On Nβ*-Closed sets in Neutrosophic Topological spaces

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Abstract: The aim of this paper is to introduce the concept of β*-closed sets in terms of neutrosophic topological spaces. We also study some of the properties of neutrosophic β*-closed sets. Further we introduce Nβ*-continuity and Nβ*-contra continuity in neutrosophic topological spaces.

Keywords: neutrosophic topology, Nβ*-closed set, Nβ*-Continuity and Nβ*-Contra Continuity.

1. Introduction

Zadeh [19] introduced and studied the fuzzy set theory. An intuitionistic fuzzy set was introduced by Atanassov [9]. Coker [10] developed intuitionistic fuzzy topology. Neutrality, the degree of indeterminacy, as an independent concept, was introduced by Smarandache [3,4] in 1998. He also defined the neutrosophic set on three components (t, f, i) = (truth, falsehood, indeterminacy). The Neutrosophic crisp set concept was converted into neutrosophic topological spaces by Salama etal. in [3]. This opened up a wide range of investigation in terms of neutosophic topology and its application in decision-making algorithms. Renu Thomas etal.[17] introduced and studied semi pre-open(or β-open) sets in neutrosophic topological spaces. R. Dhavaseelan and S. Jafari[11] introduced generalized neutrosophic closed sets. In this article, the neutrosophic β*-closed sets are introduced in neutrosophic topological space. Moreover, we introduce and investigate neutrosophic β*-continuous and neutrosophic contra β*-continuous mappings.

2. Preliminaries

Definition 2.1. [6] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x), \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ can be identified as an ordered triple $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ in $]^-0, 1^+[$ on X.

Definition 2.2. [6] Let $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ be a NS on X, then the complement C(A) may be defined as

1. $C(A) = \{\langle x, 1 - \mu_A(x), 1 - \nu_A(x)\rangle : x \in X\}$

2. $C(A) = \{\langle x, \nu_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

3. $C(A) = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Note that for any two neutrosophic sets A and B,

4. $C(A \cup B) = C(A) \cap C(B)$

5. $C(A \cap B) = C(A) \cup C(B)$.

Definition 2.3. [6] For any two neutrosophic sets $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle : x \in X\}$ we may have

1. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x) \forall\, x \in X$

2. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x) \forall\, x \in X$

3. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$

4. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)\rangle$

5. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)\rangle$

6. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)\rangle$

Definition 2.4. [6] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:
(NT1) $0_N, 1_N \in \tau$
(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(NT3) $\cup G_i \in \tau\ \forall \{G_i : i \in J\} \subseteq \tau$

Definition 2.5. [5] Let A be an Neutrosophic Set in NTS X. Then Nint(A) = ∪ {G : G is an NOS in X and G ⊆ A} is called a neutrosophic interior of A Ncl(A) = ∩ {K : K is an NCS in X and A ⊆ K} is called a neutrosophic closure of A.

Definition 2.6. A NS A of a NTS X is said to be
(1) a neutrosophic semi-open set (NSOS)[15] if A ⊆ NCl(NInt(A)) and a neutrosophic semi-closed set (NSCS) if NInt(NCl(A)) ⊆ A.
(2) a neutrosophic α-open set (NαOS)[8] if A ⊆ NInt(NCl(NInt(A))) and a neutrosophic α-closed set (NαCS) if NCl(NInt(NCl(A))) ⊆ A.
(3) a neutrosophic semi-pre open set or β-open(NβOS) [17] if A ⊆ NCl(NInt(NCl(A))) and a neutrosophic semi-pre closed set or β-closed(NβCS) if NInt(NCl(NInt(A))) ⊆ A.

Definition 2.7. [17] Consider a NS A in a NTS (X, τ). Then the neutrosophic β interior and the neutrosophic β closure are defined as
Nβint(A) = ∪{G: G is a Nβ-open set in X and G ⊆ A}
Nβcl(A) = ∩ {K: K is a Nβ-closed set in X and A ⊆ K }

Definition 2.8. [16] A subset A of a neutrosophic topological space (X, τ) is called a neutrosophic generalized closed (Ng-closed) set if Ncl(A) ⊆ U whenever A ⊆ U and U is neutrosophic open set in (X, τ).

Definition 2.9. [18] A subset A of a neutrosophic topological space (X,τ) is called a neutrosophic ω– closed (Nω-closed) set if Ncl(A) ⊆ U whenever A ⊆ U and U is neutrosophic semi - open set in (X, τ).

3. Neutrosophic β*-closed set

In this section, the new concept of neutrosophic β*-closed sets in neutrosophic topological spaces was defined and studied.

Definition 3.1: A subset A of a neutrosophic topological space (X, τ) is called a neutrosophic β*- closed (Nβ*-closed) set if Nβcl(A) ⊆ U whenever A ⊆ U and U is neutrosophic g-open set in (X, τ).

Example 3.2. Let X = {a,b,c} with τ_N = {0_N,G,1_N} where G = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2), (c,0.7,0.6,0.9)⟩. Here A = ⟨(a,0.2,0.2,0.1),(b,0,1,0.2),(c,0.9,0.4,0.7)⟩ , B = ⟨(a,0.1,0.7,0.3),(b,0.2,0,0),(c,0.8,0.3,0.9)⟩, C = ⟨(a,0.4,0.5,0.2),(b,0.2,0.1,0.4),(c,0.6,0.5,1)⟩, D = ⟨(a,0.3,0.2,0.8),(b,0.1,0.3,0.6),(c,0.7,0.2,0.8)⟩ are some examples of Nβ*-closed sets.

Theorem 3.3. Each Neutrosophic Closed Set is an Nβ*-closed set in X.
Proof. Let A ⊆ U where U is a neutrosophic g-open set in X. Since A is a neutrosophic closed set Ncl(A) = A. We have Nβcl(A) ⊆ Ncl(A) = A ⊆ U. Hence Nβcl(A) ⊆ U. Therefore A is a Nβ*-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.4. Let X = {a,b,c} with τ_N = {0_N,G,1_N} where G = ⟨(a,0.5,0.6,0.4),(b,0.4,0.5,0.2), (c,0.7,0.6,0.9)⟩. Here A = ⟨(a,0.4,0.5,0.5),(b,0.3,0.2,0.3),(c,0.5,0.4,1)⟩ is a Nβ*-closed set, however A is not a Neutrosophic Closed Set.

Theorem 3.5. Each Nβ − closed set is an Nβ*-closed set in X.
Proof. Let A ⊆ U where U is a neutrosophic g-open set in X. Let A be an Nβ–closed set in X. Hence Nβcl(A) = A ⊆ U. Hence Nβcl(A) ⊆ U. Therefore A is a Nβ*-closed set in X.

The converse of the above theorem need not be true as shown in the following example.
Example 3.6. Let X = {a,b,c} with τ_N = {0_N,A,B,1_N} where A = ⟨(a,0.5,0.5,0.4),(b,0.7,0.5,0.5), (c,0.4,0.5,0.5)⟩ and B = ⟨(a,0.3,0.4,0.4),(b,0.4,0.5,0.5),(c,0.3,0.4,0.6)⟩. Here C = ⟨(a,0.7,0.6,0.3),(b,0.9,0.7,0.2), (c,0.5,0.7,0.3)⟩ is a Nβ*-closed set, but C is not an Nβ − Closed Set.

Theorem 3.7. Each Nsemi-closed set is an Nβ*-closed set in X.
Proof. Let $A \subseteq U$ where U is a neutrosophic g-open set in X. Since A is an Nsemi-closed set in X, we have Nβcl(A) ⊆ Nscl(A) = A ⊆ U. Hence Nβcl(A) ⊆ U. Therefore A is an Nβ*-closed set in X.

The converse of the above theorem need not be true as shown in the following example.
Example 3.8. Let X = {a,b} with τ_N = {0_N,A,B,1_N} where A = ⟨(a,0.4,0.3,0.5),(b,0.1,0.2,0.5)⟩ and B = ⟨(a,0.4,0.4,0.5),(b,0.4,0.3,0.4)⟩. Here C = ⟨(a,0.4,0.6,0.5),(b,0.3,0.6,0.9)⟩ is a Nβ*-closed set, but C is not an N semi − Closed Set.

Theorem 3.9. Each Neutrosophic generalized-closed set is an Nβ*-closed set in X.

Proof. Let $A \subseteq U$ be a neutrosophic generalized closed set, where U is a neutrosophic open set in X. Since every neutrosophic open set in X is a neutrosophic g-open set, we have Ncl(A) ⊆ U. Also we have Nβcl(A) ⊆ Ncl(A) ⊆ U. Hence Nβcl(A) ⊆ U. Therefore A is an Nβ*-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.10. Let X = {a,b} with τ_N = {0_N,A,B,1_N} where A = ⟨(a,0.4,0.3,0.5),(b,0.1,0.2,0.5)⟩ and B = ⟨(a,0.4,0.4,0.5),(b,0.4,0.3,0.4)⟩. Here C = ⟨(a,0.3,0.3,0.6),(b,0.3,0.2,0.5)⟩ is a Nβ*-closed set, but C is not a Neutrosophic g– Closed Set.

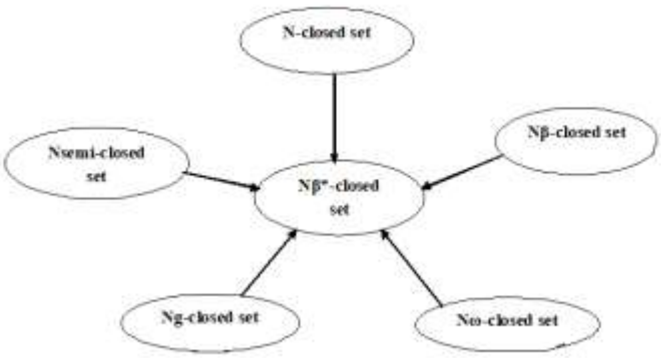
Theorem 3.11. Each Neutrosophic ω-closed set is an Nβ*-closed set in X.

Proof. Let $A \subseteq U$ be a neutrosophic ω- closed set, where U is a neutrosophic semi-open set in X. Since every neutrosophic semi-open set in X is a neutrosophic g-open set, we have Ncl(A) ⊆ U. Also we have Nβcl(A) ⊆ Ncl(A) ⊆ U. Hence Nβcl(A) ⊆ U. Therefore A is an Nβ*-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.12. Let X = {a,b,c} with τ_N = {0_N,A,B,1_N} where A = ⟨(a,0.5,0.5,0.4),(b,0.7,0.5,0.5), (c,0.4,0.5,0.5)⟩ and B = ⟨(a,0.3,0.4,0.4),(b,0.4,0.5,0.5),(c,0.3,0.4,0.6)⟩. Here C = ⟨(a,0.2,0.3,0.5),(b,0.3,0.2,0.6), (c,0.1,0.2,0.9)⟩ is an Nβ*-closed set, but C is not an Nω– Closed Set.

Remark 3.13. The following diagram shows the relationships of Nβ*-closed set with other know existing sets. A ⟶ B represents A implies B but not conversely.



Theorem 3.14. If A and B are Nβ*-closed sets in (X, τ_N), then A ∪ B is an Nβ*-closed set in (X,τ_N).

Proof: Let A and B are Nβ*-closed sets in (X,τ_N). Then Nβcl(A) ⊆ U whenever A ⊆ U and U is neutrosophic g-open set in (X, τ_N) and Nβcl(B) ⊆ U whenever B ⊆ U and U is neutrosophic g-open set in (X, τ_N). Since A ⊆ U and B ⊆ U, which implies A ∪ B ⊆ U and U is neutrosophic g-open set, then Nβcl(A) ⊆ U and Nβcl(B) ⊆ U implies Nβcl(A) ∪ Nβcl(B) ⊆ U, hence Nβcl(A ∪ B) ⊆ U. Thus A ∪ B is an Nβ*-closed set in X.

Theorem 3.15. A neutrosophic set A is Nβ*-closed set then Nβcl(A) – A does not contain any nonempty Nβ-closed sets.

Proof: Suppose that A is an Nβ*-closed set. Let F be an Nβ-closed set such that F ⊆ Nβcl(A) – A which implies F ⊆ Nβcl(A)∩A^c. Then A ⊆ F^c. Since A is Nβ*-closed set, we have Nβcl(A) ⊆ F^c . Consequently F ⊆ $(N\beta cl(A))^c$. We have F ⊆ Nβcl(A). Thus F ⊆ Nβcl(A)∩$(N\beta cl(A))^c$ = ϕ. Hence F is empty.

Theorem 3.16. If A is an Nβ*-closed set in (X,τ_N) and A ⊆ B ⊆ Nβcl(A), then B is Nβ*-closed.

Proof: Let B ⊆ U where U is a Neutrosophic g-open set in (X,τ_N). Then A ⊆ B implies A ⊆ U. Since A is an Nβ*-closed set, we have Nβcl(A)⊆U. Also A ⊆ Nβcl(B) implies Nβcl(B) ⊆ Nβcl(A). Thus Nβcl(B)⊆ U and so B is an Nβ*-closed set in (X,τ_N).

Theorem 3.17. If A is Neutrosophic g-open and Nβ*-closed, then A is Nβ-closed set.
Proof: Since A is Neutrosophic g-open and Nβ*-closed, then Nβcl(A) ⊆ A. Therefore Nβcl(A) =A. Hence A is Nβ-closed.

4. On Nβ*-Continuity and Nβ*-Contra Continuity

Definition 4.1. Let f be a mapping from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). Then f is said to be a neutrosophic β*-continuous (Nβ*-continuous) mapping if f^{-1} (A) is a Nβ*-closed set in X, for each neutrosophic-closed set A in Y.

Theorem 4.2. Consider a mapping $f: (X, \tau) \to (Y, \sigma)$. Then the following statements are equal.
(1) f is Nβ*-continuous
(2) The inverse image of each neutrosophic-closed set A in Y is Nβ*-closed set in X.
Proof. The result is obvious from the Definition 4.1.

Theorem 4.3. Consider an Nβ*-continuous mapping $f : (X, \tau) \to (Y, \sigma)$ then the following assertions hold:
(1) for all neutrosophic sets A in X, $f(N\beta^*\text{-} Ncl(A)) \subseteq Ncl(f(A))$
(2) for all neutrosophic sets B in Y, $N\beta^*Ncl\ (f^{-1}(B)) \subseteq f^{-1}\ (Ncl(B))$.
Proof. (1) Let A be a neutrosophic set in X, then Ncl(f(A)) be a neutrosophic closed set in Y and f be Nβ*-continuous, then it follows that f^{-1}(Ncl(f(A))) is Nβ*-closed in X. In view that $A \subseteq f^{-1}(Ncl(f(A)))$ and $N\beta^*cl(A) \subseteq f^{-1}(Ncl(f(A)))$. Hence, $f(N\beta^*\text{-} Ncl(A)) \subseteq Ncl(f(A))$.
(2) We get $f(N\beta^*\text{-}Ncl(f^{-1}\ (B))) \subseteq Nclf(f^{-1}(B))) \subseteq Ncl(B)$. Hence, $N\beta^*\text{-}Ncl(f^{-1}\ (B)) \subseteq f^{-1}\ (Ncl(B))$ by way of changing A with B in (1).

Definition 4.4. Let f be a mapping from a neutrosophic topological space (X, τ) to a neutrosophic topological space (Y, σ). Then f is known as neutrosophic β*-contra continuous (Nβ*-contra continuous) mapping if f^{-1} (B) is a Nβ*-closed set in X for each neutrosophic-open set B in Y.

Theorem 4.5. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$. Then the following assertions are equivalent:
(1) f is a Nβ*- contra continuous mapping
(2) f^{-1}(B) is an Nβ*-closed set in X, for each neutrosophic open set B in Y.
Proof. (1) ⟹ (2) Assume that f is Nβ*-contra continuous mapping and B is a NOS in Y. Then B^c is an NCS in Y. It follows that, f^{-1} (B^c) is an Nβ*-open set in X. For this reason, f^{-1}(B) is an Nβ*closed set in X.
(2) ⟹ (1) The converse is similar.

Theorem 4.6. Consider a bijective mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ from a NTS(X, τ) into an NTS(Y, σ). If $Ncl(f(A)) \subseteq f(N\beta^{*}int(A))$, for each NS B in X, then the mapping f is Nβ*-contra continuous.

Proof. Consider a NCS B in Y. Then $Ncl(B) = B$ and f is onto, by way of assumption, $f(N\beta^{*}int(f^{-1}(B)))$ $\subseteq Ncl(f(f^{-1}(B))) = Ncl(B) = B$. Consequently, $f^{-1}(f(N\beta^{*}int(f^{-1}(B)))) \subseteq f^{-1}(B)$. Additionally due to the fact that f is an into mapping, we have $N\beta^{*}int(f^{-1}(B)) = f^{-1}(f(N\beta^{*}int(f^{-1}(B)))) \subseteq f^{-1}(B)$. Consequently, $N\beta^{*}int(f^{-1}(B)) = f^{-1}(B)$, so $f^{-1}(B)$ is an Nβ*-open set in X. Hence, f is an Nβ*-contra continuous mapping.

5. Conclusion and Future work

In this paper we have introduced Nβ*-closed set, Nβ*- continuous function, Nβ*- contra continuous function and discussed some of its properties and derived some contradicting examples. This idea can be developed and extended in the area of homeomorphisms, compactness and connectedness and so on.

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Received: Feb 4, 2022. Accepted: Jun 19, 2022