Program

of the first international conference

"Smarandache Type Notions in Number Theory"

Thursday, August 21

Chairman: V.W.Spinadel (Argentina)

10 - 11	H.Ibstedt (Sweden)	On Smarandache's Periodic Sequences
11 - 12	C.Dumitrescu (Romania)	From the Smarandache Function to Smarandache Type Function
12 - 12 ³⁰	H.Ibstedt (Sweden)	A Few Smarandache Integer Sequences
12 ³⁰ - 13	L.Widmer (USA)	Construction of Elements of the Smarandache Square-Partial-Digital Subsequence
13 - 13 ³⁰	S.M.Ruiz (Spain)	An Algebraic Identity Leading to Wilson's Theorem
13 ³⁰ - 14	E.Radescu N.Radescu (Romania)	Some Elementary Algebraic Considerations Inspired by Generalised Smarandache Function
14 - 14 ³⁰	E.Radescu N.Radescu (Romania)	Some Considerations Concerning the Sumatory Function Associated to Generalised Smarandache Function
14 ³⁰ - 15	I.Cojocaru S.Cojocaru (Romania)	On a Function in Number Theory

15 - 15 ³⁰	M.Popescu P.Popescu V.Seleacu (Romania)	On Some Properties of a Numerical Function
15 ³⁰ - 16	C.Dumitrescu C.A.Dumitrescu (Romania)	The Fibonacci Sequence as Smarandache Function

Friday, August 22					
Chairman: H.Ibstedt (Sweden)					
9 - 10	V.W.Spinadel (Argentina)	A New Smarandache Sequence: the Family of Metallic Means			
10 - 11	P.Minut (Romania)	Reserved subject			
11 - 11 ³⁰	P.Minut (Romania)	The Function η as an Arithmetical Function			
11 ³⁰ - 12	F.Luca (USA)	Product of Factorials in Smarandache Type Expresions			
12 - 12 ³⁰	F.Luca (USA)	Perfect Powers in Smarandache Type Expresions			
12 ³⁰ - 13	Y.V.Chebrakov (Rusia)	Analytical Formulae and Algorithms for Constructing Magic Squares from an Arbitrary Set of 16 Numbers			
13 - 13 ³⁰	A.Raigorodski (Rusia)	On Borsuk's Problem			
13 ³⁰ - 14	I.Balacenoiu (Romania)	Remarkable Inequalities			
14 -14 ³⁰	N.Boboc (Romania)	The Pitagoreiques Numbers and the Last Theorem of Fermat			

14³⁰ - 15 S.Tabirca T.Tabirca (England)

Computational Aspects of Smarandache Function

Saturday, August 23					
Chairman: F. Luca (USA)					
9 - 10	N.G.Moschevitin (Rusia)	Best Linear Diophantine Approximations And Uniform Distribution			
10 - 10 ³⁰	V.V.Shmagin (Rusia)	The Analytical Formulae Yielding Some Smarandache Numbers andits Applications in Magic Squares Theory			
10 ³⁰ - 11	S.Yasinskiy (Rusia)	The System - Graphical Analysis of Some Numerical Smarandache Sequences			
11 - 11 ³⁰	H.B.Tilton (USA)	Supercommuting and a Second Distributive Law. Substraction and Division May not Commute, but They Supercommute			
11 ³⁰ -12	S.Y.Yan (England)	Perfect, Amicable and Sociable Numbers - a Computational Approach			
12 - 12 ³⁰	I.Balacenoiu (Romania)	The Semilattice with Consistent Return			
12 ³⁰ - 13	I. Balacenoiu V.Seleacu (Romania)	Properties of the Triplets \widetilde{p}^*			
13 - 13 ³⁰	V.Seleacu C.A.Dumitrescu (Romania)	The Equations $mS(n)=nS(m)$ and $mS(m)=nS(n)$ Have Infinty Many Solutions			
13 ³⁰ - 14	J.Sandor (Romania)	On Certain Inequalities and Limits for the Smarandache Function			
14-14 ³⁰	N.Bratu (Romania)	On the Quaternary Quadratic Diophantine Equations			
14 ³⁰ -15	L.Tutescu	Four Solved Diophantine Equations Involving			

(Romania)

the Smarandache Function

15-15³⁰

S.Porubsky (Czech Republic) On Smarandache's Form Of The Individual

Fermat-Euler Theorem

Remarkable Inequalities

I.Balacenoiu

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In this paper are presented inequalities between factors of canonical decomposition.

For every $n \ge 2$ holds true:

$$2^{e_2^{(n)}} > 3^{e_3^{(n)}}$$
,

where $e_2(n)$, $e_3(n)$ are Legendre's exponents.

For $p \ge 5$ and $n \ge 2$ it is true that

$$2^{e_2^{(n)}} > p^{e_p^{(n)}}$$
, p - prime number.

Let p, q be prime numbers, $n = p \cdot q \cdot x$, with $x \in \mathbb{N}^*$. If $3 \le p < q$ and $\left[\frac{q^2}{p^2}\right] > \left[\frac{q}{p}\right]$, it results $p^{e_p^{(n)}} > p^{e_q^{(n)}}$.

Reference:

- 1. Balacenoiu I., Smarandache Numerical Functions, Smarandache Function Journal Vol. 4-5, 1994.
- 2. Gronas P., A proof of the non-extince of <Samma>.Smarandache Function Journal. Vol. 4-5, 1994

The Semilanttice with Consistent Return

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Let $M=\{S_m(n)|n,m\in N^*\}$, where $S_m(n)$ is defined in [2], let A, $B\in P(N^*)\setminus \phi$ and let a=min A, b, a*= max A, b*=max B.

There are defined the set I of functions:

$$I_{A:}^{B}: N^{*} \to M \text{ with } I_{A}^{B}(n) = \begin{cases} S_{a}(b), n < \max\{a, b\} \\ S_{a_{k}}(b_{k}), \max\{a, b\} \le n \le \max\{a^{*}, b^{*}\} \end{cases}$$

It is easy to see that:

i) if
$$n \in A_1$$
 and $A_1 \subset A_2$, then $I_{A_1}^B(n) = I_{A_2}^B(n)$

ii)
$$I_{N*}^{N*}(n) = S_n(n) = S^n(n)$$

iii)
$$I_{\{m\}}^{N*} = S_m, I_{N*}^{\{m\}} = S^m$$

There is considered the map T:

 $T: I \times I \to I$ denoted as $I_A^B T I_C^D = I_{A \cup C}^{B \cup D}$ and the partial ordering relation ρ :

$$I_A^B \rho I_C^D \Leftrightarrow A \subset C \text{ and } B \subset D.$$

The structure (I, T, ρ) it is a semilattice. It is definit the return of semiattice and it is show that the return of (I, T, ρ) is consistent.

References

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- [2]. I. Balacenoiu, Smarandache Numerical Functions. Smarandache Function Journal, Vol. 4, pag. 6-13, 1994.
- [3]. I. Balacenoiu, V. Seleacu, Some properties of Smarandache Function of the type I. Smarandache Functions Journal Vol. 6, pag. 16-20, 1995.
- [4]. I. Balacenoiu, The Monotony of Smarandache Functions of First Kind. Smarandache Notions Journal, Vol. 7, pag. 39-44, 1996.

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Properties of the Triplets p*

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In this paper we define p^* as the triplets (p^{*-1},P^{*},P^{*+1}) , where p is a prime number and we study their properties. We show that the triplets (p^{*-1},P^{*},P^{*+1}) and (q^{*-1},q^{*},q^{*+1}) are separated, when p < q are two prime numbers.

The triplets (p^*-1,P^*,P^*+1) and (q^*-1,q^*,q^*+1) are F-prime if where is no $n \in \mathbb{N}, n>1$ so that $n/p^* \pm 1$ and n/q^* or $n/p^* \pm 1$.

Example:

Triplets: (5*-1,5*,5*+1) and (7*-1,7*,7*+1) are F-prime, but (7*-1,7*,7*+1) and (11*-1,11*,11*+1) are not F-prime.

$$(5*=2.3.5,7*=2.3.5.7,11*=2.3.5.7.11,p*=2.3.5.7...p)$$
.

For every $k \in \mathbb{N}^*$ where is $h \in \mathbb{N}$, h > k so that for every $s \ge h$ the triplets $(p_k *-1, p_k *, p_k *+1)$ and $(p_s *-1, p_s *, p_s *+1)$ are not F-prime.

If $q=p^*-1$ or $q=p^*+1$, then p^* and q^* are called linked triplets. We show there is no consecutive linked triplets for $5 \le p < q$. In the second part of the paper we study the Spr - function which is defined in [1].

Reference:

- 1. Charles Ashbacher A Note on the Smarandache Near To Primorial Function. Smarandache Nations Journal Vol. 7 no. 1-2-3, August 1996, p.46-49.
- 2. I. Balacenoiu The Factorial Sygnature of Natural Numbers. Numbers Theory Conference, Viena iulie 1996.
- 3. W. Sierpinsski Elementary Theory of Numbers. Warszawa 1964

The Phitagoreiques Numbers and the Last Theorem of Fermat

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This paper propose a short demonstration of the Great Theorem of Fermat that is part of a serie of affirmations formulated by Fermat about three hundreds years ago. This theorem has been formulated in 1673 and it has not been, yet, fully demostrated.

Using a study about the phitagoriques numbers that is part of the demonstration of the Great Theorem of Fermat, I'll demonstrate that the affirmation of Fermat that, using the actual algebraiques therms, says:

"There is no three integers a, b, c for which the equality $a^n + b^n = c^n$ is true, n being a natural number strictly greater than two" is true.

On the Quaternary Quadratic Diophantine Eqations

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In this paper it's trying to present a completion of some results reyarding the solutions of these equations.

It is defined the algebraical operation named "quadratic combinaison" on the complete sistem of solutions of equation: $x^2 + y^2 = z^2$.

1. Through this are recovered the known solutions for the equation:

$$x^2 + y^2 \pm z^2 = w^2 \tag{1}$$

2. One new result consists in presentation of the complet solutions of the equation:

$$x^2 + y^2 + cz^2 = w^2 (2)$$

If c is rational integer: $c = h \cdot l$, we write the solutions with four parameters:

$$w = h(p^{2} + q^{2}) + l(u^{2} + v^{2})$$

$$x = h(p^{2} - q^{2}) + l(u^{2} - v^{2})$$

$$y = 2hpq + 2luv$$

$$z = 2pv - 2uq$$
(2')

3. For the equation

$$x^2 + by^2 + cz^2 = w^2 (3)$$

Where b and c an prime numbers, the theory - Morell and Curmichreelindicates the soulutions with three parameters:

$$w = p^{2} + bq^{2} + cu^{2}$$

$$x = p^{2} - bq^{2} - cu^{2}$$

$$y = 2pq$$

$$z = 2pu$$
(3')

We given solutions with four parameters:

$$w = p^{2} + bq^{2} + cu^{2} + bcv^{2}$$

$$x = p^{2} - bq^{2} - cu^{2} + bcv^{2}$$

$$y = 2pq + 2cuv$$

$$z = 2pu - 2bqv$$
(3")

4. Another result:

Every positive can be expresed as a sum of three squares:

$$n = x^2 + y^2 + z^2$$
, or $n = u^2 + v^2 + 2w^2$ (4)

References:

- 1). L. J. Mordell: "Diophantine Eqations" London-1969.
- 2). I. Z. Borevici, I. K. Sofanevici: "Teoria Cisel" Moscova-1964
- 3). N. F. Bratu: "Note de analiza diofontica" Craiova-1997

Analytical Formulae and Algorithms for Constructing Magic Squares from an Arbitrary Set of 16 Numbers

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In the general case Magic squares represent by themselves numerical or analytical square tables, whose elements satisfy a set of definite basic and additional relations. The basic relations therewith assign some constant property for the elements located in the rows, columns and two main diagonals of a square table, and additional relations, assign additional characteristics for some other sets of its elements. Judging by the given general definition of Magic squares, there is no difficulty in understanding that, in terms of mathematics, the problem on Magic squares consists of the three interrelated problems:

- a) elucidate the possibility of choosing such a set of elements which would satisfy both the basic and all the additional characteristics of the relations;
- b) determine how many Magic squares can be constructed from the chosen set of elements;
- c) elaborate the practical methods for constructing these Magic squares.

It is a traditional way to solve all mentioned problems with taking into account concrete properties of the numerical sequences from which the Magic square numbers are generated. For instance, by using this way problems have been solved on constructing different Magic squares of natural numbers, prime numbers, Smarandache numbers of the 1st kind and so on. Smarandache type question arises: whether a possibility exists to construct the theory of Magic squares without using properties of concrete numerical sequences.

The main goal of this talk is finding an answer on this question with respect to problems of constructing Magic squares 4x4 in size. In particular, in this investigation we:

- a) describe a simple way of obtaining a general algebraic formulae of Magic squares 4x4, required no use of algebraic methods, and explain why in the general case this formula does not simplify the solution of problems on constructing Magic square 4x4;
- b) give a description of a set of invariant transformations of Magic squares 4x4;
- c) adduce a general algorithm, available for constructing Magic squares from an arbitrarily given set of 16 numbers;
- d) discuss the problems of constructing Magic squares from the structured set of 16 elements;

e) solve the problem of decomposing the general algebraic formula of Magic squares 4x4 into a complete set of the four-component formulae.

On A Function in Number Theory

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The following functions in Number Theory are well-known: the function $\mu(n)$ of Möbius, the function $\xi(s)$ of Riemann $\left(\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, s = \sigma + it \in C\right)$, the function $\Lambda(n)$ of Mangoldt $\left(\Lambda(n) = \begin{cases} \log p & \text{if } n = p^m \\ 0 & \text{if } n \neq p^m \end{cases}\right) \text{ etc.}$

The purpose of this paper is to study some series concerning the following function of the Number Theory " $S: N \to N$ such that S(n) is the smallest integer k with the property k! is divisible by n" (the Smarandache function).

We first prove the divergence of some series involving the S function, using an unitary method, and then we prove that the series $\sum_{n=2}^{\infty} \frac{1}{S(2) \cdot S(3) \cdot K \cdot S(n)}$ is convergent to a number $S \in (0,71;0,79)$ and we study some applications of this series in the Numbers Theory.

The series
$$\sum_{n=2}^{\infty} \frac{n^{\alpha}}{S(2) \cdot S(3) \cdot K \cdot S(n)}, \ \alpha \in \mathbb{R}, \alpha \ge 1 \text{ is convergent,}$$

$$S(2) \cdot S(3) \cdot K \cdot S(n) > n^{\alpha} \left(\forall \right) n \ge n_0, \ S(2) + S(3) + K + S(n) > (n-1) \cdot n^{\frac{\alpha}{n-1}} \text{ for each } n \ge n_0 \text{ etc.} \right)$$

Then we prove that series $\sum_{n=2}^{\infty} \frac{1}{S(n)!}$ is convergent to a real number $S \in (0,717;1,253)$ and

the sum of the remarcable series $\sum_{n=2}^{\infty} \frac{S(n)}{n!}$ is a irrational number.

A few Smarandache Integer Sequences

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This paper deals with the analysis of a few Smarandache Integer Sequences which first appeared in <u>Properties of the Numbers, F. Smarandache, University of Craiova Archives, 1975.</u> The first four sequences are recurrence generated sequences while the last three are concatenation sequences.

The Non-Arithmetic Progression: $\{a_i : a_i \text{ is the smallest integer such that } a_i > a_{i-1} \text{ and such that for } k \le i \text{ there are at most t-1 equal differences } a_k - a_{k_1} = a_{k_1} - a_{k_2} = \dots = a_{k_{l-2}} - a_{k_{l-1}} \}$

A strategy for building a t-term non-arithmetic progression is developed and computer implemented for 3≤t≤15 to find the first 100 terms. Results are given in tables and graphs together with some observations on the behaviour of these sequences.

The prime-Product Sequence: $\{t_n: t_n = p_n\#+1, p_n \text{ is the } nth \text{ prime number}\}$, where $p_n\#$ denotes the product of all prime numbers which are less than or equal to p_n .

The number of primes q among the first 200 terms of the prime-product sequence is given by $6 \le q \le 9$. The six confirmed primes are terms numero 1, 2, 3, 4, 5 and 11. The three terms which are either primes or pseudo primes (according to Fermat's little theorem) are terms numero 75, 171 and 172. The latter two are the terms 1019#+1 and 1021#+1.

The Square-Product Sequence: $\{t_n : t_n = (n!)^2 + 1\}$

As in the previous sequence the number of primes in the sequence is of particular interest. Complete prime factorization was carried out for the first 37 terms and the number of prime factors f was recorded. Terms 38 and 39 are composite but were not completely factorized. Complete factorization was obtained for term no 40. The terms of this sequence are in general much more time consuming to factorize than those of the prime-product sequence which accounts for the more limited results. Using the same method as for the prime-product sequence the terms t_n in the interval

40<n≤200 which may possible be primes were identified. There are only two of them, term #65: N=(65!)²+1 which is a 182 digit number and term #76: N=(76!)²+1 which has 223 digits.

The Prime-Digital Sub-Sequence: The prime-digital sub-sequence is the set $\{M=a_0+a_1\cdot 10+a_2\cdot 10^2+\ldots a_k 10^k : M \text{ is a prime and all digits } a_0, a_1, a_2\ldots a_k \text{ are primes}\}$

A proof is given for the theorem: <u>The Smarandache prime-digital sub sequence</u> is infinite, which until now has been a conjecture.

Smarandache Concatenated Sequences: Let $G=\{g_1, g_2, \ldots, g_k, \ldots\}$ be an ordered set of positive integers with a given property G. The corresponding concatenated S.G sequence is defined through $S.G=\{a_i:a_1=g_1,a_k=a_{k-1}\cdot 10^{1+\log_{10}g_k}+g_k,k\geq 1\}$.

The S.Odd Sequence: Fermat's little theorem was used to find all primes/pseudo-primes among the first 200 terms. There are only five cases which all were confirmed to be primes using the elliptic curve prime factorization program, the largest being term 49:

1357911131517192123252729313335373941434547495153555759616365 67697173757779818385878991939597

Term #201 is a 548 digit number.

The S.Even Sequence: The question how many terms are nth powers of a positive integer was investigated. It was found that there is not even a perfect square among the first 200 terms of the sequence. Are there terms in this sequence which are 2·p where p is a prime (or pseudo prime)? Strangely enough not a single term was found to be of the form 2·p.

The S.Prime Sequence: How many are primes? Again we apply the method of finding the number of primes/pseudo primes among the first 200 terms. Terms #2 and #4 are primes, namely 23 and 2357. There are only two other cases which are not proved to be composite numbers: term #128 which is a 355 digit number and term #174 which is a 499 digit number.

On Smarandache's Periodic Sequences

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This paper is based on an article in Mathematical Spectrum, Vol. 29, No 1, written by M.R. Popov. It concerns what happens when an operation applied to a n-digit integer results in a n digit number. Since the number of n-digit integers is finite a repetition must occur after applying the operation a finite number of times. It was assumed in the above article that this would lead to a periodic sequence which is not always true because the process may lead to an invariant. The second problem with the initial article is that, say, 7 is considered as 07 or 007 as the case may be in order make its reverse to be 70 or 700. However, the reverse of 7 is 7. In order not to loose the beauty of these sequences the author has introduced stringent definitions to prevent the sequences from collapse when the reversal process is carried out.

Four different operations on n-digit integers is considered. Starting points for loops (periodic sequences), loop length and the number of loops of each kind has been calculated and displayed in tabular form in all four cases. The occurrence of invariants has also been included.

Products Of Factorials In Smarandache Type Expresions

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In [3] and [5] the authors ask how many primes are of the form $x^y + y^x$, where gcd(x,y) = 1 and $x,y \ge 2$. Moreover, Jose Castillo (see[2]) asks how many primes are of the Smarandache form $x_1^{x_2} + x_2^{x_3} + K + x_n^{x_1}$, where n > 1 and $gcd(x_1, x_2, K, x_n) = 1$ (see [8]).

In this note we announce a lower bound for the size of the largest prime divisor of an expression of the type $ax^y + by^x$, where $ab \ne 0$, $x, y \ge 2$ and gcd(x, y) = 1.

For any finite extension F of Q let $d_F = [F:Q]$. For any algebraic number $\zeta \in F$ let $N_F(\zeta)$ denote the norm of ζ .

For any rational integer n let P(n) be the largest prime number P dividing n with the convention that $P(0) = P(\pm 1) = 1$.

Theorem 1 Let α and β be algebraic integers with $\alpha \cdot \beta \neq 0$. Let $K = \mathbb{Q}[\alpha, \beta]$. For any two positive integers x and y let $X = \max(x, y)$. There exist computable positive numbers C_1 and C_2 depending only on α and β such that:

$$P\left(N_{\mathbf{K}}\left(\alpha x^{y} + \beta y^{x}\right)\right) > C_{1}\left(\frac{X}{\log^{3} X}\right)^{\frac{1}{d_{\mathbf{K}}+1}}$$

whenever $x, y \ge 2$, gcd(x, y) = 1 and $X > C_2$.

The proof of Teorem 1 uses lower bounds for linear forms in logarithms of algebraic numbers (see [1] and [7]) as well as an idea of Stewart (see [9]).

Erdos and Oblath (see [4]) found all the solutions of the eqation $n! = x^p \pm y^p$ with gcd (x, y) = 1 and p > 2. Moreover, the author (see [6]) showed that in every non-degenerate binary recurrence sequence $(u_n)_n \ge 0$ there are only finitely many terms which are products of factorials.

We use Theorem 1 to show that for any two given integers a and b with $ab \ne 0$ there exist only finitely many numbers of the type $ax^y + by^x$, where $x, y \ge 2$ and gcd(x, y) = 1, which are products of factorials.

Let ρ **be the set of all positive integers which can be written as products of factorials; that is

$$\rho^{**} = \{w | w = \prod_{j=1}^{k} m_j!, \text{ for some } m_j \ge 0.$$

Theorem 2 Let $f_1, f_2, ..., f_8 \in \mathbb{Z}[X,Y]$ be $s \ge 1$ homogeneous polynomials of positive degrees. Assume that $f_i(0,Y) \cdot f_i(X,0) \equiv 0$ for i=1, ..., s. Then the equation

$$f_1(x_1^{y_1}, y_1^{x_1}) \dots f_s(x_s^{y_s}, y_s^{x_s}) \in \rho^{**},$$
 (1)

with gcd $(x_i, y_i) = 1$ and $x_i, y_i \ge 2$, for i = 1, ..., s has finitely many solutions $x_1, y_1, ..., x_s, y_s$. Moreover, there exists a computable positive number C depending only on the polynomials $f_1, ..., f_s$ such that all the solutions of equation (1) satisfy max $(x_1, y_1, ..., x_s, y_s) < C$.

We conclude with the following computational results:

Theorem 3. All solutions of the equation $x^y \pm y^x \in \rho^{**}$ with gcd(x, y) = 1 and $x, y \ge 2$, satisfy max $(x, y) < 54^{54}$.

Theorem 4. All soulutions of the equation

$$x^{y} + y^{z} + z^{x} = n!$$
 with gcd $(x,y,z) = 1$ and $x,y,z \ge 2$, satisfy max $(x,y,z) < 48^{230}$.

Bibliography

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Perfect Powers In Smarandache Type Expresions

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In [2] and [3] the authors ask how many primes are of the Smarandache form (see [7]) $x^y + y^x$, where gcd (x, y) = 1 and $x, y \ge 2$. In [5] the author announced that there are only finiely many numbers of the above form which are products of factorials.

In this note we propose the following

CONJECTURE 1. Let a, b and c be three integers with $ab \neq 0$. Then the equation

 $ax^{y} + by^{x} = cz^{n}$ with x, y, $n \ge 2$, and gcd (x, y) = 1 (1) has finitely many solutions (x, y, n).

We announce the following result:

Theorem 1. The "abc Conjecture" implies Conjecture1.

The proof of Theorem 1 is based on an idea of Lang (see [4]). We first show that, under the "abc Conjecture", the equation (1) has finitely many solutions (x, y, z, n) with y>2 and n>2. For the remaining case we use lower bounds for linear forms in logarithms of algebric numbers (see [1] and [6]), and a Liouville type argument to conclude that equation (1) has finitely many solutions (x, 2, z, 2).

We also announce the following results:

Theorem 2. The equation $x^y + y^x = z^2$ with $x, y \ge 2$, and gcd (x, y) = 1 (2) has finitely many solutions (x, y, z) with 2|xy. Moreover, all such solutions satisfy max $(x, y) < 12 \cdot 10^{169}$.

The proof of Theorem 2 uses lower bounds for linear forms in logarithms of algebraic numbers. A similar result (with probably a slightly different bound) holds for the equation $x^y - y^x = z^2$ with x, $y \ge 2$, gcd(x, y) = 1 and 2|xy.

Theorem 3. The equation $2^y + y^2 = z^n$ (3) has no solutions (y, z, n) such that y>1 is odd and n>1.

The proof of Theorem 3 is elementary and uses the fact that $\mathbb{Z}[i\sqrt{2}]$ is an UFD.

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The Function n as an Arithmetical Function

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The function η is an inversable function and consequently it can be studied like a function of the multiplicative group (in the Dirichlet sense for the inversable functions).

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Best Linear Diophantine Approximations And Uniform Distribution

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A best simultaneous approximation (b.a.) to $\alpha = (\alpha_1, K, \alpha_s)$ is defined as integer point $\zeta = (p, a_1, K, a_s) \in \mathbf{Z}^{s+1}$ for which:

$$D(\zeta) = \max_{j=1,K,s} \left| p\alpha_j - a_j \right| < \max_{j=1,K,s} \left| q\alpha_j - b_j \right|$$

$$\forall q: 1 \leq q \leq p; \ \forall (b_1, K, b_s) \in \mathbb{Z}^s \setminus \{(a_1, K, a_s)\}$$

Let $M_{\nu}[\alpha]$ be the matrix of ν -th consequative b.a. to α .

Theorem 1: Let $s \ge 3$. The one can find a continuum set of vectors (α_1, K, α_s) (each of them consists from numbers linearly independent over \mathbb{Z} together with 1, so $\dim_{\mathbb{Z}}(\alpha_1, K, \alpha_s) = s+1$) such that for each of them:

rk
$$M+\nu[\alpha] \leq 3 \ \forall \nu \in N$$
.

(And so for all ν we have $\det M_{\nu}[\alpha] = 0$.)

Theorem 1 represents a contrexample to J.Lagarias conjecture. Similar results are obtained for the best approximations in the sense of *linear forms*.

Theorem 2: For any $s \ge 3$ one can find a set of vectors (α_1, K, α_s) (of cardinality continuum) such that each of them form a s-uple linear independent over \mathbb{Z} together with 1 and for each of them there exist a linear subspace $\mathcal{L}_{\alpha} \subset \mathbb{R}^{s+1}$, $\dim \mathcal{L}_{\alpha} \subset \mathbb{R}^{s+1}$, $\dim \mathcal{L}_{\alpha} = 3$ satisfying the condition $m_v \in \mathcal{L}_{\alpha}$, $\forall v > v_0$ for best approximations $m_v \in \mathbb{Z}^{s+1}$ in the sense of linear form $\|m_1\alpha_1 + K + m_s\alpha_s\|$.

The results on best approximations can be applied to study uniform distribution of the sequence $\{\alpha_j t\}$ (mod 1). We improve the famous Weyl's theorem on mean values.

Theorem 3: Let $f: T^s \to R$ be a smooth function with zero mean value: $\int_{T^s} f \, dx = 0$. Let $\{\alpha_1, K, \alpha_s\}$ be linearly independent over Z. Then the integral $\int_0^t f(t, \alpha_1 t, K, \alpha_s t) \, dt$ oscillates.

We also suppose to introduce some results concerning multidimensional Lagrange spectra and elementary results in continued fractions theory.

On Some Properties of a Numerical Function

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The aim of the paper is to define the numerical function $S_{\min}^{-1}:N$ by $S_{\min}^{-1}(n)=\{a\in N\big|S(a)=n\}$ and to study some of its properties.

In order to give a formula for S_{\min}^{-1} we study the following particular cases: $n = p \cdot q$ and $n = 2p \cdot q, n = p^2$ and then $n = q^p$, where p and q are prime number such that p < q.

Firstly, we try to prove that $S_{\min}^{-1}(n) = p_k^{p_k - \alpha_r + 1}$, where $n \in \mathbb{N} \setminus \{1\}$ has the following decomposition in primes $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \dots \cdot p_r^{\alpha_r}$, $p_1 < p_2 < \dots < p_r$, p_k is a prime which belongs to the set $\{p_1, p_2, \dots, p_r\}$ and $e_{p_k} = \left[\frac{n}{p_r}\right] + \left[\frac{n}{p_k^2}\right] + \dots + \left[\frac{n}{p_k}\right]$ is the Lagrange exponent of p_k .

We try to show that p_k is ever the greatest prime of the above set, thus $p_k = p_r$.

Finally, we study a kind of momotony of the function S_{\min}^{-1} , some diofantine equations and we give a link with the MANGOLDT function.

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On Smarandache's Form Of The Individual Fermat-Euler Theorem

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In the paper it is shown how a form of the classical Fermat-Euler Theorem discovered by F. Smarandache fits into the generalizations found by S. Schwarz, M. Lassak and the author. Then we show how Smarandache's algorithm can be used to effective computations of the so called group membership.

Some Elementary Algebraic Considerations Inspired by Generalised Smarandache Function

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The scope of this work is to construct in an algebric way, in some situations, prolongations (surjective morhismes for an adequately choosed universal algebras) for a numerical functions of Smarandache type which are studied in the past and with known properties (see [1]).

Moreover, we obtain some generalisations for the sumatory function of a simple Smarandache function and, also, for the sumatory function of some functions of Smarandache type with a few observations about them.

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Some Consideration Concerning the Sumatory Function Associated to Generalised Smarandache Function

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Consider the lattices $N_0 = (N^*, \wedge, \vee)$, and $N_d = (N^*, \wedge, \vee)$ and $\sigma_{ij}: N_i \to N_j, i, j \in \{0, d\}$, a sequence of positive integeres defined on the set N^* . For a (dd) - sequence σ_{dd} , for example, the condition of convergence to zero is: $(c_{dd}): (\forall) n \in N^*, \exists m_n \in N^*, (\forall) m \geq m_n \Rightarrow \sigma_{dd}(m) \geq n$.

To each sequence σ_{ij} , $i, j \in \{0, d\}$, satisfying the condition (c_{ij}) , one may attach a sequence S_{ij} (a generalised Smarandache function) defined by $S_{ij}(n) = \min\{m_n | m_n \text{ is given by the condition } (c_{ij})\}$.

This paper is a survay of the part dedicated to the sumatory function $F_n = \sum_{d/n} f(d)$, where f is the generalised Smarandache function.

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On Borsuk's Problem

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Borsuk [1] conjectured that every set in \mathbb{R}^d can be partitioned into d+1 subsets of smaller diameter. Let f(d) be the minimal number so that every set in \mathbb{R}^d can be partitioned into f(d) subsets of smaller diameter. Borsuk's conjecture was proved in dimensions 2 and 3 and in arbitrary dimension for all smooth convex bodies. Using the results of Frankl and Wilson [2], Kahn and Kalai [3] showed that $f(d) \ge (1.2)^{d^{\frac{1}{2}}}$. Thereby a counterexample to the Borsuk's conjecture was obtained in the dimension 1325. It was proved in [4] that Borsuk's conjecture was false for d=946. In our paper we prove that for d=561 there exists the set in \mathbb{R}^d that cannot be partitioned into d+1 subsets of smaller diameter.

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An algebraic identity leading to Wilson's Theorem

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In most textbooks on number theory Wilson's Theorem is proved by applying Lagrange's Theorem concerning polynomial congruences. Hardy and Wright also give a proof using quadratic residues. In his note Wilson's Theorem is derived as a corollary to an algebraic identity.

On Certain Inequalities and Limits for the Smarandache Function

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The aim of this paper is to prove certain new inequalities and relations for the Smarandache function. These relations are used among others to prove limits (or inferior and superior limits) for functions connected or related to the Smarandache function. New proofs of older results are also reobtained by an unitary way.

The Analytical Formulae Yielding Some Smarandache Numbers andits Applications in Magic Squares Theory

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A joint work with Yuri Chebrakov (Sankt-Petersburg) describes a set of analytical formulae of some six Smarandache sequences and its applications for constructing Magic squares 3x3 in size from Smarandache numbers. In particular, in this talk we adduce

- a) the general recurrent expression for the terms of 6 Smarandache sequences;
- b) the analytical formula for the calculation of n-th number in the discussed Smarandache sequences;
- c) a set of analytical formulae for constructing Magic squares 3x3 in size from k-truncated Smarandache numbers;
- d) a few of concrete examples of Magic squares 3x3 in size from k-truncated Smarandache numbers.

The Equations mS(n)=nS(m) And mS(m)=nS(n) Have Infinity Many Solutions

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In this paper we prove that assertion on the title, namely the mentioned equations have infinity many solutions in the following cases:

for the first equation if:

1. m = n.

2. m > n, m = da, n = db, where d is the greatest common divisor of m and n, satisfying $m \wedge n = d$, $d \wedge a = 1$, $d \wedge b > 1$

and for the second equation if:

- 1. m = n,
- 2. m > n and $m \wedge n = 1$.

A NEW SMARANDACHE SEQUENCE: THE FAMILY OF METALLIC MEANS

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The family of Metallic Means comprises every quadratic irrational that is the positive solution of algebraic equations of the type $x^2 - nx - 1 = 0$, $x^2 - x - n = 0$ where n is a natural number. The most prominent member of this family is the Golden Mean, then it comes the Silver Mean, the Bronze Mean, the Niquel Mean, the Copper Mean, etc. They are closely related to quasiperiodic dynamics, being therefore valuable keys in the onset to chaos, but they also constitute the basis of musical and architectural proportions. Through the analysis of their common mathematical properties, it becomes evident that they interconnect different human fields of knowledge, in the sense defined by F.Smarandache ("Paradoxist Mathematics").

Any suggestion will be welcome!

Supercommuting And A Second Distributive Law. Substraction And Division May Not Commute, But They Supercommute

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This paper deals with teaching methods. Elementary textbooks tell that addition and multiplication commute but substraction and division do not. Actually they do if a simple restriction is observed. The technique is not new; but the method presented here for teaching it is believed to be new and simple enough for presentation immediately following the signed-numbers concept. The technique is dubbed "SuperCommuting" or the "shuffling" property. SuperCommuting leads directly to a new formal algebraic distributive law: one that applies to expressions of the form 1/(a*b/c/d*e...). Also, by comparison with the first distributive law, the duality concept can be surreptitiously unveiled by the dedicated instructor of beginning algebra.

Four Solved Diophantine Equations Involving the Smarandache Function

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Let S(n) be the Smarandache Function defined as the smallest integer such that S(n)! is divisible by n. The solutions of the Following equations are given

$$(1)S(x)^{x} + x^{2} = x^{S(x)} + S^{2}(x)$$

$$(2)S(y)^{x} + x^{2} = x^{S(y)} + S^{2}(y)$$

$$(3)x^{S(y)} = S(y)^{x}$$

$$(4)x^{S(x)} + S(x) = S(x)^{x} + x$$

Construction Of Elements Of The Smarandache Square-Partial-Digital Subsequence

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The Smarandeche Square-Partial-Digital Subsequence (SPDS) is the sequence of square integers which admit a partition for which each segment is a square integer. An exemple is $506^2 = 256036$ which has partition 256|0|36. Ashbacher considers these numbers on page 44 of [1] and quickly shows taht the SPDS is infinite by exhibiting two infinite "families" of elements. We will extend his results by showing how to construct infinite families of elements of SPDS containing desired patterns of digits.

<u>Theorem 1:</u> Let c be any concatenation of square numbers. There are infinitely many elements of SPDS which contain the sequence c.

<u>Proof:</u> If c forms an even integer, let N=c. Otherwise, let N be c with a digit 4 added at the right. So N is an even number. Find any factorization N=2ab. Consider the number $m=a\cdot 10^n+b$ for sufficiently large n. (Sufficiently large means that $10^n>b^2$ and $10^n>N$.) Then $m^2=a^2\cdot 10^{2n}+N\cdot 10^n+b^2\in SPDS$. q.e.d.

For example, let us construct elements of SPDS containing the string c=2514936. In the notation of our proof, we have ab=1257468 and we can use a=6 and b=209578 (among many posibilities). This gives us the numbers:

600000209578²=360000251493643922938084 6000000209578²=36000002514936043922938084 60000000209578²=3600000025149360043922938084

etc.

which all belong to SPDS.

This allows us to imbed any sequence of squares in the interior of an element of SPDS. What about the ends? Clearly we cannot put all such sequences at the end of an element of SPDS. No perfect square ends in the digits 99, for example. Our best result in this respect is the following:

<u>Theorem 2:</u> Let a be any positive integer. There are infinitely many elements of SPDS which begin and end with a^2 .

Proof: For large enough n (ie $10^n > 225 \cdot a^2$), consider

$$m = a \cdot 10^{2n} + \frac{a}{2} \cdot 10^n + a = a \cdot 10^{2n} + 5a \cdot 10^{n-1} + a$$

Then

$$m^{2} = a^{2} \cdot 10^{4n} + a^{2} \cdot 10^{3n} + \frac{9}{4}a^{2} \cdot 10^{2n} + a^{2} \cdot 10^{n} + a^{2} =$$

$$a^{2} \cdot 10^{4n} + a^{2} \cdot 10^{3n} + (15a)^{2} \cdot 10^{2n-2} + a^{2} \cdot 10^{n} + a^{2}$$

belongs to SPDS. q.e.d.

We illustrate for a=8. For succesive values of n beginning with 5, we have the following elements of SPDS:

$$80000400008^2 = 6400064001440006400064$$

$$800000400008^2 = 64000064000144000064000064$$

$$80000004000008^2 = 64000006400001440000064000064$$

We have a number of observations concerning this last result. First, an obvious debt is owed to Ashbacher's work [1], in which he gives the family $212^2 = 44944$, $20102^2 = 404090404$, ... Second, we actually have exhibited an infinite family of elements of SPDS in which a^2 appears four times. And finally, we note that an alternate proof can be given using $m = a \cdot 10^{2n+1} + \frac{a}{2} \cdot 10^n + a$ for which $m^2 = a^2 \cdot 10^{4n+2} + a^2 \cdot 10^{3n+1} + (45a)^2 \cdot 10^{2n-2} + a^2 \cdot 10^n + a^2$.

This concludes our results emphasizing the infinitude of SPDS. In addition we wish to note an instance of the square of an element of SPDS which also belong to SPDS, namely $441^2 = 194481$. Can an exemple be found of integers m, m^2 , m^4 all belonging to SPDS? It is relatively easy to find two consecutive squares in SPDS. One exemple is $12^2 = 144$ and $13^2 = 169$. Does SPDS also contain a sequence of three or more consecutive squares?

Reference:

[1] Charles Ashbacher, Collection of Problems On Smarandache Notions. Erhus University press, Vail, 1996

Perfect, Amicable And Sociable Numbers. A Comutational Approach.

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This book is about perfect, amicable and sociable numbers, with an emphasis on amicable numbers, from both a mathematical and particalarly a computational point of view. Perfect and amicable numbers have been studied since antiquity, nevertheless, many problems still remain. The book introduces the basic concepts oand results of perfect, amicable and sociable numbers and reviews the long history of the search for these numbers. It examines various methods, both numerical and algebraic, of generating these numbers, and also includes a set of important and interesting open problems in the area. The book is self-contained, and accessible to researchers, students, and even amateurs in mathematics and computing science. The only prerequisites are some familiarity with high-school algebra and basing computing techniques.

Contents:

Concepts and history
Mathematical tools
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Algebraic Assumption Methods
Algebraic Constructive Methods
Conclusions and open problems

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The System - Graphical Analysis of Some Numerical Smarandache Sequences

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A joint work with Vladimir Shmagin and Yuri Chebrakov (Sankt-Petersburg) describes the results of system - graphical analysis of some numerical Smarandache sequences. In particular, in this talk we demonstrate that system - graphical analysis results of some numerical Smarandache sequences may possess of the big aesthetic, cognitive and applied significance.

Lista cu participanții străini la prima conferință internațională

Noțiuni de tip Smarandache în teoria numerelor

- 1. Y. V. Chebrakov (Department of Mathematics, St.-Petersburg Technical University, Russia, e-mail: chebra@phdeg.hop.stu.neva.ru)
- "Algebraic Formulae Of Irregular Magic Squares 4*4 In Size"
- 2. M. Ibstedt (Glimminge 2036, 28060 Broby, Suedia, e-mail: hibstedt@swipnet.se)
- "On Smarandache's Periodic Sequences" conferință
- "A Few Smarandache Integer Sequences"
- 3. F. Luca (Department of Mathematics, Syracuse University, Syracuse, NY 13244-1150, USA, e-mail: florian@ichthus.syr.edu)
- "Products Of Factorials In Smarandache Type Expresions"
- "Perfect Powers In Smarandache Type Expresions"
- 4. N.G. Moschevitin (Rusia, e-mail: MOSH@nw.math.msu.su)
- "Best Linear Diophantine Approximations" conferință
- 5. J.C.Peral (Spania, e-mail: mtppealj@lgdx02.lg.ehu.es) subject rezervat
- 6. A. Raigorodski (Rusia, e-mail: lenok@most.ru)

[&]quot;On Borsuk's Problem"

- 7. S.M. Ruiz (Avda. De Regla, 43, Chipiona 11550 (Cadiz) Spain)
- "An Algebraic Identity Leading To Wilson's Theorem"
- 8. V.V.Schmagin (Department of Mathematics, St.-Petersburg Technical University, Rusia, e-mail: vlad@phdeg.hop.stu.neva.ru)
- "Analitical Methods For Constructing Double And Multiplicative Magic Squares"
- 9. V.W.Spinadel (Argentina, e-mail: postmast@caos.uba.ar)
- "A New Smarandache Sequence: The Family Of Metallic Means"
- 10. S.Tabirca, T.Tabirca (Technology Department, Computer Science Division, Bucks College of High Education, 6 Queen Alexandra Road HP11 2JU, High Wycombe, Bucks, U.K., fax.: 01494-605051, tel.: 01494-605050, e-mail: mtabir01@buckscol.ac.uk)
- "Computational Aspects of Smarandache Function"
- 11. Homer B. Tilton (Department of Mathematics, Physics and Astronomy, Pima Community College East 8181 East Irvington Road, Tucson, Arizona 85709-4000 U.S.A. or 8401 Desert Steppes Drive Tucson, Arizona 85710, U.S.A)
- "Supercommuting And A Second Distributive Law. Substraction And Division May Not Commute, But They Supercommute"
- 12. S.Y.Yan (York University, England, e-mail: syy@minster.cs.york.ac.uk)
- "Perfect, Amicable And Sociable Numbers A Computational Approach"
- 13. Serghey Yasinskiy (Department of Mathematics, Technical University 195251 Sankt-Petersburg, Rusia, e-mail: chebra@phdeg.hop.stu.neva.ru)
- "Constructing The Logic-Mathematical Models Of Cognitive Activity By Using Smarandache's Sequences"
- 14. Lamarr Widmer (Messiah College, Grantham, PA 17027, USA, e-mail: widmer@mcis.messiah.edu)

[&]quot;Construction Of Elements Of The Smarandache Square-Partial-Digital Subsequence"