On MBJ – Neutrosophic β – Subalgebra

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Abstract: This paper studies about the definition of MBJ – Neutrosophic set in β – algebra, and introduce the concept of MBJ – Neutrosophic β – subalgebra. Homomorphic image and inverse image of MBJ – Neutrosophic β – subalgebra is provided. Also, Cartesian product of MBJ – Neutrosophic β – subalgebra is studied.

Keywords: MBJ–Neutrosophic set; MBJ–Neutrosophic β–subalgebra; MBJ-Neutrosophic Cartesian Product.

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1 Introduction

Zadeh [35, 36] introduced the fuzzy set to discuss uncertainty in many real requitals and as a generalization, the intuitionistic fuzzy set on an universe X was brought by Atanassov [8, 9]. The concept of Neutrosophic set is given by Smarandache [28, 29] with truth, indeterminate and false membership function and is explored to various dimensions by the authors of [10,16,17,18,32]. M. A. Basset et.al [1, 2, 3, 4, 5, 6] studies various topics in Neutrosophic set and its applications. As an extension the idea of MBJ – Neutrosophic structures was introduced in [34] where the BCK/BCI – algebra deals about a single binary operation (*).

The fuzzy sets have been connected in algebraic structure begins from Rosenfeld [27]. BCK – algebra is introduced by Iseki and Tanaka [8] and it has been analysed with several branches of fuzzy settings. As a generalization of BCK – algebra, Huang [11] and Iseki [14] discussed the notion of BCI – algebra. The structure of β – algebra was introduced by Neggers and Kim [25]. Also Jun and Kim [19] dealt some related topics on β – subalgebra. Later many researchers [7, 12, 33] developed to study β – algebra by relating with different fuzzy concepts. And as generalization of that, this paper applies the MBJ – Neutrosophic set in β–algebra and some results are given. The major difference of this work is handling an algebra with binary two operations (+ and −) whereas the existing other works involved single operation. This paper also provides a homomorphic image and pre-image of MBJ – Neutrosophic β – subalgebra and the cartesian product of MBJ – Neutrosophic β – subalgebra are also disputed.

2 Preliminaries

This part provides the essential definition and examples of β – algebra and some definitions of fuzzy sets.

2.1 Definition [7] A β – algebra is a non-empty set X with a constant 0 and binary operations + and – satisfying the following axioms:

i) $x - 0 = x$

ii) $(0 - x) + x = 0$

iii) $(x - y) - z = x - (y + z)$, for all $x, y, z \in X$.

2.2 Example Let X = {0, 1, 2, 3} be a set with constant 0 and two binary operations + and – are defined on X with the Cayley's table, then (X, +,-, 0) is a β – algebra.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

-	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

2.3 Definition [7] A non – empty subset S of a β – algebra (X, +, -, 0) is called a β – subalgebra of X, if
i) $x + y \in S$
ii) $x - y \in S, \forall\, x, y \in S$.

2.4 Example [33] Let X = {(0, 1, 2, 3),+,-,0} be a β – algebra with Cayley's table given above. Consider I_1 = {0, 2} and I_2 = {0, 1}. Then I_1 is a β – subalgebra of X, whereas I_2 does not satisfy the conditions to be an a β – subalgebra of X.

2.5 Definition [33] Let (X , +, -, 0_x) and (Y, +, -, 0_y) are β – algebras. A mapping $f : X \rightarrow Y$ is said to be a β – homomorphism if
i) $f(x + y) = f(x) + f(y)$
ii) $f(x - y) = f(x) - f(y), \forall\, x, y \in X$.

2.6 Definition A fuzzy set in a universal set X is defined as $\mu : X \rightarrow [0,1]$. For each $x \in X$, $\mu(x)$ is called the membership value of x.
2.7 Definition [9] An Intuitionistic fuzzy set in a non – empty set X is defined by
$A = \{ < x, \mu(x), \nu(x) > / \ x \in X\}$ where $\mu_A : X \rightarrow [0,1]$ is a membership function of A and
$\nu_A : X \rightarrow [0,1]$ is a non – membership function of A satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$,
$\forall x \in X$.

2.8 Definition [12] An Interval valued fuzzy set on X is defined by $A = \{ (x, \bar{\mu}_A(x))\}, \forall x \in X$ where $\bar{\mu}_A : X \rightarrow D[0,1]$ and $D[0,1]$ denotes the family of all closed subintervals of [0,1]. Here $\bar{\mu}_A(x) = [\,{\mu_A}^L(x), {\mu_A}^U(x)], \forall x \in X$ and ${\mu_A}^L, {\mu_A}^U$ are fuzzy sets.

Remark: Let us define refined minimum (briefly, $rmin$) and refined maximum (briefly, $rmax$) of two elements in $D[\,0,1]$. We also define the symbols $\geq, \leq, =$ in case of two elements in $D[\,0,1]$. Consider $D_1 = [a_1, b_1]$ and $D_2 = [a_2, b_2] \in D[0,1]$ then $rmin\ (D_1, D_2) = [\min(a_1, a_2)\,, \min(b_1, b_2)]$, $rmax\ (D_1, D_2) = [\max(a_1, a_2)\,, \max(b_1, b_2)]$ $D_1 \geq D_2$ if and only if $a_1 \geq a_2$, $b_1 \geq b_2$. Likewise, $D_1 \leq D_2$ and $D_1 = D_2$. For $D_i = [a_i, b_i] \in D[\,0,1]$, for i = 1,2,3…
We define $rsup_i(D_i) = [sup_i(a_i)\,, sup_i(b_i)]$ and $rinf_i(D_i) = [inf_i(a_i)\,, inf_i(b_i)]$.
Now, $D_1 \geq D_2$ if and only if $a_1 \geq a_2$, $b_1 \geq b_2$. Similarly, $D_1 \leq D_2$ and $D_1 = D_2$.

2.9 Definition [8] An Interval valued Intuitionistic fuzzy set A on X is defined by $A = \{ < x, \bar{\mu}(x), \bar{\nu}(x) > / \ x \in X\}$. Here $\bar{\mu}_A : X \rightarrow D[0,1]$ and $\bar{\nu}_A : X \rightarrow D[0,1]$ and $D[0,1]$ is denoted as the set of all subintervals of [0,1].
Here $\bar{\mu}_A(x) = [\,{\mu_A}^L(x), {\mu_A}^U(x)]$, $\bar{\nu}_A(x) = [\,{\nu_A}^L(x), {\nu_A}^U(x)]$ with the condition
$0 \leq {\mu_A}^L(x) + {\nu_A}^L(x) \leq 1$ and $0 \leq {\mu_A}^U(x) + {\nu_A}^U(x) \leq 1$.

2.10 Definition [28, 29] An Neutrosophic fuzzy set A on X is defined by
$A = \{ < x,\ A_T(x),\ A_I(x),\ A_F(x) > / \ x \in X\}$, where $A_T : X \rightarrow [\,0,1]$ is a truth membership function, A_I :

$X \rightarrow [0,1]$ is an indeterminate membership function and $A_F : X \rightarrow [0,1]$ is a false membership function.

2.11 Definition [34] Let X be a non – empty set. MBJ – Neutrosophic set in X, is a structure of the form $A = \{< x,\ M_A(x),\ \tilde{B}_A(x),\ J_A(x) >/\ x \in X\}$ where M_A and J_A are fuzzy sets in X and M_A is a truth membership function, J_A is a false membership function and $\tilde{B}_A$ is an interval valued fuzzy set in X and is an Indeterminate Interval Valued membership function.

2.12 Definition [12] the supremum property of the fuzzy set μ for the subset T in X is defined as $\mu(x_0) = \sup_{x \in T} \mu(x)$, if there exists $x, x_0 \in T$.

2.13 Definition [33] An Intuitionistic fuzzy set A with the degree membership $\mu_A : X \rightarrow [0,1]$ and the degree of non – membership function $\nu_A : X \rightarrow [0,1]$ is said to have $sup - inf$ property if for any subset T of X there exists $x_0 \in T$ such that $\mu_A(x_0) = \sup_{x \in T} \mu_A(x)$ and $\nu_A(x_0) = \inf_{x \in T} \nu_A(x)$

2.14 Definition

An Interval valued intuitionistic fuzzy set A in any set X is said to have the $rsup - rinf$ property if for subset T of X there exists $x_0 \in T$ such that $\bar{\mu}_A(x_0) = \underset{x \in T}{rsup}\, \bar{\mu}_A(x)$ and $\bar{\nu}_A(x_0) = \underset{x \in T}{rinf}\, \bar{\nu}_A(x)$ respectively.

In fuzzy theory, subsets are assumed to satisfy sup property, in intuitionistic fuzzy theory subsets are assumed to satisfy $sup - inf$ property and in interval valued intuitionistic fuzzy subsets are assumed to satisfy $rsup - rinf$ property. Analogously, in the following we define the notion of $sup - rsup - inf$ for an MBJ – Neutrosophic set.

2.15 Definition

An MBJ – Neutrosophic fuzzy set A in any set X is said to have the $sup - rsup - inf$ property if for subset T of X there exists $x_0 \in T$ such that $M_A(x_0) = \sup_{x \in T} M_A(x)$, $\tilde{B}_A(x_0) = \underset{x \in T}{rsup}\, \bar{B}_A(x)$ and $J_A(x_0) = \inf_{x \in T} J_A(x)$ respectively.

3 MBJ – Neutrosophic Structures in β – Subalgebra

This division frames the structure of MBJ – Neutrosophic β – subalgebra of β – algebra and some relevant results are discussed.

3.1 Definition

Let X be a β – algebra. An MBJ – Neutrosophic set $A = (M_A, \tilde{B}_A, J_A)$ in X is called an MBJ – Neutrosophic β – subalgebra of X if it satisfies:

i) $M_A(x + y) \geq \min(M_A(x), M_A(y))$; and ii) $M_A(x - y) \geq \min(M_A(x), M_A(y))$;
$\tilde{B}_A(x + y) \geq \text{rmin}(\tilde{B}_A(x), \tilde{B}_A(y))$; $\tilde{B}_A(x - y) \geq \text{rmin}(\tilde{B}_A(x), \tilde{B}_A(y))$;
$J_A(x + y) \leq \max(J_A(x), J_A(y))$ $J_A(x - y) \leq \max(J_A(x), J_A(y))$

3.2 Example

1) Consider a β – algebra $X = (\{0,1,2\}, +, -)$ by the following cayley's table

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

-	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

and the MBJ – Neutrosophic set on X is defined by

$$M_A(x) = \begin{cases} 0.4 & , x = 0 \\ 0.3 & , otherwise \end{cases} \qquad \tilde{B}_A(x) = \begin{cases} [0.3, 0.8] & , x = 0 \\ [0.1, 0.5] & , otherwise \end{cases}$$

$J_A(x) = \begin{cases} 0.1 & , x = 0 \\ 0.3 & , otherwise \end{cases}$

Thus, A satisfy the terms to be an MBJ - Neutrosophic β - subalgebra of X.

2) Let $X = \{ (0, a, b, c), +, - \}$ be a β -algebra with the following cayley's table.

+	0	a	b	c
0	0	a	b	c
a	a	b	c	0
b	b	c	0	a
c	c	0	a	b

-	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Here, the MBJ – Neutrosophic set $A = \{ < x,\ M_A(x), \tilde{B}_A(x), J_A(x)) > / \ x \in X\}$ on X is defined by

$M_A(x) = \begin{cases} 0.8, & x = 0 \\ 0.5, & x = b \\ 0.3, & x = a, c \end{cases}$ $\tilde{B}_A(x) = \begin{cases} [0.4,0.7], & x = 0 \\ [0.3,0.5], & x = b \\ [0.1,0.2], & x = a, c \end{cases}$

$J_A(x) = \begin{cases} 0.2, & x = 0 \\ 0.5, & x = b \\ 0.7, & x = a, c \end{cases}$ is an MBJ Neutrosophic β - subalgebra of X.

3.3 Theorem

If A_1 and A_2 are two MBJ Neutrosophic β - subalgebras of X, then $A_1 \cap A_2$ is an MBJ – Neutrosophic β - subalgebra of X.

Proof:

Let A_1 and A_2 be two MBJ – Neutrosophic β - subalgebra of X.

Now, $M_{A_1 \cap A_2}(x + y) = \min\{ M_{A_1}(x + y), M_{A_2}(x + y)\}$

$\geq \min\{\{M_{A_1}(x), M_{A_1}(y)\}, \min\{M_{A_2}(x), M_{A_2}(y)\}\}$

$= \min\{\{M_{A_1}(x), M_{A_2}(x)\}, \{M_{A_1}(y), M_{A_2}(y)\}\}$

$\geq \min\{M_{A_1 \cap A_2}(x), M_{A_1 \cap A_2}(y)\}$

$M_{A_1 \cap A_2}(x + y) \geq \min\{M_{A_1 \cap A_2}(x), M_{A_1 \cap A_2}(y)\}$.

Similarly, $M_{A_1 \cap A_2}(x - y) \geq \min\{M_{A_1 \cap A_2}(x), M_{A_1 \cap A_2}(y)\}$.

$\tilde{B}_{A_1 \cap A_2}(x + y) = [B^L_{A_1 \cap A_2}(x + y), B^U_{A_1 \cap A_2}(x + y)]$

$= [\min\left(B^L_{A_1}(x + y), B^L_{A_2}(x + y)\right), \min\left(B^U_{A_1}(x + y), B^U_{A_2}(x + y)\right)]$

$\geq [\min(B^L_{A_1 \cap A_2}(x), B^L_{A_1 \cap A_2}(y)), \min(B^U_{A_1 \cap A_2}(x), B^U_{A_1 \cap A_2}(y))]$

$= rmin\{\tilde{B}_{A_1 \cap A_2}(x), \tilde{B}_{A_1 \cap A_2}(y)\}$

$\tilde{B}_{A_1 \cap A_2}(x + y) \geq rmin\{\tilde{B}_{A_1 \cap A_2}(x), \tilde{B}_{A_1 \cap A_2}(y)\}$

Similarly, $\tilde{B}_{A_1 \cap A_2}(x - y) \geq rmin\{\tilde{B}_{A_1 \cap A_2}(x), \tilde{B}_{A_1 \cap A_2}(y)\}$

$J_{A_1 \cap A_2}(x + y) = \max\{ J_{A_1}(x + y), J_{A_2}(x + y)\}$

$\leq \max\{\{J_{A_1}(x), J_{A_1}(y)\}, \max\{J_{A_2}(x), J_{A_2}(y)\}\}$

$= \max\{\{J_{A_1}(x), J_{A_2}(x)\}, \{J_{A_1}(y), J_{A_2}(y)\}\}$

$\leq \max\{J_{A_1 \cap A_2}(x), J_{A_1 \cap A_2}(y)\}$

$J_{A_1 \cap A_2}(x + y) \leq \max\{J_{A_1 \cap A_2}(x), J_{A_1 \cap A_2}(y)\}$.

Similarly, $J_{A_1 \cap A_2}(x - y) \leq \max\{J_{A_1 \cap A_2}(x), J_{A_1 \cap A_2}(y)\}$.

Thus, $A_1 \cap A_2$ is an MBJ – Neutrosophic β - subalgebra of X.

3.4 Lemma

Let A be an MBJ – Neutrosophic β - subalgebra of X, then

$i)\ M_A(0) \geq M_A(x), \tilde{B}_A(0) \geq \tilde{B}_A(x)\ and\ J_A(0) \leq J_A(x)$,

$ii) M_A(0) \geq M_A(x^*) \geq M_A(x), \tilde{B}_A(0) \geq \tilde{B}_A(x^*) \geq \tilde{B}_A(x)\ and\ J_A(0) \leq J_A(x^*) \leq J_A(x)$, where $x^* = 0 - x$, $\forall x \in X$.

Proof:

i) For any $x \in X$.

$M_A(0) = M_A(x - x) \geq \min\big(M_A(x), M_A(x)\big)$

$= M_A(x)$

Therefore, $M_A(0) \geq M_A(x)$.

$\tilde{B}_A(0) = [\, B_A^L(0), B_A^U(0)]$

$\geq [\, B_A^L(x), B_A^U(x)]$

$= \tilde{B}_A(x)$

$J_A(0) = J_A(x - x) \leq \max\big(J_A(x), J_A(x)\big) = J_A(x)$

Thus, $J_A(0) \leq J_A(x)$.

ii) Also for $x \in X$,

$M_A(x^*) = M_A(0 - x) \geq \min\big(M_A(0), M_A(x)\big)$

$= M_A(x)$

$Hence, M_A(x^*) \geq M_A(x)$.

$\tilde{B}_A(x^*) = [\, B_A^L(x^*), B_A^U(x^*)]$

$= [B_A^L(0 - x), B_A^U(0 - x)]$

$= [\min\big(B_A^L(0), B_A^U(x)\big), \min(B_A^L(0), B_A^U(x)\big)]$

$\geq [\, B_A^L(x), B_A^U(x)]$

$= \tilde{B}_A(x)$

$\therefore \tilde{B}_A(0) \geq \tilde{B}_A(x^*) \geq \tilde{B}_A(x)$

$J_A(x^*) = J_A(0 - x) \leq \max\big(J_A(0), J_A(x)\big) = J_A(x)$

Thus, $J_A(0) \leq J_A(x^*) \leq J_A(x)$.

3.5 Theorem

If there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} M_A(x_n) = 1$, $\lim_{n\to\infty} \tilde{B}_A(x_n) = [1,1]$, $\lim_{n\to\infty} J_A(x_n) = 0$. And A be an MBJ – Neutrosophic β - subalgebra of X. Then

$M_A(0) = 1, \tilde{B}_A(0) = [1,1], and\ J_A(0) = 0.$

Proof:

Since, $M_A(0) \geq M_A(x), \forall x \in X$,

$M_A(0) \geq M_A(x_n)$.

Similarly, $\tilde{B}_A(0) \geq \tilde{B}_A(x_n)$ and $J_A(0) \leq J_A(x_n)$ for every positive integer n.

Note that, $1 \geq M_A(0) \geq \lim_{n\to\infty} M_A(x_n) = 1$,.

Hence $M_A(0) = 1$.

$[1,1] \geq \tilde{B}_A(0) \geq \lim_{n\to\infty} \tilde{B}_A(x_n) = [1,1]$

Implies $\tilde{B}_A(0) = [1,1]$

Also $0 \leq J_A(0) \leq \lim_{n\to\infty} J_A(x_n) = 0$.

Therefore, $J_A(0) = 0$.

3.6 Theorem

Given $A = \big(M_A, \tilde{B}_A, J_A\big)$ in X such that (M_A, J_A) is an intuitionistic fuzzy subalgebra of X and B_A^L, B_A^U are fuzzy subalgebra of X, then $A = \big(M_A, \tilde{B}_A, J_A\big)$ is an MBJ – Neutrosophic β - subalgebra of X.

Proof:

To prove this it's enough to verify that $\tilde{B}_A$ satisfies the conditions:

$\forall\, x, y \in X$.

$\tilde{B}_A(x + y) \geq rmin\{\tilde{B}_A(x), \tilde{B}_A(y)\}$

$\tilde{B}_A(x - y) \geq rmin\{\tilde{B}_A(x), \tilde{B}_A(y)\}$

For any $x, y \in X$, we get

$\tilde{B}_A(x + y) = [\, B_A^L(x + y), B_A^U(x + y)]$

$\geq [\min\{\, B_A^L(x), B_A^L(y)\}, \min\{\, B_A^U(x), B_A^U(y)\}]$

$= rmin\{[\, B_A^L(x), B_A^U(x)], [\, B_A^L(y), B_A^U(y)]\}$

$= rmin\{\, \tilde{B}_A(x), \tilde{B}_A(y)\}$

$\tilde{B}_A(x + y) \geq rmin\{\, \tilde{B}_A(x), \tilde{B}_A(y)\}$

Similarly, $\tilde{B}_A(x - y) \geq rmin\{\, \tilde{B}_A(x), \tilde{B}_A(y)\}$

$\tilde{B}_A$ satisfies the condition
$\therefore\ A = \left(M_A, \tilde{B}_A, J_A\right)$ is an MBJ – Neutrosophic β - subalgebra of X.

3.7 Theorem
If $A = \left(M_A, \tilde{B}_A, J_A\right)$ is an MBJ - Neutrosophic β - subalgebra of X. Then the sets $X_{M_A} = \{\, x\ \in X / M_A(x) = M_A(0)\}$; $X_{\tilde{B}_A} = \{\, x\ \in X / \tilde{B}_A(x) = \tilde{B}_A(0)\}$ and $X_{J_A} = \{\, x\ \in X / J_A(x) = J_A(0)\}$ are subalgebra of X.
Proof:
For any $x, y\ \in\ X_{M_A}$.
Then $M_A(x) = M_A(0) = M_A(y)$
$M_A(x + y)\ \geq \min(M_A(x), M_A(y))$
$= \min\left(M_A(0), M_A(0)\right) = M_A(0)$
And $M_A(x - y) \geq \min(M_A(x), M_A(y))$
$= \min\left(M_A(0), M_A(0)\right) = M_A(0)$
$x + y\ and\ x - y\ \in\ X_{M_A}$
Therefore, X_{M_A} is a subalgebra of X.
Let $x, y\ \in\ X_{\tilde{B}_A}$, then $\tilde{B}_A(x) = \tilde{B}_A(0) = \tilde{B}_A(y)$.
Now, $\tilde{B}_A(x + y) \geq rmin\{\tilde{B}_A(x), \tilde{B}_A(y)\}$
$= rmin\{\tilde{B}_A(0), \tilde{B}_A(0)\} = \tilde{B}_A(0)$
$\therefore \tilde{B}_A(x + y) \geq\ \tilde{B}_A(0)$
Similarly, $\tilde{B}_A(x - y) \geq\ \tilde{B}_A(0)$
$\therefore X_{\tilde{B}_A}$ is a subalgebra of X.
Let $x, y\ \in\ X_{J_A}$
$J_A(x) = J_A(0) = J_A(y)$
Now, $J_A(x + y)\ \leq \max(J_A(x), J_A(y))$
$= \max\left(J_A(0), J_A(0)\right)$
$= J_A(0)$
$J_A(x - y)\ \leq \max(J_A(x), J_A(y))$
$= \max\left(J_A(0), J_A(0)\right)$
$= J_A(0)$
$\therefore\ x + y\ and\ x - y\ \in\ X_{J_A}$
X_{J_A} is a subalgebra of X.

3.8 Definition
$A = \{< x,\ M_A(x), \tilde{B}_A(x), J_A(x) >/\ x \in X\}$ be an MBJ – Neutrosophic set in X and f be mapping from X into Y then the image of A under f, $f(A)$ is defined as,
$f(A) = \left\{< x, f_{sup}(M_A), f_{rsup}\left(\tilde{B}_A\right), f_{inf}(J_A) >/\ x \in Y\right\}$ where

$$f_{sup}(M_A)(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} M_A(x), & if\ f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$

$$f_{rsup}\left(\tilde{B}_A\right)(y) = \begin{cases} \operatorname{rsup}\limits_{x\in f^{-1}(y)} \tilde{B}_A(x), & if\ f^{-1}(y) \neq \emptyset \\ [1,1] & otherwise \end{cases}$$

$$f_{inf}(J_A)(y) = \begin{cases} \inf\limits_{x\in f^{-1}(y)} J_A(x), & if\ f^{-1}(y) \neq \emptyset \\ 1 & otherwise \end{cases}$$

3.9 Definition [34]
Let $f : X\ \to Y$ be a function. Let A and B be the two MBJ – Neutrosophic β- subalgebra in X and Y respectively. Then inverse image of B under f is defined by
$f^{-1}(B) = \{x, f^{-1}\left(M_B(x)\right), f^{-1}\left(\tilde{B}_B(x)\right), f^{-1}\left(J_B(x)\right)\ /\ x\ \in X\}$ such that
$f^{-1}\left(M_B(x)\right) = M_B(f(x))$; $f^{-1}\left(\tilde{B}_B(x)\right) = \tilde{B}_B\left(f(x)\right)$ and $f^{-1}\left(J_B(x)\right) = J_B(f(x))$.

3.10 Theorem

Let $(X,+,-,0)$ and $(Y,+,-,0)$ be two β – algebras and $f: X \rightarrow Y$ be an homomorphism. If A is an MBJ – Neutrosophic β – subalgebra of X, define

$f(A) = \{< x, M_f(x) = M\big(f(x)\big), \tilde{B}_f(x) = \tilde{B}\big(f(x)\big), J_f(x) = J\big(f(x)\big) >/\ x \in X\}$. Then $f(A)$ is an MBJ – Neutrosophic β – subalgebra of Y.

Proof:

Let $x, y \in X$.

Now, $M_f(x+y) = M(f(x+y))$

$= M(f(x)+f(y))$

$\geq \min\{M\big(f(x)\big), M\big(f(y)\big)\}$

$= \min\{M_f(x), M_f(y)\}$

$M_f(x+y) \geq \min\{M_f(x), M_f(y)\}$

Similarly, $M_f(x-y) \geq \min\{M_f(x), M_f(y)\}$

$\tilde{B}_f(x+y) = \tilde{B}(f(x+y))$

$= \tilde{B}(f(x)+f(y))$

$\geq \text{rmin}\{\tilde{B}\big(f(x)\big), \tilde{B}\big(f(y)\big)\}$

$= \text{rmin}\{\tilde{B}_f(x), \tilde{B}_f(y)\}$

$\tilde{B}_f(x+y) \geq \text{rmin}\{\tilde{B}_f(x), \tilde{B}_f(y)\}$

Similarly, $\tilde{B}_f(x-y) \geq \text{rmin}\{\tilde{B}_f(x), \tilde{B}_f(y)\}$

$J_f(x+y) = J(f(x+y)) = J(f(x)+f(y))$

$\leq max\{J\big(f(x)\big), J\big(f(y)\big)\}$

$= max\{J\big(f(x)\big), J\big(f(y)\big)\}$

$J_f(x+y) \leq \max\{J_f(x), J_f(y)\}$

Similarly, $J_f(x-y) \leq \max\{J_f(x), J_f(y)\}$

Hence $f(A)$ is an MBJ – Neutrosophic β – subalgebra of Y.

3.11 Theorem

Let $f : X \rightarrow Y$ be a homomorphism of β – algebra X into a β – algebra Y. If $A = \{< x,\ M_A(x), B_A(x), J_A(x) >/\ x \in X\}$ is an MBJ – Neutrosophic β – subalgebra of X, then the image $f(A) = \big\{< x, f_{sup}(M_A), f_{rsup}\big(\tilde{B}_A\big), f_{inf}(J_A) >/\ x \in X\ \big\}$ of A under f is an MBJ – Neutrosophic β – subalgebra of Y.

Proof:

$A = \{< x, M_A(x), B_A(x), J_A(x) >/\ x \in X\}$ be an MBJ – Neutrosophic β – subalgebra of X.

Let $y_1, y_2 \in Y$

$\therefore \{x_1 + x_2 : x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2), \} \subseteq \{x \in X : x \in f^{-1}(y_1+y_2)\}$

Now,

$f_{sup}\{M_A(y_1+y_2)\} = \sup\{M_A(x)\ /\ x \in f^{-1}(y_1+y_2)\}$

$\geq \sup\{M_A(x_1+x_2)/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \sup\{\min\{M_A(x_1), M_A(x_2)\}/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$= \min\{\sup\{M_A(x_1)/x_1 \in f^{-1}(y_1)\}, \sup\{M_A(x_2)/x_2 \in f^{-1}(y_2)\}\}$

$= \min\{f_{sup}(M_A(y_1)), f_{sup}(M_A(y_2))\}$

Similarly $f_{sup}\{M_A(y_1-y_2)\} \geq \min\{f_{sup}\big(M_A(y_1)\big), f_{sup}\big(M_A(y_2)\big)\}$

$f_{rsup}\{\tilde{B}_A(y_1+y_2)\} = rsup\{\tilde{B}_A(x)/x \in f^{-1}(y_1+y_2)\}$

$\geq \text{rsup}\{\tilde{B}_A(x_1+x_2)/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\geq \text{rsup}\{\text{rmin}\{\tilde{B}_A(x_1), \tilde{B}_A(x_2)\}/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$= r\text{min}\{r\text{sup}\{\tilde{B}_A(x_1)/x_1 \in f^{-1}(y_1)\}, r\sup\{\tilde{B}_A(x_2)/x_2 \in f^{-1}(y_2)\}\}$

$\geq \text{rmin}\{f_{rsup}(\tilde{B}_A(y_1)), f_{sup}(\tilde{B}_A(y_2))\}$

$f_{rsup}\{\tilde{B}_A(y_1+y_2)\} \geq \text{rmin}\{f_{rsup}(\tilde{B}_A(y_1)), f_{sup}(\tilde{B}_A(y_2))\}$

Similarly, $f_{rsup}\{\tilde{B}_A(y_1+y_2)\} \geq \text{rmin}\{f_{rsup}(\tilde{B}_A(y_1)), f_{sup}(\tilde{B}_A(y_2))\}$

$f_{inf}\{J_A(y_1+y_2)\} = inf\{J_A(x)\ /\ x \in f^{-1}(y_1+y_2)\}$

$\leq \inf\{J_A(x_1+x_2)/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$\leq \inf\{\max\{J_A(x_1), J_A(x_2)\}/x_1 \in f^{-1}(y_1), x_2 \in f^{-1}(y_2)\}$

$= \max\{\inf\{J_A(x_1)/x_1 \in f^{-1}(y_1)\}, \inf\{J_A(x_2)/x_2 \in f^{-1}(y_2)\}\}$
$= \max\{f_{inf}(J_A(y_1)), f_{inf}(J_A(y_2))\}$

Similarly, $f_{inf}\{J_A(y_1 + y_2)\} \leq \max\{f_{inf}(J_A(y_1)), f_{inf}(J_A(y_2))\}$.

3.12 Theorem

Let $f : X \to Y$ be a homomorphism of β – algebra. If $B = (M_B, \tilde{B}_B J_B)$ is an MBJ-Neutrosophic β – subalgebra of Y. Then $f^{-1}(B) = < (f^{-1}(M_B), f^{-1}(B_B), f^{-1}(J_B)) >$ is an MBJ – Neutrosophic β – subalgebra of X, where $f^{-1}(M_B(x)) = M_B(f(x))$; $f^{-1}(\tilde{B}_B(x)) = \tilde{B}_B(f(x))$ and $f^{-1}(J_B(x)) = J_B(f(x))$, for all $x \in X$.

Proof:

Let B be an MBJ – Neutrosophic β – subalgebra of Y and let $x, y \in X$

Then $f^{-1}(M_B)(x + y) = M_B(f(x + y))$
$= M_B(f(x) + f(y))$
$\geq \min\{M_B f(x)) + M_B f(y)\}$
$= \min\{f^{-1}(M_B(x)) + f^{-1}(M_B(y))\}$

$f^{-1}(M_B)(x + y) \geq \min\{f^{-1}(M_B(x)) + f^{-1}(M_B(y))\}$.

Similarly, $f^{-1}(M_B)(x - y) \geq \min\{f^{-1}(M_B(x)) + f^{-1}(M_B(y))\}$

$f^{-1}(\tilde{B}_B)(x + y) = \tilde{B}_B(f(x + y))$
$= \tilde{B}_B(f(x) + f(y))$
$\geq rmin\{\tilde{B}_B(f(x)), \tilde{B}_B(f(y))\}$
$= r\min\{f^{-1}(\tilde{B}_B(x)), f^{-1}(\tilde{B}_B(y))\}$

$f^{-1}(\tilde{B}_B)(x + y) \geq r\min\{f^{-1}(\tilde{B}_B(x)), f^{-1}(\tilde{B}_B(y))\}$

Similarly, $f^{-1}(\tilde{B}_B)(x - y) \geq r\min\{f^{-1}(\tilde{B}_B(x)), f^{-1}(\tilde{B}_B(y))\}$

$f^{-1}(J_B)(x + y) = J_B(f(x + y))$
$= J_B(f(x) + f(y))$
$\leq \max\{J_B f(x) + J_B f(y)\}$
$= \max\{f^{-1}(J_B(x)) + f^{-1}(J_B(y))\}$

$f^{-1}(J_B)(x + y) \leq \min\{f^{-1}(J_B(x)) + f^{-1}(J_B(y))\}$.

Similarly, $f^{-1}(J_B)(x - y) \leq \min\{f^{-1}(J_B(x)) + f^{-1}(J_B(y))\}$.

Hence $f^{-1}(B) = (f^{-1}(M_B), f^{-1}(\tilde{B}_B), f^{-1}(J_B))$ is an MBJ – Neutrosophic β – subalgebra of X.

4 Product of MBJ – Neutrosophic Subalgebra

In this section the Cartesian product of the two MBJ – Neutrosophic β - subalgebra A and B of X and Y respectively is given.

4.1 Definition [12,33]

Let $A = \{< x,\ M_A(x), \tilde{B}_A(x), J_A(x) >/ x \in X\}$ and $B = \{< y,\ M_A(y), \tilde{B}_A(y), J_A(y))>/ y \in Y\}$ be two MBJ – Neutrosophic sets of X and Y respectively. The Cartesian product of A and B is denoted by $A \times B$ is defined as $A \times B = \{< (x, y),\ M_{A\times B}(x, y), \tilde{B}_{A\times B}(x, y), J_{A\times B}(x, y) >/ (x, y) \in X \times Y\}$ where

$M_{A\times B} : X \times Y \to [0,1]$, $\tilde{B}_{A\times B} : X \times Y \to D[0,1]$, $J_{A\times B} : X \times Y \to [0,1]$.

$M_{A\times B}(x, y) = \min\{M_A(x), M_A(y)\}$, $\tilde{B}_{A\times B}(x, y) = r\min\{\tilde{B}_A(x), \tilde{B}_A(y)\}$ and

$J_{A\times B}(x, y) = \max\{J_A(x), J_A(y)\}$.

4.2 Theorem

Let A *and* B be two MBJ – Neutrosophic β - subalgebra of X and Y respectively. Then $A \times B$ is also an

MBJ – Neutrosophic β - subalgebra of $X \times Y$.

Proof: Let A and B be an MBJ – Neutrosophic β - subalgebra of X and Y respectively.

Take $x = (x_1, x_2)$ and $y = (y_1, y_2) \in X \times Y$.

$$\begin{aligned} \text{Now, } M_{A\times B}(x+y) &= M_{A\times B}\big((x_1,x_2)+(y_1,y_2)\big) \\ &= M_{A\times B}\big((x_1+y_1),(y_1+y_2)\big) \\ &= \min\{M_A\big((x_1+y_1)\big), M_B\big((y_1+y_2)\big)\} \\ &\geq \min\{\min\big(M_A(x_1), M_B(y_1)\big), \min\big(M_A(x_2), M_B(y_2)\big)\} \\ &= \min\{\min\big(M_A(x_1), M_B(x_2)\big), \min\big(M_A(y_1), M_B(y_2)\big)\} \\ &= \min\{(M_{A\times B})(x_1,x_2), (M_{A\times B})(y_1,y_2)\} \\ &= \min(M_{A\times B})(x), (M_{A\times B})(y)\} \end{aligned}$$

$M_{A\times B}(x+y) \geq \min\{(M_A \times M_B)(x), (M_A \times M_B)(y)\}$.

Similarly, $M_{A\times B}(x-y) \geq \min\{(M_A \times M_B)(x), (M_A \times M_B)(y)\}$

$$\begin{aligned} \tilde{B}_{A\times B}(x+y) &= \tilde{B}_{A\times B}\big((x_1,x_2)+(y_1,y_2)\big) \\ &= \tilde{B}_{A\times B}\big((x_1+y_1),(y_1+y_2)\big) \\ &= rmin\{\tilde{B}_A(x_1+y_1), \tilde{B}_A(x_2+y_2)\} \\ &= rmin\{rmin(\tilde{B}_A(x_1), \tilde{B}_B(x_2)\}, rmin(\tilde{B}_A(y_1), \tilde{B}_B(y_2)\} \\ &= rmin\{\tilde{B}_{A\times B}(x_1,x_2), \tilde{B}_{A\times B}(y_1,y_2)\} \\ &\geq rmin\{\tilde{B}_{A\times B}(x), \tilde{B}_{A\times B}(y)\} \end{aligned}$$

$\tilde{B}_{A\times B}(x+y) \geq rmin\{\tilde{B}_{A\times B}(x), \tilde{B}_{A\times B}(y)\}$

Similarly, $\tilde{B}_{A\times B}(x-y) \geq rmin\{\tilde{B}_{A\times B}(x), \tilde{B}_{A\times B}(y)\}$

$$\begin{aligned} J_{A\times B}(x+y) &= J_{A\times B}\big((x_1,x_2)+(y_1,y_2)\big) \\ &= J_{A\times B}\big((x_1+y_1),(y_1+y_2)\big) \\ &= \max\{J\big((x_1+y_1)\big), J_B\big((y_1+y_2)\big)\} \\ &\geq \max\{\max\big(J(x_1), J(y_1)\big), \max\big(J_A(x_2), J(y_2)\big)\} \\ &= \max\{\max\big(J_A(x_1), J_B(x_2)\big), \max\big(J_A(y_1), J_B(y_2)\big)\} \\ &= \max\{(J_A \times J_B)(x_1,x_2), (J_A \times J_B)(y_1,y_2)\} \\ &= \max(J_A \times J_B)(x), (J_A \times J_B)(y)\} \end{aligned}$$

$J_{A\times B}(x+y) \leq \max(J_A \times J_B)(x), (J_A \times J_B)(y)\}$

Similarly, $J_{A\times B}(x-y) \leq \max(J_A \times J_B)(x), (J_A \times J_B)(y)\}$.

Thus, $A \times B$ is also an MBJ – Neutrosophic β - subalgebra of $X \times Y$.

4.3 Theorem

Let $A_i = \{x \in X_i: M_{A_i}(x), \tilde{B}_{A_i}(x), J_{A_i}(x)\}$ be an MBJ – Neutrosophic β - subalgebra of X_i, i=1,2,…n. Then $\prod_{i=1}^{n} A_i$ is called direct product of finite MBJ – Neutrosophic β - subalgebra of $\prod_{i=1}^{n} X_i$ if

$i)$ $\prod_{i=1}^{n} M_{A_i}(x_i+y_i) \geq \min\{\prod_{i=1}^{n} M_{A_i}(x_i), \prod_{i=1}^{n} M_{A_i}(y_i)\}$

$\prod_{i=1}^{n} \tilde{B}_{A_i}(x_i + y_i) \geq r\min\{\prod_{i=1}^{n} \tilde{B}_{A_i}(x_i), \prod_{i=1}^{n} \tilde{B}_{A_i}(y_i)\}$

$\prod_{i=1}^{n} J_{A_i}(x_i + y_i) \leq \max\{\prod_{i=1}^{n} J_{A_i}(x_i), \prod_{i=1}^{n} J_{A_i}(y_i)\}$

ii) $\prod_{i=1}^{n} M_{A_i}(x_i - y_i) \geq \min\{\prod_{i=1}^{n} M_{A_i}(x_i), \prod_{i=1}^{n} M_{A_i}(y_i)\}$

$\prod_{i=1}^{n} \tilde{B}_{A_i}(x_i - y_i) \geq rmin\{\prod_{i=1}^{n} \tilde{B}_{A_i}(x_i), \prod_{i=1}^{n} \tilde{B}_{A_i}(y_i)\}$

$\prod_{i=1}^{n} J_{A_i}(x_i - y_i) \leq \max\{\prod_{i=1}^{n} J_{A_i}(x_i), \prod_{i=1}^{n} J_{A_i}(y_i)\}$.

Proof: The prove is clear by induction and using Theorem 4.2.

4.4 Theorem

Let $A_i = \{ x \in X_i: M_{A_i}(x), \tilde{B}_{A_i}(x), J_{A_i}(x)\}$ be an MBJ – Neutrosophic β - subalgebra of X_i, respectively for i=1,2,…n. Then $\prod_{i=1}^{n} A_i$ is an MBJ – Neutrosophic β - subalgebra of $\prod_{i=1}^{n} X_i$.

Proof: Let A be an MBJ – Neutrosophic β - subalgebra of X_i

Let $(x_1, x_2, \dots x_n)$ and $(y_1, y_2, \dots y_n) \in \prod_{i=1}^{n} X_i$

Take $a = (x_1, x_2, \dots x_n)$ and $b = (y_1, y_2, \dots y_n)$

Then

$\prod_{i=1}^{n} M_{A_i}(a + b) \geq \min\{M_{A_1}(a + b), \dots \dots . M_{A_n}(a + b)\}$

$= \min\{\min\{M_{A_1}(a), M_{A_1}(bn)\}, \dots \dots \dots . \min\{M_{A_n}(a), M_{A_n}(b)\}\}$

$= \min\{\min\{M_{A_1}(a), \dots \dots M_{A_n}(a)\}, \min\{M_{A_1}(b), \dots \dots M_{A_n}(b)\}\}$

$= \min\{\prod_{i=1}^{n} M_{A_i}(a), \prod_{i=1}^{n} M_{A_i}(b)\}$

$\prod_{i=1}^{n} M_{A_i}(a + b) \geq \min\{\prod_{i=1}^{n} M_{A_i}(a), \prod_{i=1}^{n} M_{A_i}(b)\}\}$

Similarly, $\prod_{i=1}^{n} M_{A_i}(a - b) \geq \min\{\prod_{i=1}^{n} M_{A_i}(a), \prod_{i=1}^{n} M_{A_i}(b)\}\}$

$\prod_{i=1}^{n} \tilde{B}_{A_i}(a + b) \geq \min\{\tilde{B}_{A_1}(a + b), \dots \dots . \tilde{B}_{A_n}(a + b)\}$

$= r\min\{r \min\{\tilde{B}_{A_1}(a), \tilde{B}_{A_1}(b)\}, \dots \dots \dots . \min\{\tilde{B}_{A_n}(a), \tilde{B}_{A_n}(b)\}\}$

$= r\min\{\text{rmin}\{\tilde{B}_{A_1}(a), \dots \dots \tilde{B}_{A_n}(a)\}, \min\{\tilde{B}_{A_1}(b), \dots \dots \tilde{B}_{A_n}(b)\}\}$

$= r\min\{\prod_{i=1}^{n} \tilde{B}_{A_i}(a), \prod_{i=1}^{n} \tilde{B}_{A_i}(b)\}$

$\prod_{i=1}^{n} \tilde{B}_{A_i}(a + b) \geq \text{rmin}\{\prod_{i=1}^{n} \tilde{B}_{A_i}(a), \prod_{i=1}^{n} \tilde{B}_{A_i}(b)\}\}$

Similarly, $\prod_{i=1}^{n} \tilde{B}_{A_i}(a - b) \geq \text{rmin}\{\prod_{i=1}^{n} \tilde{B}_{A_i}(a), \prod_{i=1}^{n} \tilde{B}_{A_i}(b)\}\}$

$\prod_{i=1}^{n} J_{A_i}(a + b) \leq \max\{J_{A_1}(a + b), \dots \dots . J_{A_n}(a + b)\}$

$= \max\{\max\{J_{A_1}(a), J_{A_1}(b)\}, \dots \dots \dots . \max\{J_{A_n}(a), J_{A_n}(b)\}\}$

$= \max\{\max\{J_{A_1}(a), \dots \dots J_{A_n}(a)\}, \max\{J_{A_1}(b), \dots \dots J_{A_n}(b)\}\}$

$= \max\{\prod_{i=1}^{n} J_{A_i}(a), \prod_{i=1}^{n} J_{A_i}(b)\}$

$\prod_{i=1}^{n} J_{A_i}(a + b) \leq \max\{\prod_{i=1}^{n} J_{A_i}(a), \prod_{i=1}^{n} J_{A_i}(b)\}\}$

Similarly, $\prod_{i=1}^{n} J_{A_i}(a - b) \leq \max\{\prod_{i=1}^{n} J_{A_i}(a), \prod_{i=1}^{n} J_{A_i}(b)\}\}$

Thus, $\prod_{i=1}^{n} A_i$ is an MBJ – Neutrosophic β - subalgebra of $\prod_{i=1}^{n} X_i$.

Conclusion

Here, the MBJ – Neutrosophic substructure on β – algebra was introduced in double operations $+$ and $-$. Further, the study analysed the MBJ – Neutrosophic β – subalgebra using Homomorphic image, inverse image and Cartesian product. The same ideas can be extended to some other substructures like ideal, H- ideal and filters of a β – algebra for a future scope.

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