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TOPSIS Strategy for Multi-Attribute Decision Making with Trapezoidal Neutrosophic Numbers

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Abstract. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a popular strategy for Multi-Attribute Decision Making (MADM). In this paper, we extend the TOPSIS strategy of MADM problems in trapezoidal neutrosophic number environment. The attribute values are expressed in terms of single-valued trapezoidal neutrosophic numbers. The weight information of attribute is incompletely known or completely unknown. Using the maximum deviation strategy, we develop an optimization model to obtain the weight of the attributes. Then we develop an extended TOPSIS strategy to deal with MADM with single-valued trapezoidal neutrosophic numbers. To illustrate and validate the proposed TOPSIS strategy, we provide a numerical example of MADM problem.

Keywords: Single-valued trapezoidal neutrosophic number, multi-attribute decision making, TOPSIS.

1 Introduction

Multi-attribute decision making (MADM) plays an important role in decision making sciences. MADM is a process of finding the best alternative that has the highest degree of satisfaction over the predefined conflicting attributes. The preference values of alternatives are generally assessed quantitatively and qualitatively according to the nature of attributes. When the preference values are imprecise, indeterminate or incomplete, the decision maker feels comfort to evaluate the alternatives in MADM in terms of fuzzy sets [1], intuitionistic fuzzy sets [2], hesitant fuzzy sets [3], neutrosophic sets [4], etc., rather than crisp sets. A large number of strategies has been developed for MADM problems such as technique for order preference by similarity to ideal solution (TOPSIS) [5], PROMETHEE [6], VIKOR [7], ELECTRE [7, 8], AHP [9], etc. MADM problem has been studied extensively in fuzzy environment [10-14], intuitionistic fuzzy environment [15-22].

TOPSIS [5] is one of the sophisticated strategy for solving MADM. The main idea of TOPSIS is that the best alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS), simultaneously. Since its proposition, researchers have extended the TOPSIS strategy to deal with different environment. Chen [23] extended the TOPSIS strategy for solving multi-criteria decision making (MCDM) problems in fuzzy environment. Boran et al. [24] extended the TOPSIS strategy for MCDM problem in intuitionistic fuzzy environment. Zhao [25] also studied TOPSIS strategy for MADM under interval intuitionistic fuzzy environment and utilized the strategy in teaching quality evaluation. Xu [19] proposed TOPSIS strategy for hesitant fuzzy multi-attribute decision making with incomplete weight information.

However fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets have some limitations to express indeterminate and incomplete information in decision making process. Recently, single valued neutrosophic set (SVNS) [26] has been successfully applied in MADM or multi-attribute group decision [27-37]. SVNS [26] and interval neutrosophic set (INS) [38], and other hybrid neutrosophic sets have caught attention of the researchers for developing TOPSIS strategy. Biswas et al. [39] developed TOPSIS strategy for multi-attribute group decision making (MAGDM) for single valued neutrosophic environment. Sahin et al. [40] proposed another TOPSIS strategy for supplier selection in neutrosophic environment. Chi and Liu developed TOPSIS strategy to deal with interval neutrosophic sets in MADM problems. Zhang and Wu [41] proposed TOPSIS strategies for MCDM in single valued neutrosophic environment and interval neutrosophic set environment where the information about criterion weights are incompletely known or completely unknown. Ye [42] put forward TOPSIS strategy for MAGDM with single-valued neutrosophic linguistic numbers. Peng et al. [43] presented multi-attributive border approximation area comparison (MBAC), TOPSIS, and similarity measure approaches for neutrosophic MADM. Pramanik et al. [44] extended TOPSIS strategy for MADM in neutrosophic soft expert set environment. Different TOPSIS strategies [45-49] have been studied in different hybrid neutrosophic set environment.

Single valued trapezoidal neutrosophic number (SVTrNN) [50, 51] is another extension of single-valued neutrosophic sets. SVTrNN presents the situation, in which each element is characterized by trapezoidal number that has truth membership degree, indeterminate membership degree, and falsity membership degree. Recently, Deli and Şubaş [52] proposed a ranking strategy of single valued neutrosophic number and utilized this strategy in MADM problems. Biswas et al. [53] also proposed value and ambiguity based ranking strategy of single valued trapezoidal neutrosophic number and applied it to MADM.

However, TOPSIS strategy of MADM has not been studied earlier with trapezoidal neutrosophic numbers, although these numbers effectively deal with uncertain information in MADM model. In this study, our objective is to develop an MADM model, where the attribute values assume the form of SVTrNNs and the weight information of attribute is incompletely known or completely unknown. The existing TOPSIS strategy of MADM cannot handle with such situations. Therefore, we need to extend the TOPSIS strategy in SVTrNN environment.

To develop the model, we consider the following sections: Section 2 presents a preliminaries of fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, and single-valued trapezoidal neutrosophic number IFS, SVNS. Section 3 contains the extended TOPSIS strategy for MADM with SVTrNNs. Section 4 presents an illustrative example. Finally, Section 5 presents conclusion and future direction research.

2 Preliminaries

In this section, we review some basic definitions of fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, and single-valued trapezoidal neutrosophic number.

Definition 1. [1] Let X be a universe of discourse, then a fuzzy set A is defined by

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\} \quad (1)$$

which is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ is the degree of membership of the element x to the set A.

Definition 2. [54,55] A generalized trapezoidal fuzzy number A denoted by $A = (a, b, c, d; w)$ is described as a fuzzy subset of the real number $\mathbb{R}$ with membership function μ_A which is defined by

$$\mu_A(x) = \begin{cases} \frac{x-a)w}{b-a} & a \le x < b, \\ w & b \le x \le c, \\ \frac{(d-x)w}{d-c} & c < x \le d, \\ 0 & otherwise \end{cases}$$

where a, b, c, d are real number satisfying $a \le b \le c \le d$ and w is the membership degree.

Definition 3.[4] Let X be a universe of discourse. An neutrosophic sets A over X is defined by

$$A = \{x, < T_A(x), I_A(x), F_A(x) > \mid x \in X\} \quad (2)$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0,1^+[$ that is $T_A(x) : X \to]^-0,1^+[$, $I_A(x) : X \to]^-0,1^+[$ and $F_A(x) : X \to]^-0,1^+[$. The membership functions satisfy the following properties:

$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 4. [26] Let X be a universe of discourse. A single-valued neutrosophic set $\tilde{A}$ in X is given by

$$A = \{x, < T_A(x), I_A(x), F_A(x) > \mid x \in X\} \quad (3)$$

where $T_A(x) : X \to [0,1]$, $I_A(x) : X \to [0,1]$ and $F_A(x) : X \to [0,1]$ with the condition

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent, respectively, the truth membership function, the indeterminacy membership function and the falsity membership function of the element x to the set A.

Definition 5. [50, 51] Let $\tilde{a}$ is a single valued trapezoidal neutrosophic trapezoidal number (SVNTrN). Then its truth membership function is

$$T_{\tilde{a}}(x) = \begin{cases} \frac{x-a)t_{\tilde{a}}}{b-a} & a \le x < b, \\ t_{\tilde{a}} & b \le x \le c, \\ \frac{(d-x)t_{\tilde{a}}}{d-c} & c < x \le d, \\ 0 & otherwise \end{cases}$$

Its indeterminacy membership function is

$$ {}_{\tilde a}(x)=\begin{cases}\dfrac{b-x+(x-a\ i_{\tilde a}}{b-a} & a\le x<b,\\ i_{\tilde a} & b\le x\le c,\\ \dfrac{x-c+(d-x)i_{\tilde a}}{d-c} & c<x\le d,\\ 0 & otherwise\end{cases} $$

and its falsity membership function is

$$ {}_{\tilde a}(x)=\begin{cases}\dfrac{b-x+(x-a)f_{\tilde a}}{b-a} & a\le x<b,\\ f_{\tilde a} & b\le x\le c,\\ \dfrac{x-c+(d-x)f_{\tilde a}}{d-c} & c<x\le d,\\ 0 & otherwise\end{cases} $$

where $0\le T_{\tilde a}(x)\le 1$, $0\le I_{\tilde a}(x)\le 1$, $0\le F_{\tilde a}(x)\le 1$ and $0\le T_{\tilde a}(x)+I_{\tilde a}(x)+F_{\tilde a}(x)\le 3$; $a,b,c,d\in R$. Then $\tilde a=([a,b,c,d];t_{\tilde a},i_{\tilde a},f_{\tilde a})$ is called a neutrosophic trapezoidal number.

Definition 5. [50,51] Let $\tilde a_1=([a_1,b_1,c_1,d_1];t_{\tilde a_1},i_{\tilde a_1},f_{\tilde a_1})$ and $\tilde a_2=([a_2,b_2,c_2,d_2];t_{\tilde a_2},i_{\tilde a_2},f_{\tilde a_2})$ be two neutrosophic trapezoidal fuzzy numbers and $\lambda\ge 0$, then

1. $\tilde a_1\oplus\tilde a_2=([a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];t_{\tilde a_1}+t_{\tilde a_2}-t_{\tilde a_1}t_{\tilde a_2},i_{\tilde a_1}i_{\tilde a_2},f_{\tilde a_1}f_{\tilde a_2})$;
2. $\tilde a_1\otimes\tilde a_2=([a_1a_2,b_1b_2,c_1c_2,d_1d_2];t_{\tilde a_1}t_{\tilde a_2},i_{\tilde a_1}+i_{\tilde a_2}-i_{\tilde a_1}i_{\tilde a_2},f_{\tilde a_1}+f_{\tilde a_2}-f_{\tilde a_1}f_{\tilde a_2})$;
3. $\lambda\,\tilde a_1=([\lambda\,a_1,\lambda\,b_1,\lambda\,c_1,\lambda\,d_1];1-\left(1-t_{\tilde a_1}\right)^{\lambda},\left(i_{\tilde a_1}\right)^{\lambda},\left(f_{\tilde a_1}\right)^{\lambda}$;
4. $(\tilde a)^{\lambda}=([a_1^{\lambda},b_1^{\lambda},c_1^{\lambda},d_1^{\lambda}];\left(t_{\tilde a_1}\right)^{\lambda},1-\left(1-i_{\tilde a_1}\right)^{\lambda},1-\left(1-f_{\tilde a_1}\right)^{\lambda})$

Definition 6. Let $\tilde a_1=([a_1,b_1,c_1,d_1];t_{\tilde a_1},i_{\tilde a_1},f_{\tilde a_1})$ and $\tilde a_2=([a_2,b_2,c_2,d_2];t_{\tilde a_2},i_{\tilde a_2},f_{\tilde a_2})$ be two neutrosophic trapezoidal fuzzy numbers, then the normalized Hamming distance between $\tilde a_1$ and $\tilde a_2$ is defined as follows:

$$ d(\tilde a_1,\tilde a_2)=\frac{1}{12}\begin{pmatrix}\left|a_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|b_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|c_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|d_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\end{pmatrix} \quad (4) $$

Property 1 The normalized Hamming distance measure $d(.)$ of $\tilde a_1$ and $\tilde a_2$ satisfies the following properties:

i. $d(\tilde a_1,\tilde a_2)\ge 0$,
ii. $d(\tilde a_1,\tilde a_2)=d(\tilde a_2,\tilde a_1)$,
iii. $d(\tilde a_1,\tilde a_3)\le d(\tilde a_1,\tilde a_2)+d(\tilde a_2,\tilde a_3)$, where $\tilde a_3=([a_3,b_3,c_3,d_3];t_{\tilde a_3},i_{\tilde a_3},f_{\tilde a_3})$ is a SVTrNN.

Proof:

i. The distance measure $d(\tilde a_1,\tilde a_2)$ is obviously non-negative. If $\tilde a_1\approx\tilde a_2$ that is for $a_1=a_2$, $b_1=b_2$, $c_1=c_2$, $d_1=d_2$, $t_{\tilde a_1}=t_{\tilde a_2}$, $i_{\tilde a_1}=i_{\tilde a_2}$, and $f_{\tilde a_1}=f_{\tilde a_2}$ we have $d(\tilde a_1,\tilde a_1)=0$. Therefore $d(\tilde a_1,\tilde a_2)\ge 0$.
ii. The proof of straightforward.
iii. The normalized Hamming distance between $\tilde a_1$ and $\tilde a_3$ is defined as follows:

$$ d(\tilde a_1,\tilde a_3)=\frac{1}{12}\begin{pmatrix}\left|a_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-a_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ +\left|b_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-b_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ +\left|c_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-c_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ +\left|d_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-d_2(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\end{pmatrix} $$

$$ =\frac{1}{12}\begin{pmatrix}\left|\begin{matrix}a_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\\ +a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-a_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\end{matrix}\right|\\ +\left|\begin{matrix}b_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\\ +b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-b_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\end{matrix}\right|\\ +\left|\begin{matrix}c_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\\ +c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-c_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\end{matrix}\right|\\ +\left|\begin{matrix}d_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\\ +d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-d_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\end{matrix}\right|\end{pmatrix} $$

$$ =\frac{1}{12}\begin{pmatrix}\left|a_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-a_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ \left|b_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-b_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ \left|c_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|+c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-c_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ \left|d_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-d_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\end{pmatrix} $$

$$ \le\frac{1}{12}\begin{pmatrix}\left|a_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|b_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|c_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\\ +\left|d_1(2+t_{\tilde a_1}-i_{\tilde a_1}-f_{\tilde a_1})-d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})\right|\end{pmatrix} $$

$$ +\frac{1}{12}\begin{pmatrix}\left|a_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-a_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ +\left|b_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-b_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ +\left|c_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-c_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\\ +\left|d_2(2+t_{\tilde a_2}-i_{\tilde a_2}-f_{\tilde a_2})-d_3(2+t_{\tilde a_3}-i_{\tilde a_3}-f_{\tilde a_3})\right|\end{pmatrix} $$

$\le d(\tilde a_1,\tilde a_2)+d(\tilde a_2,\tilde a_3)$. □

2.1 TOPSIS Strategy for MADM

The idea behind the TOPSIS strategy [5] is to find out the optimal alternative that has the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution, simultaneously. The schematic structure of classical TOPSIS strategy is presented in the following figure (see Fig. 1)

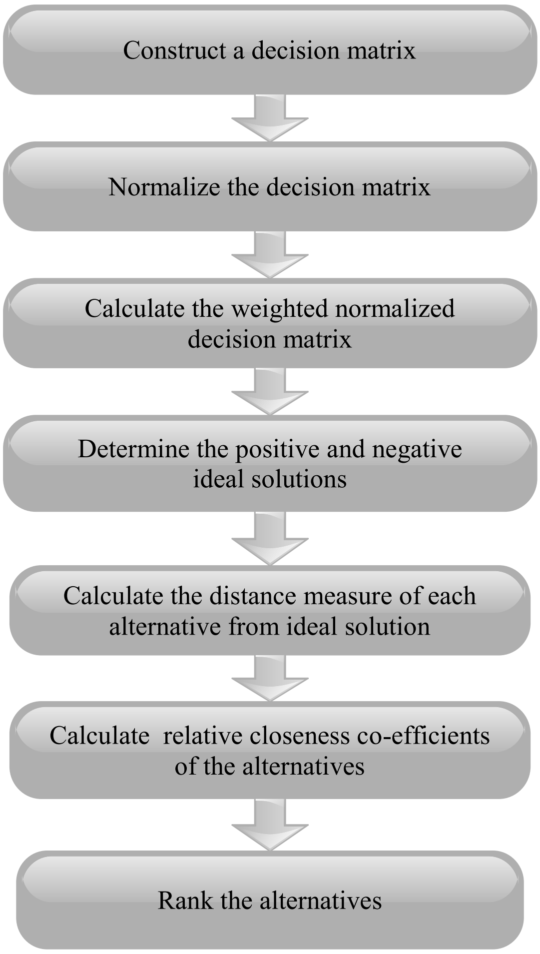

Figure 1. A schematic structure of TOPSIS strategy

3 TOPSIS strategy for multi-attribute decision making with neutrosophic trapezoidal number

In this section, we put forward a framework for determining the attribute weights and the ranking orders for all the alternatives with incomplete weight information under neutrosophic environment.

Consider a MADM problem, where $A = \{A_1, A_2, \ldots, A_m\}$ is a set of m alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ is a set of n attributes. The attribute value of alternative $A_i(i = 1,2, \ldots, m)$ over the attribute $C_j(j = 1,2, \ldots, n)$ assumes the form of neutrosophic trapezoidal number $\tilde{a}_{ij} = \left([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; t_{\tilde{a}_{ij}}, i_{\tilde{a}_{ij}}, f_{\tilde{a}_{ij}}\right)$, where $0 \le t_{\tilde{a}_{ij}} \le 1$, $0 \le i_{\tilde{a}_{ij}} \le 1$, $0 \le f_{\tilde{a}_{ij}} \le 1$ and $0 \le t_{\tilde{a}_{ij}} + i_{\tilde{a}_{ij}} + f_{\tilde{a}_{ij}} \le 3$; $a, b, c, d \in R$.

Here, $t_{\tilde{a}_{ij}}$ denotes the truth membership degree, $i_{\tilde{a}_{ij}}$ denotes the indeterminate membership degree, and $f_{\tilde{a}_{ij}}$ denotes the falsity membership degree to consider the trapezoidal number $[a_{ij}, b_{ij}, c_{ij}, d_{ij}]$ as the rating values of A_i over the attribute C_j. An MADM problem can be expressed by a decision matrix in which the entries represent the evaluation information of all alternatives with respect to the attributes. Then we construct the following neutrosophic decision matrix, whose elements are SVNTrNs:

$$D = (\tilde{a}_{ij})_{m\times n} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \ldots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \ldots & \tilde{a}_{mn} \end{pmatrix} \quad (5)$$

Due to different attribute weights, we assume that the weight vector of all attributes is given by $w = (w_1, w_2, \ldots, w_n)^T$, where $0 \le w_j \le 1$, $j = 1,2, \ldots, n$, and w_j is the weight of each attribute. The information about attribute weights is usually incomplete in decision making problems under uncertain environment. For convenience, we assume Δ be a set of the known weight information [56-59], where Δ can be constructed by the following forms, for $i \ne j$:

Form 1. A weak ranking: $\{w_i \ge w_j\}$;

Form 2. A strict ranking: $\{w_i - w_j \ge \alpha_j\}(\alpha_j > 0)$;

Form 3. A ranking of difference: $\{w_i - w_j \ge w_k - w_l\}$, for $j \ne k \ne l$;

Form 4. A ranking with multiples: $\{w_i \ge \alpha_j w_j\}$ $(0 \le \alpha_j \le 1)$;

Form 5. An interval form: $\{\alpha_i \le w_i \le \alpha_i + \epsilon_i\}(0 \le \alpha_j \le \alpha_i + \epsilon_i \le 1)$.

Now we develop a strategy for solving the MADM problems, in which the information about attribute weights is completely unknown or partially known and the attribute values are expressed by SVTrNNs.

The following steps are considered to develop the model.

3.1 Standardize the decision matrix

Let $D = (\tilde{a}_{ij})_{m\times n}$ be a neutrosophic decision matrix, where the SVTrNNs $\tilde{a}_{ij} = \left([a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4]; t_{\tilde{a}_{ij}}, i_{\tilde{a}_{ij}}, f_{\tilde{a}_{ij}}\right)$ is the rating values of alternative A_i with respect to attribute C_j. Now to eliminate the effect from different physical dimensions into decision making process, we should standardize the decision matrix $(\tilde{a}_{ij})_{m\times n}$ based on two common types of attributes such as benefit type attribute and cost type attribute. We consider the following technique to obtain the standardized decision matrix $R=\left(\tilde{r}_{ij}\right)_{m\times n}$, in which the component r_{ij}^{k} of the entry $\tilde{r}_{ij}=\left(\left[r_{ij}^{1},r_{ij}^{2},r_{ij}^{3},r_{ij}^{4}\right];t_{\tilde{r}_{ij}},i_{\tilde{r}_{ij}},f_{\tilde{r}_{ij}}\right)$ in the matrix R are considered as:

1. For benefit type attribute:

$$\tilde{r}_{ij}=\left\{\left(\left[\frac{a_{ij}^{1}}{u_j^{+}},\frac{a_{ij}^{2}}{u_j^{+}},\frac{a_{ij}^{3}}{u_j^{+}},\frac{a_{ij}^{4}}{u_j^{+}}\right];t_{\tilde{r}_{ij}},i_{\tilde{r}_{ij}},f_{\tilde{r}_{ij}}\right)\right. \quad (6)$$

2. For cost type attribute:

$$\tilde{r}_{ij}=\left\{\left(\left[\frac{u_j^{-}}{a_{ij}^{4}},\frac{u_j^{-}}{a_{ij}^{3}},\frac{u_j^{-}}{a_{ij}^{2}},\frac{u_j^{-}}{a_{ij}^{1}}\right];t_{\tilde{r}_{ij}},i_{\tilde{r}_{ij}},f_{\tilde{r}_{ij}}\right)\right., \quad (7)$$

where $u_j^{+}=\max\{a_{ij}^{4}\mid i=1,2,...m\}$ and

$u_j^{-}=\min\{a_{ij}^{1}\mid i=1,2,...m\}$ for $j=1,2,...n.$

Then we obtain the following standardized decision matrix:

$$R=\left(\tilde{r}_{ij}\right)_{m\times n}=\begin{pmatrix}\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n}\\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn}\end{pmatrix} \quad (8)$$

3.2 Determine the attribute weight

To determine the attribute weights, we use maximum deviation strategy, which was proposed by Wang [60]. According to Wang [60],

i. The attribute that has the larger deviation value among alternatives should be assigned larger weight.
ii. The attribute having deviation value among alternatives should be assigned smaller weight.
iii. The attribute having no deviation among alternatives should be assigned zero weight.

Following the idea of maximum deviation method, we construct an optimization model to determine the optimal weights of attributes with SVTrNNs. The deviation of the alternative A_i to all the other alternatives for the attribute C_j can be defined as follows:

$d_{ij}(w)=\sum_{k=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{kj})w_j$, $i=1,2,\dots,m;$ $j=1,2,\dots,n$

where

$$d(\tilde{a}_{ij},\tilde{a}_{kj})=\frac{1}{12}\begin{pmatrix}\left|a_{ij1}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj1}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\\ +\left|a_{ij2}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj2}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\\ +\left|a_{ij3}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj3}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\\ +\left|a_{ij4}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj4}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\end{pmatrix} \quad (9)$$

$$=\frac{1}{12}\sum_{p=1}^{4}\left|\begin{matrix}a_{ijp}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)\\ -a_{kjp}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\end{matrix}\right|$$

denotes the neutrosophic Hamming distance between two SVTrNNs $\tilde{a}_{ij}$ and $\tilde{a}_{kj}$.

The deviation value of all the alternatives to other alternatives for the attribute C_j can be obtained as follows:

$$D_j(w)=\sum_{i=1}^{m} d_{ij}(w)=\sum_{i=1}^{m}\sum_{k=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{kj})\,w_j$$

$$=\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{12}\sum_{p=1}^{4}\left|a_{ij}^{p}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj}^{p}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\right)w_j. \quad (10)$$

Similarly, the deviation value of all the alternatives to other alternatives for all the criteria can be taken as:

$$D(w)=\sum_{j=1}^{n} D_j(w)=\sum_{j=1}^{n}\sum_{i=1}^{m} d_{ij}(w)$$

$$=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{kj})\,w_j$$

$$=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{12}\sum_{p=1}^{4}\left|\begin{matrix}a_{ij}^{p}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)\\ -a_{kj}^{p}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\end{matrix}\right|\right)w_j$$

If the information about the attribute weights is partially known or completely unknown, then we propose two models to obtain the attribute weights.

3.2.1 *Information about the weights of attributes is partially known.*

In order to obtain the weight vector, we construct a non-linear programming model that maximizes all deviation values of attributes. The model can be presented as follows:

$$(M-2)\begin{cases}\max\ D(w)=\dfrac{1}{12}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{matrix}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\ -a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{matrix}\right|\right)w_j\\ \text{subject to } w\in\Delta,\quad \displaystyle\sum_{j=1}^{n} w_j=1,\ w_j\geq 0,\quad \text{for } j=1,2,..,n.\end{cases} \quad (11)$$

By solving the model (M-1), we obtain the optimal solution to be used as the weight vector.

3.2.2 *Information about the weights of attributes is unknown.*

If the information about attribute weight is completely unknown, then we can establish the following programming model:

$$(M-2)\begin{cases}\max\ D(w)=\dfrac{1}{12}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{matrix}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\ -a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{matrix}\right|\right)w_j\\ \text{subject to } w\in\Delta,\quad \displaystyle\sum_{j=1}^{n} w_j^{2}=1,\ w_j\geq 0,\ \text{for } j=1,2,..,n.\end{cases} \quad (12)$$

To solve the model (M-2), we develop the Lagrange function:

$$L(w,\xi)=\frac{1}{12}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)w_j+\frac{\xi}{24}\left(\sum_{j=1}^{n}w_j^2-1\right) \quad (13)$$

where ξ is a real number and denoting the Lagrange multiplier variable. Then the partial derivative of L with respect to $w_j\,(j=1,2,...,n)$ and ξ are obtained as:

$$\frac{\partial L}{\partial w_j}=\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)w_j+\xi w_j=0 \quad (14)$$

$$\frac{\partial L}{\partial \xi}=\sum_{j=1}^{n}w_j^2-1=0 \quad (15)$$

It follows from Eq. (14) that

$$w_j=\frac{-\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)}{\xi} \quad \text{for} \quad j=1,2,...,n. \quad (16)$$

Putting the values of w_j in Eq.(15), we obtain

$$\xi^2=\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)\right)^2 \quad (17)$$

$$\Rightarrow \xi=-\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)\right)^2} \quad \text{for} \quad \xi<0. \quad (18)$$

Then combining Eq.(16) and Eq.(18), we obtain the following formula for determining the weight of attribute $C_j\,(j=1,2,...,n)$:

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)\right)^2}}\,. \quad (19)$$

We make their sum into a unit by normalizing $w_j\,(j=1,2,...,n)$ and get the optimal weight of attribute $C_j\,(j=1,2,...,n)$:

$$\bar{w}_j=\frac{w_j}{\sum_{j=1}^{n}w_j}=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)} \quad (20)$$

Then we get the normalized weight vector of attributes:

$\bar{w}=\{\bar{w}_1,\bar{w}_2,...,\bar{w}_n\}$.

3.3 Determine the ideal solutions

In the normalized decision matrix $R=(\tilde{r}_{ij})_{m\times n}$, the neutrosophic trapezoidal local positive ideal solution (NTrPIS) and the neutrosophic trapezoidal local negative ideal solution (NTrNIS) are defined as follows

$$\tilde{r}^{+}=\left(\tilde{r}_1^{+},\tilde{r}_2^{+},...,\tilde{r}_n^{+}\right) \text{ and } \tilde{r}^{-}=\left(\tilde{r}_1^{-},\tilde{r}_2^{-},...,\tilde{r}_n^{-}\right) \quad (21)$$

where,

$$r_j^{+}=\left(\left[r_j^{1+},r_j^{2+},r_j^{3+},r_j^{4+}\right],t_j^{+},i_j^{+},f_j^{+}\right)=\left(\begin{array}{r}\left[\max_i(r_{ij}^1),\max_i(r_{ij}^2),\max_i(r_{ij}^3),\max_i(r_{ij}^4)\right];\\ \max_i(t_{ij}),\min_i(i_{ij}),\min_i(f_{ij})\end{array}\right) \quad (22)$$

$$r_j^{-}=\left(\left[r_j^{1-},r_j^{2-},r_j^{3-},r_j^{4-}\right],t_j^{-},i_j^{-},f_j^{-}\right)=\left(\begin{array}{r}\left[\min_i(r_{ij}^1),\min_i(r_{ij}^2),\min_i(r_{ij}^3),\min_i(r_{ij}^4)\right];\\ \min_i(t_{ij}),\max_i(i_{ij}),\max_i(f_{ij})\end{array}\right) \quad (23)$$

Moreover, the trapezoidal neutrosophic global positive ideal solution and the trapezoidal neutrosophic global trapezoidal global negative ideal solution can be directly considered as

$$r_j^{+}=\left([1,1,1,1],1,0,0\right) \text{ and } r_j^{-}=\left([0,0,0,0],0,1,1\right) \quad (24)$$

3.4 Determine the separation measures from ideal solutions to each alternative

The separation measures d_i^{+} and d_i^{-} of each alternative from the ideal solutions can be determined by Eq.(9), Eq.(20) and Eq.(21), respectively, as follows:

$$d_i^{+}=\sum_{j=1}^{n}w_j d\left(\tilde{r}_{ij},\tilde{r}_j^{+}\right)=\frac{1}{12}\sum_{j=1}^{n}w_j\sum_{p=1}^{4}\left(\left|\begin{array}{l}r_{ij}^{p}(2+t_{\tilde{r}_{ij}}-i_{\tilde{r}_{ij}}-f_{\tilde{r}_{ij}})\\-r_j^{p+}(2+t_{\tilde{r}_j^{+}}-i_{\tilde{r}_j^{+}}-f_{\tilde{r}_j^{+}})\end{array}\right|\right) \text{ for } i=1,2,...,m \quad (25)$$

$$d_i^{-}=\sum_{j=1}^{n}w_j d\left(\tilde{r}_{ij},\tilde{r}_j^{-}\right)=\frac{1}{12}\sum_{j=1}^{n}w_j\sum_{p=1}^{4}\left(\left|\begin{array}{l}r_{ij}^{p}(2+t_{\tilde{r}_{ij}}-i_{\tilde{r}_{ij}}-f_{\tilde{r}_{ij}})\\-r_j^{p-}(2+t_{\tilde{r}_j^{-}}-i_{\tilde{r}_j^{-}}-f_{\tilde{r}_j^{-}})\end{array}\right|\right) \text{ for } i=1,2,...,m \quad (26)$$

3.5 Determine the relative closeness co-efficient

The relative closeness co-efficient of an alternative A_i with respect to ideal alternative A^{+} is defined as the following formula:

$$RC(A_i)=\frac{d_i^-}{d_i^+ + d_i^-} \quad (27)$$

where $0 \le RC(A_i) \le 1$ for $i=1,2,...m$. According to the closeness co-efficient $RC(A_i)$, the ranking orders of all alternatives and the best alternative can be selected. The schematic diagram of the proposed TOPSIS is presented in Figure-2.

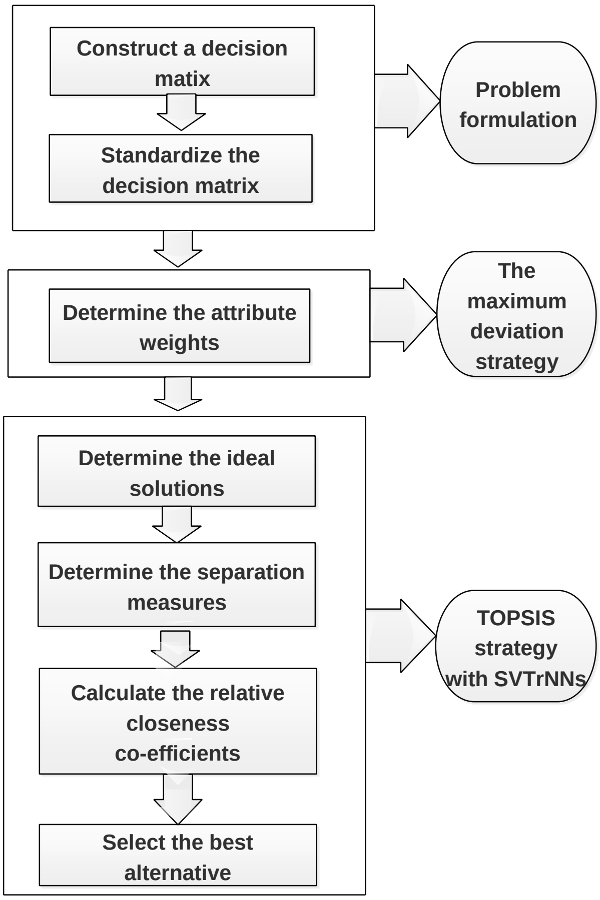

Figure 2. The schematic diagram of the proposed startegy

4 An illustrative example

In this section, we consider an illustrative example of medical representative selection problem to demonstrate and applicability of the proposed.

Consider a MADM problem, where a pharmacy company wants to recruit a medical representative. After initial scrutiny four candidates $A_i (i = 1,2,3,4)$ have been considered for further evaluation with respect to the four attributes C_j(j = 1,2,3,4) namely,

1. Oral communication skill (C_1);
2. Past experience (C_2),
3. General aptitude (C_3) and
4. Self- confidence (C_4).

The decision maker evaluates the ratings of alternatives $A_i (i=1,2,...,m)$ with respect to the attributes $C_i (i=1,2,...,n)$ with the decision matrix $D=(\tilde{a}_{ij})_{4\times 4}$ (see Table 1).

Table 1. Rating values of alternatives

	C_1	C_2
A_1	([7,8,9,10]; 0.90, 0.10, 0.05)	([5,6,7,8]; 0.65, 0.35, 0.30)
A_2	([5,6,7,8]; 0.65, 0.35, 0.30)	([6,7,8,9]; 0.80, 0.20, 0.15)
A_3	([4,5,6,7]; 0.50, 0.50, 0.45)	([5,6,7,8]; 0.65, 0.35, 0.30)
A_4	([4,5,6,7]; 0.50, 0.50, 0.45)	([5,6,7,8]; 0.65, 0.35, 0.30)
	C_3	C_4
A_1	([6,7,8,9]; 0.80, 0.20, 0.15)	([7,8,9,10]; 0.90, 0.10, 0.05)
A_2	([7,8,9,10]; 0.90, 0.10, 0.05)	([6,7,8,9]; 0.80, 0.20, 0.15)
A_3	([4,5,6,7]; 0.50, 0.50, 0.45)	([4,5,6,7]; 0.50, 0.50, 0.45)
A_4	([4,5,6,7]; 0.50, 0.50, 0.45)	([6,7,8,9]; 0.80, 0.20, 0.15)

The information of the attributes is incompletely known and the weight information is given as follows:

$$\Delta=\left\{\begin{array}{l}0.20 \le w_1 \le 0.30, 0.05 \le w_2 \le 0.20,\\ 0.20 \le w_3 \le 0.35, 0.15 \le w_4 \le 0.35; \sum_{j=1}^{4} w_j = 1\end{array}\right\} \quad (28)$$

To determine the best alternative, we use the proposed strategy involving the following steps:

Step 1. Standardize the decision matrix

Since the selective attributes are benefit type attributes, then using Eq. (6), we have the following standardized decision matrix: $R=(\tilde{r}_{ij})_{4\times 4}$ (see Table 2.)

Table 2. Standardized rating values of alternatives

	C_1	C_2
A_1	([0.7, 0.8, 0.9, 1.0]; 0.90, 0.10, 0.05)	([0.5, 0.6, 0.7, 0.8]; 0.65, 0.35, 0.30)
A_2	([0.5, 0.6, 0.7, 0.8]; 0.65, 0.35, 0.30)	([0.6, 0.7, 0.8, 0.9]; 0.80, 0.20, 0.15)

A3	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\ 0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.5,0.6,0.7,0.8];\\ 0.65,0.35,0.30\end{pmatrix}$
A4	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\ 0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.5,0.6,0.7,0.8];\\ 0.65,0.35,0.30\end{pmatrix}$
	C_3	C_4
A1	$\begin{pmatrix}[0.6,0.7,0.8,0.9];\\ 0.80,0.20,0.15\end{pmatrix}$	$\begin{pmatrix}[0.7,0.8,0.9,1.0];\\ 0.90,0.10,0.05\end{pmatrix}$
A2	$\begin{pmatrix}[0.7,0.8,0.9,1.0];\\ 0.90,0.10,0.05\end{pmatrix}$	$\begin{pmatrix}[0.6,0.7,0.8,0.9];\\ 0.80,0.20,0.15\end{pmatrix}$
A3	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\ 0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\ 0.50,0.50,0.45\end{pmatrix}$
A4	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\ 0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.6,0.7,0.8,0.9];\\ 0.80,0.20,0.15\end{pmatrix}$

Step 2. Determine the attribute weight

Case 1. Weight information is incompletely known.

Using the model (M-1), we construct the following single-objective programming problem:

$$\begin{cases}\max\ D(w)=3.2133w_1+1.1401w_2+3.4250w_3+2.9700w_4\\ \text{subject to } w\in\Delta,\quad \sum_{j=1}^{4}w_j=1,\ w_j\ge 0,\quad \text{for } j=1,2,..,4.\end{cases} \tag{29}$$

Solving this model with optimization software LINGO 13, we get the optimal weight vector as $w=(0.30,0.05,0.35,0.30)$.

Case 2. Weight information is completely unknown.
Following Eq.(20), we obtain the following optimal weight vector:
$\overline{w}=(0.2990,0.1061,0.3186,0.2763)$.

Step 3. Determine the ideal solutions

Since the chosen attributes are benefit type attribute, then following Eq.(22) we determine the neutrosophic trapezoidal positive ideal solution as

$$A^+=\begin{pmatrix}([0.7,0.8,0.9,1.0];0.90,0.10,0.05),\\ ([0.6,0.7,0.8,0.9];0.80,0.20,0.15),\\ ([0.7,0.8,0.9,1.0];0.90,0.10,0.05),\\ ([0.7,0.8,0.9,1.0];0.90,0.10,0.05)\end{pmatrix} \tag{30}$$

Similarly, using Eq.(23), we determine the neutrosophic trapezoidal negative ideal solution

$$A^-=\begin{pmatrix}([0.4,0.5,0.6,0.7];0.50,0.50,0.45),\\ ([0.5,0.6,0.7,0.8];0.65,0.35,0.30),\\ ([0.4,0.5,0.6,0.7];0.50,0.50,0.45),\\ ([0.4,0.5,0.6,0.7];0.50,0.50,0.45)\end{pmatrix} \tag{31}$$

Step 4. Determine the separation measures from ideal solutions to each alternative.

Case 1. Employing Eq.(25), we obtain the separation measures d_i^+ of each alternative $A_i (i=1,2,3,4)$ from A^+:
$d(A_1,A^+)=0.0673$, $d(A_2,A^+)=0.1538$, $d(A_3,A^+)=0.4792$, $d(A_4,A^+)=0.3807.$

Similarly, using Eq.(26), we obtain the separation measures d_i^- of each alternative $A_i (i=1,2,3,4)$ from A^-:
$d(A_1,A^-)=0.4119$, $d(A_2,A^-)=0.3254$, $d(A_3,A^-)=0$, $d(A_4,A^-)=0.0985.$

Case 2. The separation measures d_i^+ of each alternative $A_i (i=1,2,3,4)$ from A^+:
$d(A_1,A^+)=0.0721$, $d(A_2,A^+)=0.1494$, $d(A_3,A^+)=0.4615$, $d(A_4,A^+)=0.3708.$

Similarly, the separation measures d_i^- of each alternative $A_i (i=1,2,3,4)$ from A^-:
$d(A_1,A^-)=0.3894$, $d(A_2,A^-)=0.3120$, $d(A_3,A^-)=0$, $d(A_4,A^-)=0.0907.$

Step 5. Calculate the relative closeness coefficient.

Using Eq.(27), we calculate the relative closeness coefficient $RC(A_i)$ of alternative $A_i (i=1,2,3,4)$ for Case 1 and Case 2, respectively. We put the result in Table 3.

Table 3. Rating values of alternatives

RC(A$_i$)	Case 1	Case 2
RC(A$_1$)	0.8596	0.8438
RC(A$_2$)	0.6790	0.6824
RC(A$_3$)	0	0
RC(A$_4$)	0.2056	0.1965

Following Table 3, we rank the alternatives $A_i (i=1,2,3,4)$ according to the values of relative closeness coefficient $RC(A_i)$ for both cases: $A_1 \succ A_2 \succ A_4 \succ A_3$. Therefore A_1 is the best alternative.

5 Conclusions

TOPSIS strategy is a useful strategy for solving MADM problem under different environment. In this paper, we have investigated MADM problems with SVTrNNs. The weight information of attributes have been considered to be incompletely known or completely unknown. First, we have used Hamming distance measure to determine the distance measure of SVTrNNs. Second, we developed an optimization model to determine the attribute weights based on the idea of maximum deviation strategy. Third, we have extended the TOPSIS strategy for solving the MADM model with SVTrNNs. Finally, we have provided an illustrative examples to verify the feasibility and effectiveness of the proposed model. The proposed TOPSIS strategy can be extended to multi-attribute group decision making with SVTrNNs and multi-attribute decision making problem with interval trapezoidal neutrosophic numbers. The proposed TOPSIS strategy can be used in solving logistics center location selection [61, 62], weaver selection [63, 64], data mining [65], school choice [66], teacher selection [67], brick field selection [68-69), etc. under SVTrNN environment.

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