mathcal{U}$ (finite universe of discourse) characterized by three independent membership degrees as follows:

$$\tilde{\mathcal{A}}_{Bi} = \langle(\rho_1, \rho_2, \rho_3, \rho_4), (\sigma_1, \sigma_2, \sigma_3, \sigma_4), (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4)\rangle$$

Where $\rho_1 \le \rho_2 \le \rho_3 \le \rho_4$; $\sigma_1 \le \sigma_2 \le \sigma_3 \le \sigma_4$ and $\vartheta_1 \le \vartheta_2 \le \vartheta_3 \le \vartheta_4$.

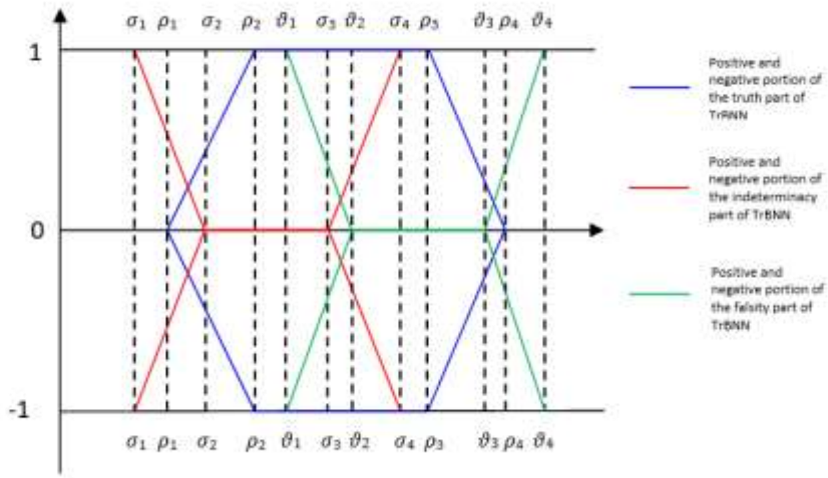

Fig. 1 Linear Trapezoidal Bipolar Neutrosophic Number

Let the positive and the negative portion of truth membership function are denoted by ${T_{\tilde{\mathcal{A}}_{Bi}}}^{+}$ and ${T_{\tilde{\mathcal{A}}_{Bi}}}^{-}$ respectively, the positive and the negative portion of indeterminacy membership function are represented by ${I_{\tilde{\mathcal{A}}_{Bi}}}^{+}$ and ${I_{\tilde{\mathcal{A}}_{Bi}}}^{-}$ respectively and the positive and the negative portion of negation membership function are denoted by ${F_{\tilde{\mathcal{A}}_{Bi}}}^{+}$ and ${F_{\tilde{\mathcal{A}}_{Bi}}}^{-}$ respectively then the validity membership degree, indeterminacy membership degree and the negation membership degree of TrBNN are written as

$$ {T_{\tilde{\mathcal{A}}_{Bi}}}^{+}(g) = \begin{cases} \frac{g-\rho_1}{\rho_2-\rho_1} & \rho_1 \le g < \rho_2 \\ 1 & \rho_2 \le g \le \rho_3 \\ \frac{\rho_4-g}{\rho_4-\rho_3} & \rho_3 < g \le \rho_4 \\ 0 & otherwise \end{cases}, \quad {T_{\tilde{\mathcal{A}}_{Bi}}}^{-}(g) = \begin{cases} \frac{\rho_2-g}{\rho_2-\rho_1} & \rho_1 \le g < \rho_2 \\ -1 & \rho_2 \le g \le \rho_3 \\ \frac{g-\rho_4}{\rho_4-\rho_3} & \rho_3 < g \le \rho_4 \\ 0 & otherwise \end{cases}; $$

$$ {I_{\tilde{\mathcal{A}}_{Bi}}}^{+}(g) = \begin{cases} \frac{\sigma_2-g}{\sigma_2-\sigma_1} & \sigma_1 \le g < \sigma_2 \\ 0 & \sigma_2 \le g \le \sigma_3 \\ \frac{g-\sigma_4}{\sigma_4-\sigma_3} & \sigma_3 < g \le \sigma_4 \\ 1 & otherwise \end{cases}, \quad {I_{\tilde{\mathcal{A}}_{Bi}}}^{-}(g) = \begin{cases} \frac{g-\sigma_2}{\sigma_2-\sigma_1} & \sigma_1 \le g < \sigma_2 \\ 0 & \sigma_2 \le g \le \sigma_3 \\ \frac{\sigma_4-g}{\sigma_4-\sigma_3} & \sigma_3 < g \le \sigma_4 \\ -1 & otherwise \end{cases} \text{ and} $$

$$ {F_{\tilde{\mathcal{A}}_{Bi}}}^{+}(g) = \begin{cases} \frac{\vartheta_2-g}{\vartheta_2-\vartheta_1} & \vartheta_1 \le g < \vartheta_2 \\ 0 & \vartheta_2 \le g \le \vartheta_3 \\ \frac{g-\vartheta_4}{\vartheta_4-\vartheta_3} & \vartheta_3 < g \le \vartheta_4 \\ 1 & otherwise \end{cases}, \quad {F_{\tilde{\mathcal{A}}_{Bi}}}^{-}(g) = \begin{cases} \frac{g-\vartheta_2}{\vartheta_2-\vartheta_1} & \vartheta_1 \le g < \vartheta_2 \\ 0 & \vartheta_2 \le g \le \vartheta_3 \\ \frac{\vartheta_4-g}{\vartheta_4-\vartheta_3} & \vartheta_3 < g \le \vartheta_4 \\ -1 & otherwise \end{cases} $$

The de-bipolarization of single typed linear TrBNN is adopted from Chakraborty et al. [12] which is written as

$$ S\left(\widetilde{De}_{\mathcal{A}_{Bi},0}\right) = \frac{\rho_1+\rho_2+\rho_3+\rho_4+\sigma_1+\sigma_2+\sigma_3+\sigma_4+\vartheta_1+\vartheta_2+\vartheta_3+\vartheta_4}{6}. \tag{1} $$

The de-bipolarization of single typed linear TBNN is adopted from Chakraborty et al. [11] which is written as

$$ S\left(\widetilde{De}_{\mathcal{A}_{Bi},0}\right) = \frac{\sigma_{11}+2\sigma_{12}+\sigma_{13}+\sigma_{21}+2\sigma_{22}+\sigma_{23}+\sigma_{31}+2\sigma_{32}+\sigma_{33}}{6}. \tag{2} $$

3. Problem development:

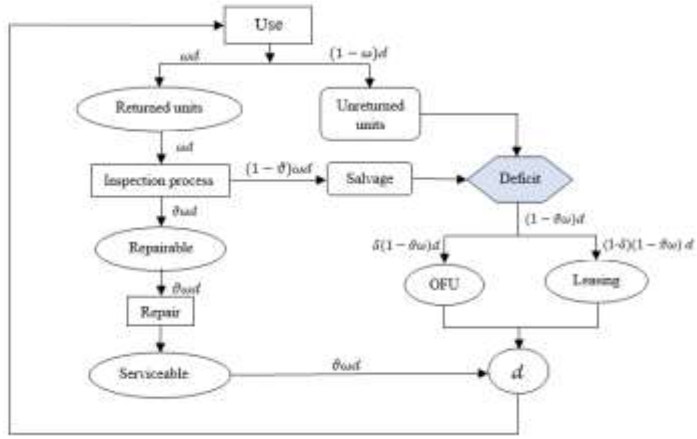

Fig.2 Diagrammatic representation of container flow per cycle

For the purpose of import and export trading, the container management organization affords the empty container to its consigner. Every CMO managing and supplies the large number of empty containers for trading. Fig. 2 clearly shows the container flow under various strategies on a closed loop supply chain. For the convenience, in this study, it is presumed that the CMO supplied the demand of d per supply chain, and it is assumed as price sensitive demand, that is $d = A - B p$, where p represents the rent acquires from customer per container, $A > 0$ and $B > 0$ are constant and price dependent coefficients respectively. Because of the imbalance flow of containers, this study scrutinizes the shortage territory. According to [16], instead of d units, ω fraction of used containers received and the balance $(1 - \omega)d$ units being unreturned in that supply chain. After receiving the used containers, they are subjected to the process of inspecting during the time τ_I at constant rate φ, as well as the process of repairing begins simultaneously at the rate R. While screening ωd units, it is observed that ϑfraction of received units is restorable but the remaining $(1 - \vartheta)\omega d$ units are not reusable which are kept as salvaged units. Here, the unreturned and the salvaged units are considered as deficit containers. The duration to mend $\vartheta\omega d$ is considered as τ_R. The number of reusable containers inspected during τ_I is $\varphi\tau_I$ and $\tau_R = \frac{\vartheta\varphi T_i}{R}$ is the repairing process duration. After the repairing process, the $\vartheta\omega d$ RCs are assigned as ready to service units. To restore the deficit containers, the OFU of container is considered and OFU vendor allows only a limited unit for one-way free use. So that, the proportion of shortfall RCs say $\delta(1 - \omega\vartheta)d$ is presumed as OFU and the remaining $(1 - \delta)(1 - \omega\vartheta)d$ RCs are leased from another CMO and stored with containers that are ready to service.

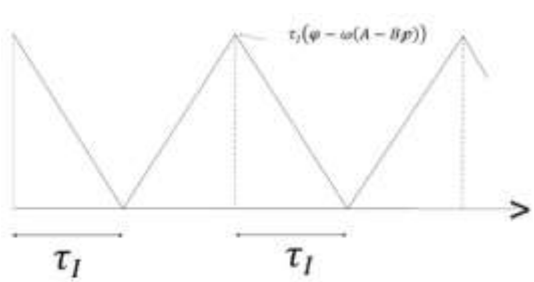

Fig. 3 (a)

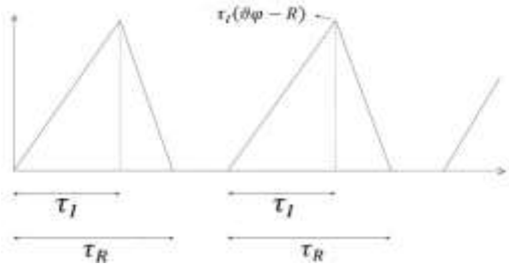

Fig. 3 (b)

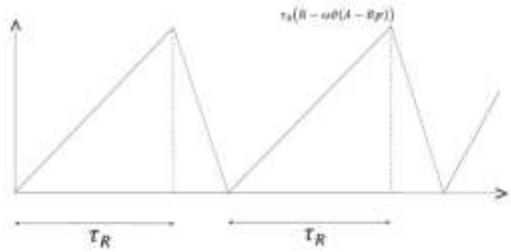

Fig. 3 (c)

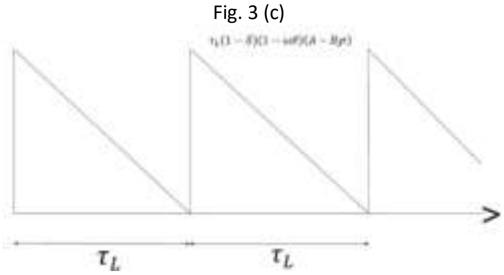

Fig. 3 (d)

Fig. 3 Inventory level of the Containers under various strategies

3.1. *Maximum inventory level*

The amount of received RCs, the number of amendable ones, the number of RCs that are ready to service and the amount of rented RCs and then the maximum units of all the above strategies are conferred as follows.

In this study, it is hypothesized that ωd reusable containers are received after the use and the left over $(1-\omega)d$ containers are not received in that cycle. Fig. 3(a) clearly indicates that the duration inspecting the received container is τ_I and the number of units examined per length is $\varphi\tau_I$. Thus, for each inspection procedure, the proportion of a year is $\frac{\varphi\tau_I}{\omega D}$, where $D = md$, is the annual demand and m denotes the total working days per annum. The idle time between the inspection procedure per unit time and the RCs stocked over that time is $m\frac{\varphi\tau_I}{\omega D} - \tau_I$. Also, by observing Figs. 3(a)-3(c), it is noted that the examining and the mending works proceeds simultaneously. Once a container is screened, it has been sent for mending process and then kept as ready to service RCs.

$$\text{The maximum unit of received RCs} = \left(m\frac{\varphi\tau_I}{\omega D} - \tau_I\right)\omega d$$
$$= \tau_I\big(\varphi - \omega(A - B\wp)\big) \quad (3)$$

Here, the inventory level reduces at a rate of $\varphi - \omega(A - B\wp)$ per day and the containers received at the fraction of $\omega(A - B\wp)$ per day when the inspection period is idle.

It is clear that the number of containers sent for repairing process is $\vartheta\varphi\tau_I = R\tau_R$. During the time τ_I, the rate of $\vartheta\varphi - R$ containers received for repairing process.

Thus, the maximum unit of amendable RCs $= \tau_I\ (\vartheta\varphi - R)$. (4)

The total days that mending process done per year is $\frac{\vartheta\varphi D}{R}$ and percentage of mending process per year is $\frac{\vartheta\varphi D}{mR}$. Clearly, the stock level reduces at the rate of R for one unit time during the period $\tau_I\ (\vartheta\varphi - R)/R$.

On observing Fig. 3(b) and Fig. 3(c), the RCs are stored as ready to service units after the completion of the mending process. Thus, the serviceable containers compile by $R - \vartheta\varphi(A - B\wp)$ for one unit time which reduces by the rate of $\vartheta\varphi(A - B\wp)$ when the mending time is idle. Hence,

The maximum stock of serviceable RCs $=\tau_R\big(R - \vartheta\varphi(A - B\wp)\big)$. (5)

To rectify the issue of unbalancing container flow, the CMO preferred some effective strategies such as OFU and leasing option. Since, the OFU vendor offers only limited number of containers for OFU, the fraction $\delta(1 - \omega\vartheta)d$ is considered as OFU and the remaining units are restored using leasing option. Therefore, the maximum number of containers for OFU are $\delta(1 - \omega\vartheta)(A - B\wp)$.

It is clearly noted that from Fig. 3(d), the time τ_L represents the successive leasing cycles. During this time period, the expected maximum number of rented RCs per cycle is given as follows.

The maximum stock of rented RCs $= \tau_L(1 - \delta)(1 - \omega\vartheta)(A - B\wp)$. (6)

Here, the inventory reduces at the rate of $(1 - \delta)(1 - \omega\vartheta)(A - B\wp)$ per leasing cycle.

3.2. *Cost model*

3.2.1. *Serviceable container cost*

The serviceable container cost is the charges for screening and repairing process of returned containers which includes the fixed charge and the variable charge of screened and repairable units and the storage charge of received, repaired and serviceable RCs. The variable charges of all the categories are computed with respect to number of RCs processed. The integrated fixed costs and the variable costs obtained from the stock of received RCs and amendable RCs is given as,

$$FC_{IR} + VC_{IR} = \frac{m\omega(A - B\wp)(W_i + W_r)}{\varphi\tau_I} + m\omega(A - B\wp)(w_i + \vartheta w_r). \quad (7)$$

The storage cost is the charge received by the empty yard for the stock of received RCs, amendable RCs and the ready to service RCs which is derived as,

$$HC_U = \frac{\tau_I}{2}\left[\big(\varphi - \omega(A - B\wp)\big)\mathcal{H}_u\ \right]. \quad (8)$$

$$HC_R = \frac{\tau_I}{2}\left[\frac{\omega\vartheta(A-B\wp)(\varphi\vartheta-R)}{R}\mathcal{H}_r\right]. \quad (9)$$

$$HC_S = \frac{\tau_I}{2}\left[\frac{\varphi\vartheta\big(R-\vartheta\omega(A-B\wp)\big)}{R}\mathcal{H}_s\right]. \quad (10)$$

The serviceable container cost for returned container is derived as

$$SCC = FC_{IR} + VC_{IR} + HC_U + HC_R + HC_S \quad (11)$$

$$= \frac{m\omega(A - B\wp)(W_i + W_r)}{\varphi\tau_I} + m\omega(A - B\wp)(w_i + \vartheta w_r) + \frac{\tau_I}{2}\left[\big(\varphi - \omega(A - B\wp)\big)\mathcal{H}_u + \frac{\omega\vartheta(A - B\wp)(\varphi\vartheta - R)}{R}\mathcal{H}_r + \frac{\varphi\vartheta\big(R - \vartheta\omega(A - B\wp)\big)}{R}\mathcal{H}_s\right]. \quad (12)$$

where W_i and w_i denotes the fixed and variable inspection charge per container; W_r and w_r represents the fixed and variable mending charge per container; $\mathcal{H}_u$, $\mathcal{H}_r$ and $\mathcal{H}_s$ are the unit storage charge for returned, repaired and serviceable containers respectively.

3.2.2. *One-way Free Use cost*

For OFU containers, the customer should pick the serviceable containers from the OFU vendor's empty depot. So that, there is no cost spent for repairing as well as carrying the OFU containers but for survey process, the inspection charge will arise. In OFU, the variable inspection charge per container is considered as $(1-\theta)$ proportion of unit variable screening charge of returned container w_i. Thus, the one-way free use cost is given as

$$OFUC = m\delta(1-\omega\vartheta)(A-B\wp)(1-\theta)w_i. \tag{13}$$

3.2.3. *Lease cost*

The fixed ordering charge and the rent for a leased container are W_ℓ and w_ℓ respectively and $\mathcal{H}_\ell$ denotes the unit storage charge per leased container. Thus, the combined fixed costs and the variable costs of leased containers is given as,

$$FC_L + VC_L = \frac{mW_\ell}{\tau_L} + m(1-\delta)(1-\omega\vartheta)(A-B\wp)w_\ell. \tag{14}$$

The carrying charge of leased containers is derived as

$$HC_L = \frac{\tau_L}{2}[(1-\delta)(1-\omega\vartheta)(A-B\wp)\mathcal{H}_\ell]. \tag{15}$$

Therefore, the lease cost is given as

$$LC = FC_L + VC_L + HC_L. \tag{16}$$

$$= \frac{mW_\ell}{\tau_L} + m(1-\delta)(1-\omega\vartheta)(A-B\wp)w_\ell + \frac{\tau_L}{2}[(1-\delta)(1-\omega\vartheta)(A-B\wp)\mathcal{H}_\ell]. \tag{17}$$

An algorithm is designed to make decision on various strategies for computing the total cost.

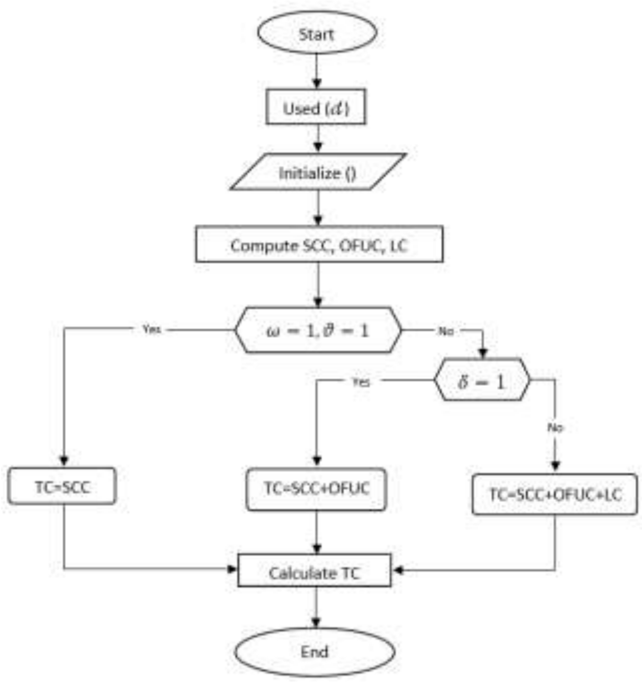

Fig. 4 Flow chart for decision making

Step 1: Initialize all parameters.

Step 2: Calculate SCC, $OFUC$ and LC.
Step 3: If $\omega = 1$ and $\vartheta = 1$, then the total cost $TC = SCC$, using Equation (12).
Step 4: If $\omega \neq 1$, $\vartheta \neq 1$ and $\delta = 1$, then the total cost $TC = SCC + OFUC$, using Equations (12) and (13).
Step 5: If $\omega \neq 1$, $\vartheta \neq 1$ and $\delta \neq 1$ then the total cost $TC = SCC + OFUC + LC$, using Equations (12), (13) and (17).
In this paper, it is considered that $\omega \neq 1$, $\vartheta \neq 1$ and $\delta \neq 1$.
Thus, the total cost, $TC = SCC + OFUC + LC$. (18)
Therefore, the total cost is obtained as

$$TC(\tau_I, \tau_L) = \frac{m\omega(A - Bp)(W_i + W_r)}{\varphi\tau_I} + \frac{mW_\ell}{\tau_L} + m(A - Bp)[\omega(w_i + \vartheta(w_r - \delta(1-\theta)w_i - (1-\delta)w_\ell)) + \delta(1-\theta)w_i + (1-\delta)w_\ell] + \frac{\tau_I}{2}\left[(\varphi - \omega(A - Bp))\mathcal{H}_u + \frac{\omega\vartheta(A - Bp)(\varphi\vartheta - R)}{R}\mathcal{H}_r + \frac{\varphi\vartheta(R - \omega\vartheta(A - Bp))}{R}\mathcal{H}_s\right] + \frac{\tau_L}{2}[(1-\delta])(1 - \omega\vartheta)(A - Bp)\mathcal{H}_\ell]. \quad (19)$$

3.3 *Bipolar Neutrosophic container inventory model*

The container inventory model under bipolar neutrosophic arena is framed by presuming the proportion of received RCs, the fraction of repairable from received RCs and the fraction of OFU RCs as TrBNN. That is,

$\widetilde{\omega} = \langle(\omega_{11}, \omega_{12}, \omega_{13}, \omega_{14}), (\omega_{21}, \omega_{22}, \omega_{23}, \omega_{24}), (\omega_{31}, \omega_{32}, \omega_{33}, \omega_{34})\rangle$;
$\widetilde{\vartheta} = \langle(\vartheta_{11}, \vartheta_{12}, \vartheta_{13}, \vartheta_{14}), (\vartheta_{21}, \vartheta_{22}, \vartheta_{23}, \vartheta_{24}), (\vartheta_{31}, \vartheta_{32}, \vartheta_{33}, \vartheta_{34})\rangle$;
$\widetilde{\delta} = \langle(\delta_{11}, \delta_{12}, \delta_{13}, \delta_{14}), (\delta_{21}, \delta_{22}, \delta_{23}, \delta_{24}), (\delta_{31}, \delta_{32}, \delta_{33}, \delta_{34})\rangle$.

Therefore, the total cost in bipolar neutrosophic sense, $TC_{Bi}(\widetilde{\tau_I, \tau_E}, \tau_L)$ is obtained as,

$$TC_{\widetilde{Bi(\tau_I}}, \tau_L) = \frac{m\widetilde{\omega}(A - Bp)(W_i + W_r)}{\varphi\tau_I} + \frac{mW_\ell}{\tau_L} + m(A - Bp)[\widetilde{\omega}\left(w_i + \widetilde{\vartheta}(w_r - \widetilde{\delta}(1-\theta)w_i - (1-\widetilde{\delta})w_\ell)\right) + \widetilde{\delta}(1-\theta)w_i + (1-\widetilde{\delta})w_\ell] + \frac{\tau_I}{2}\left[(\varphi - \widetilde{\omega}(A - Bp))\mathcal{H}_u + \frac{\widetilde{\omega}\widetilde{\vartheta}(A - Bp)(\varphi\vartheta - R)}{R}\mathcal{H}_r + \frac{\varphi\widetilde{\vartheta}\left(R - \widetilde{\omega}\widetilde{\vartheta}(A - Bp)\right)}{R}\mathcal{H}_s\right] + \frac{\tau_L}{2}[(1-\widetilde{\delta}])(1 - \widetilde{\omega}\widetilde{\vartheta})(A - Bp)\mathcal{H}_\ell]. \quad (20)$$

From equation (1), the de-bipolarization of TrBNNs $\widetilde{\omega}$, $\widetilde{\vartheta}$ and $\widetilde{\delta}$ are obtained as follows:

$$\widetilde{\omega_{BneuD}} = \frac{\omega_{11}+\omega_{12}+\omega_{13}+\omega_{14}+\omega_{21}+\omega_{22}+\omega_{23}+\omega_{24}+\omega_{31}+\omega_{32}+\omega_{33}+\omega_{34}}{6}, \quad (21)$$

$$\widetilde{\vartheta_{BneuD}} = \frac{\vartheta_{11}+\vartheta_{12}+\vartheta_{13}+\vartheta_{14}+\vartheta_{21}+\vartheta_{22}+\vartheta_{23}+\vartheta_{24}+\vartheta_{31}+\vartheta_{32}+\vartheta_{33}+\vartheta_{34}}{6}, \quad (22)$$

$$\widetilde{\delta_{BneuD}} = \frac{\delta_{11}+\delta_{12}+\delta_{13}+\delta_{14}+\delta_{21}+\delta_{22}+\delta_{23}+\delta_{24}+\delta_{31}+\delta_{32}+\delta_{33}+\delta_{34}}{6} \quad (23)$$

Hence, by de-bipolarization it is obtained as

$$\begin{aligned} TC_{\widetilde{BneuD}}(\tau_I,\tau_L) = {} & \frac{m\widetilde{\omega_{BneuD}}(A-B\wp)(W_i+W_r)}{\varphi\tau_I} + \frac{mW_\ell}{\tau_L} \\ & + m(A-B\wp)\widetilde{\omega_{BneuD}}\left(w_i + \widetilde{\vartheta_{BneuD}}\left(w_r - \widetilde{\delta_{BneuD}}(1-\theta)w_i - \left(1-\widetilde{\delta_{BneuD}}\right)w_\ell\right)\right) \\ & + \widetilde{\delta_{BneuD}}(1-\theta)w_i + \left(1-\widetilde{\delta_{BneuD}}\right)w_\ell] \\ & + \frac{\tau_I}{2}\Bigg[(\varphi - \widetilde{\omega_{BneuD}}(A-B\wp))\mathcal{H}_u + \frac{\widetilde{\omega_{BneuD}}\widetilde{\vartheta_{BneuD}}(A-B\wp)\left(\varphi\widetilde{\vartheta_{BneuD}}-R\right)}{R}\mathcal{H}_r \\ & + \frac{\varphi\widetilde{\vartheta_{BneuD}}\left(R-\widetilde{\omega_{BneuD}}\widetilde{\vartheta_{BneuD}}(A-B\wp)\right)}{R}\mathcal{H}_s\Bigg] \\ & + \frac{\tau_L}{2}\left[\left(1-\widetilde{\delta_{BneuD}}\right)\left(1-\widetilde{\omega_{BneuD}}\widetilde{\vartheta_{BneuD}}\right)(A \right. \\ & \left. - B\wp)\mathcal{H}_\ell\right]. \end{aligned} \quad (24)$$

To flourish this paper more effective, the proposed model is also computed by considering the unpredictable parameters ω, ϑ and δ as TBNN. That is,

$\widetilde{\omega} = \langle(\omega'_{11},\omega'_{12},\omega'_{13}),(\omega'_{21},\omega'_{22},\omega'_{23}),(\omega'_{31},\omega'_{32},\omega'_{33})\rangle$;
$\widetilde{\vartheta} = \langle\left(\vartheta'_{11},\vartheta'_{12},\vartheta'_{13}\right),\left(\vartheta'_{21},\vartheta'_{22},\vartheta'_{23}\right),\left(\vartheta'_{31},\vartheta'_{32},\vartheta'_{33}\right)\rangle$;
$\widetilde{\delta} = \langle\left(\delta'_{11},\delta'_{12},\delta'_{13}\right),\left(\delta'_{21},\delta'_{22},\delta'_{23}\right),\left(\delta'_{31},\delta'_{32},\delta'_{33}\right)\rangle$; .

Thus, from equation (2), the de-bipolarization of TBNNs $\widetilde{\omega}$, $\widetilde{\vartheta}$ and $\widetilde{\delta}$ are obtained as follows:

$$\widetilde{\omega_{BneuD}} = \frac{\omega'_{11}+2\omega'_{12}+\omega'_{13}+\omega'_{21}+2\omega'_{22}+\omega'_{23}+\omega'_{31}+2\omega'_{32}+\omega'_{33}}{6}, \quad (25)$$

$$\widetilde{\vartheta_{BneuD}} = \frac{\vartheta'_{11}+2\vartheta'_{12}+\vartheta'_{13}+\vartheta'_{21}+2\vartheta'_{22}+\vartheta'_{23}+\vartheta'_{31}+2\vartheta'_{32}+\vartheta'_{33}}{6}, \quad (26)$$

$$\widetilde{\delta_{BneuD}} = \frac{\delta'_{11}+2\delta'_{12}+\delta'_{13}+\delta'_{21}+2\delta'_{22}+\delta'_{23}+\delta'_{31}+2\delta'_{32}+\delta'_{33}}{6}. \quad (27)$$

The above set of equations is substituted in the equation (24) to obtain the result under TBNN environment.

Preposition:

(a) $TC_{\widetilde{BneuD}}(\tau_I,\tau_L)$ is strictly convex.

(b) The optimal inspection duration and the optimal leasing duration are

$$\tau_I^* = \sqrt{\frac{2Rm\widetilde{\omega_{BneuD}}(A-B\wp)(W_i+W_r)}{\varphi R\left(\varphi-\widetilde{\omega_{BneuD}}(A-B\wp)\right)\mathcal{H}_u+\varphi\widetilde{\omega_{BneuD}}\widetilde{\vartheta_{BneuD}}(A-B\wp)\left(\varphi\widetilde{\vartheta_{BneuD}}-R\right)\mathcal{H}_r+\varphi^2\widetilde{\vartheta_{BneuD}}\left(R-\widetilde{\omega_{BneuD}}\widetilde{\vartheta_{BneuD}}(A-B\wp)\right)\mathcal{H}_s}} \quad (28)$$

and $\tau_L^* = \sqrt{\frac{2mW_\ell}{\left(1-\widetilde{\delta_{BneuD}}\right)\left(1-\widetilde{\omega_{BneuD}}\widetilde{\vartheta_{BneuD}}\right)(A-B\wp)\mathcal{H}_\ell}}$. (29)

Proof:

Differentiating eqn. (24) partially with respect to τ_I and τ_L, it is obtained as

$$\frac{\partial TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial \tau_I} = \frac{-m\omega_{\widetilde{BneuD}}(A-B\wp)(W_i+W_r)}{\varphi {\tau_I}^2} + \frac{1}{2}\left[\left(\varphi-\omega_{\widetilde{BneuD}}(A-B\wp)\right)\mathcal{H}_u + \frac{\omega_{\widetilde{BneuD}}\vartheta_{\widetilde{BneuD}}(A-B\wp)\left(\varphi\vartheta_{\widetilde{BneuD}}-R\right)}{R}\mathcal{H}_r + \frac{\varphi\vartheta_{\widetilde{BneuD}}\left(R-\omega_{\widetilde{BneuD}}\vartheta_{\widetilde{BneuD}}(A-B\wp)\right)}{R}\mathcal{H}_s\right],$$

$$\frac{\partial TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial \tau_L} = \frac{-mW_\ell}{{\tau_L}^2} + \frac{\left(1-\delta_{\widetilde{BneuD}}\right)\left(1-\omega_{\widetilde{BneuD}}\vartheta_{\widetilde{BneuD}}\right)(A-B\wp)\mathcal{H}_\ell}{2}.$$

Again, differentiating partially with respect to τ_I, τ_E and τ_L, it follows as

$$\frac{\partial^2 TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial {\tau_I}^2} = \frac{2m\omega_{\widetilde{BneuD}}(A-B\wp)(W_i+W_r)}{\varphi {\tau_I}^3},$$

and $\frac{\partial^2 TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial {\tau_L}^2} = \frac{2mW_\ell}{{\tau_L}^3}$

Also,

$$\frac{\partial^2 TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial \tau_I \partial \tau_L} = \frac{\partial^2 TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial \tau_L \partial \tau_I} = 0.$$

Therefore, the Hessian matrix for $TC_{\widetilde{BneuD}}(\tau_I,\tau_L)$ is

$$H = \frac{\partial^2 E\left[TC_{B\iota}\widetilde{(\tau_I,\tau_E,\tau_L)}\right]}{\partial {\tau_I}^2}\frac{\partial^2 E\left[TC_{B\iota}\widetilde{(\tau_I,\tau_E,\tau_L)}\right]}{\partial {\tau_L}^2} - \left[\frac{\partial^2 TC_{\widetilde{BneuD}}(\tau_I,\tau_L)}{\partial \tau_I \partial \tau_L}\right]^2 = \frac{2m\omega_{\widetilde{BneuD}}(A-B\wp)(W_i+W_r)}{\varphi {\tau_I}^3}.\frac{2mW_\ell}{{\tau_L}^3} > 0$$

Hence, the expected total cost $TC_{\widetilde{BneuD}}(\tau_I,\tau_L)$ is strictly convex.

By setting, $\frac{\partial E\left[TC_{B\iota}\widetilde{(\tau_I,\tau_L)}\right]}{\partial \tau_I} = 0$ and $\frac{\partial E\left[TC_{B\iota}\widetilde{(\tau_I,\tau_L)}\right]}{\partial \tau_L} = 0$, the optimal inspection duration and the optimal renting period of RCs are attained.

Hence,

$${\tau_I}^* = \sqrt{\frac{2Rm\omega_{\widetilde{BneuD}}(A-B\wp)(W_i+W_r)}{\varphi R\left(\varphi-\omega_{\widetilde{BneuD}}(A-B\wp)\right)\mathcal{H}_u + \varphi\omega_{\widetilde{BneuD}}\vartheta_{\widetilde{BneuD}}(A-B\wp)(\varphi\vartheta_{\widetilde{BneuD}}-R)\mathcal{H}_r + \varphi^2\vartheta_{\widetilde{BneuD}}\left(R-\omega_{\widetilde{BneuD}}\vartheta_{\widetilde{BneuD}}(A-B\wp)\right)\mathcal{H}_s}}$$

and $${\tau_L}^* = \sqrt{\frac{2mW_\ell}{\left(1-\delta_{\widetilde{BneuD}}\right)\left(1-\omega_{\widetilde{BneuD}}\vartheta_{\widetilde{BneuD}}\right)(A-B\wp)\mathcal{H}_\ell}}.$$

This completes the proof.

4. Numerical Analysis

Some parameters below are utilized from [13]:

$A = 6000, B = 20$, $\wp = \$50$/unit, $\varphi = 8000$, $R = 6000$, $W_i = \$200$, $W_r = \$400$, $W_\ell = \$100$, $w_i = \$2$/unit, $w_r = \$4$/unit, $w_\ell = \$10$, $w_h = \$5$/unit, $w_t = \$8$/unit, $\mathcal{H}_u = \$2$/unit, $\mathcal{H}_r = \$3$/unit, $\mathcal{H}_s = \$5$/unit, $\mathcal{H}_\ell = \$5$/unit, $\theta = 0.75$, $m = 240$ days.

The proportion of the received RCs, the proportion of repairable RCs and the fraction of OFU RCs as TrBNN which are given as

$\tilde{\omega} = \langle(0.5,0.75,1,1.5),(0.05,0.1,0.15,0.2),(0.15,0.2,0.4,0.55)\rangle$;

$\tilde{\vartheta} = \langle(0.7,0.9,1,1.25),(0.05,0.075,0.1,0.125),(0.15,0.25,0.5,0.65)\rangle$;
$\tilde{\delta} = \langle(0.08,0.12,0.25,0.75),(0.04,0.1,0.24,0.34),(0.05,0.13,0.5,0.55)\rangle$.
To compute the numerical studies of the proposed model using TBNN, the following parameters are used:
$\tilde{\omega} = \langle(0.5,1,1.5),(0.075,0.1,0.15),(0.15,0.25,0.5)\rangle$;
$\tilde{\vartheta} = \langle(0.75,1,1.5),(0.05,0.075,0.1),(0.15,0.25,0.5)\rangle$;
$\tilde{\delta} = \langle(0.1,0.2,0.3),(0.2,0.25,0.4),(0.15,0.3,0.5)\rangle$.
According to the hypothesis of this study, it is considered that the values $\omega \neq 1$, $\vartheta \neq 1$ and $\delta \neq 1$. From equations (21) - (24), (28) and (29), the expected total cost under TrBNN is $TC_{\widetilde{Bi}(\tau_I,\tau_L)} = \$7,218,500$, the optimal inspection duration and the optimal leasing duration are ${\tau_I}^* = 2.8527$ days and ${\tau_L}^* = 5.9666$ days. From equations (24) - (29), the expected total cost under TBNN is $TC_{\widetilde{Bi}(\tau_I,\tau_L)} = \$7,233,000$, the optimal inspection duration and the optimal leasing duration are ${\tau_I}^* = 2.8713$ days and ${\tau_L}^* = 5.8704$ days.

4.1 Comparison study

Table 1: Comparison study of the three decision making strategy in the proposed study

Option	Condition	$TC_{\widetilde{Bi}(\tau_I,\tau_L)}$ (USD) [Using TrBNN]	$TC_{\widetilde{Bi}(\tau_I,\tau_L)}$ (USD) [Using TBNN]
I	$\omega = 1, \vartheta = 1$	7,249,400	7,249,400
II	$\omega \neq 1, \vartheta \neq 1, \delta = 1$	6,595,600	6,589,700
III	$\omega \neq 1, \vartheta \neq 1, \delta \neq 1$	7,218,500	7,233,000

The option I indicates that all the used containers are returned as well as there is no salvaged units found while inspection. In option II, it is considered that the fraction of containers are unreturned and some of the returned units are salvaged. All the shortfall RCs are one-way free used. In option III, a proportion of shortfall RCs that are unable to OFU are leased from the local dealer. From table 1, it is clear that the option II that is, all the deficit containers are replaced by OFU option is the best result when compared to other two options under both TrBNN as well as TBNN, which is shown in Fig 5. So that, the OFU is comparitively better option while leasing the containers. It is also observed that the model using TrBNN is provided the best outcomes when compared to the model under the TBNN environment.

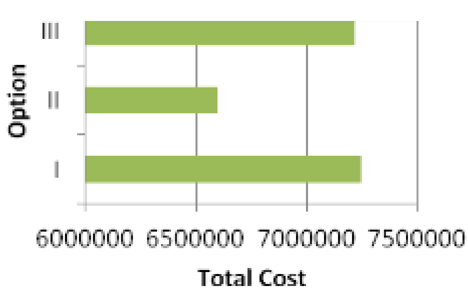

Fig. 5(a)

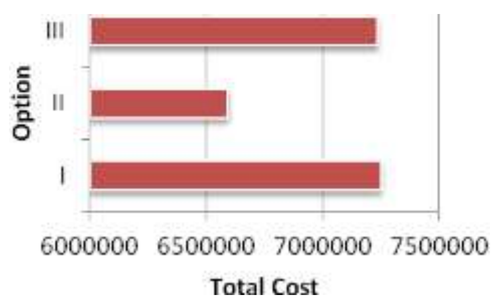

Fig. 5(b)

Fig.5 The total cost of various strategies

The below table shows the comparison of the outcome of the proposed study with the results of relative research works. As per the research works [13] and [30], the proposed research considers to sold the deficit RCs at scrap price.

Table 2: Comparison study of the proposed research with related studies

References	Implemented options to restore the deficit RC	Demand	Scrap price ($)	Total cost ($)	Result obtained in the proposed study		
					Demand	Scrap price ($)	Total cost ($) [Using TrBNN]
[13]	Purchase new Returnable Transport Items (RTIs)	8000	40	31,258,000	8000	40	8,570,100
[30]	Repositioning and leasing of RCs	5000	100	3,819,800	5000	100	2,593,500

Table 3: Descriptive comparison analysis of the proposed research with related studies

References	Implemented options to restore the deficit RC	Remarks when comparing with the proposed study
[13]	Purchase new RTIs	By setting $A = 10000, B = 40$, $\wp = \$50$/unit, $\varphi = 12000$ and $R = 10000$ and by considering the salvage RCs are sold at $40 in the proposed study, the value of the parameters in this study is similar to [13]. Thus, the total cost is $\$8,570,100$ and the optimal screening length is 2.4698 days which is the better result when compared to [13]. In [13], the total cost is $\$31,258,000$ and the optimal screening length is 2.5 days.
[30]	Repositioning and leasing of RCs	By considering the salvage RCs are sold at $100 in the proposed study, the total cost is $\$2,593,500$ but the total cost in [30] is $\$3,819,800$. Thus, the OFU option with ECR and the leasing of RCs helps the CMO to reduce the total cost when compare to [30].

The studies [15] and [22] consider purchasing new RTIs to restore the deficit RTIs. According to the present study, instead of buying new RTIs, the repositioning of RTIs and the leasing of RTIs, as well as the OFU option, lead the model to minimize the total cost. Also, the study [19] presumes the renting option to restore the deficit RCs, along with renting option, the implementation of the OFU option analyzed in the proposed study will lead the outcomes of [19] to better results.

4.2 Sensitivity analysis

The sensitivity analysis of the trapezoidal bipolar neutrosophic container inventory model is examined as follows.

Table 4: Effect of TrBNNs $\widetilde{\omega}$, $\widetilde{\vartheta}$ and $\widetilde{\delta}$ on the optimal solutions

Para meter	Value of the parameter	$\tau_I{}^*$	$\tau_L{}^*$	$\widetilde{TC_{B\iota}(\tau_I,\tau_L)}$
$\widetilde{\omega}$	⟨(0.46,0.71,0.06,1.46), (0.01,0.06,0.11,0.16), (0.11,0.16,0.36,0.51)⟩	2.5501	4.6099	7,124,800
	⟨(0.48,0.73,0.08,1.48), (0.03,0.08,0.13,0.18), (0.13,0.18,0.38,0.53)⟩	2.6955	5.1589	7,171,800
	⟨(0.5,0.75,1,1.5), (0.05,0.1,0.15,0.2), (0.15,0.2,0.4,0.55)⟩	2.8527	5.9666	7,218,500
	⟨(0.52,0.77,1.02,1.52), (0.07,0.12,0.17,0.22), (0.17,0.22,0.42,0.57)⟩	3.0240	7.3311	7,264,900
$\tilde{\vartheta}$	⟨(0.66,0.86,0.96,1.21), (0.01,0.035,0.06,0.085), (0.11,0.21,0.46,0.61)⟩	2.8723	4.6425	7,310,300
	⟨(0.68,0.88,0.98,1.23), (0.03,0.055,0.08,0.105), (0.13,0.23,0.48,0.63)⟩	2.8611	5.1817	7,264,500
	⟨(0.7,0.9,1,1.25), (0.05,0.075,0.1,0.125), (0.15,0.25,0.5,0.65)⟩	2.8527	5.9666	7,218,500
	⟨(0.72,0.92,1.02,1.27), (0.07,0.095,0.12,0.145), (0.17,0.27,0.52,0.67)⟩	2.8472	7.2970	7,172,200
$\tilde{\delta}$	⟨(0.04,0.08,0.21,0.71), (0,0.06,0.2,0.3), (0.01,0.09,0.46,0.51)⟩	2.8527	5.5198	7,322,700
	⟨(0.06,0.1,0.23,0.73), (0.02,0.08,0.22,0.32), (0.03,0.11,0.48,0.53)⟩	2.8527	5.7302	7,270,600
	⟨(0.08,0.12,0.25,0.75), (0.04,0.1,0.24,0.34), (0.05,0.13,0.5,0.55)⟩	2.8527	5.9666	7,218,500
	⟨(0.1,0.14,0.27,0.77), (0.06,0.12,0.26,0.36), (0.07,0.15,0.52,0.57)⟩	2.8527	6.2349	7,166,400

From the table 4, it is observed that when the bipolar neutrosophic proportion of the returned containers $\widetilde{\omega}$ raises, the optimal screening period $\tau_I{}^*$ and the optimal leasing duration $\tau_L{}^*$ are increase but the total cost reduces. Also, when the bipolar neutrosophic proportion of the repaired units $\tilde{\vartheta}$ raises, the optimal renting period is increases but the optimal screening period and the total cost reduce. When the bipolar neutrosophic variable $\tilde{\delta}$ raises, the optimal time length $\tau_L{}^*$ increase but the total cost reduces but the optimal screening cycle length, $\tau_I{}^*$, remains unchange.

Table 5: Effect of customers' rent price on the optimal solutions

$\wp$	$\tau_I{}^*$	$\tau_L{}^*$	$\widetilde{TC_{B\iota}(\tau_I,\tau_L)}$ \$
40	3.0106	5.8507	7,503,600
45	2.9300	5.9078	7,361,100
50	2.8527	5.9666	7,218,500
55	2.7784	6.0272	7,076,000
60	2.7069	6.0896	6,933,400

In the notion of customers' rent price for a container, without loss of generality it is obtained that the increase of customer rent price per RC leads to increase the optimal leasing period whereas the optimal screening period and the total cost decreasing, which is clearly shown in Table 5.

Table 6: Effect of container storage costs $\mathcal{H}_u$ and $\mathcal{H}_r$ on the optimal solutions

$\mathcal{H}_u$	$\mathcal{H}_r$	$\tau_I{}^*$	$\widetilde{TC_{B\iota}(\tau_I,\tau_L)}$ \$
1	3	3.1218	7211600
2	3	2.8527	7218500
3	3	2.6430	7224600
4	3	2.4737	7230100
5	3	2.3332	7235100
2	1	3.0416	7214900

2	2	2.9426	7216700
2	3	2.8527	7218500
2	4	2.7706	7220300
2	5	2.6951	7222200

The container storage cost is the major cost of any CMO for maintaining the empty containers. In this study, there is no storage charge for OFU containers because the customer used to pick the serviceable containers from the OFU vendor's empty depot but different storage cost is acquired for the used containers that are received after the usage, repairable containers, the serviceable containers and the leased units. Also, in this analysis, the storage cost for serviceable containers, ($\mathcal{H}_s$) and the storage charge for leased containers, ($\mathcal{H}_\ell$) are fixed as $5/unit. Here, the sensitive analysis is performed when the storage cost for received containers, ($\mathcal{H}_u$) and the storage charge for repairable from received units, ($\mathcal{H}_r$) are varies from $1 to $5 per container on optimal solutions. On observing from Table 6, when $\mathcal{H}_u$ varies from $1 to $5 per container and the storage cost for repairable from received container is consider as in numerical analysis, the optimal inspection length reduces but the total cost increases. The same result attains when $\mathcal{H}_r$ varies from $1 to $5 per container and the storage cost for received containers after the usage is consider as in numerical analysis.

5. Conclusion

A container inventory model under uncertain situation is performed in this research. This uncertainty condition leads the present model to utilize the notion of bipolar neutrosophic arena. The serviceable container cost, OFU cost and the lease cost are obtained under price sensitive demand by computing the expected maximum inventory of received, restorable, ready to service units, OFU units and leased units. The framed algorithm helps to make decision on various strategies for computing the total cost. The total cost is framed by presuming that the fraction of deficit units is one-way free used, and the fraction of remaining containers are leased from local dealer. The bipolar neutrosophic container inventory model is developed by which the fraction of received RCs, the fraction of repairable from received RCs and the fraction of one-way free used RCs are considered as TrBNNs. The container inventory model is performed by presuming the received rate, the repairable rate, and the OFU rate as Triangular Bipolar Neutrosophic Number (TBNN), and then the outcomes under both TrBNN as well as TBNN are compared.

The optimal time lengths of inspection process and leasing procedure in order to minimize the total cost are attained. The numerical computation on the comparison of four decision making strategies shows that, the OFU of all the deficit containers minimizes the CMO's total cost. The bipolar neutrosophic received proportion, the bipolar neutrosophic repaired proportion and the bipolar neutrosophic OFU proportion and its impact on optimal solutions are shown in sensitivity analysis. Finally, the comparison result of this study with the work of [13] on optimal screening length is given. The comparison analysis of present model and [30] is also given and then suggests the studies [15] and [22] to utilizing the OFU option in order to minimize the container management costs.

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