

**Nω–CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES**

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**Abstract.** Neutrosophic set and Neutrosophic Topological spaces has been introduced by Salama[5]. Neutrosophic Closed set and Neutrosophic Continuous Functions were introduced by Salama et. al.. In this paper, we introduce the concept of Nω-closed sets and their properties in Neutrosophic topological spaces.

**Keywords:** Intuitionistic Fuzzy set, Neutrosophic set, Neutrosophic Topology, Nω-open set, Nω-closed set, Nω-open set and Nω-closure.

1. **Introduction**

Many theories like, Theory of Fuzzy sets[10], Theory of Intuitionistic fuzzy sets[1], Theory of Neutrosophic sets[8] and The Theory of Interval Neutrosophic sets[4] can be considered as tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in[8].

In 1965, Zadeh[10] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov[1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of nonmembership of each element. The neutrosophic set was introduced by Smarandache[7] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set.

In 2012, Salama, Alblowi[5] introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of nonmembership of each element. In 2014 Salama, Smarandache and Valeri [6] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions. In this paper, we introduce the concept of Nω-closed sets and their properties in neutrosophic topological spaces.

2. **Preliminaries**

In this paper, X denote a topological space (X, τX) on which no separation axioms are assumed unless otherwise explicitly mentioned. We recall the following definitions, which will be used throughout this paper. For a subset A of X, Ncl(A), Nint(A) and A° denote the neutrosophic closure, neutrosophic interior, and the complement of neutrosophic set A respectively.

**Definition 2.1.[3]** Let X be a non-empty fixed set. A neutrosophic set(NS for short) A is an object having the form $A = \{<x, \mu_A(x), \sigma_A(x), \nu_A(x)> : \text{for all } x \in X\}$. Where $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$ which represent the degree of membership, the degree of indeterminacy and the degree of nonmembership of each element $x \in X$ to the set $A$.

**Definition 2.2.[5]** Let A and B be NSs of the form $A = \{<x, \mu_A(x), \sigma_A(x), \nu_A(x)> : \text{for all } x \in X\}$ and $B = \{<x, \mu_B(x), \sigma_B(x), \nu_B(x)> : \text{for all } x \in X\}$. Then

- **i.** $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
- **ii.** $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
- **iii.** $A^c = \{<x, \nu_A(x), 1- \sigma_A(x), \mu_A(x)> : \text{for all } x \in X\}$,
- **iv.** $A \cup B = \{<x, \mu_A(x) \lor \mu_B(x), \sigma_A(x) \land \sigma_B(x), \nu_A(x) \land \nu_B(x)> : \text{for all } x \in X\}$,
- **v.** $A \cap B = \{<x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \lor \sigma_B(x), \nu_A(x) \lor \nu_B(x)> : \text{for all } x \in X\}$.

**Definition 2.3.[5]** A neutrosophic topology(NT for short) on a non empty set $X$ is a family $\tau$ of neutrosophic subsets in $X$ satisfying the following axioms:

- **i)** $0_X, 1_X \in \tau$,
- **ii)** $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
- **iii)** $\bigcup G_i \in \tau$, for all $G_i : i \in J \subseteq \tau$

In this pair $(X, \tau)$ is called a neutrosophic topological space (NTS for short) for neutrosophic set (NOS for short) $\tau$ in $X$. The elements of $\tau$ are called open neutrosophic sets. A neutrosophic set $F$ is called closed if and only if the complement of $F$(F° for short) is neutrosophic open.

**Definition 2.4.[5]** Let $(X, \tau)$ be a neutrosophic topological space. A neutrosophic set $A$ in $(X, \tau)$ is said to be neutrosophic closed(N-closed for short) if $\text{Ncl}(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is neutrosophic open.
Definition 2.5.[5] The complement of N-closed set is N-open set.

Proposition 2.6.[6] In a neutrosophic topological space (X, T), T = Ω (the family of all neutrosophic closed sets) iff every neutrosophic subset of (X, T) is a neutrosophic closed set.

3. \( N_ω \)-closed sets

In this section, we introduce the concept of \( N_ω \)-closed set and some of their properties. Throughout this paper \( (X, τ_ω) \) represent a neutrosophic topological spaces.

Definition 3.1. Let \( (X, τ_ω) \) be a neutrosophic topological space. Then A is called neutrosophic semi open set(\( N_ω \)-open set for short) if \( A \subseteq \text{Ncl}(\text{Nint}(A)) \).

Definition 3.2. Let \( (X, τ_ω) \) be a neutrosophic topological space. Then A is called neutrosophic semi closed set(\( N_ω \)-closed set for short) if \( \text{Nint}(\text{Ncl}(A)) \subseteq A \).

Definition 3.3. Let A be a neutrosophic set of a neutrosophic topological space \( (X, τ_ω) \). Then,
   i. The neutrosophic semi closure of A is defined as \( N_ω \text{cl}(A) = \cap \{K: K \text{ is a } N_ω\text{-closed set in } X \text{ and } A \subseteq K\} \)
   ii. The neutrosophic semi interior of A is defined as \( N_ω \text{int}(A) = \cup \{G: G \text{ is a } N_ω\text{-open in } X \text{ and } G \subseteq A\} \).

Definition 3.4. Let \( (X, τ_ω) \) be a neutrosophic topological space. Then A is called neutrosophic semi closed set(\( N_ω \)-closed set for short) if \( \text{Ncl}(A) \subseteq G \) whenever \( A \subseteq G \) and G is \( N_ω \)-open set.

Theorem 3.5. Every neutrosophic closed set is \( N_ω \)-closed set, but the converse may not be true.
Proof: If A is any neutrosophic set in X and G is any \( N_ω \)-open set containing A, then \( \text{Ncl}(A) \subseteq G \). Hence A is \( N_ω \)-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let \( X = \{a,b,c\} \) and \( τ_ω = \{0_ωG_1, 1_ω\} \) is a neutrosophic topological and \( X, τ_ω \) is a neutrosophic topological spaces. Take \( G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> \) and \( A = <x, (0.2, 0.2, 0.1), (0.1, 0.2), (0.8, 0.6, 0.9)> \). Then the set A is \( N_ω \)-closed but A is not a \( N_ω \)-closed set.

Remark 3.9. The concepts of \( N_ω \)-closed sets and \( N_ω \)-closed sets are independent.

Example 3.10. Let \( X = \{a,b,c\} \) and \( τ_ω = \{0_ωG_1, 1_ω\} \) is a neutrosophic topological and \( X, τ_ω \) is a neutrosophic topological spaces. Take \( G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> \) and \( A = <x, (0.2, 0.2, 0.1), (0, 1, 0.2), (0.8, 0.6, 0.9)> \). Then the set A is \( N_ω \)-closed set but A is not a \( N_ω \)-closed set.

Theorem 3.12. If \( A \) and \( B \) are \( N_ω \)-closed sets, then \( A \cup B \) is \( N_ω \)-closed set.
Proof: If \( A \cup B \subseteq G \) and \( G \) is \( N_ω \)-open set, then \( A \subseteq G \) and \( B \subseteq G \). Since \( A \) and \( B \) are \( N_ω \)-closed sets, \( \text{Ncl}(A) \subseteq G \) and \( \text{Ncl}(B) \subseteq G \) and hence \( \text{Ncl}(A) \cup \text{Ncl}(B) \subseteq G \). This implies \( \text{Ncl}(A \cup B) \subseteq G \). Thus \( A \cup B \) is \( N_ω \)-closed set in \( X \).

Theorem 3.13. A neutrosophic set \( A \) is \( N_ω \)-closed set then \( \text{Ncl}(A) - A \) does not contain any non-empty neutrosophic closed sets.
Proof: Suppose that \( A \) is \( N_ω \)-closed set. Let \( F \) be a neutrosophic closed subset of \( \text{Ncl}(A) - A \). Then \( A \subseteq F^c \). But \( A \) is \( N_ω \)-closed set. Therefore \( \text{Ncl}(F) \subseteq F^c \).
Consequently \( F \subseteq (\text{Ncl}(A))^c \). We have \( F \subseteq \text{Ncl}(A) \). Thus \( F \subseteq \text{Ncl}(A) \cap (\text{Ncl}(A))^c \). Hence \( F \) is empty.

The converse of the above theorem need not be true as seen from the following example.

Example 3.14. Let \( X = \{a,b,c\} \) and \( τ_ω = \{0_ωG_1, 1_ω\} \) is a neutrosophic topological and \( X, τ_ω \) is a neutrosophic topological spaces. Take \( G_1 = <x, (0.5, 0.6, 0.4), (0.4, 0.5, 0.2), (0.7, 0.6, 0.9)> \) and \( A = <x, (0.2, 0.2, 0.1), (0.6, 0.6, 0.6), (0.8, 0.9, 0.9)> \). Then the set A is not a \( N_ω \)-closed set and \( \text{Ncl}(A) - A = <x, (0.2, 0.2, 0.1), (0.6, 0.6, 0.6), (0.8, 0.9, 0.9)> \) does not contain non-empty neutrosophic closed sets.

Theorem 3.15. A neutrosophic set \( A \) is \( N_ω \)-closed set if and only if \( \text{Ncl}(A) - A \) contains no non-empty \( N_ω \)-closed set.
Proof: Suppose that $A$ is $N_c$-closed set. Let $S$ be a $N_c$-closed subset of $Ncl(A) – A$. Then $A \subseteq S'$. Since $A$ is $N_c$-closed set, we have $Ncl(A) \subseteq S'$. Consequently $S \subseteq (Ncl(A))'$. Hence $S \subseteq Ncl(A) \cap (Ncl(A))' = \emptyset$. Therefore $S$ is empty.

Conversely, suppose that $Ncl(A) – A$ contains no non-empty $N_c$-closed set. Let $A \subseteq G$ and that $G$ be $N_c$-open. If $Ncl(A) \not\subseteq G$, then $Ncl(A) \cap G'$ is a non-empty $N_c$-closed subset of $Ncl(A) – A$. Hence $A$ is $N_c$-closed set.

Corollary 3.16. A $N_c$-closed set $A$ is $N_c$-closed if and only if $Ncl(A) – A$ is $N_c$-closed.

Proof: Let $A$ be any $N_c$-closed set. If $A$ is $N_c$-closed set, then $Ncl(A) – A = \emptyset$. Therefore $Ncl(A) – A$ is $N_c$-closed set. Conversely, suppose that $Ncl(A) – A$ is $N_c$-closed set. But $A$ is $N_c$-closed set and $Ncl(A) – A$ contains $N_c$-closed set. By theorem 3.15, $Ncl(A) – A = \emptyset$. Therefore $Ncl(A) = A$. Hence $A$ is $N_c$-closed set.

Theorem 3.17. Suppose that $B \subseteq A \subseteq X$, $B$ is a $N_c$-closed set relative to $A$ and $A$ is $N_c$-closed set in $X$. Then $B$ is $N_c$-closed set in $X$.

Proof: Let $B \subseteq G$, where $G$ is $N_c$-open in $X$. We have $B \subseteq A \cap G$ and $A \cap G$ is $N_c$-open in $A$. But $B$ is a $N_c$-closed set relative to $A$. Hence $Ncl(B) \subseteq A \cap G$. Since $Ncl(B) = A \cap Ncl(B)$. We have $A \cap Ncl(B) \subseteq A \cap G$. It implies $A \subseteq G(U(Ncl(B))^c$ and $G(U(Ncl(B))^c$ is a $N_c$-closed set in $X$. Since $A$ is $N_c$-closed in $X$, we have $Ncl(A) \subseteq G(U(Ncl(B))^c$. Hence $Ncl(B) \subseteq G(U(Ncl(B))^c$ and $Ncl(B) \subseteq G$. Therefore $B$ is $N_c$-closed set relative to $X$.

Theorem 3.18. If $A$ is $N_c$-closed and $A \subseteq B \subseteq Ncl(A)$, then $B$ is $N_c$-closed.

Proof: Since $B \subseteq Ncl(A)$, we have $Ncl(B) \subseteq Ncl(A)$ and $Ncl(B) \subseteq Ncl(A) – A$. But $A$ is $N_c$-closed. Hence $Ncl(A) – A$ has no non-empty $N_c$-closed subsets, neither does $Ncl(B)$ – $B$. By theorem 3.15, $B$ is $N_c$-closed.

Theorem 3.19. Let $A \subseteq Y \subseteq X$ and suppose that $A$ is $N_c$-closed in $X$. Then $A$ is $N_c$-closed relative to $Y$.

Proof: Let $A \subseteq Y \cap G$ where $G$ is $N_c$-open in $X$. Then $A \subseteq G$ and hence $Ncl(A) \subseteq G$. This implies, $Y \cap Ncl(A) \subseteq Y \cap G$. Thus $A$ is $N_c$-closed relative to $Y$.

Theorem 3.20. If $A$ is $N_c$-open and $N_c$-closed, then $Ncl(A) \subseteq A$. Therefore $Ncl(A) = A$. Hence $A$ is neutrosophic closed.

4. $N_c$-open sets

In this section, we introduce and study about $N_c$-open sets and some of their properties.

Definition 4.1. A Neutrosophic set $A$ in $X$ is called $N_c$-open in $X$ if $A^c$ is $N_c$-closed in $X$.

Theorem 4.2. Let $(X, \tau_N)$ be a neutrosophic topological space. Then

(i) Every neutrosophic open set is $N_c$-open but not conversely.

(ii) Every $N_c$-open set is $N$-open but not conversely.

The converse part of the above statements are proved by the following example.

Example 4.3. Let $X = \{a, b, c\}$ and $\tau_N = \{\emptyset, G_1, 1_N\}$ is a neutrosophic topology and $(X, \tau_N)$ is a neutrosophic topological space. Take $G_1 = \langle x, (0.7, 0.6, 0.9), (0.6, 0.5, 0.8), (0.5, 0.6, 0.4)\rangle$ and $A = \langle x, (0.8, 0.6, 0.9), (1, 0, 0.8), (0.2, 0.2, 0.1)\rangle$. Then the set $A$ is $N_c$-open set but not a neutrosophic open and $B = \langle x, (0.11, 0.25, 0.2), (0.89, 0.7, 0.9), (0.55, 0.45, 0.6)\rangle$ is $N$-open but not a $N_c$-open set.

Theorem 4.4. A neutrosophic set $A$ is $N_c$-open if and only if $F \subseteq Nint(A)$ where $F$ is $N_c$-closed and $F \subseteq A$.

Proof: Suppose that $F \subseteq Nint(A)$ where $F$ is $N_c$-closed and $F \subseteq A$. Let $A^c \subseteq G$ where $G$ is $N_c$-open. Then $G^c \subseteq A$ and $G^c$ is $N_c$-closed. Therefore $G^c \subseteq Nint(A)$. Since $G^c \subseteq Nint(A)$, we have $(Nint(A))^c \subseteq G$. This implies $Ncl((A))^c \subseteq G$. Thus $A^c$ is $N_c$-closed. Hence $A$ is $N_c$-open.

Conversely, suppose that $A$ is $N_c$-open, $F \subseteq A$ and $F$ is $N_c$-closed. Then $F^c$ is $N_c$-open and $A^c \subseteq F^c$. Therefore $Ncl((A)^c) \subseteq F^c$. But $Ncl((A)^c) = (Nint(A))^c$. Hence $F \subseteq Nint(A)$.

Theorem 4.5. A neutrosophic set $A$ is $N_c$-open in $X$ if and only if $G = X$ whenever $G$ is $N_c$-open and $(Nint(A) \cup A^c)^c \subseteq G$.

Proof: Let $A$ be a $N_c$-open, $G$ be $N_c$-open and $(Nint(A) \cup A^c)^c \subseteq G$. This implies $G^c \subseteq (Nint(A))^c \cap (A^c)^c = (Nint(A))^c – A^c = Ncl((A)^c) – A^c$. Since $A^c$ is $N_c$-closed and $G^c$ is $N_c$-closed, by Theorem 3.15, it follows that $G^c \subseteq \phi$. Therefore $X = G$.

Conversely, suppose that $F$ is $N_c$-closed and $F \subseteq X$. Then $Nint(A) \cup A^c \subseteq Nint(A) \subseteq F^c$. This implies $Nint(A) \cup F^c = X$ and hence $F \subseteq Nint(A)$. Therefore $A$ is $N_c$-open.

Theorem 4.6. If $Nint(A) \subseteq B \subseteq A$ and if $A$ is $N_c$-open, then $B$ is $N_c$-open.

Proof: Suppose that $Nint(A) \subseteq B \subseteq A$ and $A$ is $N_c$-open. Then $A^c \subseteq B \subseteq Ncl(A)^c$ and since $A^c$ is $N_c$-closed, we have by Theorem 3.15, $B^c$ is $N_c$-closed. Hence $B$ is $N_c$-open.

Theorem 4.7. A neutrosophic set $A$ is $N_c$-closed, if and only if $Ncl(A) – A$ is $N_c$-open.

Proof: Suppose that $A$ is $N_c$-closed. Let $F \subseteq Ncl(A) – A$ where $F$ is $N_c$-closed. By Theorem 3.15, $F = \emptyset$. Therefore $F \subseteq Nint(Ncl(A) – A)$ and by Theorem 4.4, we have $Ncl(A) – A$ is $N_c$-open.
Conversely, let $A \subseteq G$ where $G$ is a $N_\omega$-open set. Then $\text{Ncl}(A) \cap G \subseteq \text{Ncl}(A) \cap A^c = \text{Ncl}(A) - A$. Since $\text{Ncl}(A) \subseteq G^c$ is $N_\omega$-closed and $\text{Ncl}(A) - A$ is $N_\omega$-open. By Theorem 4.4, we have $\text{Ncl}(A) \cap G^c \subseteq \text{Nint}(\text{Ncl}(A) - A) = \phi$. Hence $A$ is $N_\omega$-closed.

**Theorem 4.8.** For a subset $A \subseteq X$ the following are equivalent:

(i) $A$ is $N_\omega$-closed.

(ii) $\text{Ncl}(A) - A$ contains no non-empty $N_\omega$-closed set.

(iii) $\text{Ncl}(A) - A$ is $N_\omega$-open set.

**Proof:** Follows from Theorem 3.15 and Theorem 4.7.

5. $N_\omega$-closure and Properties of $N_\omega$-closure

In this section, we introduce the concept of $N_\omega$-closure and some of their properties.

**Definition 5.1.** The $N_\omega$-closure (briefly $N_\omega\text{cl}(A)$) of a subset $A$ of a neutrosophic topological space $(X, \tau_N)$ is defined as follows:

$$N_\omega\text{cl}(A) = \bigcap \{ F \subseteq X / A \subseteq F \text{ and } F \text{ is } N_\omega\text{-closed in } (X, \tau_N) \}.$$

**Theorem 5.2.** Let $A$ be any subset of $(X, \tau_N)$. If $A$ is $N_\omega$-closed in $(X, \tau_N)$ then $A = N_\omega\text{cl}(A)$.

**Proof:** By definition, $N_\omega\text{cl}(A) = \bigcap \{ F \subseteq X / A \subseteq F \text{ and } F \text{ is } N_\omega\text{-closed in } (X, \tau_N) \}$ and we know that $A \subseteq A$. Hence $A = N_\omega\text{cl}(A)$.

**Remark 5.3.** For a subset $A$ of $(X, \tau_N)$, $A \subseteq N_\omega\text{cl}(A) \subseteq \text{Ncl}(A)$.

**Theorem 5.4.** Let $A$ and $B$ be subsets of $(X, \tau_N)$. Then the following statements are true:

i. $N_\omega\text{cl}(A) = \phi$ and $N_\omega\text{cl}(A) = X$.

ii. If $A \subseteq B$, then $N_\omega\text{cl}(A) \subseteq N_\omega\text{cl}(B)$

iii. $N_\omega\text{cl}(A) \cup N_\omega\text{cl}(B) = N_\omega\text{cl}(A \cup B)$

iv. $N_\omega\text{cl}(A \cap B) = N_\omega\text{cl}(A) \cap N_\omega\text{cl}(B)$

**References**


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