Neutrosophic Supra Topological Applications in Data Mining Process

G.Jayaparthasarathy[1], V.F.Little Flower[2], M.Arockia Dasan[3,*]

[1,2,3]Department of Mathematics, St.Jude's College, Thoothoor, Kanyakumari-629176, Tamil Nadu, India.

(Manonmaniam Sundaranar University Tirunelveli-627 012, Tamil Nadu, India).

[1]E-mail: jparthasarathy123@gmail.com

[2]E-mail: visjoy05796@gmail.com

[3]E-mail: dassfredy@gmail.com

*Correspondence: M.Arockia Dasan (dassfredy@gmail.com)

Abstract: The primary aim of this paper is to introduce the neutrosophic supra topological spaces. Neutrosophic subspaces and neutrosophic mappings are presented by which some contradicting examples of the statements of Abd-Monsef and Ramadan[9] in fuzzy supra topological spaces are derived. Finally, a new method is proposed to solve medical diagnosis problems by using single valued neutrosophic score function.

Keywords: Fuzzy topology, Intuitionistic topology, Neutrosophic topology, Neutrosophic subspaces, Neutrosophic supra topology.

1 Introduction

The concept of fuzzy set was introduced by A. Zadeh [1] in 1965 which is a generalization of crisp set to analyse imprecise mathematical information. Adlassnig [2] applied fuzzy set theory to formalize medical relationships and fuzzy logic to computerized diagnosis system. This theory [3, 4, 5] has been used in the fields of artificial intelligence, probability, biology, control systems and economics. C.L Chang [6] introduced the fuzzy topological spaces and further the properties of fuzzy topological spaces are studied by R. Lowen [7]. By relaxing one topological axiom, Mashhour et al. [8] introduced supra topological space in 1983 and discussed its properties. Abd-Monsef and Ramadan [9] introduced fuzzy supra topological spaces and its continuous mappings. K. Atanassov [10] considered the degree of non-membership of an element along with the degree of membership and introduced intuitionistic fuzzy sets. Dogan Coker [11] introduced intuitionistic fuzzy topology. Saadati [12] further studied the basic concept of intuitionistic fuzzy point. S.K.De et al. [13] was the first one to develop the applications of intuitionistic fuzzy sets in medical diagnosis. Several researchers [14, 15, 16] further studied intuitionistic fuzzy sets in medical diagnosis. Hung and Tuan [17] noted that the approach in [13] has some questionable results on false diagnosis of patients' symptoms. Generally it is recognized that the available information about the patient and medical relationships is inherently uncertain. There may be indeterminacy components in real life problems for data mining and neutrosophic logic can be used in this regard. Neutrosophic logic is a generalization of fuzzy, intuitionistic, boolean, paraconsistent logics etc. Compared to all other logics, neutrosophic logic introduces a percentage of "indeterminacy" and this logic allows each component t true, i indeterminate, f false to "boil over" 100 or "freeze" under 0. Here no restriction on T, I, F, or the sum $n = t + i + f$, where t, i, f are real values from the ranges T, I, F. For instance, in some tautologies $t{>}100$, called "overtrue". As a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set, Florentin Smarandache [18] introduced neutrosophic set. Neutrosophic set A consists of three independent objects called truth-membership $\mu_A(x)$, indeterminacy-membership $\sigma_A(x)$ and falsity-membership $\gamma_A(x)$ whose values are real standard or non-standard subset of unit interval $]^{-}0, 1^{+}[$. In data analysis, many methods have been introduced [19, 20, 21] to measure the similarity degree between fuzzy sets. But these are not suitable for the similarity measures of neutrosophic sets. The single-valued neutrosophic set is a neutrosophic set which can be used in real life engineering and scientific applications. The single valued neutrosophic set was first initiated by Smarandache [22] in 1998 and further studied by Wang et al. [23]. Majumdar and Samanta [24] defined some similarity measures of single valued neutrosophic sets in decision making problems. Recently many researchers [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45] introduced several similarity measures and single-valued neutrosophic sets in medical diagnosis. The notion of neutrosophic crisp sets and topological spaces were introduced by A. A. Salama and S. A. Alblowi [46,47].

In section 2 of this paper, we present some basic preliminaries of fuzzy, intuitionistic, neutrosophic sets and topological spaces. The section 3 introduces the neutrosophic subspaces with its properties. In section 4, we define the concept of neutrosophic supra topological spaces. In section 5, we introduce neutrosophic supra continuity, S^*-neutrosophic continuity and give some contradicting examples in fuzzy supra topological spaces[9]. As a real life application, a common method for data analysis under neutrosophic supra topological environment is presented in section 6. In section 7, we solve numerical examples of above proposed method and the last section states the conclusion and future work of this paper.

2 Preliminary

This section studies some of the basic definitions of fuzzy, intuitionistic, neutrosophic sets and respective topological spaces which are used for further study.

Definition 2.1. [1] Let X be a non empty set, then $A = \{(x, \mu_A(x)) : x \in X\}$ is called a fuzzy set on X, where $\mu_A(x) \in [0, 1]$ is the degree of membership function of each $x \in X$ to the set A. For X, I^X denotes the collection of all fuzzy sets of X.

Definition 2.2. [10] Let X be a non empty set, then $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$ is called an intuitionistic set on X, where $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$, $\mu_A(x), \gamma_A(x) \in [0, 1]$ are the degree of membership and non membership functions of each $x \in X$ to the set A respectively. The set of all intuitionistic sets of X is denoted by $I(X)$.

Definition 2.3. [23] Let X be a non empty set, then $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ is called a neutrosophic set on X, where $^{-}0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^{+}$ for all $x \in X$, $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x) \in]^{-}0, 1^{+}[$ are the degree of membership (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each $x \in X$ to the set A respectively. For X, $N(X)$ denotes the collection of all neutrosophic sets of X.

Definition 2.4. [18] The following statements are true for neutrosophic sets A and B on X:

(i) $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$ if and only if $A \subseteq B$.

(ii) $A \subseteq B$ and $B \subseteq A$ if and only if $A = B$.

(iii) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \min\{\sigma_A(x), \sigma_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

(iv) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \max\{\sigma_A(x), \sigma_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

More generally, the intersection and the union of a collection of neutrosophic sets $\{A_i\}_{i\in\Lambda}$, are defined by $\cap_{i\in\Lambda} A_i = \{(x, \inf_{i\in\Lambda}\{\mu_{A_i}(x)\}, \inf_{i\in\Lambda}\{\sigma_{A_i}(x)\}, \sup_{i\in\Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$ and $\cup_{i\in\Lambda} A_i = \{(x, \sup_{i\in\Lambda}\{\mu_{A_i}(x)\}, \sup_{i\in\Lambda}\{\sigma_{A_i}(x)\}, \inf_{i\in\Lambda}\{\gamma_{A_i}(x)\}) : x \in X\}$.

Notation 2.5. Let X be a non empty set. We consider the fuzzy, intuitionistic, neutrosophic empty set as $\emptyset = \{(x, 0) : x \in X\}$, $\emptyset = \{(x, 0, 1) : x \in X\}$, $\emptyset = \{(x, 0, 0, 1) : x \in X\}$ respectively and the fuzzy, intuitionistic, neutrosophic whole set as $X = \{(x, 1) : x \in X\}$, $X = \{(x, 1, 0) : x \in X\}$, $X = \{(x, 1, 1, 0) : x \in X\}$ respectively.

Definition 2.6. [24] A neutrosophic set $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ is called a single valued neutrosophic set on a non empty set X, if $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x) \in [0, 1]$ and $0 \leq \mu_A(x)+\sigma_A(x)+\gamma_A(x) \leq 3$ for all $x \in X$ to the set A. For each attribute, the single valued neutrosophic score function (shortly SVNSF) is defined as SVNSF $= \frac{1}{3m}[\sum_{i=1}^{m}[2 + \mu_i - \sigma_i - \gamma_i]]$.

Definition 2.7. [6] Let X be a non empty set. A subcollection τ_f of I^X is said to be fuzzy topology on X if the sets X and $\emptyset$ belong to τ_f, τ_f is closed under arbitrary union and τ_f is closed under finite intersection. Then (X, τ_f) is called fuzzy topological space (shortly fts), members of τ_f are known as fuzzy open sets and their complements are fuzzy closed sets.

Definition 2.8. [11] Let X be a non empty set and a subfamily τ_i of $I(X)$ is called intuitionistic fuzzy topology on X if X and $\emptyset \in \tau_i$, τ_i is closed under arbitrary union and τ_i is closed under finite intersection. Then (X, τ_i) is called intuitionistic fuzzy topological space (shortly ifts), elements of τ_i are called intuitionistic fuzzy open sets and their complements are intuitionistic fuzzy closed sets.

Definition 2.9. [46, 47] Let X be a non empty set. A neutrosophic topology on X is a subfamily τ_n of $N(X)$ such that X and $\emptyset$ belong to τ_n, τ_n is closed under arbitrary union and τ_n is closed under finite intersection. Then (X, τ_n) is called neutrosophic topological space (shortly nts), members of τ_n are known as neutrosophic open sets and their complements are neutrosophic closed sets. For a neutrosophic set A of X, the interior and closure of A are respectively defined as: $int_n(A) = \cup\{G : G \subseteq A, G \in \tau_n\}$ and $cl_n(A) = \cap\{F : A \subseteq F, F^c \in \tau_n\}$.

Corollary 2.10. [18] The following statements are true for the neutrosophic sets A, B, C and D on X:

(i) $A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$, if $A \subseteq B$ and $C \subseteq D$.

(ii) $A \subseteq B \cap C$, if $A \subseteq B$ and $A \subseteq C$. $A \cup B \subseteq C$, if $A \subseteq C$ and $B \subseteq C$.

(iii) $A \subseteq C$, if $A \subseteq B$ and $B \subseteq C$.

Definition 2.11. [48] Let $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$, $B = \{(y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y\}$ be two neutrosophic sets and $f : X \to Y$ be a function.

(i) $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)) : x \in X\}$ is a neutrosophic set on X called the pre-image of B under f.

(ii) $f(A) = \{(y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)) : y \in Y\}$ is a neutrosophic set on Y called the image of A under f, where

$$f(\mu_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \sigma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_{-}(\gamma_A)$ for $(1 - f(1 - \gamma_A))$.

3 Neutrosophic Subspaces

This section introduce differences of two fuzzy, intuitionistic and neutrosophic sets on X. We also introduce neutrosophic subspaces with its proprties.

Definition 3.1. The difference of neutrosophic sets A and B on X is a neutrosophic set on X, defined as $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|, |\sigma_A(x) - \sigma_B(x)|, 1 - |\gamma_A(x) - \gamma_B(x)|) : x \in X\}$. Clearly $X^c = X \setminus X = (x, 0, 0, 1) = \emptyset$ and $\emptyset^c = X \setminus \emptyset = (x, 1, 1, 0) = X$.

Definition 3.2. Let A, B be two intuitionistic fuzzy sets of X, then the difference of A and B is a intuitionistic fuzzy set on X, defined as $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|, 1 - |\gamma_A(x) - \gamma_B(x)|) : x \in X\}$. Clearly $X^c = X \setminus X = (x, 0, 1) = \emptyset$ and $\emptyset^c = X \setminus \emptyset = (x, 1, 0) = X$.

Definition 3.3. Let A, B be two fuzzy sets of X, then the difference of A and B is a fuzzy set on X, defined as $A \setminus B = \{(x, |\mu_A(x) - \mu_B(x)|) : x \in X\}$. Clearly $X^c = X \setminus X = (x, 0) = \emptyset$ and $\emptyset^c = X \setminus \emptyset = (x, 1) = X$.

Corollary 3.4. The following statements are true for the neutrosophic sets $\{A\}_{i=1}^{\infty}$, A, B on X:

(i) $(\bigcap_{i \in \Lambda} A_i)^c = \bigcup_{i \in \Lambda} A_i^c$, $(\bigcup_{i \in \Lambda} A_i)^c = \bigcap_{i \in \Lambda} A_i^c$.

(ii) $(A^c)^c = A$. $B^c \subseteq A^c$, if $A \subseteq B$.

Proof. : **Part(i):** $(\bigcap_{i \in \Lambda} A_i)^c = \{(x, |1 - \inf_{i \in \Lambda}\{\mu_{A_i}(x)\}|, |1 - \inf_{i \in \Lambda}\{\sigma_{A_i}(x)\}|, 1 - |0 - \sup_{i \in \Lambda}\{\gamma_{A_i}(x)\}|) : x \in X\} = \{(x, \sup_{i \in \Lambda}(|1 - \mu_{A_i}(x)|), \sup_{i \in \Lambda}(|1 - \sigma_{A_i}(x)|), \inf_{i \in \Lambda}(|1 - \gamma_{A_i}(x)|) : x \in X\} = \bigcup_{i \in \Lambda} A_i^c$. Similarly we can prove $(\bigcup_{i \in \Lambda} A_i)^c = \bigcap_{i \in \Lambda} A_i^c$ and part(ii). □

Generally, in the sense of Chang[6] every fuzzy topology is intuitionistic fuzzy topology as well as neutrosophic topology. The following lemmas show that every intuitionistic fuzzy topology τ_i induce two fuzzy topologies on X and every neutrosophic topology τ_n induce three fuzzy topologies on X.

Lemma 3.5. In an intuitionistic fuzzy topological space (X, τ_i), each of the following collections form fuzzy topologies on X:

(i) $\tau_{f_1} = \{A = (x, \mu_A(x)) : (x, \mu_A(x), \gamma_A(x)) \in \tau_i\}$.

(ii) $\tau_{f_2} = \{A = (x, 1 - \gamma_A(x)) : (x, \mu_A(x), \gamma_A(x)) \in \tau_i\}$.

Proof. : Here we shall prove part (ii) only and similarly we can prove part (i). Clearly $\emptyset = (x, 0)$ and $X = (x, 1)$ are belong to τ_{f_2}. If $\{A_j\}_{j\in\Lambda} \in \tau_{f_2}$, then $\{(x, \mu_{A_j}(x), \gamma_{A_j}(x))\}_{j\in\Lambda} \in \tau_i$ and $(x,\sup_{j\in\Lambda}\{\mu_{A_j}(x)\}, \inf_{j\in\Lambda}\{\gamma_{A_j}(x)\})\} \in \tau_i$. Therefore $(x,\sup_{j\in\Lambda}\{1 - \gamma_{A_j}(x)\}) = (x, 1-\inf_{j\in\Lambda}\{\gamma_{A_j}(x)\}) \in \tau_{f_2}$ and so $\cup_{j\in\Lambda}A_j \in \tau_{f_2}$. If $\{A_j\}_{j=1}^m \in \tau_{f_2}$, then $\{(x, \mu_{A_j}(x), \gamma_{A_j}(x))\}_{j=1}^m \in \tau_i$ and $(x,\inf_j\{\mu_{A_j}(x)\},\sup_j\{\gamma_{A_j}(x)\})\} \in \tau_i$. Therefore $(x,\inf_j\{1 - \gamma_{A_j}(x)\}) = (x, 1-\sup_j\{\gamma_{A_j}(x)\}) \in \tau_{f_2}$ and so $\cap_{j=1}^m A_j \in \tau_{f_2}$. □

Lemma 3.6. In a neutrosophic topological space (X, τ_n), each of the following collections form fuzzy topologies on X:

(i) $\tau_{f_1} = \{A = (x, \mu_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n\}$.

(ii) $\tau_{f_2} = \{A = (x, \sigma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n\}$.

(iii) $\tau_{f_3} = \{A = (x, 1 - \gamma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n\}$.

Proof. : **Part (i):** Clearly $\emptyset = (x, 0)$ and $X = (x, 1)$ are belong to τ_{f_1}. If $\{A_j\}_{j\in\Lambda} \in \tau_{f_1}$, then $\{(x, \mu_{A_j}(x), \sigma_{A_j}(x), \gamma_{A_j}(x))\}_{j\in\Lambda} \in \tau_n$ and $(x,\sup_{j\in\Lambda}\{\mu_{A_j}(x)\},\sup_{j\in\Lambda}\{\sigma_{A_j}(x)\},\inf_{j\in\Lambda}\{\gamma_{A_j}(x)\})\} \in \tau_n$. Therefore $(x,\sup_{j\in\Lambda}\{\mu_{A_j}(x)\}) \in \tau_{f_1}$ and so $\cup_{j\in\Lambda}A_j \in \tau_{f_1}$. If $\{A_j\}_{j=1}^m \in \tau_{f_1}$, then $\{(x, \mu_{A_j}(x), \sigma_{A_j}(x), \gamma_{A_j}(x))\}_{j=1}^m \in \tau_n$ and $(x,\inf_j\{\mu_{A_j}(x)\},\inf_j\{\sigma_{A_j}(x)\},\sup_j\{\gamma_{A_j}(x)\})\} \in \tau_n$. Therefore $(x,\inf_j\{\mu_{A_j}(x)\}) \in \tau_{f_1}$ and so $\cap_{j=1}^m A_j \in \tau_{f_1}$. In similar manner we can prove part (ii) and (iii). □

Corollary 3.7. Let A be a neutrosophic set of (X, τ_n), then the collection $(\tau_n)_A = \{A \cap O : O \in \tau_n\}$ is a neutrosophic topology on A, called the induced neutrosophic topology on A and the pair $(A, (\tau_n)_A)$ is called neutrosophic subspace of nts (X, τ_n). The elements of $(\tau_n)_A$ are called $(\tau_n)_A$-open sets and their complements are called $(\tau_n)_A$-closed sets.

Proof. : Obviously $\emptyset = (x,\min(\mu_A(x), 0),\min(\sigma_A(x), 0),\max(\gamma_A(x), 1)) = A \cap \emptyset \in (\tau_n)_A$ and $A = (x,\min(\mu_A(x), 1),\min(\sigma_A(x), 1),\max(\gamma_A(x), 0)) = A \cap X \in (\tau_n)_A$. Take $\{A_j\}_{j\in\Lambda} \in (\tau_n)_A$, there exist $O_j \in \tau_n$, $j \in \Lambda$, such that $A_j = A\cap O_j$ for each $j \in \Lambda$. Then $A' = \cup_{j\in\Lambda}A_j = \{(x,\sup_{j\in\Lambda}\{\mu_{A_j}(x)\},\sup_{j\in\Lambda}\{\sigma_{A_j}(x)\}, \inf_{j\in\Lambda}\{\gamma_{A_j}(x)\})\} = \{(x,\sup_{j\in\Lambda}(\min\{\mu_A(x), \mu_{O_j}(x)\}),\sup_{j\in\Lambda}(\min\{\sigma_A(x), \sigma_{O_j}(x)\}), \inf_{j\in\Lambda}(\max\{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x,\min(\sup_{j\in\Lambda}\{\mu_A(x), \mu_{O_j}(x)\}),\min(\sup_{j\in\Lambda}\{\sigma_A(x), \sigma_{O_j}(x)\}), \max(\inf_{j\in\Lambda}\{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x, \mu_{A\cap(\cup_{j\in\Lambda}O_j)}(x), \sigma_{A\cap(\cup_{j\in\Lambda}O_j)}(x), \gamma_{A\cap(\cup_{j\in\Lambda}O_j)}(x))\} \in (\tau_n)_A$. Therefore $(\tau_n)_A$ is closed under arbitrary union on A. If we take $\{A_j\}_{j=1}^m \in (\tau_n)_A$, there exist $O_j \in \tau_n$, $j = 1, 2, ..., m$, such that $A_j = A\cap O_j$ for each $j \in \Lambda$. Then $A' = \cap_{j=1}^m A_j = \{(x,\inf_j\{\mu_{A_j}(x)\},\inf_j\{\sigma_{A_j}(x)\},\sup_j\{\gamma_{A_j}(x)\})\} = \{(x,\inf_j(\min\{\mu_A(x), \mu_{O_j}(x)\}),\inf_j(\min\{\sigma_A(x), \sigma_{O_j}(x)\}),\sup_j(\max\{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x,\min(\inf_j\{\mu_A(x), \mu_{O_j}(x)\}),\min(\inf_j\{\sigma_A(x), \sigma_{O_j}(x)\}),\max(\sup_j\{\gamma_A(x), \gamma_{O_j}(x)\}))\} = \{(x, \mu_{A\cap(\cap_{j\in\Lambda}O_j)}(x), \sigma_{A\cap(\cap_{j\in\Lambda}O_j)}(x), \gamma_{A\cap(\cap_{j\in\Lambda}O_j)}(x))\} \in (\tau_n)_A$. Therefore $(\tau_n)_A$ is a neutrosophic topology on A. □

Corollary 3.8. Let A be a fuzzy set (resp. intuitionistic fuzzy set) of fts (X, τ_f) (resp. ifts (X, τ_i)), then the collection $(\tau_f)_A = \{A \cap O : O \in \tau_f\}$ (resp. $(\tau_i)_A = \{A \cap O : O \in \tau_i\}$) is a fuzzy topology (resp. intuitionistic fuzzy topology) on A, called the induced fuzzy topology (resp. induced intuitionistic fuzzy topology) on A and the pair $(A, (\tau_f)_A)$ (resp. $(A, (\tau_i)_A)$) is called fuzzy subspace (resp. intuitionistic fuzzy subspace).

Proof. : Proof follows from the above corollory. □

Lemma 3.9. Let $(A, (\tau_n)_A)$ be a neutrosophic subspace of nts (X, τ_n) and $B \subseteq A$. If B is $(\tau_n)_A$-open in $(A, (\tau_n)_A)$ and A is neutrosophic open in nts (X, τ_n), then B is neutrosophic open in (X, τ_n).

Proof. : Since B is $(\tau_n)_A$-open in $(A, (\tau_n)_A)$, $B = A \cap O$ for some neutrosophic open set O in (X, τ_n) and so B is neutrosophic open in (X, τ_n). □

Lemma 3.10. Let $(A, (\tau_f)_A)$ (resp. $(A, (\tau_i)_A)$) be a fuzzy subspace (resp. intuitionistic fuzzy subspace) of fts (X, τ_f) (resp. of ifts (X, τ_i)) and $B \subseteq A$. If B is $(\tau_f)_A$-open (resp. $(\tau_i)_A$-open) in $(A, (\tau_f)_A)$ (resp. $(A, (\tau_i)_A)$) and A is fuzzy open (resp. intuitionistic fuzzy open) in fts (X, τ_f) (resp. ifts (X, τ_i)), then B is fuzzy open (resp. intuitionistic fuzzy open) in (X, τ_f) (resp. ifts (X, τ_i)).

Proof. : Proof is similar as above lemma. □

Remark 3.11. In classical topology, we know that if (A, τ_A) is a subspace of (X, τ) and $B \subseteq A$, then

(i) $B = A \cap F$, where F is closed in X if and only if B is closed in A.

(ii) B is closed in X, if B is closed in A and A is closed in X.

The following examples illustrate that these are not true in fuzzy, intuitionistic fuzzy and neutrosophic topological spaces.

Example 3.12. Let $X = \{a, b, c\}$ with $\tau_n = \{\emptyset, X, ((1,1,1), (0,0,0), (0.7,0.7,0.7)), ((0.6,0.6,0.6), (0,0,0), (0,0,0)), ((1,1,1), (0,0,0), (0,0,0)), ((0.6,0.6,0.6), (0,0,0), (0.7,0.7,0.7))\}$. Then $(\tau_n)^c = \{X, \emptyset, ((0,0,0), (1,1,1), (0.3,0.3,0.3)), ((0.4,0.4,0.4), (1,1,1), (1,1,1)), ((0,0,0), (1,1,1), (1,1,1)), ((0.4,0.4,0.4), (1,1,1), (0.3,0.3,0.3))\}$. Let $A = ((0.6,0.6,0.2), (1,0,1), (0.8,0.7,0.6))$, then $(\tau_n)_A = \{\emptyset, A, ((0.6,0.6,0.2), (0,0,0), (0.8,0.7,0.7)), ((0.6,0.6,0.2), (0,0,0), (0.8,0.7,0.6))\}$ and $((\tau_n)_A)^c = \{A, \emptyset, ((0,0,0), (1,0,1), (1,1,0.9)), ((0,0,0), (1,0,1), (1,1,1))\}$. Clearly $B = ((0,0,0), (1,0,1), (1,1,0.9))$ is $(\tau_n)_A$-closed in $(A, (\tau_n)_A)$ and $B \neq A \cap F$ for every neutrosophic closed set F in (X, τ_n). Since $A = ((0,0,0), (1,1,1), (0.3,0.3,0.3))$ is neutrosophic closed in (X, τ_n), then $(\tau_n)_A = \{\emptyset, A, ((0,0,0), (0,0,0), (0.7,0.7,0.7)), ((0,0,0), (0,0,0), (0.3,0.3,0.3))\}$ and $((\tau_n)_A)^c = \{A, \emptyset, ((0,0,0), (1,1,1), (0.6,0.6,0.6)), ((0,0,0), (1,1,1), (1,1,1))\}$. Clearly $B = ((0,0,0), (1,1,1), (0.6,0.6,0.6))$ is $(\tau_n)_A$-closed in $(A, (\tau_n)_A)$, but it is not neutrosophic closed in (X, τ_n).

Example 3.13. Let $X = \{a, b, c\}$ with $\tau_i = \{\emptyset, X, ((0.4,0.4,0.3), (0.6,0.6,0.7)), ((0.3,0.8,0.1), (0.7,0.2,0.9)), ((0.3,0.4,0.1), (0.7,0.6,0.9)), ((0.4,0.8,0.3), (0.6,0.2,0.7))\}$. Then $(\tau_i)^c = \{X, \emptyset, ((0.6,0.6,0.7), (0.4,0.4,0.3)), ((0.7,0.2,0.9), (0.3,0.8,0.1)), ((0.7,0.6,0.9), (0.3,0.4,0.1)), ((0.6,0.2,0.7), (0.4,0.8,0.3))\}$. Since $A = ((0.7,0.2,0.9), (0.3,0.8,0.1))$ is intuitionistic fuzzy closed in (X, τ_i), then $(\tau_i)_A = \{\emptyset, A, ((0.4,0.2,0.3), (0.6,0.8,0.7)), ((0.3,0.2,0.1), (0.7,0.8,0.9))\}$ and $((\tau_i)_A)^c = \{A, \emptyset, ((0.3,0,0.6), (0.7,1,0.4)), ((0.4,0,0.8), (0.6,1,0.2))\}$. Clearly $B = ((0.3,0,0.6), (0.7,1,0.4))$ is $(\tau_i)_A$-closed in $(A, (\tau_i)_A)$, but $B \neq A \cap F$ for every intuitionistic fuzzy closed set F in (X, τ_i) and B is not intuitionistic fuzzy closed in (X, τ_i).

Example 3.14. Let $X = \{a, b, c\}$ with $\tau_f = \{\emptyset, X, (0.2, 0.3, 0.1), (0.7, 0.1, 0.8), (0.2, 0.1, 0.1), (0.7, 0.3, 0.8)\}$. Then $(\tau_f)^c = \{X, \emptyset, (0.8, 0.7, 0.9), (0.3, 0.9, 0.2), (0.8, 0.9, 0.9), (0.3, 0.7, 0.2)\}$. Since $A = (0.8, 0.7, 0.9)$ is fuzzy closed in (X, τ_f), then $(\tau_f)_A = \{\emptyset, A, (0.2, 0.3, 0.1), (0.7, 0.1, 0.8), (0.2, 0.1, 0.1), (0.7, 0.3, 0.8)\}$ and $((\tau_f)_A)^c = \{A, \emptyset, (0.6, 0.4, 0.8), (0.1, 0.6, 0.1), (0.6, 0.6, 0.8), (0.1, 0.4, 0.1)\}$. Clearly $B = (0.6, 0.6, 0.8)$ is $(\tau_f)_A$-closed in $(A, (\tau_f)_A)$, but $B \neq A \cap F$ for every fuzzy closed set F in (X, τ_f) and B is not fuzzy closed in (X, τ_f).

4 Neutrosophic Supra Topological Spaces

In this section, we introduce neutrosophic supra topological spaces and also establish its properties.

Definition 4.1. A subcollection τ_n^* of neutrosophic sets on a non empty set X is said to be a neutrosophic supra topology on X if the sets $\emptyset, X \in \tau_n^*$ and $\bigcup_{i=1}^{\infty} A_i \in \tau_n^*$, for $\{A_i\}_{i=1}^{\infty} \in \tau_n^*$. Then (X, τ_n^*) is called neutrosophic supra topological space on X (for short nsts). The members of τ_n^* are known as neutrosophic supra open sets and its complement is called neutrosophic supra closed. A neutrosophic supra topology τ_n^* on X is said to be an associated neutrosophic supra topology with neutrosophic topology τ_n if $\tau_n \subseteq \tau_n^*$. Every neutrosophic topology on X is neutrosophic supra topology on X.

Remark 4.2. The following table illustrates the combarison of fuzzy supra topological spaces, intuitionistic supra topological spaces, neutrosophic supra topological spaces.

Comparison Table

S.No	Fuzzy supra topological spaces	Intuitionistic supra topological spaces	Neutrosophic supra topological spaces
1	It deals with fuzzy sets	It deals with intuitionistic sets	It deals with neutrosophic sets
2	A subcollection τ_f^* of fuzzy sets on a non empty set X is said to be a fuzzy supra topology on X if the sets $\emptyset, X \in \tau_f^*$ and $\bigcup_{i=1}^{\infty} A_i \in \tau_f^*$, for $\{A_i\}_{i=1}^{\infty} \in \tau_f^*$.	A subcollection τ_i^* of intuitionistic sets on a non empty set X is said to be a intuitionistic supra topology on X if the sets $\emptyset, X \in \tau_i^*$ and $\bigcup_{i=1}^{\infty} A_i \in \tau_i^*$, for $\{A_i\}_{i=1}^{\infty} \in \tau_i^*$.	A subcollection τ_n^* of neutrosophic sets on a non empty set X is said to be a neutrosophic supra topology on X if the sets $\emptyset, X \in \tau_n^*$ and $\bigcup_{i=1}^{\infty} A_i \in \tau_n^*$, for $\{A_i\}_{i=1}^{\infty} \in \tau_n^*$.
3	A non empty set X together with the collection τ_f^* is called fuzzy supra topological space on X (for short fsts) denoted by the ordered pair (X, τ_f^*).	A non empty set X together with the collection τ_i^* is called intuitionistic supra topological space on X (for short ists) denoted by the ordered pair (X, τ_i^*).	A non empty set X together with the collection τ_n^* is called neutrosophic supra topological space on X (for short nsts) denoted by the ordered pair (X, τ_n^*).
4	The members of τ_f^* are known as fuzzy supra open sets.	The members of τ_i^* are known as intuitionistic supra open sets.	The members of τ_n^* are known as neutrosophic supra open sets.
5	It is a generalization of classical supra topological spaces.	It is a generalization of fuzzy supra topological spaces.	It is a generalization of intuitionistic supra topological spaces.
6	Every fuzzy topology is fuzzy supra topology.	Every intuitionistic topology is intuitionistic supra topology.	Every neutrosophic topology is neutrosophic supra topology.

Proposition 4.3. The collection $(\tau_n^*)^c$ of all neutrosophic supra closed sets in (X, τ_n^*) satisfies: $\emptyset, X \in (\tau_n^*)^c$ and $(\tau_n^*)^c$ is closed under arbitrary intersection.

Proof. : The proof is obvious. □

Lemma 4.4. As Proposition 3.4, every neutrosophic supra topology τ_n^* induce three fuzzy supra topologies $\tau_{f_1}^* = \{A = (x, \mu_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n^*\}, \tau_{f_2}^* = \{A = (x, \sigma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n^*\}$ and $\tau_{f_3}^* = \{A = (x, 1 - \gamma_A(x)) : (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \in \tau_n^*\}$ on X.

Definition 4.5. The neutrosophic supra topological interior $int_{\tau_n^*}(A)$ and closure $cl_{\tau_n^*}(A)$ operators of a neutrosophic set A are respectively defined as: $int_{\tau_n^*}(A) = \cup\{G : G \subseteq A$ and $G \in \tau_n^*\}$ and $cl_{\tau_n^*}(A) = \cap\{F : A \subseteq F$ and $F^c \in \tau_n^*\}$.

Theorem 4.6. The following are true for neutrosophic sets A and B of nsts (X, τ_n^*):

(i) $A = cl_{\tau_n^*}(A)$ if and only if A is neutrosophic supra closed.

(ii) $A = int_{\tau_n^*}(A)$ if and only if A is neutrosophic supra open.

(iii) $cl_{\tau_n^*}(A) \subseteq cl_{\tau_n^*}(B)$, if A$\subseteq B$.

(iv) $int_{\tau_n^*}(A) \subseteq int_{\tau_n^*}(B)$, if A$\subseteq B$.

(v) $cl_{\tau_n^*}(A) \cup cl_{\tau_n^*}(B) \subseteq cl_{\tau_n^*}(A \cup B)$.

(vi) $int_{\tau_n^*}(A) \cup int_{\tau_n^*}(B) \subseteq int_{\tau_n^*}(A \cup B)$.

(vii) $cl_{\tau_n^*}(A) \cap cl_{\tau_n^*}(B) \supseteq cl_{\tau_n^*}(A \cap B)$.

(viii) $int_{\tau_n^*}(A) \cap int_{\tau_n^*}(B) \supseteq int_{\tau_n^*}(A \cap B)$.

(ix) $int_{\tau_n^*}(A^c) = (cl_{\tau_n^*}(A))^c$.

Proof. : Here we shall prove parts (iii), (v) and (ix) only. The remaining parts similarly follows. Part (iii): $cl_{\tau_n^*}(B) = \cap\{G : G^c \in \tau_n^*,\ B \subseteq G\} \supseteq \cap\{G : G^c \in \tau_n^*,\ A \subseteq G\} = cl_{\tau_n^*}(A)$. Thus, $cl_{\tau_n^*}(A) \subseteq cl_{\tau_n^*}(B)$. Part (v): Since $A \cup B \supseteq A, B$, then $cl_{\tau_n^*}(A) \cup cl_{\tau_n^*}(B) \subseteq cl_{\tau_n^*}(A \cup B)$. Part (ix): $cl_{\tau_n^*}(A) = \cap\{G : G^c \in \tau_n^*,\ G \supseteq A\}, (cl_{\tau_n^*}(A))^c = \cup\{G^c : G^c$ is a neutrosophic supra open in X and $G^c \subseteq A^c\} = int_{\tau_n^*}(A^c)$. Thus, $(cl_{\tau_n^*}(A))^c = int_{\tau_n^*}(A^c)$. □

Remark 4.7. In neutrosophic topological space, we have $cl_{\tau_n}(A \cup B) = cl_{\tau_n}(A) \cup cl_{\tau_n}(B)$ and $int_{\tau_n}(A \cap B) = int_{\tau_n}(A) \cap int_{\tau_n}(B)$. These equalities are not true in neutrosophic supra topological spaces as shown in the following examples.

Example 4.8. Let $X = \{a, b, c\}$ with neutrosophic topology $\tau_n^* = \{\emptyset, X, ((0.5, 1, 0), (0.5, 1, 0), (0.5, 0, 1)), ((0.25, 0, 1), (0.25, 0, 1), (0.75, 1, 0)), ((0.5, 1, 1), (0.5, 1, 1), (0.5, 0, 0))\}$. Then $(\tau_n^*)^c = \{X, \emptyset, ((0.5, 0, 1), (0.5, 0, 1), (0.5, 1, 0)), ((0.75, 1, 0), (0.75, 1, 0), (0.25, 0, 1)), ((0.5, 0, 0), (0.5, 0, 0), (0.5, 1, 1))\}$. Let $C = ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1))$ and $D = ((0.5, 0, 0.5), (0.5, 0, 0.5), (0.5, 1, 0.5))$, then $cl_{\tau_n^*}(C) = ((0.75, 1, 0), (0.75, 1, 0), (0.25, 0, 1))$ and $cl_{\tau_n^*}(D) = ((0.5, 0, 1), (0.5, 0, 1), (0.5, 1, 0))$, so $cl_{\tau_n^*}(C) \cup cl_{\tau_n^*}(D) = ((0.75, 1, 1), (0.75, 1, 1), (0.25, 0, 0))$. But $C \cup D = ((0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5))$ and $cl_{\tau_n^*}(C \cup D) = ((1, 1, 1), (1, 1, 1), (0, 0, 0)) = X$. Therefore $cl_{\tau_n^*}(C \cup D) \neq cl_{\tau_n^*}(C) \cup cl_{\tau_n^*}(D)$.

Let $E = ((0.5, 1, 0.25), (0.5, 1, 0.25), (0.5, 0, 0.75))$ and $F = ((0.5, 0.5, 1), (0.5, 0.5, 1), (0.5, 0.5, 0))$. Then $int_{\tau_n^*}(E) = ((0.5, 1, 0), (0.5, 1, 0), (0.5, 0, 1))$ and $int_{\tau_n^*}(F) = ((0.25, 0, 1), (0.25, 0, 1), (0.75, 1, 0))$, so $int_{\tau_n^*}(E) \cap int_{\tau_n^*}(F) = ((0.25, 0, 0), (0.25, 0, 0), (0.75, 1, 1))$. But $E \cap F = ((0.5, 0.5, 0.25), (0.5, 0.5, 0.25), (0.5, 0.5, 0.75))$ and $int_{\tau_n^*}(E \cap F) = ((0, 0, 0), (0, 0, 0), (1, 1, 1)) = \emptyset$. Therefore $int_{\tau_n^*}(E \cap F) \neq int_{\tau_n^*}(E) \cap int_{\tau_n^*}(F)$.

5 Mappings of Neutrosophic Spaces

In this section, we define and establish the properties of some mappings in neutrosophic supra topological spaces and neutrosophic subspaces.

Definition 5.1. Let τ_n^* and σ_n^* be associated neutrosophic supra topologies with respect to τ_n and σ_n. A mapping f from a nts (X, τ_n) into nts (Y, σ_n) is said to be S^*-neutrosophic open if the image of every neutrosophic open set in (X, τ_n) is neutrosophic supra open in (Y, σ_n^*) and $f : X \to Y$ is said to be S^*-neutrosophic continuous if the inverse image of every neutrosophic open set in (Y, σ_n) is neutrosophic supra open in (X, τ_n^*).

Definition 5.2. Let τ_n^* and σ_n^* be associated neutrosophic supra topologies with respect to nts's τ_n and σ_n. A mapping f from a nts (X, τ_n) into a nts (Y, σ_n) is said to be supra neutrosophic open if the image of every neutrosophic supra open set in (X, τ_n^*) is a neutrosophic supra open in (Y, σ_n^*) and $f : X \to Y$ is said to be supra neutrosophic continuous if the inverse image of every neutrosophic supra open set in (Y, σ_n^*) is neutrosophic supra open in (X, τ_n^*).

A mapping f of nts (X, τ_n) into nts (Y, σ_n) is said to be a mapping of neutrosophic subspace $(A, (\tau_n)_A)$ into neutrosophic subspace $(B, (\sigma_n)_B)$ if $f(A) \subset B$.

Definition 5.3. A mapping f of neutrosophic subspace $(A, (\tau_n)_A)$ of nts (X, τ_n) into neutrosophic subspace $(B, (\sigma_n)_B)$ of nts (Y, σ_n) is said to be relatively neutrosophic continuous if $f^{-1}(O) \cap A \in (\tau_n)_A$ for every $O \in (\sigma_n)_B$. If $f(O') \in (\sigma_n)_B$ for every $O' \in (\tau_n)_A$, then f is said to be relatively neutrosophic open.

Theorem 5.4. If a mapping f is neutrosophic continuous from nts (X, τ_n) into nts (Y, σ_n) and $f(A) \subset B$. Then f is relatively neutrosophic continuous from neutrosophic subspace $(A, (\tau_n)_A)$ of nts (X, τ_n) into neutrosophic subspace $(B, (\sigma_n)_B)$ of nts (Y, σ_n).

Proof. : Let $O \in (\sigma_n)_B$, then there exists $G \in \sigma_n$ such that $O = B \cap G$ and $f^{-1}(G) \in \tau_n$. Therefore $f^{-1}(O) \cap A = f^{-1}(B) \cap f^{-1}(G) \cap A = f^{-1}(G) \cap A \in (\tau_n)_A$. □

Remark 5.5. (i) Every neutrosophic continuous (resp. neutrosophic open) mapping is S^*-neutrosophic continuous (resp. S^*-neutrosophic open), but converse need not be true.

(ii) Every supra neutrosophic continuous (resp. supra neutrosophic open) mapping is S^*-neutrosophic continuous (resp. S^*-neutrosophic open), but converse need not be true.

(iii) Supra neutrosophic continuous and neutrosophic continuous mappings are independent each other.

(iv) Supra neutrosophic open and neutrosophic open mappings are independent each other.

Proof. : The proof follows from the definition, the converse and independence are shown in the following example. □

Example 5.6. Let $Y = \{x, y, z\}$, $X = \{a, b, c\}$ with neutrosophic topologies $\tau_n = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1))\}$ and $\sigma_n = \{\emptyset, Y, ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$. Let $\tau_n^* = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)), ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$ and $\sigma_n^* = \sigma_n$ be associated neutrosophic supra topologies with respect to τ_n and σ_n. Define a mapping $f : X \to Y$ by $f(c) = z, f(b) = y, f(a) = x$, then $f^{-1}(((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))) = ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5)) \in \tau_n^*$. Clearly f is supra neutrosophic continuous and S^*-neutrosophic continuous but not neutrosophic continuous.

Let $Y = \{x, y, z\}$, $X = \{a, b, c\}$ with neutrosophic topologies $\tau_n = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1))\}$ and $\sigma_n = \{\emptyset, Y, ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1))\}$. Let $\tau_n^* = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1)), ((1, 0.5, 0), (1, 0.5, 0), (0, 0.5, 1))\}$ and $\sigma_n^* = \{\emptyset, Y, ((0.5, 0.25, 0), (0.5, 0.25, 0), (0.5, 0.75, 1)), ((0.3, 0.25, 0.5), (0.3, 0.25, 0.5), (0.7, 0.75, 0.5)), ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$ be associated neutrosophic supra topologies with respect to τ_n and σ_n. Consider a mapping $f : X \to Y$ by $f(c) = z, f(b) = y, f(a) = x$, then $f^{-1}(((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))) = ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5)) \notin \tau_n^*$. Therefore f is neutrosophic continuous and S^*-neutrosophic continuous but not supra neutrosophic continuous. If we consider a mapping $g : Y \to X$ by $g(z) = c, g(y) = b, g(x) = a$, then g is neutrosophic open and S^*-neutrosophic open but not supra neutrosophic open.

Let $Y = \{x, y, z\}$, $X = \{a, b, c\}$ with neutrosophic topologies $\tau_n = \{\emptyset, X, ((1, 0.5, 0.3), (1, 0.5, 0.3), (0, 0.5, 0.7))\}$ and $\sigma_n = \{\emptyset, Y, ((1, 0.3, 0.5), (1, 0.3, 0.5), (0, 0.7, 0.5))\}$. Let $\sigma_n^* = \{\emptyset, Y, ((1, 0.3, 0.5), (1, 0.3, 0.5), (0, 0.7, 0.5)), ((1, 0.5, 0.3), (1, 0.5, 0.3), (0, 0.5, 0.7)), ((1, 0.5, 0.5), (1, 0.5, 0.5), (0, 0.5, 0.5))\}$ and $\tau_n^* = \tau_n$ be associated neutrosophic supra topologies with respect to σ_n and τ_n. Then $f : X \to Y$ defined by $f(c) = z, f(b) = y, f(a) = x$ is S^*-neutrosophic open and supra neutrosophic open but not neutrosophic open.

Observation 5.7. The following are the examples of contradicting the statements of Abd-Monsef and Ramadan[9]. In fuzzy supra topological space, consider $Y = \{x, y, z\}$, $X = \{a, b, c\}$ with fuzzy topologies $\tau_f = \{\emptyset, X, (0.5, 0.5, 0), (0.5, 0.25, 0)\}$ and $\sigma_f = \{\emptyset, Y, (0.5, 0.25, 0)\}$. Let $\tau_f^* = \{\emptyset, X, (0.5, 0.5, 0), (0.5, 0.25, 0), (1, 0.5, 0)\}$ and $\sigma_f^* = \{\emptyset, Y, (0.5, 0.25, 0), (0.3, 0.25, 0.5), (0.5, 0.25, 0.5)\}$ be associated fuzzy supra topologies with respect to τ_f and σ_f. Consider a mapping $h : X \to Y$ by $h(c) = z, h(b) = y, h(a) = x$, then $h^{-1}((0.5, 0.25, 0.5)) = (0.5, 0.25, 0.5) \notin \tau_f^*$. Then h is fuzzy continuous but not supra fuzzy continuous. If we define a mapping $g : Y \to X$ by $g(z) = c, g(y) = b, g(x) = a$, then g is fuzzy open but not supra fuzzy open.

Theorem 5.8. The following statements are equivalent for the mapping f of nts (X, τ_n) into nts (Y, σ_n):

(i) The mapping $f : X \to Y$ is S^*-neutrosophic continuous.

(ii) The inverse image of every neutrosophic closed set in (Y, σ_n) is neutrosophic supra closed in (X, τ_n^*).

(iii) For each neutrosophic set A in Y, $cl_{\tau_n^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{\sigma_n}(A))$.

(iv) For each neutrosophic set B in X, $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n}(f(B))$.

(v) For each neutrosophic set A in Y, $int_{\tau_n^*}(f^{-1}(A)) \supseteq f^{-1}(int_{\sigma_n}(A))$.

Proof. : $(i) \Rightarrow (ii)$: Let f be a S^*-neutrosophic continuous and A be a neutrosophic closed set in (Y, σ_n), $f^{-1}(Y - A) = X - f^{-1}(A)$ is neutrosophic supra open in (X, τ_n^*) and so $f^{-1}(A)$ is neutrosophic supra closed in (X, τ_n^*).
$(ii) \Rightarrow (iii)$: $cl_{\sigma_n}(A)$ is neutrosophic closed in (Y, σ_n), for each neutrosophic set A in Y, then $f^{-1}(cl_{\sigma_n}(A))$ is neutrosophic supra closed in (X, τ_n^*). Thus $f^{-1}(cl_{\sigma_n}(A)) = cl_{\tau_n^*}(f^{-1}(cl_{\sigma_n}(A))) \supseteq cl_{\tau_n^*}(f^{-1}(A))$.
$(iii) \Rightarrow (iv)$: $f^{-1}(cl_{\sigma_n}(f(B))) \supseteq cl_{\tau_n^*}(f^{-1}(f(B))) \supseteq cl_{\tau_n^*}(B)$, for each neutrosophic set B in X and so $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n}(f(B))$.
$(iv) \Rightarrow (ii)$: Let $B = f^{-1}(A)$, for each neutrosophic closed set A in Y, then $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n}(f(B)) \subseteq cl_{\sigma_n}(A) = A$ and $cl_{\tau_n^*}(B) \subseteq f^{-1}(f(cl_{\tau_n^*}(B))) \subseteq f^{-1}(A) = B$. Therefore $B = f^{-1}(A)$ is neutrosophic supra closed in X.
$(ii) \Rightarrow (i)$: Let A be a neutrosophic open set in Y, then $X - f^{-1}(A) = f^{-1}(Y - A)$ is neutrosophic supra closed in X, since $Y - A$ is neutrosophic closed in Y. Therefore $f^{-1}(A)$ is neutrosophic supra open in X.
$(i) \Rightarrow (v)$: $f^{-1}(int_{\sigma_n}(A))$ is neutrosophic supra open in X, for each neutrosophic set A in Y and $int_{\tau_n^*}(f^{-1}(A)) \supseteq int_{\tau_n^*}(f^{-1}(int_{\sigma_n}(A))) = f^{-1}(int_{\sigma_n}(A))$.
$(v) \Rightarrow (i)$: $f^{-1}(A) = f^{-1}(int_{\sigma_n}(A)) \subseteq int_{\tau_n^*}(f^{-1}(A))$, for each neutrosophic open set A in Y and so $f^{-1}(A)$ is neutrosophic supra open in X. □

Theorem 5.9. The following statements are equivalent for the mapping f of nts (X, τ_n) into nts (Y, σ_n):

(i) A mapping $f : (X, \tau_n^*) \to (Y, \sigma_n^*)$ is neutrosophic supra continuous.

(ii) The inverse image of every neutrosophic supra closed set in (Y, σ_n^*) is neutrosophic supra closed in (X, τ_n^*).

(iii) For each neutrosophic set A in Y, $cl_{\tau_n^*}(f^{-1}(A)) \subseteq f^{-1}(cl_{\sigma_n^*}(A)) \subseteq f^{-1}(cl_{\sigma_n}(A))$.

(iv) For each neutrosophic set B in X, $f(cl_{\tau_n^*}(B)) \subseteq cl_{\sigma_n^*}(f(B)) \subseteq cl_{\sigma_n}(f(B))$.

(v) For each neutrosophic set A in Y, $int_{\tau_n^*}(f^{-1}(A)) \supseteq f^{-1}(int_{\sigma_n^*}(A)) \supseteq f^{-1}(int_{\sigma_n}(A))$.

Proof. : The proof is straightforward from theorem 5.8. □

Theorem 5.10. If $f : X \to Y$ is S^*-neutrosophic continuous and $g : Y \to Z$ is neutrosophic continuous, then $g \circ f : X \to Z$ is S^*-neutrosophic continuous.

Proof. : The proof follows directly from the definition. □

Theorem 5.11. If $f : X \to Y$ is supra neutrosophic continuous and $g : Y \to Z$ is S^*-neutrosophic continuous (or neutrosophic continuous), then $g \circ f : X \to Z$ is S^*-neutrosophic continuous.

Proof. : It follows from the definition. □

Theorem 5.12. If $f : X \to Y$ and $g : Y \to Z$ are supra neutrosophic continuous (resp. supra neutrosopic open) mappings, then $g \circ f : X \to Z$ is supra neutrosophic continuous (resp. supra neutrosophic open).

Proof. : The proof follows obviously from the definition. □

Remark 5.13. Abd-Monsef and Ramadan[9] stated that if $g : X \to Y$ is supra fuzzy continuous and $h : Y \to Z$ is fuzzy continuous, then $h \circ g : X \to Z$ is supra fuzzy continuous. But in general this is not true, for example consider $Z = \{p, q, r\}$, $Y = \{x, y, z\}$, and $X = \{a, b, c\}$ with fuzzy topologies $\tau_f = \{\emptyset, X, (1, 0.5, 0), (0.3, 0.3, 0)\}$, $\sigma_f = \{\emptyset, Y, (0.5, 0.5, 0), (0.5, 0.25, 0)\}$ and $\eta_f = \{\emptyset, Z, (0.5, 0.25, 0)\}$ on X, Y and Z respectively. Let $\tau_f^* = \{\emptyset, X, (0.5, 0.5, 0), (0.5, 0.25, 0), (1, 0.5, 0), (0.3, 0.3, 0)\}$, $\sigma_f^* = \{\emptyset, Y, (0.5, 0.5, 0), (0.5, 0.25, 0), (1, 0.5, 0)\}$ and $\eta_f^* = \{\emptyset, Z, (0.5, 0.25, 0), (0.3, 0.25, 0.5), (0.5, 0.25, 0.5)\}$ be associated fuzzy supra topologies with respect to τ_f, σ_f and η_f. Then the mapping $g : X \to Y$ defined by $g(c) = z, g(b) = y, g(a) = x$ is supra fuzzy continuous and the mapping $h : Y \to Z$ by $h(z) = r, h(y) = q, h(x) = p$ is fuzzy continuous. But $h \circ g : X \to Z$ is not supra fuzzy continuous, since $(g \circ h)^{-1}((0.3, 0.25, 0.5)) = (0.3, 0.25, 0.5) \notin \tau_f^*$.

Remark 5.14. In general the composition of two supra neutrosophic continuous mappings is again supra neutrosophic continuous, but the composition of two S^*-neutrosopic continuous mappings may not be S^*-neutrosophic continuous. Let $Z = \{p, q, r\}$, $Y = \{x, y, z\}$, and $X = \{a, b, c\}$ with neutrosophic topologies $\tau_n = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1))\}$, $\sigma_n = \{\emptyset, Y, ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$ and $\eta_n = \{\emptyset, Z, ((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5))\}$ on X, Y and Z respectively. Let $\tau_n^* = \{\emptyset, X, ((0.5, 0.5, 0), (0.5, 0.5, 0), (0.5, 0.5, 1)), ((0.5, 0.5, 0.5), (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)), ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5))\}$ and $\sigma_n^* = \{\emptyset, Y, ((0.5, 0.25, 0.5), (0.5, 0.25, 0.5), (0.5, 0.75, 0.5)), ((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5)), ((0.5, 0.7, 0.5), (0.5, 0.7, 0.5), (0.5, 0.3, 0.5))\}$ be associated neutrosophic supra topologies with respect to τ_n and σ_n. Then the mappings $f : X \to Y$ and $g : Y \to Z$ are defined respectively by $f(c) = z, f(b) = y, f(a) = x$ and $g(z) = r, g(y) = q, g(x) = p$ are S^*-neutrosophic continuous. But $g \circ f : X \to Z$ is not S^*-neutrosophic continuous, since $(g \circ f)^{-1}(((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5))) = ((0.3, 0.7, 0.5), (0.3, 0.7, 0.5), (0.7, 0.3, 0.5)) \notin \tau_n^*$.

Theorem 5.15. If mappings $f : (A, (\tau_n)_A) \to (B, (\sigma_n)_B)$ from neutrosophic subspace $(A, (\tau_n)_A)$ of nts (X, τ_n) into neutrosophic subspace $(B, (\sigma_n)_B)$ of nts (Y, σ_n) and $g : (B, (\sigma_n)_B) \to (C, (\eta_n)_C)$ from neutrosophic subspace $(B, (\sigma_n)_B)$ of nts (Y, σ_n) into neutrosophic subspace $(C, (\eta_n)_C)$ of nts (Z, η_n) are relatively neutrosophic continuous (resp. relatively neutrosopic open) mappings, then $g \circ f : (A, (\tau_n)_A) \to (C, (\eta_n)_C)$ is relatively neutrosophic continuous (resp. relatively neutrosophic open) from neutrosophic subspace $(A, (\tau_n)_A)$ of nts (X, τ_n) into neutrosophic subspace $(C, (\eta_n)_C)$ of nts (Z, η_n).

Proof. : Let $O \in (\eta_n)_C$, then $g^{-1}(O) \cap B \in (\sigma_n)_B$ and $f^{-1}(g^{-1}(O) \cap B) \cap A \in (\tau_n)_A$. Since $B \supset f(A)$, then $(g \circ f)^{-1}(O) \cap A = f^{-1}(g^{-1}(O) \cap B) \cap A$. Therefore $g \circ f$ is relatively neutrosophic continuous. Let $U \in (\tau_n)_A$, then $f(U) \in (\sigma_n)_B$ and $g(f(U)) = (g \circ f)(U) \in (\eta_n)_C$. Therefore $g \circ f$ is relatively neutrosophic open. □

6 Neutrosophic Supra Topology in Data Mining

In this section, we present a methodical approach for decision-making problem with single valued neutrosophic information. The following necessary steps are proposed the methodical approach to select the proper attributes and alternatives in the decision-making situation.

Step 1: Problem field selection:

Consider multi-attribute decision making problems with m attributes $A_1, A_2, ..., A_m$ and n alternatives $C_1, C_2, ..., C_n$ and p attributes $D_1, D_2, ..., D_p$, $(n \leq p)$.

	C_1	C_2	.	.	.	C_n
A_1	(a_{11})	(a_{12})	.	.	.	(a_{1n})
A_2	(a_{21})	(a_{22})	.	.	.	(a_{2n})
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.						
A_m	(a_{m1})	(a_{m2})	.	.	.	(a_{mn})

	A_1	A_2	.	.	.	A_m
D_1	(d_{11})	(d_{12})	.	.	.	(d_{1m})
D_2	(d_{21})	(d_{22})	.	.	.	(d_{2m})
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
D_p	(d_{p1})	(d_{p2})	.	.	.	(d_{pm})

Here all the attributes a_{ij} and d_{ki} (i = 1, 2, …, m, j = 1, 2, …, n and k = 1, 2, …, p) are single valued neutrosophic numbers.

Step 2: Form neutrosophic supra topologies for (C_j) **and** (D_k):

(i) $\tau_j^* = A \cup B$, where $A = \{1_N, 0_N, a_{1j}, a_{2j}, ..., a_{mj}\}$ and $B = \{a_{1j} \cup a_{2j}, a_{1j} \cup a_{3j}, ..., a_{m-1j} \cup a_{mj}\}$.

(ii) $\nu_k^* = C \cup D$, where $C = \{1_N, 0_N, d_{k1}, d_{k2}, ..., d_{km}\}$ and $D = \{d_{k1} \cup d_{k2}, d_{k1} \cup d_{k3}, ..., d_{km-1} \cup d_{km}\}$.

Step 3: Find Single valued neutrosophic score functions:

Single valued neutrosophic score functions (shortly SVNSF) of A, B, C, D, C_j and D_k are defined as follows.

(i) SVNSF(A) $= \frac{1}{3(m+2)}[\sum_{i=1}^{m+2}[2 + \mu_i - \sigma_i - \gamma_i]]$, and SVNSF(B) $= \frac{1}{3q}[\sum_{i=1}^{q}[2 + \mu_i - \sigma_i - \gamma_i]]$, where q is the number of elements of B. For $j = 1, 2, ..., n$,

$$\text{SVNSF}(C_j) = \begin{cases} SVNSF(A) & if SVNSF(B) = 0 \\ \frac{1}{2}[SVNSF(A) + SVNSF(B)] & otherwise \end{cases}.$$

(ii) SVNSF(C) $= \frac{1}{3(m+2)}[\sum_{i=1}^{m+2}[2 + \mu_i - \sigma_i - \gamma_i]]$ and SVNSF(D) $= \frac{1}{3r}[\sum_{i=1}^{r}[2 + \mu_i - \sigma_i - \gamma_i]]$, where r is the number of elements of D. For $k = 1, 2, ..., p$,

$$\text{SVNSF}(D_k) = \begin{cases} SVNSF(C) & if SVNSF(D) = 0 \\ \frac{1}{2}[SVNSF(C) + SVNSF(D)] & otherwise \end{cases}.$$

Step 4: Final Decision

Arrange single valued neutrosophic score values for the alternatives $C_1 \le C_2 \le ... \le C_n$ and the attributes $D_1 \le D_2 \le ... \le D_p$. Choose the attribute D_p for the alternative C_1 and D_{p-1} for the alternative C_2 etc. If $n<p$, then ignore D_k, where $k = 1, 2, ..., n - p$.

7 Numerical Example

Medical diagnosis has increased volume of information available to physicians from new medical technologies and comprises of uncertainties. In medical diagnosis, very difficult task is the process of classifying different set of symptoms under a single name of a disease. In this section, we exemplify a medical diagnosis problem for effectiveness and applicability of above proposed approach.

Step 1: Problem field selection:

Consider the following tables giving informations when consulted physicians about four patients P_1, P_2, P_3, P_4 and symptoms are Temperature, Cough, Blood Plates, Joint Pain, Insulin. We need to find the patient and to find the disease such as Tuberculosis, Diabetes, Chikungunya, Swine Flu, Dengue of the patient. The data in Table 1 are explained by the membership, the indeterminacy and the non-membership functions. From Table 2, we can observe that for tuberculosis, cough is high ($\mu = 0.9, \sigma = 0.1, \gamma = 0.1$), but for chikungunya, cough is low ($\mu = 0, \sigma = 0.1, \gamma = 0.9$).

Table 1.

Symptoms \ Patients	P1	P2	P3	P4
Temperature	(0.8,0,0.2)	(0.1,0,0.7)	(0.9,0.1,0)	(0,0.1,0.9)
Cough	(0.1,0.2,0.7)	(0.1,0.1,0.8)	(0,0.3,0.7)	(0.8,0.1,0.2)
Blood Plates	(0.8,0,0.2)	(0.2,0.1,0.6)	(0.3,0.1,0.6)	(0.3,0.1,0.6)
Joint Pain	(0.4,0.2,0.5)	(0.4,0.2,0.5)	(0.9,0,0.1)	(0.2,0.2,0.7)
Insulin	(0.3,0.2,0.5)	(0.9,0,0.1)	(0.2,0.1,0.7)	(0.4,0.3,0.2)

Table 2.

Diagnosis \ Symptoms	Temperature	Cough	Blood Plates	Joint Pain	Insulin
Tuberculosis	(0.6,0.3,0.1)	(0.9,0.1,0.1)	(0,0.2,0.8)	(0,0.1,0.8)	(0,0.1,0.9)
Diabetes	(0.1,0.1,0.8)	(0.1,0.1,0.8)	(0.2,0.2,0.1)	(0.1,0.4,0.6)	(0.9,0,0.1)
Chikungunya	(0.9,0,0.1)	(0,0.1,0.9)	(0.7,0.2,0.1)	(0.9,0.1,0.1)	(0.2,0,0.8)
Swine Flu	(0.2,0.5,0.3)	(0.1,0.4,0.3)	(0.2,0.4,0.1)	(0.1,0.3,0.5)	(0.2,0.4,0.1)
Dengue	(0.9,0,0.1)	(0.2,0.6,0.4)	(0.2,0.6,0.4)	(0.3,0.1,0.6)	(0.2,0.1,0.7)

Step 2: Form neutrosophic supra topologies for (C_j) and (D_k):

(i) $\tau_1^* = A \cup B$, where $A = \{(1,1,0),(0,0,1),(0.8,0,0.2),(0.1,0.2,0.7),(0.4,0.2,0.5),(0.3,0.2,0.5)\}$ and $B = \{(0.8,0.2,0.2)\}$.

(ii) $\tau_2^* = A \cup B$, where $A = \{(1,1,0),(0,0,1),(0.1,0,0.7),(0.1,0.1,0.8),(0.2,0.1,0.6),(0.4,0.2,0.5),(0.9,0,0.1)\}$ and $B = \{(0.1,0.1,0.7),(0.9,0.1,0.1),(0.9,0.2,0.1)\}$.

(iii) $\tau_3^* = A \cup B$, where $A = \{(1,1,0),(0,0,1),(0.9,0.1,0),(0,0.3,0.7),(0.3,0.1,0.6),(0.9,0,0.1),(0.2,0.1,0.7)\}$ and $B = \{(0.9,0.3,0),(0.3,0.3,0.6),(0.9,0.3,0.1),(0.2,0.3,0.7),(0.9,0.1,0.1)\}$.

(iv) $\tau_4^* = A \cup B$, where $A = \{(1,1,0),(0,0,1),(0,0.1,0.9),(0.8,0.1,0.2),(0.3,0.1,0.6),(0.2,0.2,0.7),(0.4,0.3,0.2)\}$ and $B = \{(0.8,0.2,0.2),(0.8,0.3,0.2),(0.3,0.2,0.6)\}$.

(i) $\nu_1^* = C \cup D$, where $C = \{(1,1,0),(0,0,1),(0.6,0.3,0.1),(0.9,0.1,0.1),(0,0.2,0.8),(0,0.1,0.8),(0,0.1,0.9)\}$ and $D = \{(0.9,0.3,0.1),(0.9,0.2,0.1)\}$.

(ii) $\nu_2^* = C \cup D$, where $C = \{(1,1,0),(0,0,1),(0.1,0.1,0.8),(0.2,0.2,0.1),(0.1,0.4,0.6),(0.9,0,0.1)\}$ and $D = \{(0.9,0.1,0.1),(0.2,0.4,0.1),(0.9,0.2,0.1),(0.9,0.4,0.1)\}$.

(iii) $\nu_3^* = C \cup D$, where $C = \{(1,1,0),(0,0,1),(0.9,0,0.1),(0,0.1,0.9),(0.7,0.2,0.1),(0.9,0.1,0.1),(0.2,0,0.8)\}$ and $D = \{(0.9,0.2,0.1),(0.2,0.1,0.8)\}$.

(iv) $\nu_4^* = C \cup D$, where $C = \{(1,1,0),(0,0,1),(0.2,0.5,0.3),(0.1,0.4,0.3),(0.2,0.4,0.3),(0.2,0.4,0.1),(0.1,0.3,0.5)\}$ and $D = \{(0.2,0.5,0.1)\}$.

(v) $\nu_k^* = C \cup D$, where $C = \{(1,1,0),(0,0,1),(0.9,0,0.1),(0.2,0.6,0.4),(0.3,0.1,0.6),(0.2,0.1,0.7)\}$ and $D = \{(0.9,0.6,0.1),(0.9,0.1,0.1),(0.3,0.6,0.4)\}$.

Step 3: Find Single valued neutrosophic score functions:

(i) SVNSF(A) $= 0.5611$ and SVNSF(B) $= 0.8$, where $q = 1$. SVNSF(C_1) $= 0.6801$.

(ii) SVNSF(A) $= 0.5524$ and SVNSF(B) $= 0.7333$, where $q = 3$. SVNSF(C_2) $= 0.6428$.

(iii) SVNSF(A) $= 0.6$ and SVNSF(B) $= 0.6933$, where $q = 5$. SVNSF(C_3) $= 0.6466$.

(iv) SVNSF(A) $= 0.5381$ and SVNSF(B) $= 0.6888$, where $q = 3$. SVNSF(C_4) $= 0.6135$.

(i) SVNSF(C) $= 0.5238$ and SVNSF(D) $= 0.85$, where $r = 2$. SVNSF(D_1) $= 0.6869$.

(ii) SVNSF(C) $= 0.5555$ and SVNSF(D) $= 0.7833$, where $r = 4$. SVNSF(D_2) $= 0.6694$.

(iii) SVNSF(C) $= 0.6333$ and SVNSF(B) $= 0.65$, where $r = 2$. SVNSF(D_3) $= 0.6416$.

(iv) SVNSF(C) $= 0.4888$ and SVNSF(B) $= 0.5333$, where $r = 1$. SVNSF(D_4) $= 0.5111$.

(v) SVNSF(C) $= 0.5555$ and SVNSF(B) $= 0.6888$, where $r = 3$. SVNSF(D_5) $= 0.6222$.

Step 4: Final Decision:

Arrange single valued neutrosophic score values for the alternatives C_1, C_2, C_3, C_4 and the attributes D_1, D_2, D_3, D_4, D_5 in acending order. We get the following sequences $C_4 \leq C_2 \leq C_3 \leq C_1$ and $D_4 \leq D_5 \leq D_3 \leq D_2 \leq D_1$. Thus the patient P_4 suffers from tuberculosis, the patient P_2 suffers from diabetes, the patient P_3 suffers from chikungunya and the patient P_1 suffers from dengue.

8 Conclusion and Future Work

Neutrosophic topological space is one of the research areas in general fuzzy topological spaces to deal the concept of vagueness. This paper introduced neutrosophic supra topological spaces and its real life application. Moreover we have discussed some mappings in neutrosophic supra topological spaces and derived some contradicting examples in fuzzy supra topological spaces. This theory can be develop and implement to other research areas of general topology such as rough topology, digital topology and so on.

References

[1] L. A. Zadeh. Probability measures of fuzzy events, Journal of Mathematical Analysis and Applications, 23 (1968), 421–427.

[2] K. P. Adlassnig. Fuzzy set theory in medical diagnosis, IEEE Transactions on Systems, Man, and Cybernetics, 16 (2) (1986), 260–265.

[3] M. Sugeno. An Introductory survey of fuzzy control, Information sciences, 36(1985), 59–83.

[4] P. R. Innocent and R. I. John. Computer aided fuzzy medical diagnosis, Information Sciences, 162(2004), 81–104.

[5] T.J. Roos. Fuzzy Logic with Engineering Applications, McGraw Hill P.C., New York, 1994.

[6] C.L.Chang. Fuzzy topological spaces, J. Math. Anal. and Appl., 24 (1968), 182–190.

[7] R.Lowen. Fuzzy topological spaces and fuzzy compactness, J. Math. Anal. Appl., 56 (1976), 621–633.

[8] A.S. Mashhour, A.A.Allam, F.S.Mohmoud and F.H.Khedr. On supra topological spaces, Indian J.Pure and Appl.Math., 14(4)(1983), 502–510.

[9] M.E. Abd El-monsef and A.E.Ramadan. On fuzzy supra topological spaces, Indian J. Pure and Appl.Math., 18(4) (1987), 322–329.

[10] K. Atanassov. Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87–96.

[11] Dogan Coker. An introduction to intuitionistic Fuzzy topological spaces, Fuzzy sets and system, 88(1)(1997), 81–89.

[12] Reza Saadati and Jin HanPark. On the intuitionistic fuzzy topological space, Chaos, Solitons and Fractals, 27(2) (2006), 331–344.

[13] S.K. De, A. Biswas and R. Roy. An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Sets and System, 117(2) (2001), 209–213.

[14] E. Szmidt and J. Kacprzyk. Intuitionistic fuzzy sets in some medical applications, In International Conference on Computational Intelligence, Springer, Berlin, Heidelberg; (2001), 148–151.

[15] P. Biswas, S. Pramanik and B. C. Giri. A study on information technology professionals' health problem based on intuitionistic fuzzy cosine similarity measure, Swiss Journal of Statistical and Applied Mathematics, 2(1)(2014), 44–50.

[16] V. Khatibi and G. A. Montazer. Intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition, Artificial Intelligence in Medicine, 47(1) (2009), 43–52.

[17] K. C. Hung and H. W. Tuan. Medical diagnosis based on intuitionistic fuzzy sets revisited, Journal of Interdisciplinary Mathematics, 16(6)(2013), 385–395.

[18] F. Smarandache. Neutrosophic set, a generialization of the intuituionistics fuzzy sets, Int. J. Pure. Appl. Math., 24 (2005), 287–297.

[19] L.K. Hyung, Y.S. Song and K.M. Lee. Similarity measure between fuzzy sets and between elements, Fuzzy Sets and Systems, 62(1994), 291–293.

[20] S.M. Chen, S.M. Yeh and P.H. Hsiao. A comparison of similarity measures of fuzzy values, Fuzzy Sets and Systems, 72(1) (1995),79–89.

[21] W.J. Wang. New similarity measures on fuzzy sets and elements, Fuzzy Sets and Systems, 85(3) (1997), 305–309.

[22] F. Smarandache. A unifying field of logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, (1998).

[23] H. Wang, F. Smarandache, Y. Zhang and R. Sunderraman. Single valued neutrosophic sets, Multi-space and Multi-structure, 4 (2010), 410–413.

[24] P. Majumdar and S.K. Samanta. On similarity and entropy of neutrosophic sets, Journal of Intelligent and Fuzzy Systems, 26(3) (2014), 1245–1252.

[25] F. Smarandache and S. Pramanik. New trends in neutrosophic theory and applications, Brussels, Belgium, EU: Pons Editions (2016).

[26] J. Ye and Q. Zhang. Single valued neutrosophic similarity measures for multiple attribute decision-making, Neutrosophic Sets and Systems, 2 (2014), 48–54.

[27] J. Ye. Neutrosophic tangent similarity measure and its application to multiple attribute decision making, Neutrosophic Sets and Systems, 9 (2015), 85–92.

[28] J. Ye and S. Ye. Medical diagnosis using distance-based similarity measures of single valued neutrosophic multisets, Neutrosophic Sets and Systems, 7 (2015), 47–54.

[29] J. Mohan, V. Krishnaveni and Y.Guo. A new neutrosophic approach of Wiener filtering for MRI denoising, Measurement Science Review, 13(4) (2013), 177–186.

[30] N. D. Thanh and M. Ali. Neutrosophic recommender system for medical diagnosis based on algebraic similarity measure and clustering, In Fuzzy Systems (FUZZ-IEEE), IEEE International Conference, (2017), 1–6.

[31] P. Biswas, S. Pramanik and B. C. Giri. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers, Neutrosophic Sets and Systems, 8 (2015),47–57.

[32] S. Broumi and F. Smarandache. Several similarity measures of neutrosophic sets, Neutrosophic Sets and Systems, 1 (2013), 54–62.

[33] S. Pramanik and K. Mondal. Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis, Journal of New Theory, 4(2015), 90–102.

[34] N.A. Nabeeh, F. Smarandache, M. Abdel-Basset, H.A. El-Ghareeb and A. Aboelfetouh. An Integrated Neutrosophic-TOPSIS Approach and Its Application to Personnel Selection: A New Trend in Brain Processing and Analysis, IEEE Access, 7(2019), 29734–29744.

[35] M. Abdel-Basset, M. Saleh, A. Gamal and F. Smarandache. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, Applied Soft Computing, 77(2019), 438–452.

[36] M. Abdel-Baset, V. Chang, A. Gamal and F. Smarandache. (2019). An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field, Computers in Industry, 106(2019), 94–110.

[37] M. Abdel-Basset, G. Manogaran, A. Gamal and F. Smarandache. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection, Journal of medical systems, 43(2)(2019), 1–13.

[38] M. Abdel-Baset, V. Chang and A. Gamal. Evaluation of the green supply chain management practices: A novel neutrosophic approach, Computers in Industry, 108(2019), 210–220.

[39] M. Abdel-Basset, G. Manogaran, A. Gamal and F. Smarandache. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria, Design Automation for Embedded Systems, (2018), 1–22.

[40] S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu. Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, (2016) , 412–416.

[41] S. Broumi, M. Talea, A. Bakali and F. Smarandache. Application of Dijkstra algorithm for solving interval valued neutrosophic shortest path problem, IEEE Symposium Series on Computational Intelligence (SSCI), (2016), 1 –6.

[42] S. Broumi, M. Talea, F. Smarandache and A. Bakali. Single Valued Neutrosophic Graphs: Degree, Order and Size, IEEE International Conference on Fuzzy Systems (FUZZ), (2016), 2444-2451.

[43] F. Karaaslan. Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making, Neutrosophic Sets and Systems, 22 (2018), 101–117.

[44] B. C. Giri, M. U. Molla and P. Biswas. TOPSIS Method for MADM based on Interval Trapezoidal Neutrosophic Number, Neutrosophic Sets and Systems, 22 (2018), 151-167.

[45] S. I. A. Aal, M. M. A. A. Ellatif and M. M. Hassan. Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality, Neutrosophic Sets and Systems, 19 (2018), 132-141.

[46] A. A. Salama and S. A. Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Mathematics, 3(4)(2012), 31–35.

[47] A. A. Salama, F. Smarandache and Valeri Kroumov. Neutrosophic Crisp Sets and Neutrosophic Crisp Topological Spaces, Neutrosophic Sets and Systems, 2 (2014), 25–30.

[48] R. Dhavaseelan, E. Roja and M. K. Uma. Generalized intuitionistic fuzzy closed sets, Advances in Fuzzy Mathematics, 5(2) (2010), 152–172.

Received: Juanary 27 2019. Accepted: March 28, 2019