New Type of Fuzzy Relational Equations and Neutrosophic Relational Equations – To analyse Customers Preference to Street Shops

Vasantha Kandasamy¹, Sekar.P and Vidhyalakshmi

¹Department of Mathematics, Indian Institute of Technology, Madras, India. E-mail: vasanthakandasamy@gmail.com
²Department of Maths, Kandaswami Naidu College, Chennai, India. E-mail: vidhusat22@yahoo.co.in

Abstract. In this paper authors study the customer’s preference of street shops to other eateries using Fuzzy Relation Equations (FREs) and Neutrosophic Relation Equations (NREs). We have constructed a new type of FRE and NRE called the new average FRE and new average NRE. This study is based on interviews/discussions taken from 32 tuck shops in and around Tambaram. This paper is organized into five sections. In section one we just recall the working of FRE and NRE.

Keywords: Fuzzy Relation Equation(FRE), Neutrosophic Relation Equation(NRE), New Average Fuzzy Relation Equation (NAFRE), New Average Neutrosophic Relation Equation(NANRE).

1 Basic Concepts

Here we just recall the basic definitions and describe the functioning of Fuzzy Relation Equations (FRE) and Neutrosophic Relation Equations (NRE). We have taken the basic definitions from [1,6]. The notion of Fuzzy Relation Equations (FREs) is associated with the concept of composition of binary relations. The FREs are based upon the max-min composition. Considering the three binary relations P(X,Y), Q(Y,Z) and R(X,Z) which are defined on the sets, X = {x₁ | i ∈ I}, Y = {yⱼ | j ∈ J} and Z = {zₖ | k ∈ K} where we assume that I = Nₙ, J = Nₘ and K = Nₖ. Let the membership matrices of P, Q and R be denoted by P = [pᵢⱼ], Q = [qⱼₖ] and R = [rᵢₖ] respectively where pᵢⱼ = P(xᵢ, yⱼ), qⱼₖ = Q(yⱼ, zₖ) and rᵢₖ = R(xᵢ, zₖ) for all i ∈ I(= Nₙ), j ∈ J(= Nₘ) and k ∈ K(= Nₖ). This means all the entries in the matrices P, Q and R are real numbers in the unit interval [0,1].

Assume now that the three relations constrain each other in such a way that

\[ P \circ Q = R \] (1)

where ‘⊙’ denotes the max-min composition. This means that

\[ \max_{j} \min_{i}(p_{i,j}q_{j,k}) = r_{i,k} \] (2)

for all i ∈ I and k ∈ K. That is, the matrix equation (1) encompasses n × s simultaneous equations of the form (2). When the two of the components in each of the equations are given and one is unknown, these equations are referred to as FREs.

We define the new notion of average FREs and average NREs and use this new model to study the problem which forms section two of this paper. Section three describes the attributes related with the customers and the types of customers based on the pilot survey made by us. The new FRE and NRE models constructed in section two of this paper is used in analysing the problem in section four. The final section gives the conclusions and suggestions made from this study.

It is pertinent to mention that in general the equation P o Q = R need not give a solution. In case when we do not have a solution to equation (1) we use neural networks to find the solution [1,3-6].

We just recall the definition of Neutrosophic Relation Equations (NREs). To this new notion we need the concept of the indeterminate I, where I² = I and I + I +...+ I = N. Here for more about these neutrosophic concept please refer [2,5]. We denote by N₁ = {a + bI / a, b ∈ [0,1]} and N₁ is defined as Fuzzy neutrosophic values.

To construct Neutrosophic Relation Equations we make use of N₁ clearly [0,1] ⊆ N₁; this is the case when b = 0. The Neutrosophic Relation Equations are based upon the max-min composition. Considering the three binary relations N₁(X,Y) Q(Y,Z) and B(X,Z) which are defined on the sets, X = {xᵢ | i ∈ I}, Y = {yⱼ | j ∈ J} and Z = {zₖ | k ∈ K} where we assume that I = Nₙ, J = Nₘ and K = Nₖ. Let the membership matrices of N₁, Q and B be denoted by N₁ = [n₁ᵢⱼ], Q = [qⱼₖ] and B = [bᵢₖ] respectively where n₁ᵢⱼ = N₁(xᵢ, yⱼ), qⱼₖ = Q(yⱼ, zₖ) and bᵢₖ = B(xᵢ, zₖ) for all i ∈ I(= Nₙ), j ∈ J(= Nₘ) and k ∈ K(= Nₖ). This means all the entries in the neutrosophic matrices N₁, Q and B are fuzzy neutrosophic values from N₁.

Assume now that the three relations constrain each other in such the way that

\[ N₁ \circ Q = B \] (3)

where ‘⊙’ denotes the max-min composition. This means that
The main motivation for construction of these new models Average Fuzzy Relation Equations and Average Neutrosophic Relation Equations arises from following factors. These models functions on the wishes of all the experts who work with the problem. If for any problem we use more than one experts opinion we may have problem of choosing the experts opinion for preference of one expert over the other may not give satisfaction to the other experts as they may feel their suggestions are ignored and this may lead to an unpleasant situation and bias in the choice.

To overcome this problem we have defined two new models called the New Average Fuzzy Relation Equations (NAFRE) and New Average Neutrosophic Relation Equations (NANRE). Here we define and describe the New Average Fuzzy Relation Equations model (NAFRE) and New Average Neutrosophic Relation Equations model (NANRE).

Suppose \( P_1(X, Y), P_2(X, Y), \ldots, P_n(X, Y) \) be the Fuzzy Relation of X on Y given by n-distinct experts, where all the n-experts agree to work with the same set of attributes from the range and domain spaces.

Let \( R_1, R_2, \ldots, R_n \) denote the related matrices of the FRE of the n-experts associated with \( P_1(X, Y), P_2(X, Y), \ldots, P_n(X, Y) \) the fuzzy relation of X on Y respectively.

We define

\[
P(X, Y) = \frac{P_1(X, Y) + \ldots + P_n(X, Y)}{n}
\]

that in terms of the Fuzzy Relation Equations, that is if R is the matrix related with P(X, Y) then R is got from

\[
\frac{1}{n} \sum_{i=1}^{n} P_i
\]

using FRE based on max-min composition is again a matrix which gives the fuzzy relation of X with Y.

The merit of using this model is that every expert is given the same preference so the experts have no disappointment when forming the final result and further this saves time and economy for we can work with one model instead of n-models. The advantages of using the average FRE model is as follows. Just like other fuzzy models the extreme values do not cancel out as the values of all the FRE matrices R related with the respective \( P_i(X, Y) \) has its entries in \([0, 1] \); \( 1 \leq i \leq n \). Hence at the outset we are justified in using this specially constructed New Average Fuzzy Relation Equations (NAFREs) model. This model also caters to the law of large numbers. So the results become more and more sensitive by increasing the number of experts and further only a single matrix represents the opinion of all these n-experts. Hence time and economy are not affected by using this new model.

Next we proceed to define the New Average NRE model. Suppose \( N_1(X, Y), N_2(X, Y), \ldots, N_n(X, Y) \) be the neutrosophic relation of X on Y given by n-distinct experts, where all the n-experts agree to work with the same set of attributes from the range and domain spaces.

Let \( B_1, B_2, \ldots, B_n \) denote the related matrices of the NREs of the n-experts associated with \( N_1(X, Y), N_2(X, Y), \ldots, N_n(X, Y) \) the neutrosophic relation of X on Y respectively. We define

\[
N(X, Y) = \frac{N_1(X, Y) + \ldots + N_n(X, Y)}{n}
\]

that in terms of the Neutrosophic Relation Equations, that is if B is the matrix related with N(X, Y) then B is got from

\[
\frac{1}{n} \sum_{i=1}^{n} N_i
\]

using NRE based on max-min composition is again a matrix which gives the neutrosophic relation of X with Y. There is no dependency between the average taken for real and indeterminacy; since as per the experts who have deterministic opinion the average of their opinion is taken separately and the experts who have indeterminacy opinion is dealt with separately. However we prefer to use NRE models mainly as certain experts express their inability to give opinion had forced us to deploy neutrosophic models.

In NRE models the extreme values do not cancel out as the values of all the NRE matrices B related with the respective \( N_i(X, Y) \) has neutrosophic values \( \{a + bI | a, b \in [0,1] \} \). Hence at the outset we are justified in using this specially constructed New Average Neutrosophic Relation Equations (NANREs) model. This model also caters to the law of large numbers. So the results become more and more sensitive by increasing the number of experts and further only a single matrix represents the opinion of all these n-experts. Hence time and economy are not affected by using this new model.
felt the following attributes can be given preference in the study of the problem. Accordingly X denotes the attributes related with the preferences of the customers which is taken as the ‘domain’ space of the Fuzzy Relation Equations. Y correspond to attributes related with the types of the customers.

We briefly describe in a line (or) two the attributes of X and Y in this section. Let S = [S₁, S₂, ..., S₇] denote the domain space.

S₁: “Cost” – The cost is reasonably fair because the street shop owners do not charge VAT, no tips for the servers and they do not charge for even hygienic water.

S₂: “Quality is good” – The view of the experts (customers) felt that the phrase “Quality is good” means that the food they get from the street shops is less in adulteration with chemical for taste and smell. They also claimed that the food is just like home made food so they prefer the tuck shops to that of big restaurant or multicuisine hotels.

S₃: “Quantity is more” – The quantity is more in comparison since for the same amount we spend on street shops, we get more and substantial amount of food which is really fulfilling the customers.

S₄: “Better Hygiene” – Since the food are instantly made we do not get left out foods. They keep the surroundings clean because they are always watched keenly by all the customers and public. They give us clean can water and they serve the food in paper plates and cups which is used only once. Added to this even sometimes they serve in fresh green banana leaves.

S₅: “Service is good” - Most of the street shop owners are themselves servers. So they take care of each customer. They are friendlier. They serve the food immediately and the customers need not wait.

S₆: “Prepared in our presence” – Since the food is prepared in our presence we can give instruction to prepare for our taste. Food are just made so hot and hygienic.

S₇: “Waiting Time” – Comparatively since the owners are themselves servers they give importance to each customer and they serve the food very quickly. The customers need not wait for long time in a long queue which very often happens in big multicuisine restaurant even to pay bills and for parcelling the food and for every service many hours are wasted.

The attributes related with the types of customers R = {C₁, C₂, ..., C₇} is taken as range space. We briefly describe in a word or two the attributes C₁, C₂, ..., C₇:

C₁: Bachelors: Most of the bachelors take food from road side shops because of so many factors like the quantity of food is large for what they pay.

C₂: Students: Both day scholars and hostellers like to have food due to the less price they charge.

C₃: LT and Call centre Employees: These type of customers give importance for hygiene food and less waiting time.

C₄: House Wives: These type of customers give much importance to better hygiene and for more quantity.

C₅: Daily Wage Labours: These labour in tambaram used to go road side eateries for various reason like more quantity of food they get for what they pay, less cost which is affordable by them and for good hospitality.

C₆: Local Employees: These type of customers mainly prefer these shops for better hygiene and for better service.

C₇: Children above 10-Years: Children prefer for some special food which is not always prepared in their home and for less cost charged for the food.

The collected data was analysed and the following limit sets are derived using the questionnaire.

S₁ ≥ 0.6 (The cost is reasonably fair so we are forced to give just 60%. S₁ < 0.6 means the cost in not reasonably fair to their expectation).

S₂ ≥ 0.5 (The quality is preferred by those who have experience in eating quality food so we are forced to give just 50% S₂ < 0.5 means the quality is not as good to their expectation).

S₃ ≥ 0.6 (Several like school students, daily wage people, etc, prefer quantity with so the expert feel after pilot survey S₃ < 0.6 is not as good as to their expectation).

S₄ ≥ 0.5 (Most of the customers like LT employees, house wives, etc, prefer better hygiene S₄ < 0.5 means the better hygiene is not as good as to their expectation).

S₅ ≥ 0.6 (The service is preferred by those who have experienced the better service when compared with the multi cuisine hotels. So we are forced to give 60% S₅ < 0.6 means the service is not good as to their expectation).

S₆ ≥ 0.4 (The expert feels that they prefer the food which is prepared in their presence so we are forced to give 40% S₆ < 0.4 means they do not give much importance for the food which is prepared in their presence).

S₇ ≥ 0.6 (The waiting time is much important and they have less waiting time compared to multiCuisine hotels S₇ < 0.6 means the waiting time is comparatively more).

In the next section we analyse the collected data using FRE and NRE.

4 Use of FRE and NRE models to analyse the problem

Here we have collected the data from 32 tuck shops. We have used five experts to work with FRE and NRE model. The FRE matrices of 5 experts P₁, P₂, P₃, P₄ and P₅ are given as follows. Now we work with first expert. Let P₁ be the membership matrix given by the first expert which is as follows:
As we get least preference is given for the food prepared in their presence and much importance is given for all the remaining constrains.

Now using
\[ Q_{11} = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6] \]
in the equation \( P_2 \circ Q_{11} = \text{MaxMin}(p_{ij}, q_{ik}) = R_{11} \) we get
\[ R_{11} = [0.8 \ 0.7 \ 0.7 \ 0.7 \ 0.6 \ 0.3 \ 0.7] \].

As analysed from \( R_{11} \) the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Now using
\[ Q_{12} = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3] \]
in the equation \( P_2 \circ Q_{12} = \text{MaxMin}(p_{ij}, q_{ik}) = R_{12} \) we get
\[ R_{12} = [0.5 \ 0.4 \ 0.4 \ 0.4 \ 0.3 \ 0.4] \].

As analysed from resultant \( R_{12} \), the expert feels much preference is given for the food prepared in their presence and gives least preference for all the other constrains.

Next we work with third expert. Let \( P_3 \) be the membership matrix given by third expert which is as follows
\[
\begin{align*}
  S_1 & = [0.1 \ 0.1 \ 0.1 \ 0.8 \ 0.9 \ 0.7 \ 0.8] \\
  S_2 & = [0.2 \ 0.3 \ 0.7 \ 0.7 \ 0.1 \ 0.4 \ 0.6] \\
  S_3 & = [0.7 \ 0.8 \ 0.1 \ 0.7 \ 0.1 \ 0.6 \ 0.6] \\
  P_3 & = [0.2 \ 0.1 \ 0.8 \ 0.8 \ 0.1 \ 0.1 \ 0] \\
  S_5 & = [0.5 \ 0.6 \ 0.4 \ 0.2 \ 0.3] \\
  S_6 & = [0.1 \ 0.2 \ 0.3 \ 0.1 \ 0.1 \ 0.1] \\
  S_7 & = [0.3 \ 0.8 \ 0.4 \ 0.4 \ 0.8] \\
\end{align*}
\]
Now using
\[ Q_{1} = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3] \]
in the equation \( P_3 \circ Q_1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{1} \) we get
\[ R_1 = [0.8 \ 0.5 \ 0.7 \ 0.7 \ 0.6 \ 0.5 \ 0.6] \].

As analysed from resultant \( R_1 \) the expert feels much preference is given for all the constrains.

Now using
\[ Q_{12} = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3] \]
in the equation \( P_3 \circ Q_{12} = \text{MaxMin}(p_{ij}, q_{ik}) = R_{12} \) we get
\[ R_{12} = [0.5 \ 0.4 \ 0.4 \ 0.4 \ 0.3 \ 0.4] \].

As analysed from resultant \( R_{12} \), the expert feels much preference is given for all the constrains.
\[ R_{31}^1 = [0.5 \ 0.4 \ 0.4 \ 0.4 \ 0.4 \ 0.4 \ 0.4]. \]

As analysed from resultant \( R_{31}^1 \), the expert feels much preference is given for the food prepared in their presence and given least importance for all the remaining constrains.

Next we work with fourth expert. Let \( P_4 \) be the membership matrix given by fourth expert which is as follows

\[
\begin{align*}
S_1 & = [0.1 \ 0.7 \ 0.7 \ 0.8 \ 0.8 \ 0.2] \\
S_2 & = [0.5 \ 0.5 \ 0.6 \ 0.7 \ 0.4 \ 0.6 \ 0.7] \\
S_3 & = [0.7 \ 0.7 \ 0.5 \ 0.4 \ 0.7 \ 0.6 \ 0.5] \\
P_4 & = S_4 [0.6 \ 0.5 \ 0.5 \ 0.6 \ 0.1 \ 0.2 \ 0.4] \\
S_5 & = [0.6 \ 0.4 \ 0.6 \ 0.7 \ 0.4 \ 0.3 \ 0.5] \\
S_6 & = [0.4 \ 0.2 \ 0.4 \ 0.5 \ 0.2 \ 0.1 \ 0] \\
S_7 & = [0.2 \ 0.1 \ 0.7 \ 0.5 \ 0 \ 0.5 \ 0.5]
\end{align*}
\]

Now using
\[ Q_{41}^1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3] \]

in the equation \( P_4 \circ Q_{41}^1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{41} \) we get
\[ R_{41}^1 = [0.8 \ 0.6 \ 0.7 \ 0.6 \ 0.4 \ 0.7]. \]

As analysed from resultant \( R_{41}^1 \), the expert feels much preference is given for all the constrains.

Now using
\[ Q_{41}^1 = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6] \]

in the equation \( P_4 \circ Q_{41}^1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{41} \) we get
\[ R_{41}^1 = [0.7 \ 0.6 \ 0.7 \ 0.6 \ 0.2 \ 0.7]. \]

As analysed from resultant \( R_{41}^1 \), the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Next we work with fifth expert. Let \( P_5 \) be the membership matrix given by fifth expert which is as follows

\[
\begin{align*}
S_1 & = [0.6 \ 0.6 \ 0.6 \ 0.7 \ 0.7 \ 0.8] \\
S_2 & = [0.3 \ 0.1 \ 0.5 \ 0.5 \ 0 \ 0.5 \ 0.7] \\
S_3 & = [0.5 \ 0.7 \ 0.1 \ 0.6 \ 0.6 \ 0.5 \ 0.6] \\
P_5 & = S_4 [0.1 \ 0 \ 0.7 \ 0.6 \ 0.1 \ 0.1 \ 0.1] \\
S_5 & = [0.1 \ 0 \ 0.6 \ 0.3 \ 0 \ 0.2 \ 0.1] \\
S_6 & = [0.1 \ 0.1 \ 0.1 \ 0.2 \ 0.1 \ 0.1 \ 0.2] \\
S_7 & = [0.3 \ 0.1 \ 0.7 \ 0.6 \ 0.1 \ 0.5 \ 0.6]
\end{align*}
\]

Now using
\[ Q_{51}^1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3] \]

in the equation \( P_5 \circ Q_{51}^1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{51} \) we get
\[ R_{51}^1 = [0.7 \ 0.5 \ 0.6 \ 0.7 \ 0.6 \ 0.2 \ 0.7]. \]

As analysed from resultant \( R_{51}^1 \), the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Now using
\[ Q_{51}^1 = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6] \]

in the equation \( P_5 \circ Q_{51}^1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{51} \) we get
\[ R_{51}^1 = [0.7 \ 0.6 \ 0.7 \ 0.6 \ 0.2 \ 0.7]. \]

As analysed from resultant \( R_{51}^1 \), the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Now using
\[ Q_{51}^1 = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3] \]

in the equation \( P_5 \circ Q_{51}^1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{51} \) we get
\[ R_{51}^1 = [0.5 \ 0.4 \ 0.5 \ 0.3 \ 0.2 \ 0.4]. \]

As analysed from resultant \( R_{51}^1 \), the expert feels least preference is given for all the constrains.

The New Average Fuzzy Relation Equations(NAFREs) defined and developed in section 2 of the paper is constructed using five experts which gives the opinion of all the 5 experts feeling. As a law of large number the average taken for all the five experts give approximately a sensitive opinion.

\[
(P_1 + P_2 + P_3 + P_4 + P_5)/5 =
\]

\[
\begin{align*}
S_1 & = [0.06 \ 0.64 \ 0.02 \ 0.68 \ 0.8 \ 0.7 \ 0.68] \\
S_2 & = [0.34 \ 0.2 \ 0.58 \ 0.62 \ 0.12 \ 0.5 \ 0.68] \\
S_3 & = [0.62 \ 0.78 \ 0.18 \ 0.64 \ 0.56 \ 0.6 \ 0.6] \\
S_4 & = [0.26 \ 0.22 \ 0.68 \ 0.64 \ 0.06 \ 0.12 \ 0.12] \\
S_5 & = [0.36 \ 0.18 \ 0.6 \ 0.44 \ 0.1 \ 0.22 \ 0.28] \\
S_6 & = [0.24 \ 0.16 \ 0.16 \ 0.32 \ 0.1 \ 0.12 \ 0.14] \\
S_7 & = [0.2 \ 0.08 \ 0.66 \ 0.54 \ 0.06 \ 0.5 \ 0.66]
\end{align*}
\]

Now using
\[ Q_{15}^1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3] \]

in the equation \( P_5 \circ Q_{15}^1 = \text{MaxMin}(p_{ij}, q_{ik}) = R_{15} \) we get
\[ R_{15}^1 = [0.8 \ 0.58 \ 0.62 \ 0.68 \ 0.6 \ 0.32 \ 0.66]. \]

As analysed from resultant the \( R_{15}^1 \), expert feels least preference for the food which is being prepared in their presence and much preference is given for all the remaining constrains.
Now using
\[ Q_{11}^1 = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6] \]
in the equation $P \circ Q_{11}^1 = \text{MaxMin}(p_i, q_{jk}) = R_{11}^1$ we get
\[ R_{11}^1 = [0.8 \ 0.67 \ 0.78 \ 0.68 \ 0.6 \ 0.32 \ 0.66] . \]

As analysed from resultant $R_{11}^1$, expert feels least preference for the food which is being prepared in their presence and much preference for all the remaining constrains.

Now using
\[ Q_{12}^1 = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3] \]
in the equation $P \circ Q_{12}^1 = \text{MaxMin}(p_i, q_{jk}) = R_{12}^1$ we get
\[ R_{12}^1 = [0.5 \ 0.4 \ 0.5 \ 0.4 \ 0.4 \ 0.5 \ 0.4] . \]

As analysed from resultant $R_{12}^1$ expert feels least preferences is given for all the constrains.

Next we consider the opinion of 5 experts who wish to use the NREs to the same problem. Now we work with first expert.

Let $N_1$ be the membership matrix given by first expert
\[
S_1 = \begin{bmatrix} 0.21 & 0.6 & 0.31 & 0.7 & 0.8 & 0.6 & 0.8 \end{bmatrix},
S_2 = \begin{bmatrix} 0.3 & 0.71 & 0.6 & 0.6 & 0.31 & 0.5 & 0.7 \end{bmatrix},
S_3 = \begin{bmatrix} 0.6 & 0.8 & 0.71 & 0.7 & 0.7 & 0.5 & 0.7 \end{bmatrix},
N_1 = S_1 = \begin{bmatrix} 0.51 & 0.2 & 0.7 & 0.6 & 0.3 & 0.2 & 0.7 \end{bmatrix},
S_4 = \begin{bmatrix} 0.7 & 0.41 & 0.6 & 0.3 & 0.21 & 0.7 & 0.5 \end{bmatrix},
S_5 = \begin{bmatrix} 0.1 & 0.5 & 0.3 & 0.2 & 0.7 & 0.1 & 0.51 \end{bmatrix},
S_6 = \begin{bmatrix} 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix},
\]
and the expert wishes to work with
\[ Q_1^1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3] . \]

Now we find solution for the following neutrosophic equations. That is $N_1 \circ Q_1^1 = \text{MaxMin}(n_i, q_{jk}) = B_{11}^1$ which gives $B_{11}^1 = [0.8 \ 0.6 \ 0.71 \ 0.7 \ 0.7 \ 0.7 \ 0.7]$.

As analysed from resultant $B_{11}^1$ expert feels that much preference is given to all the constrains and not able to express the constrain quantity of food is about 70%.

Now using
\[ Q_{11}^2 = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6] \]
in the equation $N_1 \circ Q_{11}^2 = \text{MaxMin}(n_i, q_{jk}) = B_{11}^2$ we get
\[ B_{11}^2 = [0.7 \ 0.71 \ 0.8 \ 0.7 \ 0.7 \ 0.7 \ 0.7] . \]

As analysed from resultant $B_{11}^2$ expert feels that much preference is given to all the constrains and not able to express the constrain quality of food.

Now using
\[ Q_{12}^2 = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3] \]
in the equation $N_1 \circ Q_{12}^2 = \text{MaxMin}(n_i, q_{jk}) = B_{12}^2$ we get
\[ B_{12}^2 = [0.5 \ 0.4 \ 0.5 \ 0.4 \ 0.4 \ 0.5 \ 0.4] . \]

As analysed from resultant $B_{12}^1$ the expert feels least preference is given to all the constrains expect the food prepared in their presence.

Next we work with the second expert. Let $N_2$ be the membership matrix given by second expert which is as follows:
\[
S_1 = \begin{bmatrix} 0.2 \ 0.7 \ 0.3 & 0.71 & 0.8 & 0.7 & 0.7 \end{bmatrix},
S_2 = \begin{bmatrix} 0.6 & 0.2 & 0.71 & 0.6 & 0.21 & 0.3 & 0.5 \end{bmatrix},
S_3 = \begin{bmatrix} 0.7 & 0.5 & 0.4 & 0.6 & 0.2 & 0.6 & 0.3 \end{bmatrix},
N_2 = S_4 = \begin{bmatrix} 0.4 & 0.21 & 0.6 & 0.6 & 0.1 & 0.2 & 0.1 \end{bmatrix},
S_5 = \begin{bmatrix} 0.4 & 0.11 & 0.5 & 0.4 & 0.1 & 0.3 & 0.2 \end{bmatrix},
S_6 = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.1 & 0.2 & 0.4 \end{bmatrix},
S_7 = \begin{bmatrix} 0.3 & 0.1 & 0.61 & 0.3 & 0 & 0.4 & 0.8 \end{bmatrix} . \]

Now using
\[ Q_1^2 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3] \]
in the equation $N_2 \circ Q_1^2 = \text{MaxMin}(n_i, q_{jk}) = B_2^1$ we get
\[ B_2^1 = [0.8 \ 0.71 \ 0.7 \ 0.6 \ 0.5 \ 0.4 \ 0.61] . \]

As analysed from resultant $B_2^1$ the expert feels inability to express about the quality of food and less waiting time and much preference is given for all the remaining constrains.

Now using
\[ Q_{11}^3 = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6] \]
in the equation $N_3 \circ Q_{11}^3 = \text{MaxMin}(n_i, q_{jk}) = B_{11}^3$ we get
\[ B_{11}^3 = [0.8 \ 0.6 \ 0.71 \ 0.7 \ 0.7 \ 0.7 \ 0.7] . \]

As analysed from resultant $B_{11}^3$ the expert feels inability to express about the reasonable cost, quality of food and less waiting time and less preference is given for better service and much preference for quantity of the food, better hygiene and for food prepared in their presence.

Now using
\[ Q_{12}^3 = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3] \]
in the equation $N_3 \circ Q_{12}^3 = \text{MaxMin}(n_i, q_{jk}) = B_{12}^3$ we get
\[ B_{12}^3 = [0.5 \ 0.4 \ 0.4 \ 0.4 \ 0.4 \ 0.4 \ 0.4] . \]

As analysed from resultant $B_{12}^3$ the expert feels much preference is given for food prepared in their presence and least preferences is given for all the remaining constrains.

Next we work with the third expert. Let $N_3$ be the membership matrix given by third expert which as follows:
\[
S_1 = \begin{bmatrix} 0.1 \ 0.6 \ 0 \ 0.4 \ 0.7 \ 0.8 \ 0.9 \end{bmatrix},
S_2 = \begin{bmatrix} 0.31 \ 0.1 \ 0.4 \ 0.61 \ 0.1 \ 0.5 \ 0.7 \end{bmatrix},
S_3 = \begin{bmatrix} 0.5 \ 0.9 \ 0.21 \ 0.5 \ 0.6 \ 0.5 \ 0.6 \end{bmatrix},
N_3 = S_4 = \begin{bmatrix} 0.3 \ 0.41 \ 0.6 \ 0.4 \ 0.2 \ 0.1 \ 0.2 \end{bmatrix},
S_5 = \begin{bmatrix} 0.5 \ 0.5 \ 0.6 \ 0.5 \ 0.1 \ 0.2 \ 0.5 \end{bmatrix},
S_6 = \begin{bmatrix} 0.3 \ 0.2 \ 0.3 \ 0.2 \ 0.3 \ 0.1 \ 0.3 \end{bmatrix},
S_7 = \begin{bmatrix} 0.1 \ 0.5 \ 0.2 \ 0.4 \ 0.1 \ 0.4 \ 0.5 \end{bmatrix} . \]

Now using
we get and denote it by N.

As analysed from resultant $B_3^i$ the expert feels least preference is given to the food prepared in their presence, less waiting time, and better service. Much preference is given for remaining constrains.

Now using

$$Q_{11}^i = [0.9 0.8 0.9 0.8 0.7 0.7 0.6 0.6 0.3]$$
in the equation $N_i \circ Q_{11}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_3^i$ we get

$$B_3^i = [0.7 0.5 0.6 0.6 0.6 0.3 0.5].$$

As analysed from resultant $B_3^i$ the expert feels least preference is given to the food prepared in their presence and for less waiting time and the expert is not able to express about the quality of food.

Now using

$$Q_{12}^i = [0.4 0.2 0.3 0.4 0.5 0.4 0.3]$$
in the equation $N_i \circ Q_{12}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_3^i$ we get

$$B_3^i = [0.5 0.4 0.4 0.4 0.3 0.4].$$

As analysed from resultant $B_3^i$ the expert feels least preference is given to all the constrains.

Next we work with fourth expert. Let $N_4$ be the membership matrix given by third expert which as follows:

$$N_4 = \begin{bmatrix}
S_1 & 0.1 & 0.5 & 0.2 & 0.7 & 0.6 & 0.9 & 0.5 \\
S_2 & 0.6 & 0.8 & 0.8 & 0.9 & 0.2 & 0.6 & 0.5 \\
S_3 & 0.4 & 0.2 & 0.5 & 0.4 & 0.7 & 0.6 & 0.3 \\
S_4 & 0.5 & 0.4 & 0.2 & 0.5 & 0.2 & 0.2 & 0.3 \\
S_5 & 0.5 & 0.4 & 0.5 & 0.8 & 0.6 & 0.4 & 0.5 \\
S_6 & 0.4 & 0.2 & 0.6 & 0.5 & 0.2 & 0.1 & 1 \\
S_7 & 0.2 & 0.3 & 0.6 & 0.4 & 0 & 0.2 & 0.2 
\end{bmatrix}$$

Now using

$$Q_{11}^i = [0.7 0.5 0.7 0.4 0.8 0.6 0.3]$$
in the equation $N_i \circ Q_{11}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_4^i$ we get

$$B_4^i = [0.6 0.7 0.7 0.5 0.6 0.6 0.61].$$

As analysed from the resultant $B_4^i$ the expert is not able to express about the less waiting time and given much preference to all the constrains.

Now using

$$Q_{12}^i = [0.9 0.8 0.9 0.8 0.7 0.7 0.6]$$
in the equation $N_i \circ Q_{12}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_4^i$ we get

$$B_4^i = [0.7 0.8 0.7 0.5 0.8 0.6 0.61].$$

As analysed from the resultant $B_4^i$ the expert is not able to express about the less waiting time and least preference is given to better hygiene and much preference is given to all the remaining constrains.

Now using

$$Q_{12}^i = [0.4 0.2 0.3 0.4 0.5 0.4 0.3]$$
in the equation $N_i \circ Q_{12}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_4^i$ we get

$$B_4^i = [0.5 0.4 0.4 0.5 0.4 0.4].$$

As analysed from the resultant $B_4^i$ the expert gives least preference to all the constrains expect about the food prepared in their presence.

Next we work with fourth expert. Let $N_5$ be the membership matrix given by third expert which as follows:

$$N_5 = \begin{bmatrix}
S_1 & 0.2 & 0.9 & 0.1 & 0.5 & 0.6 & 1 & 0.8 \\
S_2 & 0 & 0.2 & 0.11 & 0.6 & 0.5 & 0 & 0.5 \\
S_3 & 0.6 & 0.6 & 0.2 & 0.5 & 0.6 & 0.4 & 0.8 \\
S_4 & 0.6 & 0.1 & 0.7 & 0.6 & 0.1 & 0.1 & 0.2 \\
S_5 & 0.1 & 0.0 & 0.6 & 0.3 & 0 & 0.2 & 0.1 \\
S_6 & 0.1 & 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.2 \\
S_7 & 0.3 & 0.3 & 0.7 & 0.6 & 0.1 & 0.5 & 0.6 
\end{bmatrix}$$

Now using

$$Q_{11}^i = [0.7 0.5 0.7 0.4 0.8 0.6 0.3]$$
in the equation $N_i \circ Q_{11}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_5^i$ we get

$$B_5^i = [0.6 0.5 0.6 0.7 0.6 0.2 0.7].$$

As analysed from the resultant $B_5^i$ the expert feels least preference is given to the food prepared in their presence and quality of food.

Now using

$$Q_{12}^i = [0.9 0.8 0.9 0.8 0.7 0.7 0.6]$$
in the equation $N_i \circ Q_{12}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_5^i$ we get

$$B_5^i = [0.8 0.6 0.8 0.7 0.6 0.2 0.7].$$

As analysed from the resultant $B_5^i$ the expert feels least preference is given to the food prepared in their presence.

Now using

$$Q_{12}^i = [0.4 0.2 0.3 0.4 0.5 0.4 0.3]$$
in the equation $N_i \circ Q_{12}^i = \text{MaxMin}(n_{ij}, q_{ik}) = B_5^i$ we get

$$B_5^i = [0.5 0.5 0.4 0.3 0.2 0.4].$$

As analysed from the resultant $B_5^i$ the expert feels least preferences given to all the constrains.

The New Average Neutrosophic Relation Equation (NANRE) defined and developed in section 2 of the paper is constructed using five experts which gives the opinion of all the 5 experts feeling. As a law of large number the average taken for all the five experts gives the approximately a sensitive opinion. Using the special type of average mentioned in section two of this paper we find the average of $N_1, N_2, ..., N_5$ and denote it by N.
the expert feels least preference is given to all the constrains except the reasonable cost and expert is not able to express about the quality of food.

As analysed from the resultant $B'$ the expert feels least preference is given to better hygiene, better service, food prepared in their presence and for less waiting time and expert is not able to express the quality of food.

Now using

$$Q_{i1}^1 = [0.9 \ 0.8 \ 0.9 \ 0.8 \ 0.7 \ 0.7 \ 0.6]$$

in the equation $N \cdot Q_{i1}^1 = \text{MaxMin}(n_{ij}, q_{bk}) = B_1$ we get

$$B_1' = [0.7 \ 0.61 \ 0.6 \ 0.56 \ 0.56 \ 0.32 \ 0.56].$$

As analysed from the resultant $B_1'$ the expert feels least preference is given to better hygiene, better service, food prepared in their presence and for less waiting time and expert is not able to express the quality of food.

Now using

$$Q_{i2}^1 = [0.4 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.4 \ 0.3]$$

in the equation $N \cdot Q_{i2}^1 = \text{MaxMin}(n_{ij}, q_{bk}) = B_2$ we get

$$B_2' = [0.5 \ 0.38 \ 0.5 \ 0.4 \ 0.4 \ 0.32 \ 0.4].$$

As analysed from the resultant $B_2'$ the expert feels least preference to all the constrains.

6. Conclusions

Hence using the experts opinion for the Fuzzy Relational Equations. The performance that the most of the owners of the street shops choose this as their profession is more profit with reasonable investment and unemployment problem is solved since many owners are from 25 yrs to 35yrs. Many owners do this job as a part-time. Even retired person with less investment do this job and they have job satisfaction. These street shops even give jobs for more Handicapped persons. The following conclusion are not only derived from the five experts described here but all 32 tuck shops and their opinion are used and some of the owners whom we have interviewed are also ingrained in this analysis. Now we consolidate the opinion given by the five experts working with FRE model in the following tables and the eighth column of each of these tables gives the average of each of the $R_{ij}$‘s $1 \leq i \leq 5$ calculated for each of the five experts using the FRE model. We see readily from the table that the average $R_{ii}$ ($1 \leq i \leq 3$) so found and the resultant $R_{Xi}$ ($1 \leq i \leq 3$) calculated using NA$R$Es given in column seven of all the tables do not differ. In fact the values are very close. So we are justified in the construction of this model as it can save both time and economy.

$$Q_i = \begin{bmatrix}
15 +.21 & 0.66 & .15 +.31 & .68 & 0.8 & .7 & 0.68 \\
.38 +.31 & .33 +.71 & 0.6 +.41 & .62 & 0.12 & .54 & 0.67 \\
.56 & 0.6 & .37 +.45 & .64 & 0.62 & .6 & 0.6 \\
.33 +.51 & .23 +.31 & 0.56 & 0.54 & 0.18 & 0.16 & 0.3 \\
.04 & .34 +.25 & 0.56 & .46 & .24 +.21 & .36 & 0.36 \\
0.2 & 0.24 & 0.3 & .32 & 0.26 & 12 & 0.3 +.75 \\
0.22 & 0.24 & .53 +.1 & .46 & 0.04 & 0.4 & 0.56
\end{bmatrix}$$

Now using

$$Q_{i1}^1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3]$$

in the equation $N \cdot Q_{i1}^1 = \text{MaxMin}(n_{ij}, q_{bk}) = B'$ we get

$$B' = \begin{bmatrix}
0.7 & 0.41 & 0.56 & 0.56 & 0.56 & 0.32 & 0.533
\end{bmatrix}.$$
Hence we conclude both the new models serve not only the purpose of saving time and economy but also gives equal importance to each and every expert and avoids bias by choice which is vital.

References:


Received: January 14, 2014. Accepted: February 12, 2014.