Multi-attribute Decision Making based on Rough Neutrosophic Variational Coefficient Similarity Measure

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Abstract: The purpose of this study is to propose new similarity measures namely rough variational coefficient similarity measure under the rough neutrosophic environment. The weighted rough variational coefficient similarity measure has been also defined. The weighted rough variational coefficient similarity measures between the rough ideal alternative and each alternative are calculated to find the best alternative. The ranking order of all the alternatives can be determined by using the numerical values of similarity measures. Finally, an illustrative example has been provided to show the effectiveness and validity of the proposed approach. Comparisons of decision results of existing rough similarity measures have been provided.

Keywords: Neutrosophic set, Rough neutrosophic set; Rough variation coefficient similarity measure; Decision making.

1 Introduction

In 1965, L. A. Zadeh grounded the concept of degree of membership and defined fuzzy set [1] to represent/manipulate data with non-statistical uncertainty. In 1986, K. T. Atanassov [2] introduced the degree of non-membership as independent component and proposed intuitionistic fuzzy set (IFS). F. Smarandache introduced the degree of indeterminacy as independent component and defined the neutrosophic set [3, 4, 5]. For purpose of solving practical problems, Wang et al. [6] restricted the concept of neutrosophic set to single valued neutrosophic set (SVNS), since single value is an instance of set value. SVNS is a subclass of the neutrosophic set. SVNS consists of the three independent components namely, truth-membership, indeterminacy-membership and falsity-membership functions.

The concept of rough set theory proposed by Z. Pawlak [7] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. The hybridization of rough set theory and neutrosophic set theory produces the rough neutrosophic set [8, 9], which was proposed by Broumi, Dhar and Smarandache [8, 9]. Rough neutrosophic set theory is also a powerful mathematical tool to deal with incompleteness.

Literature review reflects that similarity measures play an important role in the analysis and research of clustering analysis, decision making, medical diagnosis, pattern recognition, etc. Various similarity measures [10, 11, 12, 13, 14, 15, 16, 17, 18] of SVNSs and hybrid SVNSs are available in the literature. The concept of similarity measures in rough neutrosophic environment [19, 20, 21] has been recently proposed.


Different methods for multiattribute decision making (MADM) and multicriteria decision making (MCDM) problems are available in the literature in different environment such as crisp environment [25, 26, 27, 28, 29], fuzzy environment [30, 31], intuitionistic fuzzy environment [32, 33, 34, 35, 36, 37, 38, 39, 40], neutrosophic environment [41, 42, 43, 44, 45, 46, 47, 48,
49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, interval neutrosophic environment [63, 65, 66, 67, 68], neutrosophic soft expert environment [69], neutrosophic bipolar environment [70, 71], neutrosophic soft environment [72, 73, 74, 75, 76], neutrosophic hesitant fuzzy environment [77, 78, 79], rough neutrosophic environment [80, 81], etc. In neutrosophic environment Biswas, Pramanik and Giri [82] studied hybrid vector similarity measure and its application in multi-attribute decision making. Getting motivation from the work of Biswas, Pramanik and Giri [82], for hybrid vector similarity measure in neutrosophic environment, we extend the concept in rough neutrosophic environment.

In this paper, a new similarity measurement is proposed, namely rough variational coefficient similarity measure under rough neutrosophic environment. A numerical example is also provided.

Rest of the paper is structured as follows. Section 2 presents neutrosophic and rough neutrosophic preliminaries. Section 3 discusses various similarity measures and varional coefficient similarity measure in crisp environment. Section 4 presents various similarity measures and variational similarity measure for single valued neutrosophic sets. Section 5 presents variational coefficient similarity measure and weighted variational coefficient similarity measure for rough neutrosophic sets and establishes their basic properties. Section 6 is devoted to present multi attribute decision making based on rough neutrosophic variational coefficient similarity measure. Section 7 demonstrates the application of rough variational coefficient similarity measures to investment problem Finally, section 8 concludes the paper with stating the future scope of research.

2 Neutrosophic preliminaries

Definition 2.1 [3, 4, 5] Neutrosophic set
Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is denoted by A = \{x\mid T_A(x), I_A(x), F_A(x): x \in X\} where, T_A(x) is the truth membership function, I_A(x) is the indeterminacy membership function and F_A(x) is the falsity membership function. The functions T_A(x), I_A(x) and F_A(x) are real standard or non-standard subsets of \([0, 1]\). There is no restriction on the sum of T_A(x), I_A(x) and F_A(x) i.e. \(0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3\).

Let X be a universal space of points (objects), with a generic element x \in X. A single-valued neutrosophic set (SVNS) \(N \subseteq X\) is denoted by

\[ N = \{\{T_N(x), I_N(x), F_N(x)\}\mid x \in X\}, \text{ when } X \text{ is continuous}; \]

\[ N = \sum_{x}^{\infty}\{T_N(x), I_N(x), F_N(x)\}\mid x \in X\], \text{ when } X \text{ is discrete}. \]

SVNS is characterized by a true membership function \(T_N(x)\), a falsity membership function \(F_N(x)\) and an indeterminacy function \(I_N(x)\) \(\text{th} T_N(x) \cdot F_N(x) + I_N(x) \in [0, 1]\) for all \(x \in X\). For each \(x \in X\), of a SVNS \(N \subseteq X\)

\[ 0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3. \]

2.1 Some operational rules and properties of SVNSs

Let \(N_A = \{T_A(x), I_A(x), F_A(x)\}\) and \(N_B = \{T_B(x), I_B(x), F_B(x)\}\) be two SVNSs in \(X\). Then the following operations are defined as follows:

I. Complement: \(N_c = \{1 - T_A(x), 1 - I_A(x), 1 - F_A(x)\}\). \(\forall x \in X\).

II. Addition: \(N_A \oplus N_B = \{T_A + T_B - T_A T_B, I_A + I_B, F_A + F_B - F_A F_B\}\).

III. Multiplication:

\[ \lambda N_A = \{1 - (1 - T_A) \lambda, I_A, (1 - (1 - F_A) \lambda)\} \text{ for } \lambda > 0. \]

IV. Scalar Multiplication:

\[ N_{\lambda A} = \{(T_A)^{\lambda}, (1 - (1 - I_A))^{\lambda}, (1 - (1 - F_A))^{\lambda}\} \text{ for } \lambda > 0. \]

Definition 2.3 [6]

Complement of a SVNS N is denoted by \(N^c\) and is defined by

\[ T_{N^c}(x) = F_N(x) ; I_{N^c}(x) = 1 - I_N(x) ; F_{N^c}(x) = T_N(x). \]

Definition 2.4 [6]

A SVNS \(N_A\) is contained in the other SVNS \(N_B\), denoted as \(N_A \subseteq N_B\), if and only if

\[ T_{N_A}(x) \leq T_{N_B}(x) ; I_{N_A}(x) \geq I_{N_B}(x) ; F_{N_A}(x) \geq F_{N_B}(x) \text{ } \forall x \in X. \]

Definition 2.5 [6]

Two SVNSs \(N_A\) and \(N_B\) are equal, i.e. \(N_A = N_B\), if and only if \(N_A \subseteq N_B\) and \(N_B \subseteq N_A\).

Definition 2.6 [6]

Union of two SVNSs \(N_A\) and \(N_B\) is a SVNS \(N_C\), written as \(N_C = N_A \cup N_B\). Its truth membership, indeterminacy-membership and falsity membership functions are related to those of \(N_A\) and \(N_B\) by

\[ T_{N_C}(x) = \max(T_{N_A}(x), T_{N_B}(x)) ; I_{N_C}(x) = \min(I_{N_A}(x), I_{N_B}(x)) ; F_{N_C}(x) = \min(F_{N_A}(x), F_{N_B}(x)) \text{ for all } x \in X. \]

Definition 2.7 [6] Intersection of two SVNSs \(N_A\) and \(N_B\) is a SVNS \(N_D\), written as \(N_D = N_A \cap N_B\), whose truth membership, indeterminacy-membership and falsity membership functions are related to those of \(N_A\) and \(N_B\) by

\[ T_{N_D}(x) = \min(T_{N_A}(x), T_{N_B}(x)) ; I_{N_D}(x) = \max(I_{N_A}(x), I_{N_B}(x)) ; F_{N_D}(x) = \max(F_{N_A}(x), F_{N_B}(x)) \text{ for all } x \in X. \]

Definition 2.8 Rough Neutrosophic Sets [8, 9]

Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the
membership function \( T_p \) indeterminacy function \( I_p \) and non-membership function \( F_p \). The lower and the upper approximations of \( P \) in the approximation \((Z, R)\) denoted by \( \underbar{N}(P) \) and \( \overline{N}(P) \) are respectively defined as follows:

\[
\begin{align*}
\underbar{N}(P) &= \{ x \in X \mid \exists z \in Z : x \in \text{dom } T_p, z \in [x]_R, z \in \text{dom } F_p \} \\
\overline{N}(P) &= \{ x \in X \mid \exists z \in Z : x \in \text{dom } T_p, z \notin [x]_R, z \in \text{dom } F_p \}
\end{align*}
\]

Here, \( T_p(x) = \wedge \{[x]_R \mid x \in \text{dom } T_p \} \),
\( I_p(x) = \vee \{[x]_R \mid x \in \text{dom } I_p \} \),
\( F_p(x) = \vee \{[x]_R \mid x \in \text{dom } F_p \} \).

\( \text{So,} \ 0 \leq \sup T_{\overline{N}(p)}(x) + \sup I_{\overline{N}(p)}(x) + \sup F_{\overline{N}(p)}(x) \leq 3 \)

Here \( \wedge \) and \( \vee \) denote “max” and “min” operators respectively. \( T(z), I(z) \) and \( F(z) \) denote respectively the membership, indeterminacy and non-membership function of \( z \) with respect to \( P \). It is easy to see that \( \underbar{N}(P) \) and \( \overline{N}(P) \) are two rough neutrosophic sets in \( Z \).

Thus, NS mappings \( \overbar{N}, \overline{N} : N(Z) \rightarrow N(Z) \) are, respectively, referred to as the lower and the upper rough NS approximation operators, and the pair \((\overbar{N}(P),\overline{N}(P))\) is called the rough neutrosophic set \([8,9]\) in \((Z, R)\).

From the above definition, it is seen that \( \overbar{N}(P) \) and \( \overline{N}(P) \) have constant membership on the equivalence classes of \( R \). If \( N(P) = \overline{N}(P) \) i.e., \( T_{\overline{N}(p)}(x) = T_{\overline{N}(p)}(x), \ i_{\overline{N}(p)}(x) = i_{\overline{N}(p)}(x) \) and \( F_{\overline{N}(p)}(x) = F_{\overline{N}(p)}(x) \) respectively.

\( P \) is said to be a definable neutrosophic set in the approximation \((Z, R)\). It can be easily proved that zero neutrosophic set \((0,0,0)\) and unit neutrosophic sets \((1,1,0)\) are definable neutrosophic sets.

**Definition 2.9** \([8,9]\)

If \( \underbar{N}(P) = (\overbar{N}(P),\overline{N}(P)) \) is a rough neutrosophic set in \((Z, R)\), then the complement \([8,9]\) of \( \underbar{N}(P) \) is the rough neutrosophic set denoted by \( \sim N(P) = (\overbar{N}(P)^t, \overline{N}(P)^t) \) where \( \overbar{N}(P)^t \) and \( \overline{N}(P)^t \) are the complements of neutrosophic sets of \( \underbar{N}(P) \) respectively.

\[
\overbar{N}(P)^t = \{ x \in X \mid F_{\overbar{N}(p)}(x), T_{\overbar{N}(p)}(x) > / x \in Z \}
\]

\[
\overline{N}(P)^t = \{ x \in X \mid F_{\overline{N}(p)}(x), T_{\overline{N}(p)}(x) > / x \in Z \}
\]

**Definition 2.10** \([8,9]\)

If \( \underbar{N}(P) \) and \( \overline{N}(P) \) are the two rough neutrosophic sets of the neutrosophic set \( P \) respectively in \( Z \), then the following definitions \([8,9]\) hold:

\[
\begin{align*}
\text{If} \left( \underbar{N}(P) \right) \text{ and } \left( \overline{N}(P) \right) \text{ are the two rough neutrosophic sets of the neutrosophic set } P \text{ respectively in } Z, \text{ then the following definitions } [8,9] \text{ hold:}
\end{align*}
\]

Let \( X = (x_1, x_2, \ldots, x_n) \) and \( Y = (y_1, y_2, \ldots, y_n) \) be two \( n \)-dimensional vectors with positive co-ordinates.

**Definition 3.1** \([83]\)

Jaccard index of two vectors (measuring the “similarity” of these vectors) can be defined as follows:

\[
J(X, Y) = \frac{XY}{\|X\|^2 + \|Y\|^2 - \sum_{i=1}^{n} x_i^2 \sum_{j=1}^{n} y_j^2}
\]

where \( \sum_{i=1}^{n} x_i^2 \) and \( \sum_{j=1}^{n} y_j^2 \) are the Euclidean norm of \( X \) and \( Y \), \( XY = \sum_{i=1}^{n} x_i y_i \) is the inner product of the vector \( X \) and \( Y \).

**Proposition 5** \([83]\)

Jaccard index satisfies the following properties:

1. \( 0 \leq J(X, Y) \leq 1 \)
2. \( J(X, Y) = J(Y, X) \)
3. \( J(X, Y) = 1 \), for \( X = Y \) i.e., \( x_i = y_i \) (\( i = 1, 2, \ldots, n \)) for every \( x_i \in X \) and \( y_i \in Y \).

**Definition 3.2** [84]

The Dice similarity measure can be defined as follows:

\[
E(X, Y) = \frac{2XY}{\|X\|^2 + \|Y\|^2} = \frac{2\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2}
\]

(2)

**Proposition 6** [84]

The Dice similarity measure satisfies the following properties:
1. \( 0 \leq E(X, Y) \leq 1 \)
2. \( E(X, Y) = E(Y, X) \)
3. \( E(X, Y) = 1 \), for \( X = Y \) i.e., \( x_i = y_i \) (\( i = 1, 2, \ldots, n \)) for every \( x_i \in X \) and \( y_i \in Y \).

**Definition 3.3** [85]

The cosine similarity measure between two vectors \( X \) and \( Y \) is the inner product of these two vectors divided by the product of their lengths and can be defined as follows:

\[
C(X, Y) = \frac{XY}{\|X\| \|Y\|} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2} \sqrt{\sum_{i=1}^{n} y_i^2}}
\]

(3)

**Proposition 7** [85]

The cosine similarity measure satisfies the following properties:
1. \( 0 \leq C(X, Y) \leq 1 \)
2. \( C(X, Y) = C(Y, X) \)
3. \( C(X, Y) = 1 \), for \( X = Y \) i.e., \( x_i = y_i \) (\( i = 1, 2, \ldots, n \)) for every \( x_i \in X \) and \( y_i \in Y \).

These three formulas are similar in the sense that they take values in the interval \([0, 1]\). Jaccard and Dice similarity measures are undefined when \( x_i = 0 \) and \( y_i = 0 \) for \( i = 1, 2, \ldots, n \) and cosine similarity measure is undefined when \( x_i = 0 \) or \( y_i = 0 \) for \( i = 1, 2, \ldots, n \).

**Definition 3.4** [86]

Variational co-efficient similarity measure can be defined as follows:

\[
V(X, Y) = \lambda \frac{2XY}{\|X\| + \|Y\|} + (1-\lambda) \frac{XY}{\sqrt{\sum_{i=1}^{n} x_i^2} \sqrt{\sum_{i=1}^{n} y_i^2}}
\]

(4)

**Proposition 8** [86]

Variational co-efficient similarity measure satisfies the following properties:
1. \( 0 \leq V(X, Y) \leq 1 \)
2. \( V(X, Y) = V(Y, X) \)
3. \( V(X, Y) = 1 \), for \( X = Y \) i.e., \( x_i = y_i \) (\( i = 1, 2, \ldots, n \)) for every \( x_i \in X \) and \( y_i \in Y \).

4. Various similarity measures for single valued neutrosophic sets.

Assume \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) be two SVNSs in a universe of discourse \( X = \{x_1, x_2, \ldots, x_n\} \). \( T_A, I_A, F_A \in [0, 1] \) for any \( x_i \in X \) in \( N_A \) or \( T_B, I_B, F_B \in [0, 1] \) for any \( x_i \in X \) in \( N_B \) can be considered as a vector representation with three elements. Let \( w_i \in [0, 1] \) be the weight of each element \( x_i \) for \( i = 1, 2, \ldots, n \) such that \( \sum_{i=1}^{n} w_i = 1 \), then Jaccard, Dice and cosine similarity measures can be presented as follows:

**Definition 4.1** [10]

Jaccard similarity measure between \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) can be defined as follows:

\[
Jac(N_A, N_B) = \frac{(T_A(x_i) + I_A(x_i) + F_A(x_i)) + (T_A(x_i) + I_A(x_i) + F_A(x_i))}{\sum_{i=1}^{n} \left[ (T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2 \right]}
\]

**Proposition 9** [10]

Jaccard similarity measure satisfies the following properties:
1. \( 0 \leq Jac(N_A, N_B) \leq 1 \)
2. \( Jac(N_A, N_B) = Jac(N_B, N_A) \)
3. \( Jac(N_A, N_B) = 1 \) if \( N_A = N_B \) i.e., \( T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i) \) and \( F_A(x_i) = F_B(x_i) \) for every \( x_i \) (\( i = 1, 2, \ldots, n \)) in \( X \).

**Definition 4.1** [10]

Weighted Jaccard similarity measure between \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) can be defined as follows:

\[
Jac_w(N_A, N_B) = \left( T_A(x_i) + I_A(x_i) + F_A(x_i) \right) + \left( T_A(x_i) + I_A(x_i) + F_A(x_i) \right)
\]

**Proposition 10** [10]

Weighted Jaccard similarity measure satisfies the following properties:
1. \( 0 \leq Jac_w(N_A, N_B) \leq 1 \)
2. \( Jac_w(N_A, N_B) = Jac_w(N_B, N_A) \)
3. \( Jac_w(N_A, N_B) = 1 \) if \( N_A = N_B \) i.e., \( T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i) \) and \( F_A(x_i) = F_B(x_i) \) for every \( x_i \) (\( i = 1, 2, \ldots, n \)) in \( X \).
Definition 4.2 [11]

Dice similarity measure between \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) is defined as:

\[
\text{Dic}(N_A, N_B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{T_A(x_i) T_B(x_i) + I_A(x_i) I_B(x_i)}{T_A(x_i)^2 + I_A(x_i)^2 + F_A(x_i)^2} \right]
\]

Proposition 11 [11]

Dice similarity measure satisfies the following properties:
1. \( 0 \leq \text{Dic}(N_A, N_B) \leq 1 \);
2. \( \text{Dic}(N_A, N_B) = \text{Dic}(N_B, N_A) \);
3. \( \text{Dic}(N_A, N_B) = 1 \) if \( N_A = N_B \) i.e., \( T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i), \) and \( F_A(x_i) = F_B(x_i) \), for every \( x_i \) in \( X \).

Definition 4.2.1 [11]

Weighted Dice similarity measure between \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) can be defined as follows:

\[
\text{Dic}_w(N_A, N_B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{T_A(x_i) T_B(x_i) + I_A(x_i) I_B(x_i)}{T_A(x_i)^2 + I_A(x_i)^2 + F_A(x_i)^2} \right]
\]

Proposition 12 [11]

Weighted Dice similarity measure satisfies the following properties:
1. \( 0 \leq \text{Dic}_w(N_A, N_B) \leq 1 \);
2. \( \text{Dic}_w(N_A, N_B) = \text{Dic}_w(N_B, N_A) \);
3. \( \text{Dic}_w(N_A, N_B) = 1 \) if \( N_A = N_B \) i.e., \( T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i), \) and \( F_A(x_i) = F_B(x_i) \), for every \( x_i \) in \( X \).

Definition 4.3 [12]

Cosine similarity measure between \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) can be defined as follows:

\[
\text{Cos}(N_A, N_B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{T_A(x_i) F_A(x_i) + I_A(x_i) F_A(x_i)}{\sqrt{T_A(x_i)^2 + I_A(x_i)^2 + F_A(x_i)^2}} \right]
\]

Proposition 13 [12]

Cosine similarity measure satisfies the following properties:
1. \( 0 \leq \text{Cos}_w(N_A, N_B) \leq 1 \);
2. \( \text{Cos}_w(N_A, N_B) = \text{Cos}_w(N_B, N_A) \)

3. \( \text{Cos}_w(N_A, N_B) = 1 \) if \( N_A = N_B \) i.e., \( T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i), \) and \( F_A(x_i) = F_B(x_i) \), for every \( x_i \) in \( X \).

Definition 4.3.1 [12]

Weighted cosine similarity measure between \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) can be defined as follows:

\[
\text{Cos}_w(N_A, N_B) = \frac{\sum_{i=1}^{n} w_i (T_A(x_i) F_B(x_i) + I_A(x_i) F_B(x_i))}{\sqrt{\sum_{i=1}^{n} w_i (T_A(x_i)^2 + I_A(x_i)^2 + F_A(x_i)^2)}} \]

Proposition 14 [12]

Weighted cosine similarity measure satisfies the following properties:
1. \( 0 \leq \text{Cos}_w(N_A, N_B) \leq 1 \);
2. \( \text{Cos}_w(N_A, N_B) = \text{Cos}_w(N_B, N_A) \);
3. \( \text{Cos}_w(N_A, N_B) = 1 \) if \( N_A = N_B \) i.e., \( T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i), \) and \( F_A(x_i) = F_B(x_i) \), for every \( x_i \) in \( X \).

Jaccard and Dice similarity measures between two
neutrosophic sets \( N_A = \{T_A, I_A, F_A\} \) and \( N_B = \{T_B, I_B, F_B\} \) are undefined when \( T_A(x_i) = I_A(x_i) = F_A(x_i) = 0 \) and \( T_B(x_i) = I_B(x_i) = F_B(x_i) = 0 \) for all \( i = 1, 2, \ldots, n \).

5 Variational similarity measures for rough

neutrosophic sets

The notion of rough neutrosophic set (RNS) is used as vector representations in 3D-vector space. Assume that \( X = (x_1, x_2, \ldots, x_a) \) and \( Y = (y_1, y_2, \ldots, y_a) \) be two \( n \)-dimensional vectors with positive co-ordinates. Jaccard, Dice, cosine and cotangent similarity measures between two vectors are stated as follows.

Definition 5.1 [21]

Jaccard similarity measure under rough

neutrosophic environment

Assume that \( A = \{T_A(x_i), I_A(x_i), F_A(x_i)\} \) and \( B = \{T_B(x_i), I_B(x_i), F_B(x_i)\} \) in \( X = (x_1, x_2, \ldots, x_a) \) be any two rough neutrosophic sets. Jaccard similarity measure [21] between rough neutrosophic sets \( A \) and \( B \) can be defined as follows:

\[
\text{Jacc}(A, B) = \]
Then, let $\delta T_{\alpha}(x_i) = \delta T_{\alpha}(x_i) + \delta \alpha(x_i)$ and $\delta \alpha(x_i)$ be any two rough neutrosophic sets in $X = \{x_1, x_2, \ldots, x_n\}$. Dice similarity measure between rough neutrosophic sets $A$ and $B$ can be defined as follows:

$$
\text{DIC}_{\text{RNS}}(A, B) = \frac{1}{n} \sum_{i=1}^{n} w_i \left( \frac{2(\delta T_{\alpha}(x_i) + \delta T_{\alpha}(x_i))}{(\delta T_{\alpha}(x_i))^2 + (\delta T_{\alpha}(x_i))^2 + (\delta F_{\sigma}(x_i))^2} \right)
$$

Proposition 17 [21]
Dice similarity measure [21] satisfies the following properties.
1. $0 \leq \text{DIC}_{\text{RNS}}(A, B) \leq 1$;
2. $\text{DIC}_{\text{RNS}}(A, B) = \text{DIC}_{\text{RNS}}(B, A)$;
3. $\text{DIC}_{\text{RNS}}(A, B) = 1$; if $A = B$
4. If $C$ is a RNS in $Y$ and $A \subset B \subset C$ then, $\text{DIC}_{\text{RNS}}(A, C) \leq \text{DIC}_{\text{RNS}}(A, B)$ and $\text{DIC}_{\text{RNS}}(A, C) \leq \text{DIC}_{\text{RNS}}(B, C)$.

For proofs of the above mentioned four properties, see [21].

Definition 5.2.1 [21]
If we consider the weights of each element $x_i$ weighted rough Jaccard similarity measure [21] between rough neutrosophic sets $A$ and $B$ can be defined as follows:

$$
\text{Jac}_{\text{WRNS}}(A, B) = \frac{\sum_{i=1}^{n} w_i \left( \frac{2(\delta T_{\alpha}(x_i) + \delta T_{\alpha}(x_i))}{(\delta T_{\alpha}(x_i))^2 + (\delta T_{\alpha}(x_i))^2 + (\delta F_{\sigma}(x_i))^2} \right)}{\sum_{i=1}^{n} w_i}
$$

where $w_i \in [0, 1]$, $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, $i = 1, 2, \ldots, n$, then $\text{Jac}_{\text{WRNS}}(A, B) = \text{Jac}_{\text{RNS}}(A, B)$

Proposition 18 [21]
The weighted rough Dice similarity [21] measure between two rough neutrosophic sets $A$ and $B$ also satisfies the following properties:
1. $0 \leq \text{DIC}_{\text{WRNS}}(A, B) \leq 1$;
2. $\text{DIC}_{\text{WRNS}}(A, B) = \text{DIC}_{\text{WRNS}}(B, A)$;
3. $\text{DIC}_{\text{WRNS}}(A, B) = 1$; if $A = B$
4. If $C$ is a WRNS in $Y$ and $A \subset B \subset C$ then, $\text{DIC}_{\text{WRNS}}(A, C) \leq \text{DIC}_{\text{WRNS}}(A, B)$ and $\text{DIC}_{\text{WRNS}}(A, C) \leq \text{DIC}_{\text{WRNS}}(B, C)$.

For proofs of the above mentioned four properties, see [21].

Definition 5.3 [20]
Cosine similarity measure can be defined as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Pramanik and Mondal [20]
defined cosine similarity measure between rough
neutrosophic sets in 3-D vector space.

Assume that

\[
A = \left( \sum_{i=1}^{n} (\bar{T}_a(x_i) - \bar{T}_b(x_i)) \right) \left( \sum_{i=1}^{n} (\bar{F}_a(x_i) - \bar{F}_b(x_i)) \right) \quad \text{and}
B = \left( \sum_{i=1}^{n} (\bar{F}_a(x_i) - \bar{F}_b(x_i)) \right) \left( \sum_{i=1}^{n} (\bar{T}_a(x_i) - \bar{T}_b(x_i)) \right)
\]

in \( X = (x_1, x_2, \ldots, x_n) \) be any rough neutrosophic sets. Pramanik and Mondal [20] defined cosine similarity measure between rough neutrosophic sets \( A \) and \( B \) as follows:

\[
C_{RNS}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \frac{\delta T_a(x_i) \delta T_b(x_i) + \delta F_a(x_i) \delta F_b(x_i)}{\sqrt{\left( \delta T_a(x_i) \right)^2 + \left( \delta F_a(x_i) \right)^2} \sqrt{\left( \delta T_b(x_i) \right)^2 + \left( \delta F_b(x_i) \right)^2}}
\]

Here, \( \delta T_a(x_i) = \frac{\bar{T}_a(x_i) - \bar{T}_b(x_i)}{2} \), \( \delta T_b(x_i) = \frac{\bar{T}_b(x_i) - \bar{T}_b(x_i)}{2} \), \( \delta F_a(x_i) = \frac{\bar{F}_a(x_i) - \bar{F}_b(x_i)}{2} \), \( \delta F_b(x_i) = \frac{\bar{F}_b(x_i) - \bar{F}_b(x_i)}{2} \).

**Proposition 19** [20]

Let \( A \) and \( B \) be rough neutrosophic sets. Cosine similarity measure [20] between \( A \) and \( B \) satisfies the following properties.

1. \( 0 \leq C_{RNS}(A, B) \leq 1 \);
2. \( C_{RNS}(A, B) = C_{RNS}(B, A) \);
3. \( C_{RNS}(A, B) = 1 \) if \( A = B \);
4. If \( C \) is a RNS in \( Y \) and \( A \subset B \subset C \) then, \( C_{RNS}(A, C) \leq C_{RNS}(A, B) \), and \( C_{RNS}(B, C) \leq C_{RNS}(A, C) \).

**Definition 5.3.1** [20]

If we consider the weights of each element \( x_i \), a weighted rough cosine similarity measure between rough neutrosophic sets \( A \) and \( B \) can be defined as follows:

\[
C_{W_{RNS}}(A, B) = \frac{1}{n} \sum_{i=1}^{n} w_i \frac{\delta T_a(x_i) \delta T_b(x_i) + \delta F_a(x_i) \delta F_b(x_i)}{\sqrt{\left( \delta T_a(x_i) \right)^2 + \left( \delta F_a(x_i) \right)^2} \sqrt{\left( \delta T_b(x_i) \right)^2 + \left( \delta F_b(x_i) \right)^2}}
\]

\( w_i \in \{0, 1\} \), \( i = 1, 2, \ldots, n \), and \( \sum_{i=1}^{n} w_i = 1 \). If we take \( w_i = \frac{1}{n} \), \( i = 1, 2, \ldots, n \), then \( C_{RNS}(A, B) = C_{W_{RNS}}(A, B) \).

**Proposition 20** [20]

The weighted rough cosine similarity measure [20] between two rough neutrosophic sets \( A \) and \( B \) also satisfies the following properties:

1. \( 0 \leq C_{W_{RNS}}(A, B) \leq 1 \);
2. \( C_{W_{RNS}}(A, B) = C_{W_{RNS}}(B, A) \);
3. \( C_{W_{RNS}}(A, B) = 1 \) if \( A = B \);
4. If \( C \) is a WRNS in \( Y \) and \( A \subset B \subset C \) then, \( C_{W_{RNS}}(A, C) \leq C_{W_{RNS}}(A, B) \), and \( C_{W_{RNS}}(B, C) \leq C_{W_{RNS}}(B, C) \).

For proofs of the above mentioned four properties, see [20].

**Definition 5.4** [19] Cotangent similarity measures of rough neutrosophic sets

Assume that

\[
A = \left( \sum_{i=1}^{n} (\bar{F}_a(x_i) - \bar{F}_b(x_i)) \right) \left( \sum_{i=1}^{n} (\bar{T}_a(x_i) - \bar{T}_b(x_i)) \right) \quad \text{and}
B = \left( \sum_{i=1}^{n} (\bar{T}_a(x_i) - \bar{T}_b(x_i)) \right) \left( \sum_{i=1}^{n} (\bar{F}_a(x_i) - \bar{F}_b(x_i)) \right)
\]

in \( X = (x_1, x_2, \ldots, x_n) \) be any two rough neutrosophic sets. Pramanik and Mondal [19] defined cotangent similarity measure between rough neutrosophic sets \( A \) and \( B \) as follows:

\[
C_{T_{RNS}}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \frac{3 + \delta I_a(x_i) - \delta I_b(x_i) + \delta I_a(x_i) - \delta I_b(x_i) + \delta I_a(x_i) - \delta I_b(x_i)}{\sqrt{\left( \delta T_a(x_i) \right)^2 + \left( \delta F_a(x_i) \right)^2} \sqrt{\left( \delta T_b(x_i) \right)^2 + \left( \delta F_b(x_i) \right)^2}}
\]

Here, \( \delta I_a(x_i) = \frac{\bar{I}_a(x_i) - \bar{I}_b(x_i)}{2} \), \( \delta I_b(x_i) = \frac{\bar{I}_b(x_i) - \bar{I}_b(x_i)}{2} \), \( \delta F_a(x_i) = \frac{\bar{F}_a(x_i) - \bar{F}_b(x_i)}{2} \), \( \delta F_b(x_i) = \frac{\bar{F}_b(x_i) - \bar{F}_b(x_i)}{2} \).

**Proposition 21** [19]

Cotangent similarity measure satisfies the following properties:

1. \( 0 \leq C_{T_{RNS}}(A, B) \leq 1 \);
2. \( C_{T_{RNS}}(A, B) = C_{T_{RNS}}(B, A) \);
3. \( C_{T_{RNS}}(A, B) = 1 \) if \( A = B \).

4. If \( C \) is a RNS in \( Y \) and \( A \subset B \subset C \) then, \( C_{T_{RNS}}(A, C) \leq C_{T_{RNS}}(A, B) \), and \( C_{T_{RNS}}(B, C) \leq C_{T_{RNS}}(B, C) \).

**Definition 5.4.1**

If we consider the weights of each element \( x_i \), a weighted rough cotangent similarity measure [19] between rough neutrosophic sets \( A \) and \( B \) can be defined as follows:

\[
C_{W_{T_{RNS}}}(A, B) = \frac{1}{n} \sum_{i=1}^{n} w_i \frac{3 + \delta I_a(x_i) - \delta I_b(x_i) + \delta I_a(x_i) - \delta I_b(x_i) + \delta I_a(x_i) - \delta I_b(x_i)}{\sqrt{\left( \delta T_a(x_i) \right)^2 + \left( \delta F_a(x_i) \right)^2} \sqrt{\left( \delta T_b(x_i) \right)^2 + \left( \delta F_b(x_i) \right)^2}}
\]

\( w_i \in \{0, 1\} \), \( i = 1, 2, \ldots, n \), and \( \sum_{i=1}^{n} w_i = 1 \). If we take \( w_i = \frac{1}{n} \), \( i = 1, 2, \ldots, n \), then \( C_{T_{RNS}}(A, B) = C_{W_{T_{RNS}}}(A, B) \).

**Proposition 22** [19]

The weighted rough cosine similarity measure between two rough neutrosophic sets \( A \) and \( B \) also satisfies the following properties:

1. \( 0 \leq C_{W_{T_{RNS}}}(A, B) \leq 1 \);
2. \( C_{W_{T_{RNS}}}(A, B) = C_{W_{T_{RNS}}}(B, A) \);
3. \( C_{W_{T_{RNS}}}(A, B) = 1 \) if \( A = B \).

4. If \( C \) is a WRNS in \( Y \) and \( A \subset B \subset C \) then, \( C_{W_{T_{RNS}}}(A, C) \leq C_{W_{T_{RNS}}}(A, B) \), and \( C_{W_{T_{RNS}}}(B, C) \leq C_{W_{T_{RNS}}}(B, C) \).
Definition 5.5 (Variational co-efficient similarity measure between rough neutrosophic sets)
Let $A = \left( \left( T_A(x_i), L_A(x_i), F_A(x_i) \right), \left( \overline{T}_A(x_i), \overline{L}_A(x_i), \overline{F}_A(x_i) \right) \right)$ and $B = \left( \left( T_B(x_i), L_B(x_i), F_B(x_i) \right), \left( \overline{T}_B(x_i), \overline{L}_B(x_i), \overline{F}_B(x_i) \right) \right)$ be two rough neutrosophic sets. Variational co-efficient similarity measure between rough neutrosophic sets can be presented as follows:
$$\text{Var}_{\text{RNS}}(A, B) = \frac{1}{n} \left[ \sum_{i=1}^{n} \lambda \left( \frac{1}{2} \left( \delta T_A(x_i) \delta T_B(x_i) + \delta L_A(x_i) \delta L_B(x_i) + \delta F_A(x_i) \delta F_B(x_i) \right) + \delta F_A(x_i) \delta F_B(x_i) \right) \right]$$

Equation (19)

Proposition 23
The variational co-efficient similarity measure $\text{Var}_{\text{RNS}}(A, B)$ between two rough neutrosophic sets $A$ and $B$, satisfies the following properties:

1. $0 \leq \text{Var}_{\text{RNS}}(A, B) \leq 1$;
2. $\text{Var}_{\text{RNS}}(A, B) = \text{Var}_{\text{RNS}}(B, A)$;
3. $\text{Var}_{\text{RNS}}(A, B) = 1$ if $A = B$, i.e.,
   $$\delta T_A(x_i) = \delta T_B(x_i), \; \delta L_A(x_i) = \delta L_B(x_i), \; \text{and} \; \delta F_A(x_i) = \delta F_B(x_i), \; \text{for every} \; x_i (i = 1, 2, \ldots, n) \; \text{in} \; X.$$

Proof.

(1) It is obvious that $\text{Var}_{\text{RNS}}(A, B) \geq 0$. Thus it is required to prove that $\text{Var}_{\text{RNS}}(A, B) \leq 1$.

From rough neutrosophic dice similarity measure it can be written that
$$0 \leq \frac{1}{n} \sum_{i=1}^{n} \left[ \lambda \left( \frac{1}{2} \left( \delta T_A(x_i) \delta T_B(x_i) + \delta L_A(x_i) \delta L_B(x_i) + \delta F_A(x_i) \delta F_B(x_i) \right) + \delta F_A(x_i) \delta F_B(x_i) \right) \right] \leq 1$$

Combining Eq. (20) and Eq. (21), we obtain
$$\text{Var}_{\text{RNS}}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \lambda \left( \frac{1}{2} \left( \delta T_A(x_i) \delta T_B(x_i) + \delta L_A(x_i) \delta L_B(x_i) + \delta F_A(x_i) \delta F_B(x_i) \right) + \delta F_A(x_i) \delta F_B(x_i) \right) \right]$$

(21)
Thus it is (25), then Eq.(23) is reduced to Eq.(19).

\[
\sum_{n}^{+} \frac{1}{n} \left[ 2 \left( \delta T(x_i) \delta T(y_i) + \delta I(x_i) \delta I(y_i) \right) \right. \\
\left. + \delta F(x_i) \delta F(y_i) \right] \\
+ \left( 1 - \lambda \right) \sum_{n}^{+} \left[ \left( \delta T(x_i) \delta T(y_i) + (\delta I(x_i) \delta I(y_i) + (\delta F(x_i) \delta F(y_i) \right) \right] \\
= \frac{1}{n} \left[ n \delta \lambda + (n - \lambda) \right] = 1
\]

These results show the completion of the proofs of the three properties.

**Definition 5.6** (Weighted variational co-efficient similarity measure between rough neutrosophic sets)

Let \( A = \{ T_A(x_i), I_A(x_i), F_A(x_i) \} \) and \( B = \{ T_B(x_i), I_B(x_i), F_B(x_i) \} \) be any two rough neutrosophic sets. Rough variational co-efficient similarity measure between rough neutrosophic sets \( A \) and \( B \) in 3-D vector space can be presented as follows:

\[
 Var_{RWRNS}(A, B) = \sum_{n}^{+} \frac{1}{n} \left[ 2 \left( \delta T(x_i) \delta T(y_i) + \delta I(x_i) \delta I(y_i) \right) \right. \\
\left. + \delta F(x_i) \delta F(y_i) \right] \\
+ \left( 1 - \lambda \right) \sum_{n}^{+} \left[ \left( \delta T(x_i) \delta T(y_i) + (\delta I(x_i) \delta I(y_i) + (\delta F(x_i) \delta F(y_i) \right) \right] \\
\]

If \( w = \frac{1}{n} \), then Eq.(23) is reduced to Eq.(19).

**Proposition 24**

The weighted variational co-efficient similarity measure also satisfies the following properties:

1. \( 0 \leq Var_{RWRNS}(A, B) \leq 1 \);

2. \( Var_{RWRNS}(A, B) = Var_{RWRNS}(B, A) \);

3. \( Var_{RWRNS}(A, B) = 1 \); if \( A = B \) i.e., \( \delta T_A(x_i) = \delta T_B(x_i) \), \( \delta I_A(x_i) = \delta I_B(x_i) \), and \( \delta F_A(x_i) = \delta F_B(x_i) \), for every \( x_i (i = 1, 2, \ldots, n) \) in \( X \).

**Proof:**

(1.) It is obvious that \( Var_{RWRNS}(A, B) \geq 0 \). Thus it is required to prove that \( Var_{RWRNS}(A, B) \leq 1 \).

From rough neutrosophic weighted dice similarity measure, it can be written that

\[
0 \leq Var_{RWRNS}(A, B) = \sum_{n}^{+} \frac{1}{n} \left[ 2 \left( \delta T(x_i) \delta T(y_i) + \delta I(x_i) \delta I(y_i) \right) \right. \\
\left. + \delta F(x_i) \delta F(y_i) \right] \\
+ \left( 1 - \lambda \right) \sum_{n}^{+} \left[ \left( \delta T(x_i) \delta T(y_i) + (\delta I(x_i) \delta I(y_i) + (\delta F(x_i) \delta F(y_i) \right) \right] \\
\leq \lambda + (1 - \lambda) = 1
\]

Thus, \( 0 \leq Var_{RWRNS}(A, B) \leq 1 \).

(2.) \( Var_{RWRNS}(A, B) = \)

\[
\sum_{n}^{+} \frac{1}{n} \left[ 2 \left( \delta T(x_i) \delta T(y_i) + \delta I(x_i) \delta I(y_i) \right) \right. \\
\left. + \delta F(x_i) \delta F(y_i) \right] \\
+ \left( 1 - \lambda \right) \sum_{n}^{+} \left[ \left( \delta T(x_i) \delta T(y_i) + (\delta I(x_i) \delta I(y_i) + (\delta F(x_i) \delta F(y_i) \right) \right] \\
\leq \lambda + (1 - \lambda) = 1
\]

(3.) If \( A = B \) i.e.,

\[
Var_{RWRNS}(B, A) = \]
\[ \delta T_{j}(x_{i}) = \delta T_{y}(x_{i}), \quad \delta F_{j}(x_{i}) = \delta F_{y}(x_{i}), \text{ for every } x_{i} (i = 1, 2, \ldots, n) \text{ in } X, \]

\[ \lambda \sum_{j=1}^{n} w_{j} \delta T_{j}(x_{i}) \delta F_{j}(x_{i}) \]

\[ = \left[ \lambda \sum_{j=1}^{n} w_{j} \right] (1 - \lambda) \sum_{j=1}^{n} w_{j} = 1 \]

These results show the completion of the proofs of the three propositions.

6. Multi attribute decision making based on rough neutrosophic variational coefficient similarity measure

In this section, a rough variational co-efficient similarity measure is employed to multi-attribute decision making in rough neutrosophic environment. Assume that \( A = \{A_{1}, A_{2}, \ldots, A_{m}\} \) be the set of alternatives and \( C = \{C_{1}, C_{2}, \ldots, C_{n}\} \) be the set of attributes in a multi-attribute decision making problem. Assume that \( w_{j} \) be the weight of the attribute \( C_{j} \) provided by the decision maker such that each \( w_{j} \in [0,1] \) and \( \sum_{j=1}^{n} w_{j} = 1 \). However, in real situation decision maker may often face difficulty to evaluate alternatives over the attributes due to vague or incomplete information about alternatives in a decision making situation. Rough neutrosophic set can be used in MADM to deal with incomplete information of the alternatives. In this paper, the assessment values of all the alternatives with respect to attributes are considered as the rough neutrosophic values (see Table 1).

| \( A_{i} \) | \( d_{11}, \overline{d}_{11} \) | \( d_{12}, \overline{d}_{12} \) | \( \ldots \) | \( d_{1n}, \overline{d}_{1n} \) |
| \( A_{2} \) | \( d_{21}, \overline{d}_{21} \) | \( d_{22}, \overline{d}_{22} \) | \( \ldots \) | \( d_{2n}, \overline{d}_{2n} \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( A_{m} \) | \( d_{m1}, \overline{d}_{m1} \) | \( d_{m2}, \overline{d}_{m2} \) | \( \ldots \) | \( d_{mn}, \overline{d}_{mn} \) |

Here \( \langle d_{ij}, \overline{d}_{ij} \rangle \) is the rough neutrosophic number for the \( i \)-th alternative and the \( j \)-th attribute.

**Definition 6.1:** Transforming operator for SVNSs [80]

The rough neutrosophic decision matrix (27) can be transformed to single valued neutrosophic decision matrix whose \( ij \)-th element \( \alpha_{ij} \) can be presented as follows:

\[ \alpha_{ij} = \left\{ \frac{d_{ij} + \overline{d}_{ij}}{2} \right\}_{mn}, \text{ for } i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n. \]  

(28)

**Step 1.** Determine the neutrosophic relative positive ideal solution

- In multi-criteria decision-making environment, the concept of ideal point has been used to help identify the best alternative in the decision set.

**Definition 6.2** [51].

Let \( H \) be the collection of two types of attributes, namely, benefit type attribute \( (P) \) and cost type attribute \( (L) \) in the MADM problems. The relative positive ideal neutrosophic solution (RPINS) \( Q_{s}^{+} = \{\delta_{x_{1}}^{+}, \delta_{x_{2}}^{+}, \ldots, \delta_{x_{n}}^{+}\} \) is the solution of the decision matrix \( D_{s} = \{\delta_{x_{1}}, \delta_{x_{2}}, \ldots, \delta_{x_{n}}\}_{mn} \) where, every component of \( Q_{s}^{+} \) has the following form:

- for benefit type attribute, every component of \( Q_{s}^{+} \) has the following form:

\[ q_{x_{j}}^{+} = \{\delta_{x_{j}}^{+}, \delta_{x_{j}}^{+}, \delta_{x_{j}}^{+}\} \]

(29)

\[ = \{\max \{\delta_{x_{j}}^{+}\}, \min \{\delta_{x_{j}}^{+}\}, \min \{\delta_{x_{j}}^{+}\}\} \text{ for } j \in P \]

- and for cost type attribute, every component of \( Q_{s}^{+} \) has the following form:

\[ q_{x_{j}}^{+} = \{\delta_{x_{j}}^{+}, \delta_{x_{j}}^{+}, \delta_{x_{j}}^{+}\} \]

(30)

\[ = \{\min \{\delta_{x_{j}}^{+}\}, \max \{\delta_{x_{j}}^{+}\}, \max \{\delta_{x_{j}}^{+}\}\} \text{ for } j \in L \]

**Step 2.** Determine the weighted variational co-efficient similarity measure between ideal alternative and each alternative.

The variational co-efficient similarity measure between ideal alternative \( Q_{s}^{+} \) and each alternative \( A_{i} \) for \( i = 1, 2, \ldots, m \) can be determined by the following equation as follows:

\[ \text{Var}_{W_{BNS}}(Q_{s}, D_{s}) = \frac{2\delta T_{j} \delta T_{j} + \delta I_{j} \delta I_{j} \delta F_{j} \delta F_{j}}{\left[ \left( \delta T_{j} \right)^{2} + \left( \delta I_{j} \right)^{2} + \left( \delta F_{j} \right)^{2} \right]^{2}} \]

\[ + (1 - \lambda) \sum_{j=1}^{n} w_{j} \left[ \left( \delta T_{j} \right)^{2} + \left( \delta I_{j} \right)^{2} + \left( \delta F_{j} \right)^{2} \right] \]

(28)

**Step 3. Rank the alternatives.**

According to the values obtained from Eq.(31), the ranking order of all the alternatives can be easily determined. Highest value indicates the best alternative.

**Table 1:** Rough neutrosophic decision matrix

\[ D_{BNS} = \{d_{ij}, \overline{d}_{ij}\}_{mn} \]

| \( A_{i} \) | \( d_{11}, \overline{d}_{11} \) | \( d_{12}, \overline{d}_{12} \) | \( \ldots \) | \( d_{1n}, \overline{d}_{1n} \) |
| \( A_{2} \) | \( d_{21}, \overline{d}_{21} \) | \( d_{22}, \overline{d}_{22} \) | \( \ldots \) | \( d_{2n}, \overline{d}_{2n} \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( A_{m} \) | \( d_{m1}, \overline{d}_{m1} \) | \( d_{m2}, \overline{d}_{m2} \) | \( \ldots \) | \( d_{mn}, \overline{d}_{mn} \) |

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Step 4. End.

7 Numerical example

In this section, rough neutrosophic MADM regarding investment problem is considered to demonstrate the applicability and the effectiveness of the proposed approach. However, investment problem is not easy to solve. It not only requires oodles of patience and discipline, but also a great deal of research and a sound understanding of the market, mathematical tools, among others. Suppose an investment company wants to invest a sum of money in the best option. Assume that there are four possible alternatives to invest the money: (1) $A_1$ is a computer company; (2) $A_2$ is a garment company; (3) $A_3$ is a telecommunication company; and (4) $A_4$ is a food company. The investment company must take a decision based on the following three criteria: (1) $C_1$ is the growth factor; (2) $C_2$ is the risk factor; (3) $C_3$ is the environmental impact. The four possible alternatives are to be evaluated under the criteria $C_1$, $C_2$, and $C_3$.

Table 2. Rough neutrosophic decision matrix

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>(0.1,0.2,0.2)</td>
<td>(0.6,0.4,0.3)</td>
<td>(0.3,0.2,0.3)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.3,0.2,0.2)</td>
<td>(0.8,0.2,0.3)</td>
<td>(0.5,0.2,0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.2,0.4,0.3)</td>
<td>(0.6,0.3,0.3)</td>
<td>(0.1,0.4,0.3)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.4,0.2,0.3)</td>
<td>(0.8,0.1,0.1)</td>
<td>(0.3,0.2,0.3)</td>
</tr>
</tbody>
</table>

Eq. (32) can be obtained as:

\[ \text{Maximize } \sum_{i=1}^{3} \sum_{j=1}^{4} w_i D^{ij} \]

The known weight information is given as follows:

\[ W = [w_1, w_2, w_3]^T = [0.3, 0.3, 0.4] \]

Step 1. Determine the types of criteria.

First two types i.e. $C_1$ and $C_2$ of the given criteria are benefit type criteria and the last one criterion i.e. $C_3$ is the cost type criteria.

Step 2. Determine the relative neutrosophic positive ideal solution

Using Eq. (29), Eq.(30), the relative positive ideal neutrosophic solution for the given matrix defined in Eq.(32) can be obtained as:

\[ Q^+ = ([0.4,0.2,0.2],[0.7,0.2,0.2],[0.1,0.3,0.3]) \]

Step 3. Determine the weighted variational similarity measure

The weighted variational co-efficient similarity measure is determined by using Eq.(28), Eq.(31) and Eq.(32). The results obtained for different values of $\lambda$ have been shown in the Table 3.

### Table 3. Results of rough variational similarity measure for different values of $\lambda$

<table>
<thead>
<tr>
<th>Similarity measure method</th>
<th>Values of $s$</th>
<th>Measure values</th>
<th>Ranking order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Var_{R,G}(Q^+_3, D_3)$</td>
<td>0.10</td>
<td>0.8769; 0.9741; 0.9917; 0.8107</td>
<td>$A_1 &gt; A_2 &gt; A_3 &gt; A_4$</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.8740; 0.9739; 0.9905; 0.8078</td>
<td>$A_1 &gt; A_3 &gt; A_4 &gt; A_2$</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>0.8692; 0.9735; 0.9887; 0.8028</td>
<td>$A_1 &gt; A_3 &gt; A_2 &gt; A_4$</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>0.8643; 0.9730; 0.9868; 0.7979</td>
<td>$A_3 &gt; A_2 &gt; A_4 &gt; A_1$</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>0.8614; 0.9728; 0.9857; 0.7949</td>
<td>$A_2 &gt; A_4 &gt; A_1 &gt; A_3$</td>
</tr>
</tbody>
</table>

Step 4. Rank the alternatives.

According to the different values of $\lambda$, the results obtained in Table-3 reflects that $A_3$ is the best alternative.

8. Comparisons of different rough similarity measure with rough variation similarity measure

In this section, four existing rough similarity measures - namely: rough cosine similarity measure, rough dice similarity measure, rough cotangent similarity measure and rough Jaccard similarity measure - have been compared with proposed rough variational co-efficient similarity measure for different values of $\lambda$. The comparison results are listed in the Table 3 and Table 4.
Table-4. Results of existing rough neutrosophic similarity measure methods.

<table>
<thead>
<tr>
<th>Rough similarity method</th>
<th>Values of s</th>
<th>Measure values</th>
<th>Ranking order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{JAC}<em>{\text{WRNS}}(Q_7, D</em>{3})$ [21]</td>
<td>...</td>
<td>0.7870, 0.9471; 0.9739; 0.6832</td>
<td>$A_3 &gt; A_1 &gt; A_1 &gt; A_4$</td>
</tr>
<tr>
<td>$\text{DJC}<em>{\text{WRNS}}(Q_7, D</em>{3})$ [21]</td>
<td>...</td>
<td>0.8595; 0.9726; 0.9873; 0.7929</td>
<td>$A_2 &gt; A_1 &gt; A_1 &gt; A_1$</td>
</tr>
<tr>
<td>$\text{C}_{\text{WRNS}}(Q_7, D_2)$ [20]</td>
<td>...</td>
<td>0.8788; 0.9738; 0.9920; 0.9132</td>
<td>$A_2 &gt; A_1 &gt; A_1 &gt; A_1$</td>
</tr>
<tr>
<td>$\text{COT}_{\text{WRNS}}(Q_7, D_3)$ [19]</td>
<td>...</td>
<td>0.8472; 0.9358; 0.9643; 0.8013</td>
<td>$A_3 &gt; A_1 &gt; A_1 &gt; A_4$</td>
</tr>
</tbody>
</table>


eutrosophic environment.
other multi attribute decision making problems in rough

References


Conclusion

In this paper, we have proposed rough variational coefficient similarity measures. We also proved some of their basic properties. We have presented an application of rough neutrosophic variational coefficient similarity measure for a decision making problem on investment. The concept presented in the paper can be applied to deal with other multi attribute decision making problems in rough neutrosophic environment.
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