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On Pythagoras Triples in Symbolic 3-Plithogenic Rings

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Abstract:

The objective of this paper is to find necessary and sufficient conditions for a symbolic 3-plithogenic triple

$(t_0 + t_1P_1 + t_2P_2 + t_3P_3, s_0 + s_1P_1 + s_2P_2 + s_3P_3, k_0 + k_1P_1 + k_2P_2 + k_3P_3)$ to be a Pythagoras triple, i.e. to be a solution for the non-linear Diophantine equation

$X^2 + Y^2 = Z^2$. Also, many examples will be illustrated and presented to explain how the theorems work.

Keywords: symbolic 3-plithogenic ring, Pythagoras triple, Pythagoras Diophantine equation

Introduction and Preliminaries.

Symbolic n-plithogenic sets were defined by Smarandache in [1-3], where these sets were used in generalizing classical algebraic structures such as symbolic 2-plithogenic and symbolic 3-plithogenic structures [4-9], with many applications in other fields [10-12].

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It is useful to refer that symbolic n-plithogenic algebraic structures are very similar to neutrosophic and refined neutrosophic structures, see [13-21].

In this paper, we continue other efforts to study Pythagoras triples in many different rings [22-25].

We present the concept of Pythagoras triple in a symbolic 3-plithogenic commutative ring with many clear examples that clarify the validity of our work.

Definition.

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:

$3-SP_R=\{a_0+a_1P_1+a_2P_2+a_3P_3;\ a_i\in R, {P_j}^2=P_j, P_i\times P_j=P_{max(i,j)}\}$.

Smarandache has defined algebraic operations on $3-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2+a_3P_3]+[b_0+b_1P_1+b_2P_2+b_3P_3]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2+(a_3+b_3)P_3$.

Multiplication:

$[a_0+a_1P_1+a_2P_2+a_3P_3].[b_0+b_1P_1+b_2P_2+b_3P_3]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_0b_3P_3+a_1b_0{P_1}^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2{P_2}^2+a_1b_3P_3P_1+a_2b_3P_2P_3+a_3b_3(P_3)^2+a_3b_0P_3+a_3b_1P_3P_1+a_3b_2P_2P_3+a_1b_1P_1P_1=a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2+(a_0b_3+a_1b_3+a_2b_3+a_3b_3+a_3b_0+a_3b_1+a_3b_2)P_3$.

Main Discussion

Definition.

Let R be a ring, then (t,s,k) is called a Pythagoras triple if and only if

$t^2+s^2=k^2; t,s,k\in R$..

Theorem.

Let $T=t_0+t_1P_1+t_2P_2+t_3P_3, S=s_0+s_1P_1+s_2P_2+s_3P_3, K=k_0+k_1P_1+k_2P_2+k_3P_3$ are three arbitrary symbolic 3-plithogenic elements $T,S,K\in 3-SP_R$, then (T,S,K) are Pythagoras triple in $3-SP_R$ if and only if:

$$\begin{cases}(t_0,s_0,k_0),(t_0+t_1,s_0+s_1,k_0+k_1)\ are\ pythagoras\ triples\ in\ R\\(t_0+t_1+t_2,s_0+s_1+s_2,k_0+k_1+k_2),(t_0+t_1+t_2+t_3,s_0+s_1+s_2+s_3,k_0+k_1+k_2+k_3)\ are\ pythagoras\ triples\ in\ R\end{cases}$$

Proof.

According to [ ], we have:

$$T^2 = {t_0}^2 + [(t_0 + t_1)^2 - {t_0}^2]P_1 + [(t_0 + t_1 + t_2)^2 - (t_0 + t_1)^2]P_2 + [(t_0 + t_1 + t_2 + t_3)^2 - (t_0 + t_1 + t_2)^2]P_3$$

$$S^2 = {s_0}^2 + [(s_0 + s_1)^2 - {s_0}^2]P_1 + [(s_0 + s_1 + s_2)^2 - (s_0 + s_1)^2]P_2 + [(s_0 + s_1 + s_2 + s_3)^2 - (s_0 + s_1 + s_2)^2]P_3$$

$$T^2 = {k_0}^2 + \left[(k_0 + k_1)^2 - {k_0}^2\right]P_1 + [(k_0 + k_1 + k_2)^2 - (k_0 + k_1)^2]P_2 + [(k_0 + k_1 + k_2 + k_3)^2 - (k_0 + k_1 + k_2)^2]P_3$$

The equation $T^2 + S^2 = K^2$ is equivalent to:

${t_0}^2 + {s_0}^2 = {k_0}^2$ $(equation\ 1)$,

$(t_0 + t_1)^2 + (s_0 + s_1)^2 = (k_0 + k_1)^2$ $(equation\ 2)$,

$(t_0 + t_1 + t_2)^2 + (s_0 + s_1 + s_2)^2 = (k_0 + k_1 + k_2)^2$ $(equation\ 3)$,

$(t_0 + t_1 + t_2 + t_3)^2 + (s_0 + s_1 + s_2 + s_3)^2 = (k_0 + k_1 + k_2 + k_3)^2$ $(equation\ 4)$

Equation (1) implies that (t_0, s_0, k_0) is a Pythagoras triple in R.

Equation (2) implies that $(t_0 + t_1, s_0 + s_1, k_0 + k_1)$ is a Pythagoras triple in R.

Equation (3) implies that $(t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2)$ is a Pythagoras triple in R.

Equation (4) implies that $(t_0 + t_1 + t_2 + t_3, s_0 + s_1 + s_2 + s_3, k_0 + k_1 + k_2 + k_3)$ is a Pythagoras triple in R.

Thus, the proof is complete.

Theorem.

Let $(t_0, s_0, k_0), (t_1, s_1, k_1), (t_2, s_2, k_2), (t_3, s_3, k_3)$ be four Pythagoras triples in the ring R, then (T, S, K) Pythagoras triple in $3 - SP_R$, where:

$T = t_0 + [t_1 - t_0]P_1 + [t_2 - t_1]P_2 + [t_3 - t_2]P_3$

$S = s_0 + [s_1 - s_0]P_1 + [s_2 - s_1]P_2 + [s_3 - s_2]P_3$

$K = k_0 + [k_1 - k_0]P_1 + [k_2 - k_1]P_2 + [k_3 - k_2]P_3$

Proof.

We have: $t_0 + (t_1 - t_0) = t_1, t_0 + (t_1 - t_0) + (t_2 - t_1) = t_2, t_0 + (t_1 - t_0) + (t_2 - t_1) + (t_3 - t_2) = t_3$

$$s_0 + (s_1 - s_0) = s_1, s_0 + (s_1 - s_0) + (s_2 - s_1) = s_2, s_0 + (s_1 - s_0) + (s_2 - s_1) + (s_3 - s_2) = s_3$$

$k_0 + (k_1 - k_0) = k_1, k_0 + (k_1 - k_0) + (k_2 - k_1)$

$= k_2, k_0 + (k_1 - k_0) + (k_2 - k_1) + (k_3 - k_2) = k_3$

This implies that (T, S, K) Pythagoras triple in $3 - SP_R$ according to the theorem.

Examples.

We have:

$$\begin{cases} (t_0, s_0, k_0) = (3,4,5) \\ (t_1, s_1, k_1) = (6,8,10) \\ (t_2, s_2, k_2) = (4,3,5) \\ (t_3, s_3, k_3) = (5,12,13) \end{cases}$$

Are four Pythagoras triples in Z.

The corresponding symbolic 3-plithogenic Pythagoras triple is (T, S, K), where:

$T = 3 + [6 - 3]P_1 + [4 - 6]P_2 + [5 - 4]P_3 = 3 + 3P_1 - 2P_2 + P_3$

$S = 4 + [8 - 4]P_1 + [3 - 8]P_2 + [12 - 3]P_3 = 4 + 4P_1 - 5P_2 + 9P_3$

$K = 5 + [10 - 5]P_1 + [5 - 10]P_2 + [13 - 5]P_3 = 5 + 5P_1 - 5P_2 + 8P_3$

Example.

Find all Pythagoras triples in $3 - SP_{Z_2}$, where Z_2 I the ring of integers module 2.

First, we find all Pythagoras triples in Z.

$L_1 = (0,0,0), L_2 = (1,0,1), L_3 = (0,1,1), L_4 = (1,1,0)$

Remark that for every permutation of the set $\{L_1, L_2, L_3, L_4\}$, we get a different symbolic 3-plithogenic Pythagoras triple.

We discuss all possible cases:

Permutation (1).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_1 = P_1 - P_2 + P_3 = P_1 + P_2 + P_3 \\ \acute{Y}_1 = P_2 \\ Y_1'' = P_1 - P_3 = P_1 + P_3 \end{cases}$$

Permutation (2).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_2 = P_2 \\ \acute{Y}_2 = P_1 - P_2 + P_3 = P_1 + P_2 + P_3 \\ Y_2'' = P_1 - P_3 = P_1 + P_3 \end{cases}$$

Permutation (3).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_3 = P_1 + P_3 \\ \acute{Y}_3 = P_1 + P_2 + P_3 \\ Y_3'' = P_2 \end{cases}$$

Permutation (4).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_4 = P_1 + P_2 + P_3 \\ \acute{Y}_4 = P_1 + P_3 \\ Y_4{}'' = P_2 \end{cases}$$

Permutation (5).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_5 = P_1 + P_3 \\ \acute{Y}_5 = P_2 \\ Y_5{}'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (6).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_6 = P_2 \\ \acute{Y}_6 = P_1 + P_3 \\ Y_6{}'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (7).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_7 = 1 + P_2 + P_3 \\ \acute{Y}_7 = P_1 \\ Y_7{}'' = 1 + P_1 + P_2 + P_3 \end{cases}$$

Permutation (8).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_8 = P_2 \\ \acute{Y}_8 = 1 + P_1 + P_2 \\ Y_8{}'' = 1 + P_1 \end{cases}$$

Permutation (9).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_9 = 1 + P_1 + P_3 \\ \acute{Y}_9 = P_1 + P_2 + P_3 \\ Y_9{}'' = 1 + P_2 \end{cases}$$

Permutation (10).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{10} = P_1 + P_2 + P_3 \\ \acute{Y}_{10} = 1 + P_1 + P_3 \\ Y_{10}{}'' = 1 + P_2 \end{cases}$$

Permutation (11).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{11} = 1 + P_2 \\ \acute{Y}_{11} = 1 + P_1 + P_3 \\ Y_{11}{}'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (12).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{12} = 1 + P_1 + P_2 + P_3 \\ \acute{Y_{12}} = 1 + P_1 + P_3 \\ Y_{12}'' = P_2 \end{cases}$$

Permutation (13).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{13} = 1 + P_1 + P_2 + P_3 \\ \acute{Y_{13}} = 1 + P_2 \\ Y_{13}'' = P_1 + P_3 \end{cases}$$

Permutation (14).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{14} = 1 + P_1 + P_3 \\ \acute{Y_{14}} = 1 + P_2 \\ Y_{14}'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (15).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{15} = 1 + P_1 + P_3 \\ \acute{Y_{15}} = 1 + P_1 + P_2 + P_3 \\ Y_{15}'' = P_2 \end{cases}$$

Permutation (16).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{16} = 1 + P_2 \\ \acute{Y_{16}} = 1 + P_1 + P_2 + P_3 \\ Y_{16}'' = P_1 + P_3 \end{cases}$$

Permutation (17).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{17} = 1 + P_2 \\ \acute{Y_{17}} = P_1 + P_2 + P_3 \\ Y_{17}'' = 1 + P_1 + P_3 \end{cases}$$

Permutation (18).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{18} = 1 + P_2 \\ \acute{Y_{18}} = P_1 + P_3 \\ Y_{18}'' = 1 + P_1 + P_2 + P_3 \end{cases}$$

Permutation (19).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{19} = P_1 + P_3 \\ \acute{Y_{19}} = 1 + P_1 + P_2 + P_3 \\ Y_{19}'' = 1 + P_2 \end{cases}$$

Permutation (20).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{20} = P_1 + P_3 \\ \acute{Y_{20}} = 1 + P_1 + P_2 + P_3 \\ Y_{20}'' = 1 + P_2 \end{cases}$$

Permutation (21).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{21} = P_1 + P_2 + P_3 \\ Y_{21}' = 1 + P_2 + P_3 \\ Y_{21}'' = 1 + P_1 \end{cases}$$

Permutation (22).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{22} = 1 + P_1 + P_3 \\ Y_{22}' = P_1 + P_2 + P_3 \\ Y_{22}'' = 1 + P_2 \end{cases}$$

Permutation (23).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{23} = P_1 + P_3 \\ Y_{23}' = 1 + P_2 \\ Y_{23}'' = 1 + P_1 + P_2 + P_3 \end{cases}$$

Permutation (24).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{24} = P_1 + P_2 + P_3 \\ Y_{24}' = 1 + P_1 + P_3 \\ Y_{24}'' = 1 + P_2 \end{cases}$$

Also, other quadraples $(L_i, L_j, L_k, L_s); 1 \le i, j, k, s \le 4$ give Pythagoras triples with i, j, k, s are not distinct at all.

We continuo our discussions.

Permutation (25).

$$(L_1, L_1, L_1, L_1): \begin{cases} Y_{25} = (0,0,0) \\ Y_{25}' = (0,0,0) \\ Y_{25}'' = (0,0,0) \end{cases}$$

Permutation (26).

$$(L_1, L_1, L_1, L_2): \begin{cases} Y_{26} = P_3 \\ Y_{26}' = 0 \\ Y_{26}'' = P_3 \end{cases}$$

Permutation (27).

$$(L_1, L_1, L_1, L_3): \begin{cases} Y_{27} = 0 \\ Y_{27}' = P_3 \\ Y_{27}'' = P_3 \end{cases}$$

Permutation (28).

$$(L_1, L_1, L_1, L_4): \begin{cases} Y_{28} = P_3 \\ Y_{28}' = P_3 \\ Y_{28}'' = 0 \end{cases}$$

Permutation (29).

$$(L_1, L_2, L_1, L_1): \begin{cases} Y_{29} = P_1 + P_2 \\ \acute{Y_{29}} = 0 \\ Y_{29}'' = P_1 + P_2 \end{cases}$$

Permutation (30).

$$(L_1, L_3, L_1, L_1): \begin{cases} Y_{30} = 0 \\ \acute{Y_{30}} = P_1 + P_2 \\ Y_{30}'' = P_1 + P_2 \end{cases}$$

Permutation (31).

$$(L_1, L_4, L_1, L_1): \begin{cases} Y_{31} = P_1 + P_2 \\ \acute{Y_{31}} = P_1 + P_2 \\ Y_{31}'' = 0 \end{cases}$$

Permutation (32).

$$(L_1, L_1, L_2, L_1): \begin{cases} Y_{32} = P_2 + P_3 \\ \acute{Y_{32}} = 0 \\ Y_{32}'' = P_2 + P_3 \end{cases}$$

Permutation (33).

$$(L_1, L_1, L_3, L_1): \begin{cases} Y_{33} = 0 \\ \acute{Y_{33}} = P_2 + P_3 \\ Y_{33}'' = P_2 + P_3 \end{cases}$$

Permutation (34).

$$(L_1, L_1, L_4, L_1): \begin{cases} Y_{34} = P_2 + P_3 \\ \acute{Y_{34}} = P_2 + P_3 \\ Y_{34}'' = 0 \end{cases}$$

Permutation (35).

$$(L_2, L_2, L_2, L_2): \begin{cases} Y_{35} = 1 \\ \acute{Y_{35}} = 0 \\ Y_{35}'' = 1 \end{cases}$$

Permutation (36).

$$(L_2, L_2, L_2, L_1): \begin{cases} Y_{36} = 1 + P_3 \\ \acute{Y_{36}} = 0 \\ Y_{36}'' = 1 + P_3 \end{cases}$$

Permutation (37).

$$(L_2, L_2, L_2, L_1): \begin{cases} Y_{37} = 1 + P_3 \\ Y_{37}' = P_3 \\ Y_{37}'' = 1 \end{cases}$$

Permutation (38).

$$(L_2, L_2, L_2, L_4): \begin{cases} Y_{38} = 1 \\ Y_{38}' = P_3 \\ Y_{38}'' = 1 + P_3 \end{cases}$$

Permutation (39).

$$(L_2, L_2, L_1, L_2): \begin{cases} Y_{39} = 1 + P_2 + P_3 \\ Y_{39}' = 0 \\ Y_{39}'' = 1 + P_2 + P_3 \end{cases}$$

Permutation (40).

$$(L_2, L_2, L_3, L_2): \begin{cases} Y_{40} = 1 + P_2 + P_3 \\ Y_{40}' = P_2 + P_3 \\ Y_{40}'' = 1 \end{cases}$$

Permutation (41).

$$(L_2, L_2, L_4, L_2): \begin{cases} Y_{41} = 1 \\ Y_{41}' = P_2 + P_3 \\ Y_{41}'' = 1 + P_2 + P_3 \end{cases}$$

Permutation (42).

$$(L_2, L_1, L_2, L_2): \begin{cases} Y_{42} = 1 + P_1 \\ Y_{42}' = 0 \\ Y_{42}'' = 1 + P_1 \end{cases}$$

Permutation (43).

$$(L_2, L_4, L_2, L_2): \begin{cases} Y_{43} = 1 \\ Y_{43}' = P_1 + P_2 \\ Y_{43}'' = 1 + P_2 + P_1 \end{cases}$$

Permutation (44).

$$(L_2, L_3, L_2, L_2): \begin{cases} Y_{44} = 1 + P_2 + P_3 \\ Y_{44}' = P_1 + P_2 \\ Y_{44}'' = 1 + P_1 + P_3 \end{cases}$$

Permutation (45).

$$(L_3, L_3, L_3, L_3): \begin{cases} Y_{45} = 0 \\ Y_{45}' = 1 \\ Y_{45}'' = 1 \end{cases}$$

Permutation (46).

$$(L_3, L_3, L_3, L_1): \begin{cases} Y_{46} = 0 \\ Y_{46}' = P_3 \\ Y_{46}'' = P_3 \end{cases}$$

Permutation (47).

$$(L_3, L_3, L_3, L_2): \begin{cases} Y_{47} = P_3 \\ Y_{47}' = 1 + P_3 \\ Y_{47}'' = 1 \end{cases}$$

Permutation (48).

$$(L_3, L_3, L_3, L_4): \begin{cases} Y_{48} = P_3 \\ Y_{48}' = 1 \\ Y_{48}'' = 1 + P_3 \end{cases}$$

Permutation (49).

$$(L_3, L_3, L_1, L_3): \begin{cases} Y_{49} = 0 \\ Y_{49}' = 1 + P_2 + P_3 \\ Y_{49}'' = 1 + P_2 + P_3 \end{cases}$$

Permutation (50).

$$(L_3, L_3, L_2, L_3): \begin{cases} Y_{50} = P_2 + P_3 \\ Y_{50}' = 1 + P_2 + P_3 \\ Y_{50}'' = 1 \end{cases}$$

Permutation (51).

$$(L_3, L_3, L_4, L_3): \begin{cases} Y_{51} = P_2 + P_3 \\ Y_{51}' = 1 \\ Y_{51}'' = 1 + P_2 + P_3 \end{cases}$$

Permutation (52).

$$(L_3, L_1, L_3, L_3): \begin{cases} Y_{52} = 0 \\ Y_{52}' = 1 + P_1 + P_2 \\ Y_{52}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (53).

$$(L_3, L_2, L_3, L_3): \begin{cases} Y_{53} = P_1 + P_2 \\ Y_{53}' = 1 + P_1 + P_2 \\ Y_{53}'' = 1 \end{cases}$$

Permutation (54).

$$(L_3, L_4, L_3, L_3): \begin{cases} Y_{54} = P_1 + P_2 \\ Y_{54}' = 1 \\ Y_{54}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (55).

$$(L_4, L_4, L_4, L_4): \begin{cases} Y_{55} = 1 \\ Y_{55}' = 1 \\ Y_{55}'' = 0 \end{cases}$$

Permutation (56).

$$(L_4, L_4, L_4, L_1): \begin{cases} Y_{56} = 1 + P_3 \\ Y_{56}' = 1 + P_3 \\ Y_{56}'' = 0 \end{cases}$$

Permutation (57).

$$(L_4, L_4, L_4, L_2): \begin{cases} Y_{57} = 1 \\ Y_{57}' = 1 + P_3 \\ Y_{57}'' = P_3 \end{cases}$$

Permutation (58).

$$(L_4, L_4, L_4, L_3): \begin{cases} Y_{58} = 1 + P_3 \\ Y_{58}' = 1 \\ Y_{58}'' = P_3 \end{cases}$$

Permutation (59).

$$(L_4, L_4, L_1, L_4): \begin{cases} Y_{59} = 1 + P_2 + P_3 \\ Y_{59}' = 1 + P_2 + P_3 \\ Y_{59}'' = 0 \end{cases}$$

Permutation (60).

$$(L_4, L_4, L_2, L_4): \begin{cases} Y_{60} = 1 \\ Y_{60}' = 1 + P_2 + P_3 \\ Y_{60}'' = P_2 + P_3 \end{cases}$$

Permutation (61).

$$(L_4, L_4, L_3, L_4): \begin{cases} Y_{61} = 1 + P_2 + P_3 \\ Y_{61}' = 1 \\ Y_{61}'' = P_2 + P_3 \end{cases}$$

Permutation (62).

$$(L_4, L_1, L_4, L_4): \begin{cases} Y_{62} = 1 + P_1 + P_2 \\ Y_{62}' = 1 + P_1 + P_2 \\ Y_{62}'' = 0 \end{cases}$$

Permutation (63).

$$(L_4, L_2, L_4, L_4): \begin{cases} Y_{63} = 1 \\ Y_{63}' = 1 + P_1 + P_2 \\ Y_{63}'' = P_1 + P_2 \end{cases}$$

Permutation (64).

$$(L_4, L_3, L_4, L_4): \begin{cases} Y_{64} = 1 + P_1 + P_2 \\ Y_{64}' = 1 \\ Y_{64}'' = P_1 + P_2 \end{cases}$$

By continuing this argument, we can get all Pythagoras triples in $3 - SP_{Z_2}$

Conclusion.

In this paper, we have studied Pythagoras triples in symbolic 3-plithogenic commutative rings, where necessary and sufficient conditions for a symbolic 3-plithogenic triple (x, y, z) to be a Pythagoras triple.

Also, we have presented some related examples that explain how to find 3-plithogenic triples from classical triples.

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