Neutrosophic Logic - A Unifying Field in Logics

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Abstract: In this paper one generalizes fuzzy, paraconsistent, and intuitionistic logic to neutrosophic logic. Many examples are presented, and a survey to the evolution of logics up to neutrosophic logic.

Keywords and Phrases: Fuzzy Logic, Paraconsistent Logic, Intuitionistic Logic, Neutroosphic Logic.

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2. NEUTROSOPHIC LOGIC:

2.1. Introduction.

As an alternative to the existing logics we propose the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction. It is a non-classical logic.

Eksioglu (1999) explains some of them:

"Imprecision of the human systems is due to the imperfection of knowledge that humain receives (observation) from the external world. Imperfection leads to a doubt about the value of a variable, a decision to be taken or a conclusion to be drawn for the actual system. The sources of uncertainty can be stochasticity (the case of intrinsic imperfection where a typical and single value does not exist), incomplete knowledge (ignorance of the totality, limited view on a system because of its complexity) or the acquisition errors (intrinsically imperfect observations, the quantitative errors in measures)."

"Probability (called sometimes the objective probability) process uncertainty of random type (stochastic) introduced by the chance. Uncertainty of the chance is clarified by the time or by events' occurrence. The probability is thus connected to the frequency of the events' occurrence."

"The vagueness which constitutes another form of uncertainty is the character of those with contours or limits lacking precision, clearness. […]For certain objects, the fact to be in or out of a category is difficult to mention. Rather, it is possible to express a partial or gradual membership."

Indeterminacy means degrees of uncertainty, vagueness, imprecision, undefined, unknown, inconsistency, redundancy.

A question would be to try, if possible, to get an axiomatic system for the neutrosophic logic. Intuition is the base for any formalization, because the postulates and axioms derive from intuition.

2.2. Definition:

A logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are defined above, is called *Neutrosophic Logic*.

We use a subset of truth (or indeterminacy, or falsity), instead of a number only, because in many cases we are not able to exactly determine the percentages of truth and of falsity but to approximate them: for example a proposition is between 30-40% true and between 60-70% false, even worst: between 30-40% or 45-50% true (according to various analysers), and 60% or between 66-70% false.
The subsets are not necessary intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.) in accordance with the given proposition.
A subset may have one element only in special cases of this logic.

Constants: (T, I, F) truth-values, where T, I, F are standard or non-standard subsets of the non-standard interval $]\!]^-0, 1^+[\![$, where $n_{inf} = \inf T + \inf I + \inf F \geq {}^-0$, and $n_{sup} = \sup T + \sup I + \sup F \leq 3^+$.
Atomic formulas: a, b, c, … .
Arbitrary formulas: A, B, C, … .

The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy, and falsehood.
My hypothesis is that **no theory is exempted from paradoxes**, because of the language imprecision, metaphoric expression, various levels or meta-levels of understanding/interpretation which might overlap.

2.3. History:

The Classical Logic, also called Bivalent Logic for taking only two values {0, 1}, or Boolean Logic from British mathematician George Boole (1815-64), was named by the philosopher Quine (1981) “sweet simplicity”.
Peirce, before 1910, developed a semantics for three-valued logic in an unpublished note, but Emil Post’s dissertation (1920s) is cited for originating the three-valued logic. Here “1” is used for truth, “1/2” for indeterminacy, and “0” for falsehood. Also, Reichenbach, leader of the logical empiricism, studied it.
The three-valued logic was employed by HalldJn (1949), K`rner (1960), Tye (1994) to solve Sorites Paradoxes. They used truth tables, such as Kleene’s, but everything depended on the definition of validity.
A three-valued paraconsistent system (LP) has the values: ‘true’, ‘false’, and ‘both true and false’. The ancient Indian metaphysics considered four possible values of a statement: ‘true (only)’, ‘false (only)’, ‘both true and false’, and ‘neither true nor false’; J. M. Dunn (1976) formalized this in a four-valued paraconsistent system as his First Degree Entailment semantics;
The Buddhist logic added a fifth value to the previous ones, ‘none of these’ (called *catushkoti*).

In order to clarify the anomalies in science, Rugina (1949, 1981) proposed an original method, starting first from an economic point of view but generalizing it to any science, to study the equilibrium and disequilibrium of systems. His Orientation Table comprises seven basic models:

Model M_1 (which is 100% stable)
Model M_2 (which is 95% stable, and 5% unstable)
Model M_3 (which is 65% stable, and 35% unstable)
Model M_4 (which is 50% stable, and 50% unstable)
Model M_5 (which is 35% stable, and 65% unstable)
Model M_6 (which is 5% stable, and 95% unstable)
Model M_7 (which is 100% unstable)

He gives Orientation Tables for Physical Sciences and Mechanics (Rugina 1989), for the Theory of Probability, for what he called Integrated Logic, and generally for any Natural or Social Science (Rugina 1989). This is a Seven-Valued Logic.

The $\{0, a_1, ..., a_n, 1\}$ Multi-Valued, or Plurivalent, Logic was develop by °ukasiewicz, while Post originated the m-valued calculus.
The many-valued logic was replaced by Goguen (1969) and Zadeh (1975) with an Infinite-Valued Logic (of continuum power, as in the classical mathematical analysis and classical probability) called Fuzzy Logic, where the truth-value can be any number in the closed unit interval [0, 1]. The Fuzzy Set was introduced by Zadeh in 1975.

Rugina (1989) defines an anomaly as "a deviation from a position of stable equilibrium represented by Model M_1", and he proposes a Universal Hypothesis of Duality:
"The physical universe in which we are living, including human society and the world of ideas, all are composed in different and changeable proportions of stable (equilibrium) and unstable (disequilibrium) elements, forces, institutions, behavior and value"
and a General Possibility Theorem:
"there is an unlimited number of possible combinations or systems in logic and other sciences".
According to the last assertions one can extend Rugina's Orientation Table in the way that any system in each science is s% stable and u% unstable, with s+u=100 and both parameters
$0 \le s, u \le 100$, somehow getting to a fuzzy approach.
But, because each system has hidden features and behaviors, and there would always be unexpected occurring conditions we are not able to control - we mean the indeterminacy plays a role as well, a better approach would be the *Neutrosophic Model*:
Any system in each science is s% stable, i% indeterminate, and u% unstable, with s+i+u=100 and all three parameters $0 \le s, i, u \le 100$.

Therefore, we finally generalize the fuzzy logic to a transcendental logic, called "neutrosophic logic": where the interval [0, 1] is exceeded, i.e. , the percentages of truth, indeterminacy, and falsity are approximated by non-standard subsets – not by single numbers, and these subsets may overlap and exceed the unit interval in the sense of the non-standard analysis; also the superior sums and inferior sum, $n_{sup} = \sup T + \sup I + \sup F \in]^-0, 3^+[$, may be as high as 3 or 3^+, while $n_{inf} = \inf T + \inf I + \inf F \in]^-0, 3^+[$, may be as low as 0 or $^-0$.

Generally speaking, passing from the attribute "classical" (traditional) to the attribute "modern" (in literature, arts, and philosophy today one says today "postmodern") one

invalidates many theorems. Voltaire (1694-1778), a French writer and philosopher, asserted that "the laws in arts are made in order to encroach upon them". Therefore, in neutrosophic logic most of the classical logic laws and its properties are not preserved.
Although at a first look neutrosophic logic appears counter-intuitive, maybe abnormal, because the neutrosophic-truth values of a proposition A, NL(A), may even be (1,1,1), i.e. a proposition can completely be true and false and indeterminate at the same time, studying the paradoxes one soon observes that it is intuitive.

The idea of tripartition (truth, falsehood, indeterminacy) appeared in 1764 when J. H. Lambert investigated the credibility of one witness affected by the contrary testimony of another. He generalized Hooper's rule of combination of evidence (1680s), which was a Non-Bayesian approach to find a probabilistic model. Koopman in 1940s introduced the notions of lower and upper probability, followed by Good, and Dempster (1967) gave a rule of combining two arguments. Shafer (1976) extended it to the Dempster-Shafer Theory of Belief Functions by defining the Belief and Plausibility functions and using the rule of inference of Dempster for combining two evidences proceeding from two different sources. Belief function is a connection between fuzzy reasoning and probability. The Dempster-Shafer Theory of Belief Functions is a generalization of the Bayesian Probability (Bayes 1760s, Laplace 1780s); this uses the mathematical probability in a more general way, and is based on probabilistic combination of evidence in artificial intelligence.
In Lambert "there is a chance p that the witness will be faithful and accurate, a chance q that he will be mendacious, and a chance 1-p-q that he will simply be careless" [apud Shafer (1986)]. Therefore three components: accurate, mendacious, careless, which add up to 1.
Van Fraassen introduced the supervaluation semantics in his attempt to solve the sorites paradoxes, followed by Dummett (1975) and Fine (1975). They all tripartitioned, considering a vague predicate which, having border cases, is undefined for these border cases. Van Fraassen took the vague predicate 'heap' and extended it positively to those objects to which the predicate definitively applies and negatively to those objects to which it definitively doesn't apply. The remaining objects border was called penumbra. A sharp boundary between these two extensions does not exist for a soritical predicate. Inductive reasoning is no longer valid too; if S is a sorites predicate, the proposition "$\exists n(Sa_n \& \neg Sa_{n+1})$" is false. Thus, the predicate Heap (positive extension) = true, Heap (negative extension) = false, Heap (penumbra) = indeterminate.

Narinyani (1980) used the tripartition to define what he called the "indefinite set", and Atanassov (1982) continued on tripartition and gave five generalizations of the fuzzy set, studied their properties and applications to the neural networks in medicine:
a) Intuitionistic Fuzzy Set (IFS):
Given an universe E, an IFS A over E is a set of ordered triples <universe_element, degree_of_membership_to_A(M), degree_of_non-membership_to_A(N)> such that $M+N \leq 1$ and $M, N \in [0, 1]$. When $M + N = 1$ one obtains the fuzzy set, and if $M + N < 1$ there is an indeterminacy $I = 1-M-N$.
b) Intuitionistic L-Fuzzy Set (ILFS):
Is similar to IFS, but M and N belong to a fixed lattice L.
c) Interval-valued Intuitionistic Fuzzy Set (IVIFS):
Is similar to IFS, but M and N are subsets of [0, 1] and $\sup M + \sup N \leq 1$.

d) Intuitionistic Fuzzy Set of Second Type (IFS2):
Is similar to IFS, but $M^2 + N^2 \leq 1$. M and N are inside of the upper right quarter of unit circle.

e) Temporal IFS:
Is similar to IFS, but M and N are functions of the time-moment too.

This neutrosophic logic is the (first) attempt to unify many logics in a single field. However, sometimes a too large generalization may have no practical impact. Such unification theories, or attempts, have been done in the history of sciences:

a) Felix Klein (1872), in his Erlangen programme, in geometry, has proposed a common standpoint from which various branches of geometries could be re-organized, interpreted, i.e.:
Given a manifold and a group of transformations of the manifold, to study the manifold configurations with respect to those features that are not altered by the transformations of the group (Klein 1893, p. 67; apud Torretti 1999).

b) Einstein tried in physics to build a Unifying Field Theory that seeks to unite the properties of gravitational, electromagnetic, weak, and strong interactions so that a single set of equations can be used to predict all their characteristics; whether such a theory may be developed it is not known at the present (Illingworth 1991, p. 504).

c) Also, one mentions the Grand Unified Theory, which is a unified quantum field theory of the electromagnetic, weak, and strong interactions (Illingworth 1991, p. 200).

But generalizations become, after some levels, “very general”, and therefore not serving at much and, if dealing with indeterminacy, underlying the infinite improbability drive. Would the gain of such total generality offset the losses in specificity? A generalization may be done in one direction, but not in another, while gaining in a bearing but loosing in another.
How to unify, not too much generalizing? Dezert (1999) suggested to develop the less limitative possible theory which remains coherent with certain existing theories. The rules of inferences in this general theory should satisfy many important mathematical properties.
“Neutrosophic Logic could permit in the future to solving certain practical problems posed in the domain of research in Data/Information fusion. So far, almost all approaches are based on the Bayesian Theory, Dempster-Shafer Theory, Fuzzy Sets, and Heuristic Methods” (Dezert 1999). Theoretical and technical advances for Information Fusion are probability and statistics, fuzzy sets, possibility, evidential reasoning, random sets, neural networks and neuro-mimetic approaches, and logics (Dezert 2000).
The confidence interval <Bel, Pl> in Dempster-Shafer Theory is the truth subset (T) in the neutrosophic set (or logic). The neutrosophic logic, in addition to the it, contains an indeterminacy set (say indeterminacy interval) and falsehood set (say in-confidence internal).

ºukasiewicz, together with Kotarbi½ski and LeÑniewski from the Warsaw Polish Logic group (1919-1939), questioned the status of truth: eternal, sempiternal (everlasting, perpetual), or both? Also, ºukasiewicz had the idea of *logical probability* in between the two world wars.

Let’s borrow from the modal logic the notion of “world”, which is a semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is ‘true in all possible worlds’ (syntagme first used by Leibniz) and all conjunctures, that one may call “absolute truth” (in the modal logic it was named *necessary truth*, Dinulescu-C>mpina (2000) names it ‘intangible absolute truth’), whereas

$NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a 'specific' world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each 'world' is dynamic, depending on an ensemble of parameters, we introduce the sub-category 'conjuncture' within it to reflect a particular state of the world.

How can we differentiate <the truth behind the truth>? What about the <metaphoric truth>, which frequently occurs in the humanistic field? Let's take the proposition "99% of the politicians are crooked" (Sonnabend 1997, Problem 29, p. 25). "No," somebody furiously comments, "100% of the politicians are crooked, *even more*!" How do we interpret this "even more" (than 100%), i. e. more than the truth?

One attempts to formalize. For $n \geq 1$ one defines the "n-level relative truth" of the statement A if the statement is true in at least n distinct worlds, and similarly "countably-" or "uncountably-level relative truth" as gradual degrees between "first-level relative truth" (1) and "absolute truth" (1^+) in the monad $\mu(1^+)$. Analogue definitions one gets by substituting "truth" with "falsehood" or "indeterminacy" in the above.

In *largo sensu* the notion "world" depends on parameters, such as: space, time, continuity, movement, modality, (meta)language levels, interpretation, abstraction, (higher-order) quantification, predication, complement constructions, subjectivity, context, circumstances, etc. Pierre d'Ailly upholds that the truth-value of a proposition depends on the sense, on the metaphysical level, on the language and meta-language; the auto-reflexive propositions (with reflection on themselves) depend on the mode of representation (objective/subjective, formal/informal, real/mental).

In a formal way, let's consider the world W as being generated by the formal system FS.

One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it "relative falsehood". Also, the neutrosophic indeterminacy-value $NL_i(A) = 1^+$ if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".

On the other hand, $NL_t(A) = {}^-0$ if A is false in all possible world, whereas $NL_t(A) = 0$ if A is false in at least one world; $NL_f(A) = {}^-0$ if A is true in all possible world, whereas $NL_f(A) = 0$ if A is true in at least one world; and $NL_i(A) = {}^-0$ if A is indeterminate in no possible world, whereas $NL_i(A) = 0$ if A is not indeterminate in at least one world.

The ${}^-0$ and 1^+ monads leave room for degrees of super-truth (truth whose values are greater than 1), super-falsehood, and super-indeterminacy.

Here there are some corner cases:

There are tautologies, some of the form "B is B", for which $NL(B) = (1^+, {}^-0, {}^-0)$, and contradictions, some of the form "C is not C", for which $NL(B) = ({}^-0, {}^-0, 1^+)$.

While for a paradox, P, $NL(P) = (1,1,1)$. Let's take the Epimenides Paradox, also called the Liar Paradox, "This very statement is true". If it is true then it is false, and if it is false then it is true. But the previous reasoning, due to the contradictory results, indicates a high

indeterminacy too. The paradox is the only proposition true and false in the same time in the same world, and indeterminate as well!
Let's take the Grelling's Paradox, also called the heterological paradox [Suber, 1999], "If an adjective truly describes itself, call it 'autological', otherwise call it 'heterological'. Is 'heterological' heterological? " Similarly, if it is, then it is not; and if it is not, then it is.
For a not well-formed formula, nwff, i.e. a string of symbols which do not conform to the syntax of the given logic, NL(nwff) = n/a (undefined). A proposition which may not be considered a proposition was called by the logician Paulus Venetus *flatus voci*. NL(*flatus voci*) = n/a.

2.4. An Attempt of Classification of Logics

(upon the following, among many other, criteria):

a) The way the connectives, or the operators, or the rules of inferences are defined.
b) The definitions of the formal systems of axioms.
c) The number of truth-values a proposition can have: two, three, finitely many-values, infinitely many (of continuum power).
d) The partition of the interval [0, 1] in propositional values: bi-partition (in degrees of truth and falsehood), or tri-partition (degrees of truth, falsehood, and indeterminacy).
e) The distinction between conjunctural (relative) true, conjunctural (relative) false, conjunctural (relative) indeterminacy - designed by 1, with respect to absolute true (or super-truth), absolute false (super-falshood), absolute indeterminacy – designed by 1^+. Then, if a proposition is absolute true, it is underfalse ($^-0$), i.e. NL(P)=(1^+, I, $^-0$).

   For example, the neutrosophic truth-value of the proposition "The number of planets of the Sun is divisible by three" is 1 because the proposition is necessary *de re*, i.e. relates to an actual individual mentioned since its truth depends upon the number nine, whereas the neutrosophic truth-value of the proposition "The number of planets of the Sun is the number of its satellites" is 1^+ because the second proposition is necessary *de dicto*, i.e. relates to the expression of a belief, a possibility since its truth is not dependent upon which number in fact that is. The first proposition might not be true in the future if a new planet is discovered or an existing planet explodes in an asteroid impact, while the second one is always true as being a tautology. This is the difference between the truth-value "1" (dependent truth) and the truth-value "1^+" (independent truth).
f) The components of the truth values of a proposition summing up to 1 (in boolean logic, fuzzy logic, intuitionistic fuzzy logic), being less than 1 (in intuitionistic logic), or being greater than 1 (in paraconsistent logic, neutrosophic logic). The maximum sum may be 3 in neutrosophic logic, where NL(paradox)=(1,1,1).
g) Parameters that influence the truth-values of a proposition. For example in temporal logic the time is involved. A proposition may be true at a time t_1, but false at a time t_2, or may have some degree of truth in the open interval (0, 1) at a time t_3.
h) Using approximations of truth-values, or exact values.

   For example, the probabilistic logic, interval-valued fuzzy logic, interval-valued intuitionistic fuzzy logic, possibility logic (Dubois, Prade) use approximations.

   The boolean logic uses exact values, either 0 or 1.
i) Studying the paradoxes or not.

   In the neutrosophic logic one can treat the paradoxes, because NL(paradox)=(1,1,1), and in dialetheism. In fuzzy logic FL(paradox)=(1,0) or (0,1)? Because FL(paradox)≠(1,1), due to the fact that the sum of the components should be 1 not greater.

j) The external or internal structure of propositions: Sentential (or Propositional) Calculus, which is concerned with logical relations of propositions treated only as a whole, and Predicate (or Functional) Calculus which is concerned besides the logical relations treated as a whole with their internal structure in terms of subject and predicate.

k) Quantification: First-Order (or Lower) Predicate Calculus (quantification is restricted to individuals only, and predicates take only individuals as arguments), Second-Order Predicate Calculus (quantification over individuals and over some classes as well), Higher-Order Calculus (n-predicates take, and quantifiers bind, order n-1 predicates as arguments, for n>1).

l) In proof-theoretic terms:

- Monotonic Logic: let Γ be a collection of statements, ν_1, ν_2, …, ν_n, and ϖ, φ other statements; if $\Gamma \vdash \varphi$ then also $(\Gamma, \varpi) \vdash \varphi$.
- Non-Monotonic Weak Logic: For some Γ, ϖ, φ one has

  $\Gamma \vdash_{NML} \varphi$ but from (Γ, ϖ) does not $\vdash_{NML} \varphi$;
- Non-Monotonic Strong Logic: For some Γ, ϖ, φ, where Γ and $\Gamma \wedge \varphi$ are consistent, one has

  $(\Gamma, \varpi) \vdash_{NML} \neg\varphi$.

m) From a traditional standpoint: Classical or Non-classical.

n) Upon inclusion or exclusion of empty domains (and defining the logical validity accordingly), there are Inclusive Predicate Logic, and (Standard) Predicate Logic respectively.

o) Upon the number of arguments the predicates can take, there are Monadic Predicate Logic (predicates take only one argument), Dyadic Predicate Logic (predicates take two arguments), Polyadic Predicate Logic or Logic of Relations (predicates take n>1 arguments).

p) Upon formalization again: Formal Logic, and Informal Logic.

q) Upon types of formalization, there are: Number-Theoretic Predicate Calculus (system with function symbols and individual constants), Pure Predicate Calculus (system without function symbols nor individual constants).

r) Upon standardization: Standard Logic, and Non-Standard Logic.

s) Upon identity: Predicate Logic With Identity (with the axiom $(\forall x)(x=x)$, and the axiom schema $[(x=y)\rightarrow(A\rightarrow A')]^c$, where A' is obtained from A by replacing any free occurrence of x in A with y, and B^c is an arbitrary closure of B), Predicate Logic Without Identity.

t) According to the *ex contradictione quodlibet* (ECQ) principle, from contradictory premises follows anything, there are:

- Explosive logics, which validates it (classical logic, intuitionistic logic);
- Non-Explosive Logics, which invalidate it (paraconsistent logic, neutrosophic logic).

u) According to the Law of Excluded Middle (LEM), either A or ¬A, there are:

- Constructive Logic, which invalidate it (intuitionistic logic, paraconsistent logic, neutrosophic logic);
- Non-Constructive Logics (classical logic).

The criteria are not exhausted. There are sub-classifications too.

Let's take the <u>Modal Logic</u> which is an extension of the Propositional Calculus but with

operators that express various modes of truth, such as: necessarily A, possible A, probably A, it is permissible that A, it is believed that A, it has always been true that A.
The Modal Logic comprises:

- Alethic Logic (which formalizes the concepts of pertaining to truth and falsehood simultaneously, such as *possibly true* and *necessarily true*); only for this case there are more than two hundred systems of axioms!
- Deontic Logic (which seeks to represent the concepts of *obligatoriness* and *permissibility*); it is sub-divided into:
    - Standard Deontic Logic, which has two monadic operators added to the classical propositional calculus: "O" = it ought to be that, and "P" = it is permissible that;
    - Dyadic Deontic Logic, which has two similar dyadic operators added to the classical propositional calculus: "O(/)" = it ought tobe that …, given that …, and P(/) = is it permisible that …, given that … ;
    - Two-sorted Deontic Logic (CastaÁeda 1975) , which distinguishes between *propositions* (which bear truth-values) and *practitions* (which content imperatives, commands, requests). The deontic operators in this case are: Oi = it is obligatory I that, Pi = it is permissible i that, Wi = it is wrong I that, and Li = it is optional i that. A deontic operator applied to a practition yields a proposition.
- Epistemic Logic (which seeks to represent to concepts of *knowledge*, *belief*, and *ignorance*);
- and Doxastic Logic (which studies the concept of *belief*); it is included in the Epistemic Logic, which is the investigation of epistemic concepts, the main ones being: knowledge, reasonable belief, justification, evidence, certainty.

Dynamic Logic (1970), as a generalization of the modal logic, has a category of expressions interpretable as *propositions* and another category of expressions interpretable as *actions*, with two operators:

[α]A = after every terminating computation according to α it is the case that A;

<α>A = after some terminating computation according to α it is the case that A,

and it is used in the verification of the computer programs.

Combinatory Logic (Schoenfinkel, Haskell Curry, 1920s) is a system for reducing the operational notation of logic, mathematics, or functional language to a sequence of modifications to the input data structure.

Temporal Logic is an extension of Predicate Calculus that includes notation for arguing about when (at what time) statements are true, and employs prefix operators such as:

◯x = x is true at the next time;

□x = x is true from now on;

◇x = x is eventually true;

or infix operators such as:

xUy = x is true until y is true;

xPy = x precedes y;

xWy = x is weak until y is true.

Temporal Logic studies the Linear Time, which considers only one possible future, and Branching Time, which has two extra operators:

"A" = all futures,

and "E" = some futures.

Default Logic (Raymond Reiter 1980) is a formal system with two default operators:

P:MQ/Q = if P is believed, and Q is consistent with this believe, then Q must be believed;

P:M¬Q/¬Q = if P is believed, and Q is not consistent with this believe, then Q must not be believed.

Tense Logic (Arthur Prior 1967), which is related to the Modal Logic, introduces in the classical logic two operators:

P = it was the case that … (past tense);

F = it will be the case that … (future tense).

The truth-value is not static as in classical logic, but changing in time.

Deviant Logics are logics which treat the same classical logic subjects, but in a different way (either by interpreting the connectives and quantifiers non-classically, or rejecting some classical laws): intuitionistic logic, paraconsistent logic, free logic, multi-valued logic.

Free Logic is a system of quantification theory which allows non-denoting singular terms (free variables and individual constants).

In Webster's dictionary (1988) denotation of a term means the class of all particular objects to which the term refers, and connotation of a term means the properties possessed by all the objects in the term's extension.

Erotetic Logic is the logic of questions, answers, and the relations between them. There are (1) imperative approaches (A. Cqvist, J. Hintikka, et al.), epistemic sentences embedded in an imperative sentence system, and (2) interrogative approaches (N. Belnap, T. Kubi½ski, and others), system of interrogative expressions and their answers.

Relational Logic (Pierce 1870, 1882) is a formal study of the properties of the (binary) relations and the operations on relations.

Because the neutrosphic logic is related to intuitionistic logic, paraconsistent logic, and dialetheism we'll focus more on these types of logics.

Intuitionistic Logic (Brouwer 1907) is a deviant logic from the classic, where the Law of Excluded Middle of Aristotle ($A \vee \neg A$) is invalidated. In this logic: a proof of existence, $\exists x P(x)$, does not count unless a method/algorithm of constructing a such x is giving (the interpretation of 'there exists' as 'we can construct' distinguishes between *classical mathematics* and *constructive mathematics* respectively); and a proof of $A \vee B$ counts only if a proof of A exhibits or a proof of B. Similarly (Bridges 1997), a proof of $A \wedge B$ counts if both a proof of A and a proof of B exibit, a proof of $A \rightarrow B$ counts if an algorithm is constructed that converts a proof of A into a proof of B, a proof of $\neg A$ means to show that A implies a contradiction, and a proof of $\forall x P(x)$ means to construct an algorithm that applied to any x proves that P(x) holds. As a consequence, the axiom of choice also fails. Brouwer considered some unsolved problem from number theory as proposition A, which is not -- with our present knowledge -- proved true, neither $\neg A$ is proved true. Thus, neutrosophically $NL_t(A \vee \neg A) < 1$, $NL_f(A \vee \neg A) < 1$, and $NL_t(A \vee B) < 1$, $NL_f(A \vee B) < 1$, for some propositions A, B.

Paraconsistent Logic is a logic in which the principle that anything follows from contradictory premises, for all formulas A and B one has $A \wedge \neg A \supset B$, fails. Therefore, $A \wedge \neg A$ is not always false, i.e. for some A $NL_t(A \wedge \neg A) > 0$ or NL(A) = (t, i, f) where $t+f>1$.

It is motivated by dialetheists who support the idea that some contradictions are true, by automated reasoning (information processing) due to inconsistent data stored in computers, and by the fact that people impart opposite beliefs. There are four types of propositional paraconsistent logics (Priest and Tanaka, 1996):

- Non-Adjunctive Systems (Jaskowski's discussive logic), where the inference

$\{A, B\} \supset A \wedge B$ fails; in a discourse a participant's opinion A may be inconsistent with other participant's opinion B on the same subject;

- Non-Truth-Functional Logics (da Costa), which maintains the mechanism of positive logics (classical, intuiitonistic) but the value of the negation, $\neg A$, is interpreted independently of that of A;
- Many-Valued Systems (Asenjo), many-valued logic which allows both A and $\neg A$ to be designated (to function as the analogue of truth in a two-valued logic); for example a three-valued paraconsistent system (LP) has the values: 'true', 'false', and 'both true and false', while in a four-valued system (J. M. Dunn 1976) one adds another value 'neither true nor false';
- Relevance Logic (or Relevant Logic) (Wilhelm Ackermann 1956, Alan Anderson and Nuel Belnap 1959-1974) promulgates that the premises of a valid inference must be relevant to the conclusion. The disjunctive syllogism, which states that 'if $A \vee B$ and $\neg A$ are true then so is B', is not admitted in relevance logic, neither in neutrosophic logic. However, Ackermann's rule Gamma, that 'if $A \vee B$ and $\neg A$ are theses then so is B', is admitted.

Dialetheism asserts that some contradictions are true, encroaching upon the Aristotle's Law of Non-Contradiction (LNC) that not both A and $\neg A$ are true. The dialetheism distinguishes from the trivialism, which views all contradictions as being true. Neither neutrosophic logic is trivialist.

There is a duality (Mortensen 1996) between paraconsistency and intuitionism (i.e. between inconsistency and incompleteness respectively), the Routley * operation (1972) between inconsistent theories and incomplete theories.

Linear Logic (J. Y. Girard 1987) is a resource sensitive logic that emphasizes on state. It employs the central notions of truth from classical logic and of proof construction from intuitionistic logic. Assumptions are considered resources, and conclusions as requirements; A implies B means that the resource A is spent to meet the requirement B. In the deductions there are two structural rules (Scedrov 1999), that allow us to discard or duplicate assumptions (distinguishing linear logic from classical and intuitionistic logics): *contraction*, which stipulates that any assumption once stated may be reused as often as desired, and *weakening* which stipulates that it's possible to carry out a deduction without using all the assumptions. They are replaced by explicit modal logical rules such as: "storage" or "reuse" operator, !A, which means unlimited creation of A, and its dual, ?B, which means unlimited consumption of B.

2.5. Comments on Neutrosophic Logic in Comparison with Other Logics.

How to adopt the G`del-Gentzen *negative translation*, which transforms a formula A of a language L into an equivalent formula A` with no $\vee$ or $\exists$, in the neutrosophic predicate logic?

In the Boolean logic a *contingent statement* is a statement which is true under certain conditions and false under others. Then a neutrosophic contingent statement is a statement which has the truth value (T_1, I_1, F_1) under certain conditions and (T_2, I_2, F_2) under others.

The Medieval paradox, called Buridan's Ass after Jean Buridan (near 1295-1356), is a perfect example of complete indeterminacy. An ass, equidistantly from two quantitatively

and qualitatively heaps of grain, starves to death because there is no ground for preferring one heap to another.
The neutrosophic value of ass's decision, NL = (0, 1, 0).

In a two-valued system one regards all the designated values as species of truth and all the anti-designated values as species of falsehood, with truth-value (or falsehood-value) gaps between designated and anti-designated values. In the neutrosophic system one stipulates the non-designated values as species of indeterminacy and, thus, each neutrosophic consequence has degrees of designated, non-designated, and anti-designated values.

Of course, the Law of Excluded Middle (a proposition is either true or false) does not hold in a neutrosophic system.

The Contradiction Law, that no <A> is <Non-A> does not hold too. NL(<A>) may be equivalent with NL(<Non-A>) and often they at least overlap. Neither the law of *Reductio ad absurdum* (or method of indirect proof): $(A \supset \neg A) \supset \neg A$ and $(\neg A \supset A) \supset A$.

Some tautologies (propositions logically necessary, or *true in virtue of form*) in the classical logic might not be tautologies (absolute truth-value propositions) in the neutrosophic logic and, *mutatis mutandis*, some contradictions (propositions logically impossible, or *false in virtue of form*) in the classical logic might not be contradictions (absolute falsehood-value propositions) in the neutrosophic logic.

The mixed hypothetical syllogism Modus Ponens,

If P then Q
P

Q

The mixed hypothetical syllogism Modus Tollens,

If P then Q
Non Q

Non P

The Inclusive (Weak) Disjunctive Syllogism:

If (P or Q)
Non P

Q

The Exclusive (Strong) Disjunctive Syllogism:

If (either P or Q)
Non P

Q

Hypothetical Syllogism,

If P then Q
If Q then R

If P then R

Constructive Dilemma,

P or Q
If P then R
If Q then R

R

Destructive Dilemma,

P or Q
Non P

Q

The Polysyllogism, which is formed by many syllogisms such that the conclusion of one becomes a premise of another,

and the Nested Arguments, a chainlike where the conclusion of an argument forms the premise of another where intermediate conclusions are typically left out,

are not valid anymore in the neutrosophic logic, but they acquire a more complex form.

Also, the classical *entailment*, which is the effect that a proposition Q is a necessary consequence of another proposition P, P $\rightarrow$ Q, partially works in the neutrosopohic logic. Neither
the informal *fish-hook* symbol, ---], used to show that a proposition Q is an accidental consequence of a proposition P, P ---] Q, works.

Is it possible in the neutrosophic predicate calculus to transform each formula into an equivalent in prenex form one using the prenex operations?
Prenex (normal) form means a formula formalized as follows:

$(Qx_1)(Qx_2)\ldots(Qx_n)S$,

where "Q" is a universal or existential quantifier, the variables x_1, x_2, …, x_n are distinct, and S is an open sentence (a well-formed expression containing a free variable). Prenex operation is any operation which transforms any well-formed formula into equivalent in

prenex form formula; for example, $(\exists x)Ax \rightarrow B \equiv (\forall x)(Ax \rightarrow B)$.
In the classical predicate calculus any well-formed formula can be transformed into a prenex form formula.

The double negation, $\neg(\neg A) \equiv A$, which is not valid in intuitionistic logic, is not valid in the neutrosophic logic if one considers the negation operator $\eta_1(A)=1\ominus NL(A)$, but it is valid for the negation operator $\eta_2(A)=(F, I, T)$, where $NL(A)=(T, I, F)$.

Neutrosophic Logic admits non-trivial inconsistent theories.

In stead of saying "a sentence holds (or is assertible)" as in classical logic, one extends to "a sentence p% holds (or is p% assertible)" in neutrosophic logic. In a more formalized way, a sentence (T, I, F)% holds [or is (T, I, F)% assertible].

A neutrosophic predicate is a vague, incomplete, or not well known attribute, property or function of a subject. It is a kind of three-valued set function. If a predicate is applied to more than one subject, it is called neutrosophic relation.
An example: 'Andrew is tall'.
The predicate "tall" is imprecise. Andrew is maybe tall according to Linda, but short in Jack's opinion, however his tallness is unknown to David. Everybody judges him in terms of his/her own tallness and acquaintance of him.

The neutrosophic set and logic attempt to better model the non-determinism. They try:

- to represent the paradoxical results even in science not talking in the humanistic where the paradox is very common;
- to evaluate the peculiarities;
- to illustrate the contradictions and conflicting theories, each true from a specific point of view, false from another one, and perhaps indeterminate from a third perspective;
- to catch the mysterious world of the atom, where the determinism fails; in quantum mechanics we are dealing with systems having an infinite number of degrees of freedom;
- to study submicroscopic particles which behave non-Newtonianly, and some macroscopic phenomena which behave in nearly similar way.

In physics, the light is at once a wave and a particle (photon). Two contradictory theories were both proven true:
The first one, Wave Theory (Maxwell, Huygens, Fresnel), says that light is a wave due to the interference: two beams of light could cross each other without suffering any damage.
The second one, Particle Theory (Newton, Hertz, Lenard, Planck, Einstein), says that light is corpuscular, due to the photoelectric effect that ultraviolet light is able to evaporate electrons from metal surfaces and to the manner in which light bounces off electrons.
De Broglie reconciled both theories proving that light is a matter wave! Matter and radiation are at the same time waves and particles.
Let L1(x) be the predicate: "X is of corpuscular nature",
and L2(x) the predicate: "X is of wave nature".
L2(x) is the opposite of L1(x), nonetheless L1(light) = true and L2(light) = true simultaneously.

Also, there exist four different Atom Theories: of Bohr, Heisenberg, Dirac, and Schr`dinger respectively, each of them plausibly true in certain conditions (hypotheses).

Another example, from Maxwell's equations an electron does radiate energy when orbits the nucleus, from Bohr's theory an electron does not radiate energy when orbits the nucleus, and both propositions are proved true with our today's knowledge.

Falsehood is infinite, and truthhood quite alike; in between, at different degrees, indeterminacy as well.

In the neutrosophic theory:
between being and nothingness
existence and nonexistence
geniality and mediocrity
certainty and uncertainty
value and nonvalue
and generally speaking <A> and <Non-A>
there are infinitely many transcendental states.
And not even 'between', but even beyond them.
An infinitude of infinitudes.
They are degrees of neutralities <Neut-A> combined with <A> and <Non-A>.

In fact there also are steps:
between being and being
existence and existence
geniality and geniality
possible and possible
certainty and certainty
value and value
and generally speaking between <A> and <A>.
The notions, in a pure form, last in themselves only (intrinsicalness), but outside they have an interfusion form.

Infinitude of shades and degrees of differentiation: between white and black there exists an unbounded palette of colors resulted from thousands of combinations among them.

All is alternative: progress alternates with setback, development with stagnation and underdevelopment.

In between objective and subjective there is a plurality of shades.
In between good and bad...
In between positive and negative...
In between possible and impossible
In between true and false...
In between "A" and "Anti-A"...

As a neutrosophic ellipse:

```
                         |
                         |
                   Good | Neighborhood
                         .
                     .   |    .
                   .     |      .
                  .      |       .
                 .       |        .
 Indeterminate .         |         . Indeterminate
      ------------.---------|----------.--------------
Neighborhood .          |          . Neighborhood
                 .       |        .
                  .      |       .
                   .     |      .
                     .   |    .
                         .
                    Bad | Neighborhood
                         |
                         |
```

Everything is g% good, i% indeterminate, and b% bad,
where g varies in the subset G, i varies in the subset I, and b varies in the subset B, and G, I, B are included in ╟ $^{-}0, 1^{+}$ ╢.

Besides Diderot's dialectics on good and bad ("Rameau's Nephew", 1772), any act has its “god", "indeterminate", and of "bad" as well incorporated.

Rodolph Carnap said:
"Metaphysical propositions are neither true nor false, because they assert nothing, they contain neither knowledge nor error (...)".
Hence, there are infinitely many states between "Good" and "Bad", and generally speaking between "A" and "Anti-A" (and even beyond them), like on the real number line:

```
  -∞                 f  0   1  t            +∞
 <----------------|--|---|--|---------------->
                 False    True
                   Bad    Good
            Non-sense    Sense
                Anti-A    A
```

f is the absolute falsity ($f<0$), t the absolute truth ($t>1$). In between each oppositing pair, normally in a vicinity of 0.5, are being set up the neutralities.

There exist as many states in between "True" and "False" as in between "Good" and "Bad". Irrational and transcendental standpoints belong to this interval.
Even if an act apparently looks to be only good, or only bad, other hidden sides might be sought. The ratios

$$\frac{\text{Anti-A}}{A}, \quad \frac{\text{Non-A}}{A}$$

vary indefinitely. Are they transfinite?
If a statement is 30%T (true) and 60%I (indeterminate), then it maybe, for example, 15%F (false) . This is somehow alethic, meaning to simultaneously pertain to truthhood and falsehood, or to truthhood and indeterminacy, or to falsehood and indeterminacy, or even to all three components.
More general, if a statement is 30%T and 60%I, it may be between 5-20%F or 25%F.

2.6. Comparison between Fuzzy Logic and Neutrosophic Logic:

The neutrosophic connectives have a better truth-value definition approach to the real-world systems than the fuzzy connectives. They are defined on triple non-standard subsets included in the non-standard unit interval $\Vdash {}^-0,\ 1^+ \dashv\Vert$, while in fuzzy logic they are defined on the closed interval [0, 1]. n_sup is not restricted to 1, but it's enlarged to a monad ∶(3^{+}), i.e. a set of hyper-real numbers; similarly, n_inf may be as low as ∶$({}^-0)$, not as 0.
A paradox, which is true and false in the same time, can not be evaluate in fuzzy logic, because the sum of components should add up to 1, but it is allowed in neutrosophic logic because NL(paradox) maybe (1, 1, 1).

In opposition to Fuzzy Logic, if a proposition <A> is t% true, doesn't necessarily mean it is (100-t)% false. A better approach is t% true, f% false, and i% indeterminate as in Intuitionistic Fuzzy Logic (Atanassov), where $t \in T$, $i \in I$, $f \in F$,
even more general, with $n\_sup \leq 3^+$ and $n\_inf \geq {}^-0$.
One considers subsets of truth, indeterminacy, and falsity instead of single numbers because of imprecision, uncertainty, and vagueness.
The neutrosophic logical value of <A> is noted by
NL(A) = (T, I, F). On components one writes:

- for the truth value $NL_t(A) = T$;
- for the indeterminacy value $NL_i(A) = I$;
- for the falsity value $NL_t(A) = F$.

Neutrosophic Logic means the study of neutrosophic logical values of the propositions.
There exist, for each individual one, PRO parameters, CONTRA parameters, and NEUTER parameters which influence the above values.
Indeterminacy results from any hazard which may occur, from unknown parameters, or from new arising conditions. This resulted from practice.

2.7. Neutrosophical Modal Logic:

In modal logic, the primitive operators $\Diamond$ 'it is possible that' and $\Box$ 'it is necessary that' can be defined by:

$t\_inf(\Diamond A) > 0$,

and, because $\Box A$ could be regarded as $\neg(\Diamond\neg A)$,

$$t\_sup(\Box A) < 1.$$

2.8. Applications:

Neutrosophic logic is useful in the real-world systems for designing control logic, and may work in quantum mechanics.

The candidate C, who runs for election in a metropolis M of p people with right to vote, will win.

This proposition is, say, 20-25% true (percentage of people voting for him), 35-45% false (percentage of people voting against him), and 40% or 50% indeterminate (percentage of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Tomorrow it will rain.

This proposition is, say, 50% true according to meteorologists who have investigated the past years' weather, between 20-30% false according to today's very sunny and droughty summer, and 40% undecided.

This is a heap.

As an application to the sorites paradoxes, we may now say that this proposition is 80% true, 40% false, and 25-35% indeterminate (the neutrality comes for we don't know exactly where is the difference between a heap and a non-heap; and, if we approximate the border, our 'accuracy' is subjective). Vagueness plays here an important role.

We are not able to distinguish the difference between yellow and red as well if a continuum spectrum of colors is painted on a wall imperceptibly changing from one into another.

We would be able to say at a given moment that a section is both yellow and red in the same time, or neither one!

A paradox within a *sorites paradox*: a frontal bald man, with a hair high density on the remaining region of his head, may have more hairs on his head that another man who is not bald but the skin surface of his head and the hair density are smaller than the previous one.

2.9. Definition of Neutrosophic Logical Connectives:

The connectives (rules of inference, or operators), in any non-bivalent logic, can be defined in various ways, giving rise to lots of distinct logics. For example, in three-valued logic, where three possible values are possible: true, false, or undecided, there are 3072 such logics! (Weisstein, 1998) A single change in one of any connective's truth table is enough to form a (completely) different logic.

> The rules are hypothetical or factual. How to choose them? The philosopher Van Fraassen (1980) [see Shafer, 1986] commented that such rules may always be controvertible "for it always involves the choice of one out of many possible but nonactual worlds". There are general rules of combination, and ad hoc rules.

For an applied logic to artificial intelligence, a better approach, the best way would be to define the connectives recursively (Dubois, Prade), changing/adjusting the definitions after

each step in order to improve the next result. This might be comparable to approximating the limit of a convergent sequence, calculating more and more terms, or by calculating the limit of a function successively substituting the argument with values closer and closer to the critical point. The recurrence allows evolution and self-improvement.
Or to use *greedy algorithms*, which are combinatorial algorithms that attempt at each iteration as much improvement as possible unlike myopic algorithms that look at each iteration only at very local information as with steepest descent method.
As in non-monotonic logic, we make assumptions, but we often err and must jump back, revise our assumptions, and start again. We may add rules which don't preserve monotonicity.
In bio-mathematics Heitkoetter and Beasley (1993-1999) present the *evolutionary algorithms* which are used "to describe computer-based problem solving systems which employ computational models of some of the known mechanisms of evolution as key elements in their design and implementation". They simulate, via processes of selection, mutation, and reproduction, the evolution of individual structures. The major evolutionary algorithms studied are: genetic algorithm (a model of machine learning based on genetic operators), evolutionary programming (a stochastic optimization strategy based on linkage between parents and their offspring; conceived by L. J. Fogel in 1960s), evolution strategy, classifier system, genetic programming.
Pei Wang devised a Non-Axiomatic Reasoning System as an intelligent reasoning system, where intelligence means working and adopting with insufficient knowledge and resources.

The inference mechanism (endowed with rules of transformation or rules of production) in neutrosophy should be non-monotonic and should comprise ensembles of recursive rules, with preferential rules and secondary ones (priority order), in order to design a good expert system. One may add new rules and eliminate old ones proved unsatisfactory. There should be strict rules, and rules with exceptions. Recursivity is seen as a computer program that learns from itself. The statistical regression method may be employed as well to determine a best algorithm of inference.
Non-monotonic reasoning means to make assumptions about things we don't know. Heuristic methods may be involved in order to find successive approximations.
In terms of the previous results, a default neutrosophic logic may be used instead of the normal inference rules. The distribution of possible neutrosophic results serves as an orientating frame for the new results. The flexible, continuously refined, rules obtain iterative and gradual approaches of the result.
A comparison approach is employed to check the result (conclusion) p by studying the opposite of this: what would happen if a non-p conclusion occurred? The inconsistence of information shows up in the result, if not eliminated from the beginning. The data bases should be stratified. There exist methods to construct preferable coherent sub-bases within incoherent bases. In Multi-Criteria Decision one exploits the complementarity of different criteria and the complementarity of various sources.
For example, the Possibility Theory (Zadeh 1978, Dubois, Prade) gives a better approach than the Fuzzy Set Theory (Yager) due to self-improving connectives. The Possibility Theory is proximal to the Fuzzy Set Theory, the difference between these two theories is the way the fusion operators are defined.

One uses the definitions of neutrosophic probability and neutrosophic set operations.
Similarly, there are many ways to construct such connectives according to each particular

problem to solve; here we present the easiest ones:

One notes the neutrosophic logical values of the propositions A_1 and A_2 by
$NL(A_1) = (T_1, I_1, F_1)$ and $NL(A_2) = (T_2, I_2, F_2)$.
If, after calculations, in the below operations one obtains values < 0 or > 1, then one replaces them with $^-0$ or 1^+ respectively.

2.9.1. Negation:

$NL(\neg A_1) = (\{1^+\}\ominus T_1, \{1^+\}\ominus I_1, \{1^+\}\ominus F_1)$.

2.9.2. Conjunction:

$NL(A_1 \wedge A_2) = (T_1\odot T_2, I_1\odot I_2, F_1\odot F_2)$.
(And, in a similar way, generalized for n propositions.)

2.9.3. Weak or inclusive disjunction:

$NL(A_1 \vee A_2) = (T_1\oplus T_2\ominus T_1\odot T_2, I_1\oplus I_2\ominus I_1\odot I_2, F_1\oplus F_2\ominus F_1\odot F_2)$.
(And, in a similar way, generalized for n propositions.)

2.9.4. Strong or exclusive disjunction:

$$NL(A_1 \veebar A_2) =$$
$$(T_1\odot (\{1^+\}\ominus T_2) \oplus T_2\odot (\{1^+\}\ominus T_1) \ominus T_1\odot T_2\odot (\{1^+\}\ominus T_1) \odot (\{1^+\}\ominus T_2),$$
$$I_1 \odot (\{1^+\}\ominus I_2) \oplus I_2 \odot (\{1^+\}\ominus I_1) \ominus I_1 \odot I_2 \odot (\{1^+\}\ominus I_1) \odot (\{1^+\}\ominus I_2),$$
$$F_1\odot (\{1^+\}\ominus F_2) \oplus F_2\odot (\{1^+\}\ominus F_1) \ominus F_1\odot F_2 \odot (\{1^+\}\ominus F_1) \odot (\{1^+\}\ominus F_2)).$$
(And, in a similar way, generalized for n propositions.)

2.9.5. Material conditional (implication):

$NL(A_1 \mapsto A_2) = (\{1^+\}\ominus T_1\oplus T_1\odot T_2, \{1^+\}\ominus I_1\oplus I_1\odot I_2, \{1^+\}\ominus F_1\oplus F_1\odot F_2)$.

2.9.6. Material biconditional (equivalence):

$$NL(A_1 \leftrightarrow A_2) = ((\{1^+\}\ominus T_1\oplus T_1\odot T_2) \odot (\{1^+\}\ominus T_2\oplus T_1\odot T_2),$$
$$(\{1^+\}\ominus I_1\oplus I_1\odot I_2) \odot (\{1^+\}\ominus I_2\oplus I_1 \odot I_2),$$
$$(\{1^+\}\ominus F_1\oplus F_1\odot F_2) \odot (\{1^+\}\ominus F_2\oplus F_1\odot F_2)).$$

2.9.7. Sheffer's connector:

$NL(A_1 \mid A_2) = NL(\neg A_1 \vee \neg A_2) = (\{1^+\}\ominus T_1\odot T_2, \{1^+\}\ominus I_1\odot I_2, \{1^+\}\ominus F_1\odot F_2)$.

2.9.8. Peirce's connector:

$$NL(A_1 \downarrow A_2) = NL(\neg A_1 \wedge \neg A_2) =$$
$$= ((\{1^+\}\ominus T_1) \odot (\{1^+\}\ominus T_2), (\{1^+\}\ominus I_1) \odot (\{1^+\}\ominus I_2), (\{1^+\}\ominus F_1) \odot (\{1^+\}\ominus F_2)).$$

2.10. Comments on Neutrosophic Operators:

The conjunction is well defined, associative, commutative, admits a unit element *U* with t(*U*) = 1, but no element whose truth-component is different from 1 is inversable.
The conjunction is not absorbent, i.e. $t(\ A\wedge(A\wedge B)\ ) \wedge t(A)$,
except for the cases when t(A) = 0, or t(A) = t(B) = 1.

The disjunction is well-defined, associative, commutative, admits a unit element *O* with t(*O*) = 0, but no element whose truth-component is different from 0 is inversable.
The disjunction is not absorbent, i.e. $t(\ A\vee(A\vee B)\ ) \vee t(A)$,
except for the cases when one of t(A) = 1, or t(A) = t(B) = 0.

None of them is distributive with respect to the other.

De Morgan laws do not apply either.

Therefore (NL, $\wedge$, $\vee$, $\neg$), where NL is the set of neutrosophic logical propositions, is not an algebra.
Nor ($\mathcal{P}(\|\!-\,{}^{-}0,\ 1^{+}\,-\!\|)$, $\cap$, $\cup$, *C*), where $\mathcal{P}(\|\!-\,{}^{-}0,\ 1^{+}\,-\!\|)$ is the set of all subsets of $\|\!-\,{}^{-}0,\ 1^{+}\,-\!\|$, and *C*(A) is the neutrosophic complement of A.

One names a set N, endowed by two associative unitary internal laws, * and #, which are not inversable except for their unit elements respectively, and not distributive with respect to each other, *Ninversity*.
If both laws are commutative, then N is called a *Commutative Ninversity*.

For a better understanding of the neutrosophic logic one needs to study the commutative ninversity.

2.11. Other Types of Neutrosophic Logical Connectors:

There are situations when we have to more focus on the percentage of falsity or of indeterminacy than on the percentage of truth.

Thus, we define in a similar way the logical connectors, but the main component will then be considered the last or second one respectively. An intriguing idea would be to take the arithmetic average of the corresponding components of the truth-, indeterminacy-, and false-neutrosophic connectors.
Let's reconsider the previous notations.

2.12. Neutrosophic Topology.

A) Definitions:
Let's construct a Neutrosophic Topology on NT = $\|\!-\,{}^{-}0,\ 1^{+}\,-\!\|$,
considering the associated family of standard or non-standard subsets included in NT, and the empty set $\emptyset$, called *open sets*, which is closed under set union and finite intersection.

Let A, B be two such subsets.
The union is defined as: $A\cup B = A\oplus B\ominus A\odot B$,
and the intersection as: $A\cap B = A\odot B$.
The complement of A, $C(A) = \{1^{+}\}\ominus A$, which is a closed set. {When a non-standard

number occurs at an extremity of an internal, one can write "╟" instead of "(" and "╢" instead of ")".}
The interval NT, endowed with this topology, forms a *neutrosophic topological space*.

2.13. Neutrosophic Sigma-Algebra.

The collection of all standard or non-standard subsets of ╟ $^{-}0$, 1^{+} ╢, constitute a *neutrosophic sigma-algebra* (or *neutrosophic* *F-algebra*), because the set itself, the empty set ∅, the complements in the set of all members, and all countable unions of members belong to the power set $\mathcal{P}$(╟ $^{-}0$, 1^{+}╢).
The complement of a subset is defined above.
The interval NT, endowed with this sigma-algebra, forms a *neutrosophic measurable space*.

2.14. Generalizations:

When the sets are reduced to an element only respectively, then
t_sup = t_inf = t, i_sup = i_inf = i, f_sup = f_inf = f,
and n_sup = n_inf = n = t+i+f)

Hence, the neutrosophic logic generalizes:

- the *intuitionistic logic*, which supports incomplete theories (for $0 < n < 1$ and $i=0$, $0 \leq t, i, f \leq 1$);
- the *fuzzy logic* (for $n = 1$ and $i = 0$, and $0 \leq t, i, f \leq 1$);

from "CRC Concise Concise Encyclopedia of Mathematics", by Eric W. Weisstein, 1998, the fuzzy logic is "an extension of two-valued logic such that statements need not to be True or False, but may have a degree of truth between 0 and 1";

- the *Boolean logic* (for $n = 1$ and $i = 0$, with t, f either 0 or 1);
- the *multi-valued logic* (for $0 \leq t, i, f \leq 1$);

definition of <many-valued logic> from "The Cambridge Dictionary of Phylosophy", general editor Robert Audi, 1995, p. 461: "propositions may take many values beyond simple truth and falsity, values functionally determined by the values of their components"; Lukasiewicz considered three values (1, 1/2, 0). Post considered m values, etc. But they varied in between 0 and 1 only. In the neutrosophic logic a proposition may take values even greater than 1 (in percentage greater than 100%) or less than 0.

- the *paraconsistent logic* (for $n > 1$ and $i = 0$, with both $t, f < 1$);
- the *dialetheism*, which says that some contradictions are true (for $t = f = 1$ and $i = 0$; some paradoxes can be denoted this way too);
- the *faillibilism*, which says that uncertainty belongs to every proposition (for $i > 0$);
- the *paradoxist logic*, based on study of paradoxes ($i > 1$) only;
- the *pseudoparadoxist logic*, based on pseudoparadoxes ($0 < i < 1$, $t + f > 1$);
- the *tautologic logic*, based on study of tautologies ($i < 0$, $t > 1$) only.

Compared with all other logics, the neutrosophic logic and intuitionistic fuzzy logic introduce a percentage of "indeterminacy" - due to unexpected parameters hidden in some propositions, or unknowness, or God's will, but only neutrosophic logic let each component t, i, f be even boiling *over 1* (overflooded) or freezing *under 0* (underdried): to be able to make distinction between relative truth and absolute truth, and between relative falsity and absolute falsity.

For example: in some tautologies $t > 1$, called "overtrue". Similarly, a proposition may be "overindeterminate" (for $i > 1$, in some paradoxes), "overfalse" (for $f > 1$, in some unconditionally false propositions); or "undertrue" (for $t < 0$, in some unconditionally false propositions), "underindeterminate" (for $i < 0$, in some unconditionally true or false propositions), "underfalse" (for $f < 0$, in some unconditionally true propositions).

This is because we should make a distinction between unconditionally true ($t > 1$, and $f < 0$ or $i < 0$) and conditionally true propositions ($t \leq 1$, and $f \leq 1$ or $i \leq 1$).

While in classical true/false logic it is possible to define precisely 2m different m-ary operators for each m>0 (Charles D. Ashbacher),

the *neutrosophic m-ary operators* may be defined in uncountably infinite different ways. The good operatorial selection would lead to applications in neural networks, automated reasoning, quantum physics, and probabilistic models.

Dempster-Shafer Theory doesn't work for some classes of examples:

1) Assume the universe U = {A, B, C}. If $m_1(A) = a$, $m_1(B) = 0$, $m_1(C)=1-a$, where $0 < a < 1$ and a is very close to 1, and $m_2(A) = 0$, $m_2(B) = b$, $m_2(C)=1-b$, where $0 < b <1$ and b is very close to 1, then $m_1+m_2(C) = 1$! This example generalizes Zadeh's (1984). Dezert (2000) defends the theory because, as he asserts, in this case the mass fusion is impossible for the sources of evidences are entirely conflictive.
2) Even more, the previous example can be extended to k > 2 masses $m_1, \ldots, m_k$ weighting k+1 exclusive events of the universe $U = \{A_1, \ldots, A_k, A_{k+1}\}$, such that for all $i \neq j$, $1 \leq i, j \leq k$, $m_i(A_i)=a_i$ and $m_i(A_j)=0$, where $0 < a_I < 1$ and a_i is very close to 1, and $m_i(A_{k+1})=1-a_i$.
3) In the following particular example, $m_1(A)=.11$, $m_1(B)=.11$, $m_1(C)=.11$, $m_1(D)=.67$, and $m_2(A)=.11$, $m_2(B)=.11$, $m_2(C)=.11$, $m_2(D)=.67$, using the Dempster's rule of combining evidences one gets m1+m2(D)=.925185, which is a 38.0873% increment jump from the two equal evidences of .67, and it looks counter-intuitive. Why not a smaller jump?

In the paraconsistent logic one cannot derive all statements from a contradiction, *ex contradictione quodlibet* fails. In the neutrosophic logic from a given contradiction one can derive a specific statement only, depending on the neutrosophic operator used and the given particular contradiction.

In the *dialetheism* it works the metaphysical thesis that some contradictions are true. In the neutrosophic logic there are contradictions denoted by t=f=1, which means 100% true and 100% false in the same time; even more, it is possible to have propositions which are, say, 70% true and 60% false (considering different sources or criteria) - the truth- and falsehood-components overlap (especially in pseudo paradoxes), while in the fuzzy logic it is not - because the components should sum to 100%, i.e. 70% true and 30% false.

What is the logic of the logic? We study the apparently illogic of the logic, as well as the logic of the illogic.

There exist two main types of truth: the true truth and the false truth, besides the intermediate shades of truth. And similarly for the falsity: the true falsity and the false falsity, beside the intermediate shades of falsity.

The neutrosophic logic unifies many logics; it is like Felix Klein's program in geometry, or Einstein's unified field in physics.

In *Propositional Calculus* a statement may be decidable or undecidable. In the First-Order Logic, due to the quantifiers, a statement may be semi-decidable. In Neutrosophic Logic a statement may be p%-decidable, q%-undecidable, $^{-}0 \leq p, q \leq 100^{+}$.