Neutrosophic Sets and Systems, Vol. 28, 2019
University of New Mexico

Neutrosophic soft cubic Subalgebras of G-algebras

Mohsin Khalid [1], Rakib Iqbal [2] and Said Broumi [3,*]

[1]The University of Lahore, 1Km Raiwind Road, Lahore, 54000, Pakistan. E-mail: mk4605107@gmail.com
[2]University of Management and Technology (UMT), C-II, Johar Town, Lahore, 54000, Pakistan
E-mail: rakibiqbal2012@gmail.com
[3]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco.
E-mail: s.broumi@flbenmsik.ma

* Correspondence; Said Broumi; s.broumi@flbenmsik.ma

Abstract: In this paper, neutrosophic soft cubic G-subalgebra is studied through P-union, P-intersection, R-union and R-intersection etc. furthermore we study the notion of homomorphism on G-algebra with some results.

Keywords: G-algebra, Neutrosophic soft cubic set, Neutrosophic soft cubic G-subalgebra, Homomorphism of neutrosophic soft cubic subalgebra.

1 Introduction

Zadeh was the introducer of the fuzzy set and interval-valued fuzzy theory [2] in 1965. Many researchers afterward followed the notions of Zadeh. The cubic set was defined by Jun et al. [9, 10] They used the notion of cubic sets in group and initiated the idea of cubic subgroups. The algebraic structures like BCK/BCI-algebra was introduced by Imai et al. [1] in 1966. This algebra was a field of propositional calculus. Many algebraic structures like G-algebra, BG-algebra, etc. [19, 4] are structured as an extension of Q-algebra. Quadratic B-algebra was investigated by Park et al. [22]. Molodstov gave the concept of soft sets [14] in 1999. Cubic soft set with application and subalgebra in BCK/BCI-algebra were studied by Muhiuddin et al. [15,16]. Senapati et al. [13] generalized the concept of cubic set to B-subalgebra with cubic subalgebra and cubic closed ideals. Subalgebra, ideal are studied with the help of cubic set by Jun et al. [12]. The intuitionistic fuzzy G-subalgebra is studied by Jana et al. [18]. L-fuzzy G-subalgebra was studied by Senapati et al. [7]. As an extension of B-algebra, lots of work on BG-algebra have been done by the Senapati et al. [8]. The idea of a neutrosophic set which was the extension of intuitionistic fuzzy set theory and neutrosophic probability were introduced by Smarandache [20,21]. The notion of neutrosophic cubic set introduced truth-internal and truth-external were extended by Jun et al. [11] and related properties were also investigated by him. Rosenfeld's fuzzy subgroup with an interval-valued membership function was studied by Biswas [3]. The characteristics of the neutrosophic cubic soft set were studied by Pramanik et al. [5]. Cubic G-subalgebra with significent results were investigated by jana et al. [17]. The bipolar fuzzy structure of BG-algebra was interrogated by Senapati [6]. Neutrosophic cubic soft expert sets were studied for its applications in games by Gulistan M et al. [23]. Neutrosophic cubic graphs and find out the applications of neutrosophic cubic graphs in the industry by defining the model which are based on the present time and future predictions was studied by Gulistan M et al. [24]. Complex neutrosophic subsemigroups with the Cartesian product, complex neutrosophic (left, right, interior, ideal, characteristic function and direct product was observed by Gulistan M et al. [25]. Results showed the most preferred and the lowest preferred metrics in order to evaluate the sustainability of the supply chain strategy are studied by Abdel-Basset et al. [26]. Hybrid combination between analytical hierarchical process (AHP) as an MCDM method and neutrosophic theory to successfully detect and handle the uncertainty and inconsistency challenges proposed by Abdel-Basset et al. [27].

In this paper, the notion of neutrosophic soft cubic subalgebras (**NSCSU)** of G-algebras is introduced. And some relevant properties are studied. This study also discussed the homomorphism of neutrosophic soft cubic subalgebras and several related properties.

2 Preliminaries

Definition 2.1 [13] A nonempty set Y with a constant 0 and a binary operation $*$ is said to be G-algebra if it fulfills these axioms.

G1: $t_1 * t_1 = 0.$

G2: $t_1 * (t_1 * t_2) = t_2,$ for all $t_1, t_2 \in Y.$

A G-algebra is denoted by $(Y, *, 0)$.

Definition 2.2 [3] A nonempty subset S of G-algebra Y is called a subalgebra of Y if $t_1 * t_2 \in S\ \forall\ t_1, t_2 \in S$.

Definition 2.3 [3] Mapping $\tau|Y \to X$ of G-algebras is called homomorphism if $\tau(t_1 * t_2) = \tau(t_1) * \tau(t_2)\ \forall\ t_1, t_2 \in Y.$

Note that if $\tau|Y \to X$ is a g-homomorphism, then $\tau(0) = 0$.

Definition 2.4 [11] A nonempty set A in Y of the $A = \{< t_1, \vartheta_A(t_1) > |t_1 \in Y\}$, is called fuzzy set, where $\vartheta_A(t_1)$ is called the existence value of t_1 in A and $\vartheta_A(t_1) \in [0,1]$.

For a family $A_i = \{< t_1, \vartheta_{A_i}(t_1) > |t_1 \in Y\}$ of fuzzy sets in Y, where $i \in h$ and h is index set, we define the join (∨) meet (∧) operations like this:

$$\bigvee_{i\in h} A_i = (\bigvee_{i\in h} \vartheta_{A_i})(t_1) = \sup\{\vartheta_{A_i}|i \in h\},$$

and

$$\bigwedge_{i\in h} A_i = (\bigwedge_{i\in h} \vartheta_{A_i})(t_1) = \inf\{\vartheta_{A_i}|i \in h\} \text{ respectively, } \forall\ t_1 \in Y.$$

Definition 2.5 [11] A nonempty set A over Y of the form $A = \{< t_1, \tilde{\vartheta}_A(t_1) > \mid t_1 \in Y\}$, is called IVFS where $\tilde{\vartheta}_A|Y \to D[0,1]$, here $D[0,1]$ is the collection of all subintervals of [0,1].

The intervals $\tilde{\vartheta}_A t_1 = [\vartheta_A^-(t_1), \vartheta_A^+(t_1)]\ \forall\ t_1 \in Y$ denote the degree of existence of the element t_1 to the set A. Also $\tilde{\vartheta}_A^c = [1 - \vartheta_A^-(t_1), 1 - \vartheta_A^+(t_1)]$ represents the complement of $\tilde{\vartheta}_A$.

For a family $\{A_i|i \in k\}$ of IVFSs in Y where h is an index set, the union $G = \bigcup_{i\in h} \tilde{\vartheta}_{A_i}(t_1)$ and the intersection $F = \bigcap_{i\in h} \tilde{\vartheta}_{A_i}(t_1)$ are defined below:

$$G(t_1) = (\cup_{i\in h} \tilde{\vartheta}_{A_i})(t_1) = \mathrm{rsup}\{\tilde{\vartheta}_{A_i}(t_1)|i \in h\}$$

and

$$F(t_1) = (\cap_{i\in h} \tilde{\vartheta}_{A_i})(t_1) = \mathrm{rinf}\{\tilde{\vartheta}_{A_i}(t_1)|i \in k\}, \text{ respectively, } \forall\ t_1 \in Y.$$

Definition 2.6 [12] Consider two elements $K_1, K_2 \in D[0,1]$. If $K_1 = [f_1^-, f_1^+]$ and $K_2 = [f_2^-, f_2^+]$, then $\mathrm{rmax}(K_1, K_2) = [\max(f_1^-, f_2^-), \max(f_1^+, f_2^+)]$ which is denoted by $K_1 \vee^r K_2$ and $\mathrm{rmin}(K_1, K_2) = [\min(f_1^-, f_2^-), \min(f_1^+, f_2^+)]$ which is denoted by $K_1 \wedge^r K_2$. Thus, if $K_i = [f_i^-, f_i^+] \in K[0,1]$ for $i = 1,2,3, \ldots,$ then we define $\mathrm{rsup}_i(K_i) = [\sup_i(f_i^-), \sup_i(f_i^+)]$, i.e., $\vee_i^r K_i = [\vee_i (f_i^-), \vee_i (f_i^+)]$. Similarly we define $\mathrm{rinf}_i(K_i) = [\inf_i(f_i^-), \inf_i(f_i^+)]$, i.e., $\wedge_i^r K_i = [\wedge_i (f_i^-), \wedge_i (f_i^+)]$. Now $K_1 \geq K_2 \Leftarrow f_1^- \geq f_2^-$ and $f_1^+ \geq f_2^+$. Similarly the relations $K_1 \leq K_2$ and $K_1 = K_2$ are defined.

Definition 2.7 [13] A fuzzy set $A = \{< t_1, \vartheta_A(t_1) > | t_1 \in Y\}$ is called a fuzzy subalgebra of Y if $\vartheta_A(t_1 * t_2) \geq \min\{\vartheta_A(t_1), \vartheta_A(t_2)\}$ $\forall$ $t_1, t_2 \in Y$.

Definition 2.8 [22] A pair $\tilde{\mathcal{P}}_k = (\mathbf{A}, \Lambda)$ is called NCS where $\mathbf{A} = \{\langle t_1; A_T(t_1), A_I(t_1), A_F(t_1)\rangle \mid t_1 \in Y\}$ is an INS in Y and $\Lambda = \{\langle t_1; \lambda_T(t_1), \lambda_I(t_1), \lambda_F(t_1)\rangle \mid t_1 \in Y\}$ is a neutrosophic set in Y.

Definition 2.9 [3] Let $C = \{\langle t_1, A(t_1), \lambda(t_1)\rangle\}$ be a cubic set, where $A(t_1)$ is an IVFS in Y, $\lambda(t_1)$ is a fuzzy set in Y and Y is subalgebra. Then A is cubic subalgebra under binary operation * if it fulfills these axioms:

C1: $A(t_1 * t_2) \geq \text{rmin}\{A(t_1), A(t_2)\}$,

C2: $\lambda(t_1 * t_2) \leq \max\{\lambda(t_1), \lambda(t_2)\}$ $\forall$ $t_1, t_2 \in Y$.

Definition 3.0 [14] Let U be an universe set. Let $NC(U)$ represents the set of all neutrosophic cubic sets and E be the collection of parameters. Let $K \subset E$ then $\tilde{P}_K = \{\langle t_1, A_{e_i}(t_1), \lambda_{e_i}(t_1)\rangle \mid t_1 \in U, e_i \in K\}$, where $A_{e_i}(t_1) = \{\langle A^T_{e_i}(t_1), (A)^I_{e_i}(t_1), (A)^F_{e_i}(t_1)\rangle \mid t_1 \in U\}$, is an interval neutrosophic soft set, $\lambda_{e_i}(t_1) = \{\langle \lambda^T_{e_i}(t_1), (A)^I_{e_i}(t_1)(t_1), (\lambda)^F_{e_i}(t_1)\rangle \mid t_1 \in U\}$ is a neutrosophic soft set. $\tilde{P}_k$ is named as the neutrosophic soft cubic set over U where $\tilde{P}$ is a mapping given by $\tilde{P}|K \to NC(U)$. The sets of all neutrosophic soft cubic sets over U will be denoted by C^N_U.

3 Neutrosophic Soft Cubic Subalgebras of G-Algebra

Definition 3.1 Let $\tilde{\mathcal{P}}_k = (\mathbf{A}_{\mathbf{e_i}}, \Lambda_{e_i})$ be a neutrosophic soft cubic set, where Y is subalgebra. Then $\tilde{\mathcal{P}}_k$ is **NSCSU** under binary operation $*$ if it holds the following conditions:

N1:

$A^T_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^T_{e_i}(t_1), A^T_{e_i}(t_2)\}$

$A^I_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^I_{e_i}(t_1), A^I_{e_i}(t_2)\}$

$A^F_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^F_{e_i}(t_1), A^F_{e_i}(t_2)\}$,

N2:

$\Lambda^T_{e_i}(t_1 * t_2) \leq \max\{\Lambda^T_{e_i}(t_1), \Lambda^T_{e_i}(t_2)\}$

$\Lambda^I_{e_i}(t_1 * t_2) \leq \max\{\Lambda^I_{e_i}(t_1), \Lambda^I_{e_i}(t_2)\}$

$\Lambda^F_{e_i}(t_1 * t_2) \leq \max\{\Lambda^F_{e_i}(t_1), \Lambda^F_{e_i}(t_2)\}$.

For simplicity we introduced new notation for neutrosophic soft cubic set as

$$\tilde{\mathcal{P}}_k = (A^{T,I,F}_{e_i}, \lambda^{T,I,F}_{e_i}) = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}) = \{\langle t_1, A^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_1)\rangle\}$$

and for conditions N1, N2 as

N1: $A^{\varrho}_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$,

N2: $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$.

Example 3.2 Let $Y = \{0, c_1, c_2, c_3, c_4, c_5\}$ be a G-algebra with the following Cayley table.

$*$	0	c_1	c_2	c_3	c_4	c_5
0	0	c_5	c_4	c_3	c_2	c_1
c_1	c_1	0	c_5	c_4	c_3	c_2
c_2	c_2	c_1	0	c_5	c_4	c_3
c_3	c_3	c_2	c_1	0	c_5	c_4
c_4	c_4	c_3	c_2	c_1	0	c_5
c_5	c_5	c_4	c_3	c_2	c_1	0

A NSCS $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y is defined by

$*$	0	c_1	c_2	c_3	c_4	c_5
$A^T_{e_i}$	[0.6,0.8]	[0.5,0.7]	[0.6,0.8]	[0.5,0.7]	[0.6,0.8]	[0.5,0.7]
$A^I_{e_i}$	[0.5,0.4]	[0.4,0.3]	[0.5,0.4]	[0.4,0.3]	[0.5,0.4]	[0.4,0.3]
$A^F_{e_i}$	[0.5,0.7]	[0.3,0.6]	[0.5,0.7]	[0.3,0.6]	[0.5,0.7]	[0.3,0.6],

and

$*$	0	c_1	c_2	c_3	c_4	c_5
$\lambda^T_{e_i}$	0.3	0.5	0.3	0.5	0.3	0.5
$\Lambda^I_{e_i}$	0.5	0.7	0.5	0.7	0.5	0.7
$\Lambda^F_{e_i}$	0.7	0.8	0.7	0.8	0.7	0.8.

Definition 3.1 is satisfied by the set $\tilde{\mathcal{P}}_k$. Thus $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is a **NSCSU** of Y.

Proposition 3.3 Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_1)\rangle\}$ is a **NSCSU** of Y, then $\forall\ t_1 \in Y,\ A^{\varrho}_{e_i}(t_1) \geq A^{\varrho}_{e_i}(0)$ and $\lambda^{\varrho}_{e_i}(0) \leq \lambda^{\varrho}_{e_i}(t_1)$. Thus, $A^{\varrho}_{e_i}(0)$ and $\lambda^{\varrho}_{e_i}(0)$ are the upper bounds and lower bounds of $A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}(t_1)$ respectively.

Proof. For all $t_1 \in Y$, we have $A^{\varrho}_{e_i}(0) = A^{\varrho}_{e_i}(t_1 * t_1) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_1)\} = A^{\varrho}_{e_i}(t_1) \Rightarrow A^{\varrho}_{e_i}(0) \geq A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}(0) = \lambda^{\varrho}_{e_i}(t_1 * t_1) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_1)\} = \lambda^{\varrho}_{e_i}(t_1) \Rightarrow \lambda^{\varrho}_{e_i}(0) \leq \lambda^{\varrho}_{e_i}(t_1)$.

Theorem 3.4 Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_1)\rangle\}$ be a **NSCSU** of Y. If there exists a sequence $\{(t_1)_n\}$ of Y such that $\lim_{n\to\infty} A^{\varrho}_{e_i}((t_1)_n) = [1,1]$ and $\lim_{n\to\infty} \lambda^{\varrho}_{e_i}((t_1)_n) = 0$. Then $A^{\varrho}_{e_i}(0) = [1,1]$ and $\lambda^{\varrho}_{e_i}(0) = 0$.

Proof. Using Proposition 3.3, $A^{\varrho}_{e_i}(0) \geq A^{\varrho}_{e_i}(t_1)\ \forall\ t_1 \in Y$, $\therefore\ A^{\varrho}_{e_i}(0) \geq A^{\varrho}_{e_i}((t_1)_n)$ for $n \in Z^+$. Consider, $[1,1] \geq A^{\varrho}_{e_i}(0) \geq \lim_{n\to\infty} A^{\varrho}_{e_i}((t_1)_n) = [1,1]$. Hence, $A^{\varrho}_{e_i}(0) = [1,1]$. Again, using Proposition 3.3, $\lambda^{\varrho}_{e_i}(0) \leq \lambda^{\varrho}_{e_i}(t_1)\ \forall\ t_1 \in Y$, $\therefore\ \lambda^{\varrho}_{e_i}(0) \leq \lambda^{\varrho}_{e_i}((t_1)_n)$ for $n \in Z^+$. Consider, $0 \leq \lambda^{\varrho}_{e_i}(0) \leq \lim_{n\to\infty} \lambda^{\varrho}_{e_i}((t_1)_n) = 0$. Hence, $\lambda^{\varrho}_{e_i}(0) = 0$.

Theorem 3.5 The R-intersection of any set of **NSCSU** of Y is also a **NSCSU** of Y.

Proof. Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$, be set of **NSCSU** of Y and $t_1, t_2 \in Y$. Then

$$
\begin{aligned}
&(\cap A^{\varrho}_{e_i})(t_1 * t_2) = \mathrm{rinf} A^{\varrho}_{e_i}(t_1 * t_2) \\
&\geq \mathrm{rinf}\{\mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} \\
&= \mathrm{rmin}\{\mathrm{rinf} A^{\varrho}_{e_i}(t_1), \mathrm{rinf} A^{\varrho}_{e_i}(t_2)\} \\
&= \mathrm{rmin}\{(\cap A^{\varrho}_{e_i})(t_1), (\cap A^{\varrho}_{e_i})(t_2)\} \\
&\Rightarrow (\cap A^{\varrho}_{e_i})(t_1 * t_2) \geq \mathrm{rmin}\{(\cap A^{\varrho}_{e_i})(t_1), (\cap A^{\varrho}_{e_i})(t_2)\}
\end{aligned}
$$

and

$$
\begin{aligned}
&(\vee \lambda^{\varrho}_{e_i})(t_1 * t_2) = \sup \lambda^{\varrho}_{e_i}(t_1 * t_2) \\
&\leq \sup\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} \\
&= \max\{\sup \lambda^{\varrho}_{e_i}(t_1), \sup \lambda^{\varrho}_{e_i}(t_2)\} \\
&= \max\{(\vee \lambda^{\varrho}_{e_i})(t_1), (\vee \lambda^{\varrho}_{e_i})(t_2)\} \\
&\Rightarrow (\vee \lambda^{\varrho}_{e_i})(t_1 * t_2) \leq \max\{(\vee \lambda^{\varrho}_{e_i})(t_1), (\vee \lambda^{\varrho}_{e_i})(t_2)\},
\end{aligned}
$$

which show that R-intersection of $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y.

Remark 3.6 This is not compulsary that R-union, P-intersection and P-union of **NSCSU** are also the **NSCSU**.

Example 3.7 Let $\mathrm{Y} = \{0, c_1, c_2, c_3, c_4, c_5\}$ be a G-algebra with the following Cayley table.

$*$	0	c_1	c_2	c_3	c_4	c_5
0	0	c_2	c_1	c_3	c_4	c_5
c_1	c_1	0	c_2	c_5	c_3	c_4
c_2	c_2	c_1	0	c_4	c_5	c_3
c_3	c_3	c_4	c_5	0	c_1	c_2
c_4	c_4	c_5	c_3	c_2	0	c_1
c_5	c_5	c_3	c_4	c_1	c_2	0.

Let $\mathcal{A}_{e_1} = (A^{\varrho}_{e_1}, \lambda^{\varrho}_{e_1})$ and $\mathcal{A}_{e_2} = (A^{\varrho}_{e_2}, \lambda^{\varrho}_{e_2})$ are neutrosophic soft cubic sets of Y defined by

	0	c_1	c_2	c_3	c_4	c_5
$A_{e_1}T$	[0.5,0.4]	[0.1,0.2]	[0.1,0.2]	[0.5,0.4]	[0.1,0.2]	[0.1,0.2]
$A_{e_1}I$	[0.6,0.7]	[0.2,0.3]	[0.2,0.3]	[0.6,0.7]	[0.2,0.3]	[0.2,0.3]
$A_{e_1}F$	[0.7,0.8]	[0.3,0.4]	[0.3,0.4]	[0.7,0.8]	[0.3,0.4]	[0.3,0.4]
$A_{e_2}T$	[0.6,0.7]	[0.2,0.3]	[0.2,0.3]	[0.6,0.7]	[0.2,0.3]	[0.2,0.3]
$A_{e_2}I$	[0.5,0.4]	[0.1,0.2]	[0.1,0.2]	[0.1,0.2]	[0.5,0.4]	[0.1,0.2]
$A_{e_2}F$	[0.4,0.3]	[0.2,0.4]	[0.2,0.4]	[0.2,0.4]	[0.4,0.5]	[0.2,0.4]

and

	0	c_1	c_2	c_3	c_4	c_5
$\lambda_{e_1}T$	0.2	0.8	0.8	0.3	0.8	0.8
$\lambda_{e_1}I$	0.3	0.7	0.7	0.4	0.7	0.7
$\lambda_{e_1}F$	0.5	0.6	0.6	0.5	0.6	0.6
$\lambda_{e_2}T$	0.3	0.5	0.5	0.5	0.4	0.5
$\lambda_{e_2}I$	0.4	0.7	0.7	0.7	0.5	0.7
$\lambda_{e_2}F$	0.5	0.9	0.9	0.9	0.6	0.9

Then $\mathcal{A}_{e_1}$ and $\mathcal{A}_{e_2}$ are neutrosophic soft cubic subalgebras of Y but R-union, P-union and P-intersection of $\mathcal{A}_{e_1}$ and $\mathcal{A}_{e_2}$ are not neutrosophic soft cubic subalgebras of Y. $\left(\cup A^{\varrho}_{e_i}\right)(c_3 * c_4) = ([0.2,0.5],[0.2,0.3],[0.3,0.4])_{\varrho} = \mathrm{rmin}\{\left(\cup A^{\varrho}_{e_i}\right)(c_3), \left(\cup A^{\varrho}_{e_i}\right)(c_4)\}$ *and* $\left(\wedge\lambda^{\varrho}_{e_i}\right)(c_3 * c_4) = (0.7,0.6,0.8)_{\varrho} \nleq (0.1,0.2,0{,}3)_{\varrho} = \max\{(\wedge\lambda^{\varrho}_{e_i})(c_3), (\wedge\lambda^{\varrho}_{e_i})(c_4)\}$.

We give the conditions that R-union, P-union and P-intersection of **NSCSU** are also **NSCSU**. Which are at Theorem 3.8, 3.9, 3.10.

Theorem 3.8 Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$ be set of **NSCSU** of Y, where $i \in k$. If $\inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} = \max\{\inf\lambda^{\varrho}_{e_i}(t_1), \inf\lambda^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1 \in Y$. Then the P-intersection of $\tilde{\mathcal{P}}_k$ is also a **NSCSU** of Y.

Proof. Suppose that $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$ be set of **NSCSU** of Y such that $\inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} = \max\{\inf\lambda^{\varrho}_{e_i}(t_1), \inf\lambda^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in Y$. Then for $t_1, t_2 \in Y$. Then $(\cap A^{\varrho}_{e_i})(t_1 * t_2) = r\inf A^{\varrho}_{e_i}(t_1 * t_2) \geq r\inf\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{r\inf A^{\varrho}_{e_i}(t_1), r\inf A^{\varrho}_{e_i}(t_2)\} = r\min\{(\cap A^{\varrho}_{e_i})(t_1), (\cap A^{\varrho}_{e_i})(t_2)\} \Rightarrow (\cap A^{\varrho}_{e_i})(t_1 * t_2) \geq r\min\{(\cap A^{\varrho}_{e_i})(t_1), \left(\cap A^{\varrho}_{e_i}\right)(t_2)$ and $(\wedge\lambda^{\varrho}_{e_i})(t_1 * t_2) = \inf\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} = \max\{\inf\lambda^{\varrho}_{e_i}(t_1), \inf\lambda^{\varrho}_{e_i}(t_2)\} = \max\{(\wedge\lambda^{\varrho}_{e_i})(t_1), (\wedge\lambda^{\varrho}_{e_i})(t_2)\} \Rightarrow (\wedge\lambda^{\varrho}_{e_i})(t_1 * t_2) \leq \max\{(\wedge\lambda^{\varrho}_{e_i})(t_1), (\wedge\lambda^{\varrho}_{e_i})(t_2)\}$, which show that P - intersection of $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y.

Theorem 3.9 Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$ be set of **NSCSU** of Y . If $\sup\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{\sup A^{\varrho}_{e_i}(t_1), \sup A^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in Y$. Then the P-union of $\tilde{\mathcal{P}}_k$ is also a **NSCSU** of Y.

Proof. Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$ be set of **NSCSU** of Y such that $\sup\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{\sup A^{\varrho}_{e_i}(t_1), \sup A^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1 \in Y$. Then for $t_1, t_2 \in Y$, $(\cup A^{\varrho}_{e_i})(t_1 * t_2) = r\sup A^{\varrho}_{e_i}(t_1 * t_2) \geq r\sup\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{r\sup A^{\varrho}_{e_i}(t_1), r\sup A^{\varrho}_{e_i}(t_2)\} = r\min\{(\cup A^{\varrho}_{e_i})(t_1), (\cup A^{\varrho}_{e_i})(t_2)\} \Rightarrow (\cup A^{\varrho}_{e_i})(t_1 * t_2) \geq r\min\{(\cup A^{\varrho}_{e_i})(t_1), (\cup A^{\varrho}_{e_i})(t_2)\}$

an $\left(\vee\lambda^{\varrho}_{e_i}\right)(t_1 * t_2) = \sup\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \sup\left\{\max\left\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\right\}\right\} = \max\left\{\sup\lambda^{\varrho}_{e_i}(t_1), \sup\lambda^{\varrho}_{e_i}(t_2)\right\} =$

$\max\{(\vee\lambda^{\varrho}_{e_i})(t_1), (\vee\lambda^{\varrho}_{e_i})(t_2)\} \Rightarrow (\vee\lambda^{\varrho}_{e_i})(t_1 * t_2) \leq \max\{(\vee\lambda^{\varrho}_{e_i})(t_1), (\vee\lambda^{\varrho}_{e_i})(t_2)\}$, which show that P-union of $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y.

Theorem 3.10 Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$ be set of **NSCSU** of Y. If $\inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} = \max\{\inf\lambda^{\varrho}_{e_i}(t_1), \inf\lambda^{\varrho}_{e_i}(t_1)\}$ and $\sup\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{\sup A^{\varrho}_{e_i}(t_1), \sup A^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in Y$. Then the R-union of $\tilde{\mathcal{P}}_k$ is also a **NSCSU** of Y.

Proof. Let $\tilde{\mathcal{P}}_k = \{\langle t_1, A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i}\rangle | t_1 \in Y\}$ where $i \in k$ be set of **NSCSU** of Y such that $\inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} = \max\{\inf\lambda^{\varrho}_{e_i}(t_1), \inf\lambda^{\varrho}_{e_i}(t_1)\}$ and $\sup\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{\sup\lambda^{\varrho}_{e_i}(t_1), \sup\lambda^{\varrho}_{e_i}(t_1)\}$ $\forall$ $t_1 \in Y$. Then for $t_1, t_2 \in Y$, $(\cup A^{\varrho}_{e_i})(t_1 * t_2) = r\sup A^{\varrho}_{e_i}(t_1 * t_2) \geq r\sup\{r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = r\min\{r\sup A^{\varrho}_{e_i}(t_1), r\sup A^{\varrho}_{e_i}(t_2)\} = r\min\{(\cup A^{\varrho}_{e_i})(t_1), (\cup A^{\varrho}_{e_i})(t_2)\} \Rightarrow (\cup A^{\varrho}_{e_i})(t_1 * t_2) \geq r\min\{(\cup A^{\varrho}_{e_i})(t_1), (\cup A^{\varrho}_{e_i})(t_2)\}$ and $(\wedge\lambda^{\varrho}_{e_i})(t_1 * t_2) = \inf\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\} = \max\{\inf\lambda^{\varrho}_{e_i}(t_1), \inf\lambda^{\varrho}_{e_i}(t_2)\} = \max\{(\wedge\lambda^{\varrho}_{e_i})(t_1), (\wedge\lambda^{\varrho}_{e_i})(t_2)\} \Rightarrow (\wedge\lambda^{\varrho}_{e_i})(t_1 * t_2) \leq \max\{(\wedge\lambda^{\varrho}_{e_i})(t_1), (\wedge\lambda^{\varrho}_{e_i})(t_2)\}$, which show that R-union of $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y.

Proposition 3.11 If a neutrosophic soft cubic set $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y is a subalgebra. Then $\forall$ $t_1 \in Y$, $A^{\varrho}_{e_i}(0 * t_1) \geq A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}(0 * t_1) \leq \lambda^{\varrho}_{e_i}(t_1)$.

Proof. For all $t_1 \in Y$, $A^{\varrho}_{e_i}(0 * t_1) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(0), A^{\varrho}_{e_i}(t_1)\} = \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1 * t_1), A^{\varrho}_{e_i}(t_1)\} \geq \mathrm{rmin}\{\mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_1)\}, A^{\varrho}_{e_i}(t_1)\} = A^{\varrho}_{e_i}(t_1)$ and similarly $\lambda^{\varrho}_{e_i}(0 * t_1) \leq \max\{\lambda^{\varrho}_{e_i}(0), \lambda^{\varrho}_{e_i}(t_1)\} = \lambda^{\varrho}_{e_i}(t_1)$.

Lemma 3.12 If a netrosophic soft cubic set $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y is a subalgebra. Then $\tilde{\mathcal{P}}_k(t_1 * t_2) = \tilde{\mathcal{P}}_k(t_1 * (0 * (0 * t_2)))$ $\forall$ $t_1, t_2 \in Y$.

Proof. Let Y be a G-algebra and $t_1, t_2 \in Y$. Then $t_2 = 0 * (0 * t_2)$ by ([9], Lemma 3.1). Hence $A^{\varrho}_{e_i}(t_1 * t_2) = A^{\varrho}_{e_i}(t_1 * (0 * (0 * t_2)))$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) = \lambda^{\varrho}_{e_i}(t_1 * (0 * (0 * t_2)))$. Therefore, $\tilde{\mathcal{P}}_k(t_1 * t_2) = \tilde{\mathcal{P}}_k(t_1 * (0 * (0 * t_2)))$

Proposition 3.13 If a NSCS $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y is **NSCSU**. Then $\forall$ $t_1, t_2 \in Y$, $A^{\varrho}_{e_i}(t_1 * (0 * t_2)) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * (0 * t_2)) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$.

Proof. Let $t_1, t_2 \in Y$. Then we have $A^{\varrho}_{e_i}(t_1 * (0 * t_2)) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(0 * t_2)\} \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * (0 * t_2)) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(0 * t_2)\} \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$ by Definition 3.1 and Proposition 3.11. Hence proof is completed.

Theorem 3.14 If a NSCS $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y satisfies the following conditions. Then $\tilde{\mathcal{P}}_k$ refers to a **NSCSU** of Y.

1. $A^{\varrho}_{e_i}(0 * t_1) \geq A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}(0 * t_1) \leq \lambda^{\varrho}_{e_i}(x)$ $\forall$ $t_1 \in Y$.
2. $A^{\varrho}_{e_i}(t_1 * (0 * t_2)) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * (0 * t_2)) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in Y$.

Proof. Assume that the neutrosophic soft cubic set $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y satisfies the above conditions. Then by Lemma 3.12, $A^{\varrho}_{e_i}(t_1 * t_2) = A^{\varrho}_{e_i}(t_1 * (0 * (0 * t_2))) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(0 * t_2)\} \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) = \lambda^{\varrho}_{e_i}(t_1 * (0 * (0 * t_2))) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(0 * t_2)\} \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in Y$. Hence $\tilde{\mathcal{P}}_k$ is **NSCSU** of Y.

Theorem 3.15 A neutrosophic soft cubic set $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ of Y is **NSCSU** of Y iff $(A^{\varrho}_{e_i})^-, (A^{\varrho}_{e_i})^+$ and $\lambda^{\varrho}_{e_i}$ are fuzzy subalgebras of Y.

Proof. Let $(A^{\varrho}_{e_i})^-, (A^{\varrho}_{e_i})^+$ and $\lambda^{\varrho}_{e_i}$ are fuzzy subalgebra of Y and $t_1, t_2 \in Y$ then $(A^{\varrho}_{e_i})^-(t_1 * t_2) \geq \min\{(A^{\varrho}_{e_i})^-(t_1), (A^{\varrho}_{e_i})^-(t_2)\}$, $(A^{\varrho}_{e_i})^+(t_1 * t_2) \geq \min\{(A^{\varrho}_{e_i})^+(t_1), (A^{\varrho}_{e_i})^+(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$. Now, $A^{\varrho}_{e_i}(t_1 * t_2) = [(A^{\varrho}_{e_i})^-(t_1 * t_2), (A^{\varrho}_{e_i})^+(t_1 * t_2)] \geq [\min\{(A^{\varrho}_{e_i})^-(t_1), (A^{\varrho}_{e_i})^-(t_2)\}, \min\{(A^{\varrho}_{e_i})^+(t_1), (A^{\varrho}_{e_i})^+(t_2)\}] \geq \mathrm{rmin}\{[(A^{\varrho}_{e_i})^-(t_1), (A^{\varrho}_{e_i})^+(t_1)], [(A^{\varrho}_{e_i})^-(t_2), (A^{\varrho}_{e_i})^+(t_2)]\} = \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$. Therefore, $\tilde{\mathcal{P}}_k$ is **NSCSU** of Y.

Conversely, assume that $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y. For any $t_1, t_2 \in Y$, $[(A^{\varrho}_{e_i})^-(t_1 * t_2), (A^{\varrho}_{e_i})^+(t_1 * t_2)] = A^{\varrho}_{e_i}(t_1 * t_2) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\} = \mathrm{rmin}\{[(A^{\varrho}_{e_i})^-(t_1), (A^{\varrho}_{e_i})^+(t_1)], [(A^{\varrho}_{e_i})^-(t_2), (A^{\varrho}_{e_i})^+(t_2)]\} = [\min\{(A^{\varrho}_{e_i})^-(t_1), (A^{\varrho}_{e_i})^-(t_2)\}, \min\{(A^{\varrho}_{e_i})^+(t_1), (A^{\varrho}_{e_i})^+(t_2)\}]$. Thus, $(A^{\varrho}_{e_i})^-(t_1 *$

$t_2) \geq \min\{(A^{\varrho}_{e_i})^-(t_1), (A^{\varrho}_{e_i})^-(t_2)\}$, $(A^{\varrho}_{e_i})^+(t_1 * t_2) \geq \min\{(A^{\varrho}_{e_i})^+(t_1), (A^{\varrho}_{e_i})^+(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$. Hence $(A^{\varrho}_{e_i})^-, (A^{\varrho}_{e_i})^+$ and $\lambda^{\varrho}_{e_i}$ are fuzzy subalgebras of Y.

Theorem 3.16 Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be a **NSCSU** of Y and let $n \in \mathbb{Z}^+$. Then

i) $A^{\varrho}_{e_i}(\coprod n\, t_1 * t_1) \geq A^{\varrho}_{e_i}(t_1)$ for $n \in \mathbb{O}$.

ii) $\lambda^{\varrho}_{e_i}(\coprod n\, t_1 * t_1) \leq A^{\varrho}_{e_i}(t_1)$ for $n \in \mathbb{O}$.

iii) $A^{\varrho}_{e_i}(\coprod n\, t_1 * t_1) = A^{\varrho}_{e_i}(t_1)$ for $n \in \mathbb{E}$.

iv) $\lambda^{\varrho}_{e_i}(\coprod n\, t_1 * t_1) = A^{\varrho}_{e_i}(t_1)$ for $n \in \mathbb{E}$.

Proof. Let $t_1 \in Y$ and suppose that n is odd. Then $n = 2p - 1$ for some $p \in Z^+$. We prove the theorem by induction.

Now $A^{\varrho}_{e_i}(t_1 * t_1) = A^{\varrho}_{e_i}(0) \geq A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}(t_1 * t_1) = \lambda^{\varrho}_{e_i}(0) \leq \lambda^{\varrho}_{e_i}(t_1)$. Suppose that $A^{\varrho}_{e_i}((\coprod_{2p-1})(t_1 * t_1)) \geq A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}((\coprod_{2p-1})(t_1 * t_1)) \leq \lambda^{\varrho}_{e_i}(t_1)$. Then by assumption, $A^{\varrho}_{e_i}((\coprod_{2(p+1)-1}(t_1 * t_1)) = A^{\varrho}_{e_i}((\coprod_{2p+1})(t_1 * t_1)) = A^{\varrho}_{e_i}(\coprod_{2p-1} t_1 * (t_1 * (t_1 * t_1))) = A^{\varrho}_{e_i}((\coprod_{2p-1})(t_1 * t_1)) \geq A^{\varrho}_{e_i}(t_1)$ and $\lambda^{\varrho}_{e_i}((\coprod_{2(p+1)-1}(t_1 * t_1)) = \lambda^{\varrho}_{e_i}((\coprod_{2p+1}(t_1 * t_1)) = \lambda^{\varrho}_{e_i}(\coprod_{2p-1} t_1 * (t_1 * (t_1 * t_1))) = \lambda^{\varrho}_{e_i}(\coprod_{2p-1} t_1 * t_1) \leq \lambda^{\varrho}_{e_i}(t_1)$, which proves (1) and (2). Similarly, cases (3) and (4) has the same proofs.

These sets denoted by $I_{A^{\varrho}_{e_i}}$ and $I_{\lambda^{\varrho}_{e_i}}$ are subalgebras of Y. Which were defined as

$$I_{A^{\varrho}_{e_i}} = \{t_1 \in Y | A^{\varrho}_{e_i}(t_1) = A^{\varrho}_{e_i}(0)\},\ I_{\lambda^{\varrho}_{e_i}} = \{t_1 \in Y | \lambda^{\varrho}_{e_i}(t_1) = \lambda^{\varrho}_{e_i}(0)\}.$$

Theorem 3.17 Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be a **NSCSU** of Y. Then the sets $I_{A^{\varrho}_{e_i}}$ and $I_{\lambda^{\varrho}_{e_i}}$ are subalgebras of Y.

Proof. Let $t_1, t_2 \in I_{A^{\varrho}_{e_i}}$. Then $A^{\varrho}_{e_i}(t_1) = A^{\varrho}_{e_i}(0) = A^{\varrho}_{e_i}(t_2)$ and so, $A^{\varrho}_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ $= A^{\varrho}_{e_i}(0)$. By using Proposition 3.3, we know that $A^{\varrho}_{e_i}(t_1 * t_2) = A^{\varrho}_{e_i}(0)$ or equivalently $t_1 * t_2 \in I_{A^{\varrho}_{e_i}}$.

Again suppose $t_1, t_2 \in I_{A^{\varrho}_{e_i}}$. Then $\lambda^{\varrho}_{e_i}(t_1) = \lambda^{\varrho}_{e_i}(0) = \lambda^{\varrho}_{e_i}(t_2)$ and so, $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\} = \lambda^{\varrho}_{e_i}(0)$. Again by using Proposition 3.3, we know that $\lambda^{\varrho}_{e_i}(t_1 * t_2) = \lambda^{\varrho}_{e_i}(0)$ or equivalently $t_1 * t_2 \in I_{A^{\varrho}_{e_i}}$. Hence the sets $I_{A^{\varrho}_{e_i}}$ and $\lambda_{A^{\varrho}_{e_i}}$ are subalgebras of Y.

Theorem 3.18 Assume B is a nonempty subset of Y and $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be a neutrosophic soft cubic set of Y defined by $A^{\varrho}_{e_i}(t_1) = \begin{cases} [\xi_{T,I,F_1}, \xi_{T,I,F_2}], & \text{if } t_1 \in B \\ [\beta_{T,I,F_1}, \beta_{T,I,F_2}], & \text{otherwise,} \end{cases}$ $\Lambda^{T}_{e_i}(t_1) = \begin{cases} \gamma_{\varrho}, & \text{if } t_1 \in B \\ \delta_{\varrho}, & \text{otherwise,} \end{cases}$

$\forall\ [\xi_{T,I,F_1}, \xi_{T,I,F_2}], [\beta_{T,I,F_1}, \beta_{T,I,F_2}] \in D[0,1]$ and $\gamma_{\varrho}, \delta_{\varrho} \in [0,1]$ with $[\xi_{T,I,F_1}, \xi_{T,I,F_2}] \geq [\beta_{T,I,F_1}, \beta_{T,I,F_2}]$ and $\gamma_{\varrho} \leq \delta_{\varrho}$. Then $\tilde{\mathcal{P}}_k$ is a nuetrosophic soft cubic subalgebra of Y $\Leftarrow$ B is a subalgebra of Y. Moreover, $I_{A^{\varrho}_{e_i}} = B = I_{\lambda^{\varrho}_{e_i}}$.

Proof. Let $\tilde{\mathcal{P}}_k$ be a **NSCSU** of Y. Let $t_1, t_2 \in Y$ such that $t_1, t_2 \in B$. Then $A^{\varrho}_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\} = \text{rmin}\{[\xi_{T,I,F_1}, \xi_{T,I,F_2}], [\xi_{T,I,F_1}, \xi_{T,I,F_2}]\} = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\} = \max\{\gamma_{\varrho}, \gamma_{\varrho}\} = \gamma_{\varrho}$. Therefore $t_1 * t_2 \in B$. Hence, B is a subalgebra of Y.

Conversely, assume that B is a subalgebra of Y. Let $t_1, t_2 \in Y$. Now take two cases.

Case 1: If $t_1, t_2 \in B$, then $t_1 * t_2 \in B$, thus $A^{\varrho}_{e_i}(t_1 * t_2) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}] = \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) = \gamma_{\varrho} = \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$.

Case 2: If $t_1 \notin B$ or $t_2 \notin B$, then $A^{\varrho}_{e_i}(t_1 * t_2) \geq [\beta_{T,I,F_1}, \beta_{T,I,F_2}] = \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \delta_{\varrho} = \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$. Hence $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y.

Now, $I_{A^{\varrho}_{e_i}}$ $=\{t_1 \in Y, A^{\varrho}_{e_i}(t_1) = A^{\varrho}_{e_i}(0)\}=$ $\{t_1 \in Y, A^{\varrho}_{e_i}(t_1) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]\} = B$ and $I_{\lambda^{\varrho}_{e_i}}$ $=\{t_1 \in Y, \lambda^{\varrho}_{e_i}(t_1) = \lambda^{\varrho}_{e_i}(0)\}$=$\{t_1 \in Y, \lambda^{\varrho}_{e_i}(t_1) = \gamma_{\varrho}\}$=B.

Definition 3.19 Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be a neutrosophic soft cubic set of Y. For $[w_{T_1}, w_{T_2}], [w_{I_1}, w_{I_2}], [w_{F_1}, w_{F_2}] \in D[0,1]$ and $t_{T_1}, t_{I_1}, t_{F_1} \in [0,1]$, the set $U(A^{\varrho}_{e_i}|([w_{T_1}, w_{T_2}], [w_{I_1}, w_{I_2}], [w_{F_1}, w_{F_2}])) = \{t_1 \in Y | A^{T}_{e_i}(t_1) \geq [w_{T_1}, w_{T_2}], A^{I}_{e_i}(t_1) \geq [w_{I_1}, w_{I_2}], A^{F}_{e_i}(t_1) \geq [w_{F_1}, w_{F_2}]\}$ is called upper $([w_{T_1}, w_{T_2}], [w_{I_1}, w_{I_2}], [w_{F_1}, w_{F_2}])$-level of $\tilde{\mathcal{P}}_k$ and $L(\lambda^{\varrho}_{e_i}|(t_{T_1}, t_{I_1}, t_{F_1}))$ $=\{t_1 \in Y | \Lambda^{T}_{e_i}(t_1) \leq t_{T_1}, \Lambda^{I}_{e_i}(t_1) \leq t_{I_1}, \Lambda^{F}_{e_i}(t_1) \leq t_{F_1}\}$ is called lower $(t_{T_1}, t_{I_1}, t_{F_1})$-level of $\tilde{\mathcal{P}}_k$.

For convenience, we introduced the new notions for upper level and lower level of $\tilde{\mathcal{P}}_k$ as, $U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}]$=$\{t_1 \in Y | A^{\varrho}_{e_i}(t_1) \geq [w_{T,I,F_1}, w_{T,I,F_2}]\}$ is called upper $([w_{T,I,F_1}, w_{T,I,F_2}])$-level of $\tilde{\mathcal{P}}_k$ and $L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$=$\{t_1 \in Y | \lambda^{\varrho}_{e_i}(t_1) \leq t_{T,I,F_1}\}$ is called lower t_{T,I,F_1}-level of $\tilde{\mathcal{P}}_k$.

Theorem 3.20 If $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is neutrosophic soft cubic subalgebra of Y, then the upper $[w_{T,I,F_1}, w_{T,I,F_2}]$-level and lower t_{T,I,F_1}-level of $\tilde{\mathcal{P}}_k$ are subalgebras of Y.

Proof. Let $t_1, t_2 \in U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}])$. Then $A^{\varrho}_{e_i}(t_1) \geq [w_{T,I,F_1}, w_{T,I,F_2}]$ and $A^{\varrho}_{e_i}(t_2) \geq [w_{T,I,F_1}, w_{T,I,F_2}]$. It follows that $A^{\varrho}_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\} \geq [w_{T,I,F_1}, w_{T,I,F_2}] \Rightarrow t_1 * t_2 \in U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}])$. Hence, $U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}]$ is a subalgebra of Y.
Let $t_1, t_2 \in L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$. Then $\lambda^{\varrho}_{e_i}(t_1) \leq t_{T,I,F_1}$ and $\lambda^{\varrho}_{e_i}(t_2) \leq t_{T,I,F_1}$. It follows that $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\} \leq t_{T,I,F_1} \Rightarrow t_1 * t_2 \in L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$. Hence $L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$ is a subalgebra of Y.

Corollary 3.21 Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is NSCSU of Y. Then $A([w_{T,I,F_1}, w_{T,I,F_2}], t_{T,I,F_1}) = U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}]) \cap L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$=$\{t_1 \in Y | A^{\varrho}_{e_i}(t_1) \geq [w_{T,I,F_1}, w_{T,I,F_2}], \lambda^{\varrho}_{e_i}(t_1) \leq t_{T,I,F_1}\}$ is a subalgebra of Y.

Proof. We can prove it by using Theorem 3.20.
This example shows that the converse of Corollary 3.21 is not true
Example 3.22 Let $Y = \{0, c_1, c_2, c_3, c_4, c_5\}$ be a G-algebra in Remark 3.6 and $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is a neutrosophic soft cubic set defined by

	0	c_1	c_2	c_3	c_4	c_5
$A^{T}_{e_i}$	[0.3,0.5]	[0.3,0.4]	[0.3,0.4]	[0.3,0.4]	[0.1,0.2]	[0.1,0.2]
$A^{I}_{e_i}$	[0.5,0.7]	[0.2,0.3]	[0.2,0.3]	[0.5,0.7]	[0.1,0.1]	[0.1,0.1]
$A^{F}_{e_i}$	[0.4,0.6]	[0.2,0.5]	[0.2,0.5]	[0.2,0.5]	[0.1,0.2]	[0.1,0.2],

and

	0	c_1	c_2	c_3	c_4	c_5
$\Lambda^T_{e_i}$	0.1	0.4	0.4	0.6	0.4	0.6
$\Lambda^I_{e_i}$	0.2	0.5	0.5	0.7	0.5	0.7
$\Lambda^F_{e_i}$	0.3	0.6	0.6	0.8	0.6	0.8

We take $[w_{T,I,F_1}, w_{T,I,F_2}] = ([0.41,0.48],[0.30,0.36],[0.13,0.17])$ and $t_{T,I,F_1} = (0.3,0.4,0.5)$. Then $A([w_{T,I,F_1}, w_{T,I,F_2}], t_{T,I,F_1}) = U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}]) \cap L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1}) = \{t_1 \in Y | A^{\varrho}_{e_i}(t_1) \geq [w_{T,I,F_1}, w_{T,I,F_2}], \lambda^{\varrho}_{e_i}(t_1) \leq t_{T,I,F_1}\} = \{0, c_1, c_2, c_3\} \cap \{0, c_1, c_2, c_4\} = \{0, c_1, c_2\}$ is a subalgebra of Y, but $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is not a **NSCSU**, since $A^T_{e_i}(c_1 * c_3) = [0.2,0.3] \ngeq [0.4,0.5] = r\min\{A^T_{e_i}(c_1), A^T_{e_i}(c_3)\}$ and $\Lambda^T_{e_i}(c_2 * c_4) = 0.4 \nleq 0.3 = \max\{\Lambda^T_{e_i}(c_2), \Lambda^T_{e_i}(c_4)\}$.

Theorem 3.23 Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be a neutrosophic soft cubic set of Y, such that the sets $U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}])$ and $L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$ are subalgebras of Y for every $[w_{T,I,F_1}, w_{T,I,F_2}] \in D[0,1]$ and $t_{T,I,F_1} \in [0,1]$. Then $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is NSCSU of Y.

Proof. Let $U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}])$ and $L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$ are subalgebras of Y for every $[w_{T,I,F_1}, w_{T,I,F_2}] \in D[0,1]$ and $t_{T,I,F_1} \in [0,1]$. On the contrary, let $(t_1)_0, (t_2)_0 \in Y$ be such that $A^{\varrho}_{e_i}((t_1)_0 * (t_2)_0) < r\min\{A^{\varrho}_{e_i}((t_1)_0), A^{\varrho}_{e_i}((t_2)_0)\}$. Let $A^{\varrho}_{e_i}((t_1)_0) = [\phi_1, \phi_2]$, $A^{\varrho}_{e_i}((t_2)_0) = [\phi_3, \phi_4]$ and $A^{\varrho}_{e_i}((t_1))_0 * (t_2)_0) = [w_{T,I,F_1}, w_{T,I,F_2}]$. Then $[w_{T,I,F_1}, w_{T,I,F_2}] < r\min\{[\phi_1, \phi_2], [\phi_3, \phi_4]\} = [\min\{\phi_1, \phi_3\}, \min\{\phi_2, \phi_4\}]$. So, $w_{T,I,F_1} < r\min\{\phi_1, \phi_3\}$ and $w_{T,I,F_2} < \min\{\phi_2, \phi_4$. Let us consider, $[\rho_1, \rho_2] = \frac{1}{2}[A^{\varrho}_{e_i}((t_1)_0 * (t_2)_0) + r\min\{A^{\varrho}_{e_i}((t_1)_0), A^{\varrho}_{e_i}((t_2)_0)\}] = \frac{1}{2}[[w_{T,I,F_1}, w_{T,I,F_2}] + [\min\{\phi_1, \phi_3\}, \min\{\phi_2, \phi_4\}]] = [\frac{1}{2}(w_{T,I,F_1} + \min\{\phi_1, \phi_3\}), \frac{1}{2}(w_{T,I,F_2} + \min\{\phi_2, \phi_3\})]$. Therefore, $\min\{\phi_1, \phi_3\} > \rho_1 = \frac{1}{2}(w_{T,I,F_1} + \min\{\phi_1, \phi_3\}) > w_{T,I,F_1}$ and $\min\{\phi_2, \phi_4\} > \rho_2 = \frac{1}{2}(w_{T,I,F_2} + \min\{\phi_2, \phi_4\}) > w_{T,I,F_2}$. Hence, $[\min\{\phi_1, \phi_3\}, \min\{\phi_2, \phi_4\}] > [\rho_1, \rho_2] > [w_{T,I,F_1}, w_{T,I,F_2}]$ so that $(t_1)_0 * (t_2)_0 \notin U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}])$ which is a contradiction since $A^{\varrho}_{e_i}((t_1)_0) = [\phi_1, \phi_2] \geq [\min\{\phi_1, \phi_3\}, \min\{\phi_2, \phi_4\}] > [\rho_1, \rho_2]$ and $A^{\varrho}_{e_i}((t_2)_0) = [\phi_3, \phi_4] \geq [\min\{\phi_1, \phi_3\}, \min\{\phi_2, \phi_4\}] > [\rho_1, \rho_2]$. This implies $(t_1)_0 * (t_2)_0 \in U(A^{\varrho}_{e_i}|[w_{T,I,F_1}, w_{T,I,F_2}])$. Thus $A^{\varrho}_{e_i}(t_1 * t_2) \geq r\min\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\ \forall\ t_1, t_2 \in Y$.

Again, let $(t_1)_0, (t_2)_0 \in Y$ be such that $\lambda^{\varrho}_{e_i}((t_1)_0 * (t_2)_0) > \max\{\lambda^{\varrho}_{e_i}((t_1)_0), \lambda^{\varrho}_{e_i}(0)\}$. Let $\lambda^{\varrho}_{e_i}((t_1)_0) = \eta_{T,I,F_1}$, $\lambda^{\varrho}_{e_i}((t_2)_0) = \eta_{T,I,F_2}$ and $\lambda^{\varrho}_{e_i}((t_1)_0 * (t_2)_0) = t_{T,I,F_1}$. Then $t_{T,I,F_1} > \max\{\zeta_{T,I,F_1}. \zeta_{T,I,F_2}\}$. Let us consider $t_{T,I,F_2} = \frac{1}{2}[\lambda^{\varrho}_{e_i}((t_1)_0 * \hat{v}_0) + \max\{\lambda^{\varrho}_{e_i}((t_1)_0), \lambda^{\varrho}_{e_i}(0)\}]$. We get that $t_{T,I,F_2} = \frac{1}{2}(t_{T,I,F_1} + \max\{\zeta_{T,I,F_1}, \zeta_{T,I,F_2}\})$. Therefore, $\zeta_{T,I,F_1} < t_{T,I,F_2} = \frac{1}{2}(t_{T,I,F_1} + \max\{\zeta_{T,I,F_1}, \zeta_{T,I,F_2}\}) < t_{T,I,F_1}$ and $\zeta_{T,I,F_2} < t_{T,I,F_2} = \frac{1}{2}(t_{T,I,F_1} + \max\{\zeta_{T,I,F_1}, \zeta_{T,I,F_2}\}) < t_{T,I,F_1}$. Hence, $\max\{\zeta_{T,I,F_1}, \zeta_{T,I,F_2}\} < t_{T,I,F_2} < t_{T,I,F_1} = \lambda^{\varrho}_{e_i}((t_1)_0, (t_2)_0)$, so that $(t_1)_0 * (t_2)_0 \notin L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$ which is a contradiction since $\lambda^{\varrho}_{e_i}((t_1)_0) = \zeta_{T,I,F_1} \leq \max\{\zeta_{T,I,F_1}, \zeta_{T,I,F_2}\} < t_{T,I,F_2}$ and $\lambda^{\varrho}_{e_i}((t_2)_0) = \zeta_{T,I,F_2} \leq \max\{\zeta_{T,I,F_1}, \zeta_{T,I,F_2}\} < t_{T,I,F_2}$. This implies $(t_1)_0, (t_2)_0 \in L(\lambda^{\varrho}_{e_i}|t_{T,I,F_1})$. Thus $\lambda^{\varrho}_{e_i}(t_1 *$

$t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}\ \forall\ t_1, t_2 \in Y$. Therefore, $U(A^{\varrho}_{e_i} | [w_{T,I,F_1}, w_{T,I,F_2}])$ and $L(\lambda^{\varrho}_{e_i} | t_{T,I,F_1})$ are subalgebras of Y. Hence, $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is **NSCSU** of Y.

Theorem 3.24 Any subalgebra of Y can be consider as both the upper $[w_{T,I,F_1}, w_{T,I,F_2}]$- level and lower t_{T,I,F_1}-level of some **NSCSU** of Y.

Proof. Let $\tilde{\mathcal{N}}_k$ be a **NSCSU** of Y, and $\tilde{\mathcal{P}}_k$ be a neutrosophic soft cubic set on Y defined by

$$A^{\varrho}_{e_i} = \begin{cases} [\xi_{T,I,F_1}, \xi_{T,I,F_2}] & \text{if } t_1 \in \tilde{\mathcal{N}}_k \\ [0,0] & \text{otherwise}. \end{cases}, \lambda^{\varrho}_{e_i} = \begin{cases} \beta_{T,I,F_1} & \text{if } t_1 \in \tilde{\mathcal{N}}_k \\ 0 & \text{otherwise}. \end{cases}$$

$\forall\ [\xi_{T,I,F_1}, \xi_{T,I,F_2}] \in D[0,1]$ and $\beta_{T,I,F_1} \in [0,1]$. We consider the following cases.

Case1 : If $\forall\ t_1, t_2 \in \tilde{\mathcal{N}}_k$ then $A^{\varrho}_{e_i}(t_1) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$, $\lambda^{\varrho}_{e_i}(t_1) = \beta_{T,I,F_1}$ and $A^{\varrho}_{e_i}(t_2) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$, $\lambda^{\varrho}_{e_i}(t_2) = \beta_{T,I,F_1}$. Thus $A^{\varrho}_{e_i}(t_1 * t_2) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}] = \text{rmin}\{[\xi_{T,I,F_1}, \xi_{T,I,F_2}], [\xi_{T,I,F_1}, \xi_{T,I,F_2}]\} = \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) = \beta_{T,I,F_1} = \max\{\beta_{T,I,F_1}, \beta_{T,I,F_1}\} = \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$.

Case2: If $t_1 \in \tilde{\mathcal{N}}_k$ and $t_2 \notin \tilde{\mathcal{N}}_k$, then $A^{\varrho}_{e_i}(t_1) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$, $\lambda^{\varrho}_{e_i}(t_1) = \beta_{T,I,F_1}$ and $A^{\varrho}_{e_i}(t_2) = [0,0]$, $\lambda^{\varrho}_{e_i}(t_2) = 1$. Thus $A^{\varrho}_{e_i}(t_1 * t_2) \geq [0,0] = \text{rmin}\{[\xi_{T,I,F_1}, \xi_{T,I,F_2}], [0,0]\} = \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq 1 = \max\{\beta_{T,I,F_1}, 1\} = \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$.

Case3 : If $t_1 \notin \tilde{\mathcal{N}}_k$ and $t_2 \in \tilde{\mathcal{N}}_k$, then $A^{\varrho}_{e_i}(t_1) = [0,0]$, $\lambda^{\varrho}_{e_i}(t_1) = 1$ and $A^{\varrho}_{e_i}(t_2) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$, $\lambda^{\varrho}_{e_i}(t_2) = \beta_{T,I,F_1}$. Thus $A^{\varrho}_{e_i}(t_1 * t_2) \geq [0,0] = \text{rmin}\{[0,0], [\xi_{T,I,F_1}, \xi_{T,I,F_2}]\} = \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq 1 = \max\{1, \beta_{T,I,F_1}\} = \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$.

Case4: If $t_1 \notin \tilde{\mathcal{N}}_k$ and $t_2 \notin \tilde{\mathcal{N}}_k$, then $A^{\varrho}_{e_i}(t_1) = [0,0]$, $\lambda^{\varrho}_{e_i}(t_1) = 1$ and $A^{\varrho}_{e_i}(t_2) = [0,0]$, $\lambda^{\varrho}_{e_i}(t_2) = 1$. Thus $A^{\varrho}_{e_i}(t_1 * t_2) \geq [0,0] = \text{rmin}\{[0,0], [0,0]\} = \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq 1 = \max\{1,1\} = \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2)\}$. Therefore, $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y.

Theorem 3.25 Let $\tilde{\mathcal{N}}_k$ be a subset of Y and $\tilde{\mathcal{P}}_k$ be a neutrosophic soft cubic set on Y which is given in the proof of Theorem 3.24. If $\tilde{\mathcal{P}}_k$ is realized as lower level subalgebra and upper level subalgebra of some **NSCSU** of Y, then $\tilde{\mathcal{N}}_k$ is a neutrosophic soft cubic one of Y.

Proof. Let $\tilde{\mathcal{P}}_k$ be a **NSCSU** of Y, and $t_1, t_2 \in \tilde{\mathcal{N}}_k$. Then $A^{\varrho}_{e_i}(t_1) = A^{\varrho}_{e_i}(t_2) = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$ and $\lambda^{\varrho}_{e_i}(t_1) = \lambda^{\varrho}_{e_i}(t_2) = \beta_{T,I,F_1}$. Thus $A^{\varrho}_{e_i}(t_1 * t_2) \geq \text{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\} = \text{rmin}\{[\xi_{T,I,F_1}, \xi_{T,I,F_2}], [\xi_{T,I,F_1}, \xi_{T,I,F_2}]\} = [\xi_{T,I,F_1}, \xi_{T,I,F_2}]$ and $\lambda^{\varrho}_{e_i}(t_1 * t_2) \leq \max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_2) = \max\{\beta_{T,I,F_1}, \beta_{T,I,F_1}\} = \beta_{T,I,F_1} \Rightarrow t_1 * t_2 \in \tilde{\mathcal{N}}_k$. Hence $\tilde{\mathcal{N}}_k$ is a neutrosophic soft cubic one of Y.

4 Homomorphism of Neutrosophic Soft Cubic Subalgebras

Suppose τ be a mapping from a set Y into a set Y and $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be a neurosophic soft cubic set in Y. Then the inverse-image of $\tilde{\mathcal{P}}_k$ is defined as $\tau^{-1}(\tilde{\mathcal{P}}_k) = \{\langle t_1, \tau^{-1}(A^{\varrho}_{e_i}), \tau^{-1}(\lambda^{\varrho}_{e_i})\rangle | t_1 \in Y\}$ and $\tau^{-1}(A^{\varrho}_{e_i})(t_1) = A^{\varrho}_{e_i}(\tau(t_1))$ and $\tau^{-1}(\lambda^{\varrho}_{e_i})(t_1) = \lambda^{\varrho}_{e_i}(\tau(t_1))$. It is clear that $\tau^{-1}(\tilde{\mathcal{P}}_k)$ is a neutrosophic soft cubic set.

Theorem 4.1 Let $\tau \mid Y \to X$ is a homomorphic mapping of G-algebra. If $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is a **NSCSU** of X. Then the pre-image $\tau^{-1}(\tilde{\mathcal{P}}_k) = \{\langle t_1, \tau^{-1}(A^{\varrho}_{e_i}), \tau^{-1}(\lambda^{\varrho}_{e_i})\rangle | t_1 \in X\}$ of $\tilde{\mathcal{P}}_k$ under τ is a **NSCSU** of Y.

Proof. Assume that $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is a **NSCSU** of Y and $t_1, t_2 \in Y$. Then $\tau^{-1}(A^{\varrho}_{e_i})(t_1 * t_2) = A^{\varrho}_{e_i}(\tau(t_1 * t_2)) = A^{\varrho}_{e_i}(\tau(t_1) * \tau(t_2)) \geq \mathrm{rmin}\{A^{\varrho}_{e_i}(\tau(t_1)), A^{\varrho}_{e_i}(\tau(t_2))\} = \mathrm{rmin}\{\tau^{-1}(A^{\varrho}_{e_i})(t_1), \tau^{-1}(A^{\varrho}_{e_i})(t_2)\}$ and $\tau^{-1}(\lambda^{\varrho}_{e_i})(t_1 * t_2) = \lambda^{\varrho}_{e_i}(\tau(t_1 * t_2)) = \lambda^{\varrho}_{e_i}(\tau(t_1) * \tau(t_2)) \leq \max\{\lambda^{\varrho}_{e_i}(\tau(t_1)), \lambda^{\varrho}_{e_i}(\tau(t_2))\} = \max\{\tau^{-1}(\lambda^{\varrho}_{e_i})(t_1), \tau^{-1}(\lambda^{\varrho}_{e_i})(t_2)\}$. Hence $\tau^{-1}(\tilde{\mathcal{P}}_k) = \{\langle t_1, \tau^{-1}(A^{\varrho}_{e_i}), \tau^{-1}(\lambda^{\varrho}_{e_i})\rangle | t_1 \in Y\}$ is **NSCSU** of Y.

Theorem 4.2 Let $\tau \mid Y \to X$ is a homomorphic mapping of G-algebra and $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_j}, \lambda^{\varrho}_{e_j})$ is a **NSCSU** of X where $j \in k$. If $\inf\{\max\{\lambda^{\varrho}_{e_j}(t_2), \lambda^{\varrho}_{e_j}(t_2)\}\} = \max\{\inf \lambda^{\varrho}_{e_j}(t_2), \inf \lambda^{\varrho}_{e_j}(t_2)\}$ $\forall$ $t_2 \in Y$. Then $\tau^{-1}(\bigcap_{R\, j\in k} \tilde{\mathcal{P}}_k)$ is a **NSCSU** of Y.

Proof. Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_j}, \lambda^{\varrho}_{e_j})$ be a **NSCSU** of Y where $j \in k$ satisfying $\inf\left\{\max\left\{\lambda^{\varrho}_{e_j}(t_2), \lambda^{\varrho}_{e_j}(t_2)\right\}\right\} = \max\{\inf \lambda^{\varrho}_{e_j}(t_2), \inf \lambda^{\varrho}_{e_j}(t_2)\}$ $\forall$ $t_2 \in Y$. Then by Theorem 3.8, $\bigcap_{R\, j\in k} \tilde{\mathcal{P}}_k$ is a **NSCSU** of Y. Hence $\tau^{-1}(\bigcap_{R\, j\in k} \tilde{\mathcal{P}}_k)$ is also a **NSCSU** of Y.

Definition 4.3 A neutrosophic soft cubic set $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ in Y is said to have sup-property and inf-property if for any subset S of Y, $\exists$ $s_0 \in T$ such that $A^{\varrho}_{e_i}(s_0) = \underset{s_0\in S}{\mathrm{rsup}} A^{\varrho}_{e_i}(s_0)$ and $\lambda^{\varrho}_{e_i}(s_0) = \inf_{t_0\in T} \lambda^{\varrho}_{e_i}(t_0)$ respectively.

Definition 4.4 Let τ be the mapping from the set Y to the set X. If $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is neutrosphic cubic set of Y, then the image of $\tilde{\mathcal{P}}_k$ under τ denoted by $\tau(\tilde{\mathcal{P}}_k)$ and is defined as $\tau(\tilde{\mathcal{P}}_k)=\{\langle t_1, \tau_{\mathrm{rsup}}(A^{\varrho}_{e_i}), \tau_{\inf}(\lambda^{\varrho}_{e_i})\rangle | t_1 \in Y\}$, where

$$\tau_{\mathrm{rsup}}(A^{\varrho}_{e_i})(t_2) = \begin{cases} \underset{t_1\in\tau^{-1}(t_2)}{A^{\varrho}_{e_i}}(t_1), & \text{if } \tau^{-1}(t_2) \neq \phi \\ [0,0], & \text{otherwise}, \end{cases}$$

and

$$\tau_{\inf}(\lambda^{\varrho}_{e_i})(t_2) = \begin{cases} \underset{t_1\in\tau^{-1}(t_2)}{\lambda^{\varrho}_{e_i}}(t_1), & \text{if } \tau^{-1}(t_2) \neq \phi \\ 1, & \text{otherwise}. \end{cases}$$

Theorem 4.5 Assume $\tau \mid Y \to X$ is a homomorphic mapping of $G-$algebra and $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ is a **NSCSU** of Y, where $i \in k$. If $\inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_1)\}\} = \max\{\inf \lambda^{\varrho}_{e_i}(t_1), \inf \lambda^{\varrho}_{e_i}(t_1)\}$ $\forall$ $t_1 \in Y$. Then $\tau(\bigcap_{P\, i\in k} \tilde{\mathcal{P}}_k)$ is a **NSCSU** of Y.

Proof. Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be **NSCSU** of Y where $i \in k$ satisfying $\inf\{\max\{\lambda^{\varrho}_{e_i}(t_1), \lambda^{\varrho}_{e_i}(t_1)\}\} = \max\{\inf \lambda^{\varrho}_{e_i}(t_1), \inf \lambda^{\varrho}_{e_i}(t_1)\}$ $\forall$ $t_1 \in Y$. Then by Theorem 3.8, $\bigcap_{P\, i\in k} \tilde{\mathcal{P}}_k$ is a **NSCSU** of Y. Hence $\tau(\bigcap_{P\, i\in k} \tilde{\mathcal{P}}_{ki})$ is a **NSCSU** of Y.

Theorem 4.6 Suppose $\tau \mid Y \to X$ is a homomorphic mapping of G-algebra. Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be **NSCSU** of Y where $i \in k$. If $\mathrm{rsup}\{\mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\} = \mathrm{rmin}\{\mathrm{rsup}A^{\varrho}_{e_i}(t_1), \mathrm{rsup}A^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in X$. Then $\tau(\bigcup_{P\, i\in k} \tilde{\mathcal{P}}_k)$ is a **NSCSU** of X.

Proof. Let $\tilde{\mathcal{P}}_k = (A^{\varrho}_{e_i}, \lambda^{\varrho}_{e_i})$ be **NSCSU** of Y where $i \in k$ satisfying $\mathrm{rsup}\{\mathrm{rmin}\{A^{\varrho}_{e_i}(t_1), A^{\varrho}_{e_i}(t_2)\}\}$=$\mathrm{rmin}\{\mathrm{rsup}A^{\varrho}_{e_i}(t_1), \mathrm{rsup}A^{\varrho}_{e_i}(t_2)\}$ $\forall$ $t_1, t_2 \in Y$. Then by Theorem 3.8, $\bigcup_{\substack{P \\ i\in k}} \tilde{\mathcal{P}}_k$ is a **NSCSU** of Y. Hence $\tau(\bigcup_{\substack{P \\ i\in k}} \tilde{\mathcal{P}}_k)$ is a **NSCSU** of X.

Corollary 4.7 For a homomorphism $\tau \mid Y \to X$ of G-algebras, these results hold:

1. If $\forall$ $i \in k$, $\tilde{\mathcal{P}}_k$ are **NSCSU** of Y, then $\tau(\bigcap_{\substack{R \\ i\in k}} (\tilde{\mathcal{P}}_k))$ is **NSCSU** of X
2. If $\forall$ $i \in k$, $\tilde{\mathcal{N}}_k$ are **NSCSU** of X, then $\tau^{-1}(\bigcap_{\substack{R \\ i\in k}} (\tilde{\mathcal{N}}_k))$ is **NSCSU** of Y.

Proof. Straigtforward.

Theorem 4.8 Let τ be an isomorphic mapping from a G-algebra Y to a G-algebra X. If $\tilde{\mathcal{P}}_k$ is a **NSCSU** of Y. Then $\tau^{-1}(\tau(\tilde{\mathcal{P}}_k)) = \tilde{\mathcal{P}}_k$.

Proof. For any $t_1 \in Y$, let $\tau(t_1) = t_2$. Since τ is an isomorphism, $\tau^{-1}(t_2) = \{t_1\}$. Thus $\tau(\tilde{\mathcal{P}}_k)(\tau(t_1)) = \tau(\tilde{\mathcal{P}}_k)(t_2) = \bigcup_{t_1\in\tau^{-1}(t_2)} \tilde{\mathcal{P}}_k(t_1) = \tilde{\mathcal{P}}_k(t_1)$. For any $t_2 \in Y$, since τ is an isomorphism, $\tau^{-1}(t_2) = \{t_1\}$ so that $\tau(t_1) = t_2$. Thus $\tau^{-1}(\tilde{\mathcal{P}}_k)(t_1) = \tilde{\mathcal{P}}_k(\tau(t_1)) = \tilde{\mathcal{P}}_k(t_2)$. Hence, $\tau^{-1}(\tau(\tilde{\mathcal{P}}_k)) = \tau^{-1}(\tilde{\mathcal{P}}_k) = \tilde{\mathcal{P}}_k$.

5. Conclusions

In this paper, the concept of neutrosophic soft cubic subalgebra of G-algebra was investigated through several useful results. Homomorhic properties of NSCSU were also investigated. For future work this study will provide base for t-soft cubic subalgebra, t-neutrosophic soft cubic subalgebra.

References

1. Imai, Y. Iseki, K. On axiom systems of propositional Calculi, XIV proc. Jpn. Academy, 1966, 42, 19-22.
2. Zadeh, L. A. Fuzzy sets, Information and control, 1965, 8, 338-353.
3. Biswas, R. Rosenfeld's fuzzy subgroup with interval valued membership function, Fuzzy Sets and Systems, 1994, 63, 87-90.
4. Bandru, R. K. Rafi, N. On G-algebras, Sci. Magna, 2012, 8(3), 17.
5. Pramanik, S. Dalapati, S. Alam, S. Roy, T. K. Some Operations and Properties of Neutrosophic Cubic Soft Set. Glob J Res Rev 2017.
6. Senapati, T. Bipolar fuzzy structure of BG-algebras, The Journal of Fuzzy Mathematics, 2015, 23, 209-220.
7. Senapati, T. Jana, C. Bhowmik, M. Pal, M. L-fuzzy G-subalgebra of G-algebras, Journal of the Egyptian Mathematical Society, 2014, http://dx.doi.org/10.1016/j.joems. 2014.05.010.
8. Senapati, T. Bhowmik, M. Pal, M. Interval-valued intuitionistic fuzzy BG-subalgebras, The Journal of Fuzzy Mathematics, 2012, 20, 707-720.
9. Jun, Y. B. Kim, C. S. Yang, K. O. Cubic sets, Annuals of Fuzzy Mathematics and Informatics, 2012 4, 83-98.
10. Jun, Y. B. Jung, S. T. Kim, M. S. Cubic subgroup, Annals of Fuzzy Mathematics and Infirmatics, 2011, 2, 9-15.
11. Jun, Y. B. Smarandache, F. Kim, C. S. Neutrosophic Cubic Sets, New Math. and Cho, J. R. Kim, H. S Natural Comput, (2015) 8-41.
12. Jun, Y. B. Kim, C. S. Kang, M. S. Cubic Subalgebras and ideals of BCK/BCI-algebra, Far East Journal of Mathematical Sciences, (2010) 44, 239-250.
13. Senapati , T. Kim, C. H. Bhowmik, M. Pal, M. Cubic subalgebras and cubic closed ideals of B-algebras, Fuzzy. Inform. Eng., 7 (2015) 129-149.

14. D. Molodtsov, Soft set theory - First results, Comput. Math. Appl. 37 (1999), 19-31.
15. Muhiuddin, G. Feng, F. Jun, Y. B. Subalgerbas of BCK=BCI-algebras based on cubic soft sets, The Scientfic World Journal, Volume 2014, Article ID 458638, 9 pages.
16. Muhiuddin, G. Al-roqi, A. M. Cubic soft sets with applications in BCK=BCI-algebras, Ann. Fuzzy Math. Inform. 8(2) (2014), 291-304.
17. Jana, C. Senapati, T. Cubic G-subalgebras of G-algebras, jounral of Pure and Applied Mathematics (2015), Vol. 10, No.1, 105-115 ISSN: 2279-087X (P), 2279-0888(online).
18. ana, C. Senapati, T. Bhowmik, M. and Pal, M. On Intuitionistic Fuzzy G-subalgebras of G-algebras, Fuzzy Information and Engineering, 7, (2015), 195-209.
19. Kim, C. B. Kim, H. S. On BG-algebra, Demonstration Mathematica, (2008) 41, 497-505.
20. Smarandache, F. Neutrosophic set-a generalization of the intuitionistic fuzzy set, Int. J. Pure Appl. Math. (2005) 24(3), 287-297.
21. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, (American Reserch Press), (1999) Rehoboth.
22. Park, H. K. Kim, H.S. On quadratic B-algebras, Qausigroups and Related System, (2001) 7, 67-72.
23. Gulistan M, Hassan N. A Generalized Approach towards Soft Expert Sets via Neutrosophic Cubic Sets with Applications in Games. Symmetry. 2019 Feb;11(2):289.
24. Muhiuddin, G. Yaqoob, N. Rashid, Z. Smarandache, F. Wahab, H. A study on neutrosophic cubic graphs with real life applications in industries. Symmetry. 2018 Jun;10(6):203.
25. Muhiuddin, G. Khan, A. Abdullah, A. Yaqoob, N. Complex neutrosophic subsemigroups and ideals. International Journal of Analysis and Applications. 2018 Jan 1;16(1):97-116.
26. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics. Symmetry, 11(7), 903.
27. Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems, 1-21.

Received: 25 March, 2019; Accepted: 22 August, 2019