Neutrosophic Set - A Unifying Field in Sets

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Abstract: In this paper one generalizes fuzzy, paraconsistent, and intuitionistic sets to neutrosophic set. Many examples are presented.

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3. **NEUTROSOPHIC SET**:

3.1. Definition:

Let T, I, F be real standard or non-standard subsets of $]^{-}0, 1^{+}[$,

with sup T = t_sup, inf T = t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup+i_sup+f_sup,
n_inf = t_inf+i_inf+f_inf.

Let U be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as x(T, I, F) and belongs to M in the following way:
it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F.

3.2. General Examples:

Let A and B be two neutrosophic sets.
One can say, by language abuse, that any element neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between 0 and 1 or even less than 0 or greater than 1.
Thus: x(50,20,30) belongs to A (which means, with a probability of 50% x is in A, with a probability of 30% x is not in A, and the rest is undecidable); or y(0,0,100) belongs to A (which normally means y is not for sure in A); or z(0,100,0) belongs to A (which means one does know absolutely nothing about z's affiliation with A).
More general, x((20-30), (40-45)∪[50-51], {20,24,28}) belongs to the set A, which means:
- with a probability in between 20-30% x is in A (one cannot find an exact approximate because of various sources used);
- with a probability of 20% or 24% or 28% x is not in A;
- the indeterminacy related to the appurtenance of x to A is in between 40-45% or between 50-51% (limits included);

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and n_sup = 30+51+28 > 100 in this case.

3.3. Physics Examples:

a) For example the Schrodinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time, which translated to the neutrosophic set means that an element (quantum state) belongs and does not belong to a set (one place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory.

In Schroedinger's Equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function Psi which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Don't we better describe, using the attribute "neutrosophic" than "fuzzy" or any others, a quantum particle that neither exists nor non-exists?

b) How to describe a particle ζ in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\zeta \in P_1$ and $\zeta \notin P_1$ as a true contradiction, or $\zeta \in P_1$ and $\zeta \in \neg P_1$.

3.4. Philosophical Examples:

Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?

In Eastern Philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.

How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

There are many ways to construct them, in terms of the practical problem we need to simulate or approach. Below there are mentioned the easiest ones:

3.5. Application:

A cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set (e.g. there are a kind of separated water drops, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud).

Also, we are not sure where the cloud ends nor where it begins, neither if some elements are or are not in the set. That's why the percent of indeterminacy is required and the neutrosophic probability (using subsets - not numbers - as components) should be used for better modeling: it is a more organic, smooth, and especially accurate estimation. Indeterminacy is the zone of ignorance of a proposition's value, between truth and falshood.

3.6. Neutrosophic Set Operations:

One notes, with respect to the sets A and B over the universe U,

$x = x(T_1, I_1, F_1) \in A$ and $x = x(T_2, I_2, F_2) \in B$, by mentioning x's *neutrosophic membership appurtenance*.

And, similarly, $y = y(T', I', F') \in B$.
If, after calculations, in the below operations one obtains values < 0 or > 1, then one replaces them with $^{-}0$ or 1^{+} respectively.

3.6.1. Complement of A:

If $x(T_1, I_1, F_1) \in A$,
then $x(\{1^+\}\ominus T_1, \{1^+\}\ominus I_1, \{1^+\}\ominus F_1) \in C(A)$.

3.6.2. Intersection:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1\odot T_2, I_1\odot I_2, F_1\odot F_2) \in A \cap B$.

3.6.3. Union:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1\oplus T_2\ominus T_1\odot T_2, I_1\oplus I_2\ominus I_1\odot I_2, F_1\oplus F_2\ominus F_1\odot F_2) \in A \cup B$.

3.6.4. Difference:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1\ominus T_1\odot T_2, I_1\ominus I_1\odot I_2, F_1\ominus F_1\odot F_2) \in A \setminus B$,
because $A \setminus B = A \cap C(B)$.

3.6.5. Cartesian Product:

If $x(T_1, I_1, F_1) \in A$, $y(T', I', F') \in B$,
then $(x(T_1, I_1, F_1), y(T', I', F')) \in A \times B$.

3.6.6. M is a subset of N:

If $x(T_1, I_1, F_1) \in M \Rightarrow x(T_2, I_2, F_2) \in N$,
where $\inf T_1 \le \inf T_2$, $\sup T_1 \le \sup T_2$, and $\inf F_1 \ge \inf F_2$, $\sup F_1 \ge \sup F_2$.

3.6.7. Neutrosophic n-ary Relation:

Let $A_1, A_2, \ldots, A_n$ be arbitrary non-empty sets.
A Neutrosophic n-ary Relation R on $A_1 \times A_2 \times \ldots \times A_n$ is defined as a subset of the cartesian product $A_1 \times A_2 \times \ldots \times A_n$, such that for each ordered n-tuple $(x_1, x_2, \ldots, x_n)(T, I, F)$, T represents the degree of validity, I the degree of indeterminacy, and F the degree of non-validity respectively of the relation R.
It is related to the definitions for the *Intuitionistic Fuzzy Relation* independently given by Atanassov (1984, 1989), Toader Buhaescu (1989), Darinka Stoyanova (1993), Humberto Bustince Sola and P. Burillo Lopez (1992-1995).

3.7. Generalizations and Comments:

From the intuitionistic logic, paraconsistent logic, dialetheism, faillibilism, paradoxes, pseudoparadoxes, and tautologies we transfer the "adjectives" to the sets, i.e. to intuitionistic set (set incompletely known), paraconsistent set, dialetheist set, faillibilist set (each element has a percenatge of indeterminacy), paradoxist set (an element may belong and may not belong in the same time to the set), pseudoparadoxist set, and tautologic set respectively.

Hence, the neutrosophic set generalizes:

- the *intuitionistic set*, which supports incomplete set theories (for $0 < n < 1$ and $i = 0$, $0 \leq t, i, f \leq 1$) and incomplete known elements belonging to a set;
- the *fuzzy set* (for $n = 1$ and $i = 0$, and $0 \leq t, i, f \leq 1$);
- the *classical set* (for $n = 1$ and $i = 0$, with t, f either 0 or 1);
- the *paraconsistent set* (for $n > 1$ and $i = 0$, with both $t, f < 1$);

there is at least one element x(T,I,F) of a paraconsistent set M which belongs at the same time to M and to its complement set C(M);

- the *faillibilist set* ($i > 0$);
- the *dialethist set*, which says that the intersection of some disjoint sets is not empty (for $t = f = 1$ and $i = 0$; some paradoxist sets can be denoted this way too);

every element x(T,I,F) of a dialethist set M belongs at the same time to M and to its complement set C(M);

- the *paradoxist set*, each element has a part of indeterminacy if it is or not in the set ($i > 1$);
- the *pseudoparadoxist set* ($0 < i < 1$, $t + f > 1$);
- the *tautological set* ($i < 0$).

Compared with all other types of sets, in the neutrosophic set each element has three components which are subsets (not numbers as in fuzzy set) and considers a subset, similarly to intuitionistic fuzzy set, of "indeterminacy" - due to unexpected parameters hidden in some sets, and let the superior limits of the components to even boil *over 1* (overflooded) and the inferior limits of the components to even freeze *under 0* (underdried).

For example: an element in some tautological sets may have $t > 1$, called "overincluded". Similarly, an element in a set may be "overindeterminate" (for $i > 1$, in some paradoxist sets), "overexcluded" (for $f > 1$, in some unconditionally false appurtenances); or "undertrue" (for $t < 0$, in some unconditionally false appurtenances), "underindeterminate" (for $i < 0$, in some unconditionally true or false appurtenances), "underfalse" (for $f < 0$, in some unconditionally true appurtenances).

This is because we should make a distinction between unconditionally true ($t > 1$, and $f < 0$ or $i < 0$) and conditionally true appurtenances ($t \leq 1$, and $f \leq 1$ or $i \leq 1$).

In a *rough set* RS, an element on its boundary-line cannot be classified neither as a member of RS nor of its complement with certainty. In the neutrosophic set a such element may be characterized by x(T, I, F), with corresponding set-values for $T, I, F \subseteq \,]^-0, 1^+[$.