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On The Dual Symbolic 2-Plithogenic Numbers

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Abstract:

The objective of this paper is to combine dual numbers with symbolic 2-plithogenic numbers in one algebraic structure called dual symbolic 2-plithogenic real numbers.

Also, many elementary properties of the suggested system such as inverses and idempotents will be handled by many related theorems and examples.

Keywords: Symbolic 2-plithogenic number, dual number, dual symbolic 2-plithogenic number.

Introduction and preliminaries.

Dual numbers are considered as a generalization of real numbers, where they are defined as follows:

$D = \{a + bt;\ t^2 = 0, a, b \in R\}$[1]. Dual numbers make together a commutative ring with many interesting properties.

Addition on D is defined as follows:

$(a_0 + b_0 t) + (a_1 + b_1 t) = (a_0 + a_1) + (b_0 + b_1)t$

Multiplication on D is defined as follows:

$(a_0 + b_0 t).(a_1 + b_1 t) = (a_0 a_1) + (a_0 b_1 + b_0 a_1)t$

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In [2-4], Smarandache presented symbolic n-plithogenic sets, then they were used in generalizing many famous algebraic structures such as rings, matrices, and other structures [6-11].

We refer to many similar numerical systems that generalize real number, such as neutrosophic numbers, split-complex number, and weak fuzzy numbers [12-18]. We refer to some applications of these generalized number systems in matrix theory and cryptography [19-24].

Through this paper, we use symbolic 2-plithogenic real numbers to build a new generalization of real numbers, and we present some of its elementary algebraic properties.

Main concepts.

Definition.

The set of symbolic 2-plithogenic dual numbers I defined as follows:

$$2-SP_D=\{(x_0+x_1t)+(y_0+y_1t)P_1+(z_0+z_1t)P_2; x_i,y_i,z_i\in R, t^2=0\}$$

Definition.

Addition of $2-SP_D$ is defined:

$$\begin{aligned}[(m_0+m_1t)+(k_0+k_1t)P_1+(s_0+s_1t)P_2]&\\ +[(n_0+n_1t)+(l_0+l_1t)P_1+(q_0+q_1t)P_2]&\\ =(m_0+n_0)+(m_1+n_1)t+[(k_0+l_0)+(k_1+l_1)t]P_1&\\ +[(s_0+q_0)+(s_1+q_1)t]P_2&\end{aligned}$$

$(2-SP_D,+)$ is an abelian group.

Remark.

A symbolic 2-plithogenic dual number $X=(x_0+x_1t)+(y_0+y_1t)P_1+(z_0+z_1t)P_2$ can be written:

$$X=(x_0+y_0P_1+z_0P_2)+t(x_1+y_1P_1+z_1P_2)$$

Definition.

Let $X=(x_0+x_1P_1+x_2P_2)+t(\acute{x}_0+\acute{x}_1P_1+\acute{x}_2P_2)=M_1+M_2t$,

$Y=(y_0+y_1P_1+y_2P_2)+t(\acute{y}_0+\acute{y}_1P_1+\acute{y}_2P_2)=N_1+N_2t\in 2-SP_D$,

then:

Multiplication on $2-SP_D$ is defined as follows:

$X.Y = M_1N_1 + t(M_1N_2 + N_1M_2)$

Example.

Consider $X = (1 + P_1 + P_2) + t(2 - P_2), Y = P_1 + t(1 - P_1)$, we have:

$X + Y = (1 + 2P_1 + P_2) + t(3 - P_1 - P_2)$

$X.Y = (1 + P_1 + P_2)P_1 + t[(1 + P_1 + P_2)(1 - P_1) + (2 - P_2)P_1] = (2P_1 + P_2) +$
$t[(1 - P_1 + P_1 - P_1 + P_2 - P_2) + 2P_1 - P_1] = (2P_1 + P_2) + t(1 + P_1 - P_2)$.

Remark.

$(2 - SP_D, +, .)$ Is a commutative ring.

Invertibility:

Theorem.

Let $X = (m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2) \in 2 - SP_D$, then X is invertible if and only if $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0$ and:

$$X^{-1} = \frac{1}{X} = \left[\frac{1}{m_0} + \left(\frac{1}{m_0 + m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0 + m_1 + m_2} - \frac{1}{m_0 + m_1}\right)P_2\right] - t\left[\frac{n_0}{(m_0)^2} + \left(\frac{n_0 + n_1}{(m_0 + m_1)^2} - \frac{n_0}{(m_0)^2}\right)P_1 + \left(\frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2} - \frac{n_0 + n_1}{(m_0 + m_1)^2}\right)P_2\right]$$

Proof.

X is invertible if and only if $\frac{1}{X}$ is defined as follows:

$$\frac{1}{X} = \frac{1}{(m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2)}$$
$$= \frac{(m_0 + m_1P_1 + m_2P_2) - t(n_0 + n_1P_1 + n_2P_2)}{[(m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2)][(m_0 + m_1P_1 + m_2P_2) - t(n_0 + n_1P_1 + n_2P_2)]}$$
$$== \frac{(m_0 + m_1P_1 + m_2P_2) - t(n_0 + n_1P_1 + n_2P_2)}{(m_0 + m_1P_1 + m_2P_2)^2}$$

So that $m_0 + m_1P_1 + m_2P_2$ is invertible in $2 - SP_R$.

This is equivalent to $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0$.

On the other hand, $\frac{1}{X} = \frac{1}{m_0+m_1P_1+m_2P_2} - t\frac{(n_0+n_1P_1+n_2P_2)^2}{(m_0+m_1P_1+m_2P_2)^2}$

Put $Y = \frac{1}{m_0} + \left(\frac{1}{m_0+m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0+m_1+m_2} - \frac{1}{m_0+m_1}\right)P_2 - t\left[\frac{n_0}{(m_0)^2} + \left(\frac{n_0+n_1}{(m_0+m_1)^2} - \frac{n_0}{(m_0)^2}\right)P_1 + \left(\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2} - \frac{n_0+n_1}{(m_0+m_1)^2}\right)P_2\right]$

Compute the result of XY as follows:

$$XY = (m_0 + m_1P_1 + m_2P_2)\left[\frac{1}{m_0} + \left(\frac{1}{m_0+m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0+m_1+m_2} - \frac{1}{m_0+m_1}\right)P_2\right]$$
$$+ t\left[-(m_0 + m_1P_1 + m_2P_2)\left[\frac{n_0}{(m_0)^2} + \left(\frac{n_0+n_1}{(m_0+m_1)^2} - \frac{n_0}{(m_0)^2}\right)P_1 + \left(\frac{n_0+n_1+n_2}{(m_0+m_1+m_2)^2} - \frac{n_0+n_1}{(m_0+m_1)^2}\right)P_2\right]\right]$$
$$+ (n_0 + n_1P_1 + n_2P_2)\left[\frac{1}{m_0} + \left(\frac{1}{m_0+m_1} - \frac{1}{m_0}\right)P_1 + \left(\frac{1}{m_0+m_1+m_2} - \frac{1}{m_0+m_1}\right)P_2\right]$$
$$= 1 + t\left[\frac{-n_0}{m_0} + \left(-m_0\frac{(n_0+n_1)}{m_0+m_1} + \frac{n_0}{m_0} - \frac{m_1n_0}{(m_0)^2} - \frac{m_1(n_0+n_1)}{(m_0+m_1)^2} + \frac{m_1n_0}{(m_0)^2}\right)P_1\right.$$
$$+ \left(-m_0\frac{(n_0+n_1+n_2)}{(m_0+m_1+m_2)^2} + \frac{m_0(n_0+n_1)}{(m_0+m_1)^2} - m_1\frac{(n_0+n_1+n_2)}{(m_0+m_1+m_2)^2}\right.$$
$$+ \frac{m_1(n_0+n_1)}{(m_0+m_1)^2} - \frac{m_2n_0}{(m_0)^2} - \frac{m_2(n_0+n_1)}{(m_0+m_1)^2} + \frac{m_2n_0}{(m_0)^2} - m_2\frac{(n_0+n_1+n_2)}{(m_0+m_1+m_2)^2}$$
$$\left.\left. + \frac{m_2(n_0+n_1)}{(m_0+m_1)^2}\right)P_2\right] + \frac{n_0}{m_0} + \left(\frac{n_0}{m_0+m_1} - \frac{n_0}{m_0} + \frac{n_1}{m_0} + \frac{n_1}{m_0+m_1} - \frac{n_1}{m_0}\right)P_1$$
$$+ \left(\frac{n_0}{m_0+m_1+m_2} - \frac{n_0}{m_0+m_1} + \frac{n_1}{m_0+m_1+m_2} - \frac{n_1}{m_0+m_1} + \frac{n_2}{m_0}\right.$$
$$\left. + \frac{n_2}{m_0+m_1+m_2} - \frac{n_2}{m_0} + \frac{n_2}{m_0+m_1+m_2} - \frac{n_2}{m_0+m_1}\right)P_2 = 1$$

So that, $X^{-1} = \frac{1}{X} = Y$

Example.

Take $X = (1 + P_1 + P_2) + t(2 + P_1 - P_2) \in 2 - SP_D$:

$$X^{-1} = \frac{1}{1} + \left(\frac{1}{2} - 1\right)P_1 + \left(\frac{1}{3} - \frac{1}{2}\right)P_2 - t\left[\frac{2}{1} + \left(\frac{3}{4} - \frac{2}{1}\right)P_1 + \left(\frac{2}{9} - \frac{3}{4}\right)P_2\right]$$
$$= 1 - \frac{1}{2}P_1 - \frac{1}{6}P_2 - t\left(2 - \frac{5}{4}P_1 - \frac{21}{36}P_2\right)$$

Natural power.

Theorem.

Let $X = (m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2) \in 2 - SP_D$, then:

$X^n = (m_0)^n + ((m_0 + m_1)^n - (m_0)^n)P_1 + ((m_0 + m_1 + m_2)^n - (m_0 + m_1)^n)P_2 + n(n_0 + n_1P_1 + n_2P_2)[(m_0)^{n-1} + ((m_0 + m_1)^{n-1} - (m_0)^{n-1})P_1 + ((m_0 + m_1 + m_2)^{n-1} - (m_0 + m_1)^{n-1})P_2]$ for $n \in N$.

Proof.

Let $X = A + Bt; A, B \in 2 - SP_D$, then:

$A^n = A^n + nA^{n-1}Bt$, we get:

$A^n = (m_0)^n + ((m_0 + m_1)^n - (m_0)^n)P_1 + ((m_0 + m_1 + m_2)^n - (m_0 + m_1)^n)P_2$, then the proof holds.

Example.

Take $X = (1 + P_1 + P_2) + t(2 - P_1 + P_2) \in 2 - SP_D$

$X^3 = 1 + (8 - 1)P_1 + (1 - 8)P_2 + 3t(2 - P_1 + P_2)[1 + (4 - 1)P_1 + (1 - 4)P_2] = 1 + 7P_1 - 7P_2 + 3t[(2 - P_1 + P_2)(1 + 3P_1 - 3P_2)] = 1 + 7P_1 - 7P_2 + t(6 + 6P_1 - 6P_2)$.

Idempotency.

Definition.

Let $X \in 2 - SP_D$, then X is called idempotent if and only if $X^2 = X$.

Theorem.

Let $X = (m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2) \in 2 - SP_D$, then X is called idempotent if and only if:

1. $m_0 + m_1P_1 + m_2P_2$ is idempotent.
2. $(n_0 + n_1P_1 + n_2P_2)[2m_0 - 1 + 2m_1P_1 + 2m_2P_2] = 0$

Proof.

$X = M + Nt$ is idempotent if and only if:

$$X^2 = X \Rightarrow \begin{cases} M^2 = M \\ 2MN = N \Rightarrow N(2M - 1) = 0 \end{cases}$$

For $M = m_0 + m_1P_1 + m_2P_2, N = n_0 + n_1P_1 + n_2P_2 \in 2 - SP_R$.

This implies the proof.

Remark.

The idempotency of M is equivalent to $m_0{}^2 = m_0, (m_0 + m_1)^2 = (m_0 + m_1), (m_0 + m_1 + m_2)^2 = (m_0 + m_1 + m_2)$, hence $m_0, m_0 + m_1, m_0 + m_1 + m_2 \in \{0,1\}$.

The equation $N(2M - 1) = 0$ means that $N, 2M - 1$ are zero divisors in $2 - SP_R$, so they should not invertible in $2 - SP_R$.

Now, let's compute the result of $N(2M - 1) = 0$.

$N(2M - 1) = (n_0 + n_1P_1 + n_2P_2)(2m_0 - 1 + m_1P_1 + m_2P_2) = 0$, thus:

$$\begin{cases} n_0(2m_0 - 1) = 0 \quad (1) \\ (n_0 + n_1)(2m_0 - 1 + 2m_1) = 0 \quad (2) \\ (n_0 + n_1 + n_2)(2m_0 - 1 + 2m_1 + 2m_2) = 0 \quad (3) \end{cases}$$

Since $m_0 \in \{0,1\}$, then $2m_0 - 1 \neq 0$ and $n_0 = 0$.

Equation (2) has two possible cases:

$$\begin{cases} n_1 = 0 \\ or \\ 2m_0 + 2m_1 = 1 \end{cases}$$

Equation (3) has two possible cases:

$$\begin{cases} n_1 + n_1 = 0 \\ or \\ 2m_0 + 2m_1 + 2m_2 = 1 \end{cases}$$

We discuss all possible cases.

Case1.

$m_0 = 0, m_0 + m_1 = 0, m_0 + m_1 + m_2 = 0$, thus $m_1 = m_2 = 0$, $n_0 = 0$, $n_1 = 0$, $n_2 = 0$, thus $X = 0$.

Case2.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } m_1 = 1, m_2 = -1, n_0 = 0, n_1 = n_2 = 0$$

thus $X = P_1 - P_2$

Case3.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } m_1 = 0, m_2 = 1, n_0 = 0, n_1 = n_2 = 0$$

thus $X = P_2$

Case4.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } m_1 = 1, m_2 = 0, n_0 = 0, n_1 = n_2 = 0$$

thus $X = P_1$

Case5.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } m_1 = -1, m_2 = 0, n_0 = 0, n_1 = n_2 = 0$$

thus $X = 1 - P_1$

Case6.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } m_1 = 0, m_2 = -1, n_0 = 0, n_1 = n_2 = 0$$

thus $X = 1 - P_2$

Case7.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } m_1 = -1, m_2 = 1, n_0 = 0, n_1 = n_2 = 0$$

thus $X = 1 - P_1 + P_2$

Case8.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } m_1 = 0, m_2 = 0, n_0 = 0, n_1 = n_2 = 0$$

thus $X = 1$

remark.

The idempotent in $2 - SP_D$ are:

$$\{0,1,1 - P_1 + P_2, 1 - P_2, 1 - P_1, P_1, P_2, P_1 - P_2\}$$

Definition.

Let $X = M + Nt \in 2 - SP_D$, we say that X is 3-potent element if and only if $X^3 = X$.

Remark.

X is 3-potent if and only if $X^3 = X$ which is equivalent to:

$$\begin{cases} M^3 = M \\ 3M^2N = N \Longrightarrow N(3M^2 - 1) = 0 \end{cases}$$

$M^3 = M$ implies:

$$\begin{cases} {m_0}^3 = m_0 \\ (m_0 + m_1)^3 = m_0 + m_1 \\ (m_0 + m_1 + m_2)^3 = m_0 + m_1 + m_2 \end{cases}$$

So that $m_0, m_0 + m_1, m_0 + m_1 + m_2 \in \{0,1 - 1\}$.

$N(3M^2 - 1) = 0$ implies:

$$\begin{cases} n_0(3m_0 - 1) = 0 \quad (1) \\ (n_0 + n_1)(3m_0 - 1 + 3m_1) = 0 \quad (2) \\ (n_0 + n_1 + n_2)(3m_0 - 1 + 3m_1 + 3m_2) = 0 \quad (3) \end{cases}$$

Equation (1) means that $n_0 = 0$, that is because $m_0 \neq \frac{1}{3}$.

Equation (2) means that:

$$\begin{cases} n_0 + n_1 = 0 \\ or \\ 3m_0 + 3m_1 = 1 \end{cases}$$, since $m_0 + m_1$ is integer, then $n_1 = 0$.

Equation (3) means that:

$$\begin{cases} n_0 + n_1 + n_2 = 0 \\ or \\ 3m_0 + 3m_1 + 3m_2 = 1 \end{cases}$$, since $m_0 + m_1 + m_2$ is integer, then $n_2 = 0$.

This implies that:

Case1.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}$$, then $X = 0$.

Case2.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}$$, then $X = P_1 - P_2$

Case3.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 0 \end{cases}$$, then $X = -P_1 + P_2$

Case4.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}$$, then $X = P_2$

Case5.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{then } X = -P_2$$

Case6.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } X = P_1$$

Case7.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{then } X = P_1 - 2P_2$$

Case8.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } X = -P_1 + 2P_2$$

Case9.

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{then } X = -P_1$$

Case10.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } X = 1 - P_1$$

Case11.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } X = 1 - P_2$$

Case12.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } X = 1 - 2P_1 + P_2$$

Case13.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } X = 1 - P_1 + P_2$$

Case14.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{then } X = 1 - P_1 - P_2$$

Case15.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } X = 1$$

Case16.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{then } X = 1 - 2P_2$$

Case17.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } X = 1 - 2P_1 - 2P_2$$

Case18.

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{then } X = 1 - 2P_1$$

Case19.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } X = -1 + P_1$$

Case20.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } X = -1 + 2P_1 - P_2$$

Case21.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{then } X = -1 + P_2$$

Case22.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{then } X = -1 + P_1 + P_2$$

Case23.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -1 + P_1 - P_2$$

Case24.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = -1 + 2P_1$$

Case25.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -1$$

Case26.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -1 + 2P_1 - 2P_2$$

Case27.

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = -1 + 2P_2$$

Conclusion

In this paper, we have studied for the first time the combination of symbolic 2-plithogenic numbers with dual numbers. The novel algebraic structure generated by them is called dual symbolic 2-plithogenic numbers.

We have determined the invertibility condition and the formula of the inverse for dual symbolic 2-plithogenic numbers. Also, all idempotent elements in the ring of dual symbolic 2-plithogenic numbers were presented and computed.

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