Abstract—R-unions and R-intersections of T-external (I-
external, F-external) neutrosophic cubic sets are considered.
Examples to show that the R-intersection and R-union of T-
external (I-external, F-external) neutrosophic cubic sets may not
be a T-external (I-external, F-external) neutrosophic cubic set are
provided. Conditions for the R-union and R-intersection of T-
external (I-external, F-external) neutrosophic cubic sets to be a T-
external (I-external, F-external) neutrosophic cubic set are
discussed.

Index Terms—Truth-internal (indeterminacy-internal, falsity-
internal) neutrosophic cubic set, truth-external (indeterminacy-
external, falsity-external) neutrosophic cubic set, R-union, R-
intersection.

I. INTRODUCTION

MARANDACHE ([5], [6]) developed the concept of neu-
rosophic set as a more general platform which extends
the concepts of the classic set and fuzzy set, intuitionistic fuzzy
set and interval valued intuitionistic fuzzy set. We know
that neutrosophic set theory is applied to various part (refer
to the site http://fs.gallup.unm.edu/neutrosophy.htm). Ali and
Smarandache [1] introduced complex neutrosophic sets to
handle imprecise, indeterminate, inconsistent, and incomplete
information of periodic nature. Deli et al. [2] introduced the
concept of bipolar neutrosophic set and its some operations.

In [3], Jun et al. introduced the notion of (internal, external)
cubic sets, and investigated several properties. Jun et al. [4]
extended the concept of cubic sets to the neutrosophic sets, and
investigated/investigated the notions/properties of T-internal (I-
internal, F-external) neutrosophic cubic sets and T-external (I-
external, F-external) neutrosophic cubic sets. As a continuation
of the paper [4], we consider R-unions and R-intersections of
T-external (I-external, F-external) neutrosophic cubic sets. We
provide examples to show that the R-intersection and the R-
union of T-external (resp. I-external and F-external) neutrosophic
cubic sets may not be a T-external (resp. I-external and F-
external) neutrosophic cubic set. We discuss conditions for the R-
union of T-external (resp. I-external and F-external) neutrosophic
cubic sets to be a T-external (resp. I-external and F-external)
neutrosophic cubic set. We consider a condition for the R-
intersection of T-external (resp. I-external and F-external)
eutrosophic cubic sets to be a T-external (resp. I-external and F-
external) neutrosophic cubic set.

II. R-INTERSECTIONS AND R-UNIONS OF
NEUTROSOPHIC CUBIC SETS

Jun et al. [4] considered the notion of neutrosophic cubic
sets as an extension of cubic sets. Let X be a non-empty set. A
neutrosophic cubic set (NCS) in X is a pair A = (A, Λ) where
A := \{ (x; A_T(x), A_I(x), A_F(x)) \mid x \in X \}
is an interval neutrosophic set in X and
Λ := \{ (x; λ_T(x), λ_I(x), λ_F(x)) \mid x \in X \}
is a single-valued neutrosophic set in X.

For further particulars on the notions of T (resp., I, F)-
ternal neutrosophic cubic sets, T (resp., I, F)-external neu-
rosophic cubic sets, R-union and R-intersection of neutrosophic
cubic sets, we refer the reader to the the paper [4].

We know that R-intersection and R-union of T-external
(resp., I-external and F-external) neutrosophic cubic sets may not
be a T-external (resp., I-external and F-external) neutrosophic
cubic sets as seen in the following example.

Example 2.1: Let A = (A, Λ) and B = (B, Ψ) be neutrosophic cubic sets in X = [0, 1] where
A = \{ (x; 0.5, 0.7, [0.2, 0.4], [0.3, 0.5]) \mid x \in [0, 1] \},
Λ = \{ (x; 0.6, 0.7, 0.8) \mid x \in [0, 1] \},
B = \{ (x; 0.6, 0.7, [0.6, 0.8], [0.7, 0.9]) \mid x \in [0, 1] \},
Ψ = \{ (x; 0.5, 0.9, 0.8) \mid x \in [0, 1] \}.

Then A = (A, Λ) and B = (B, Ψ) are I-external neutro-
sophic cubic sets in X = [0, 1]. The R-union A ∪R B = (A ∪ B, Λ ∪ Ψ) of A = (A, Λ) and B = (B, Ψ) is given as follows:
A ∪ B = \{ (x; 0.6, 0.7), [0.6, 0.8], [0.7, 0.9]) \mid x \in [0, 1] \},
Λ ∪ Ψ = \{ (x; 0.5, 0.7, 0.8) \mid x \in [0, 1] \},
and it is not an I-external neutrosophic cubic set in X = [0, 1].

We provide a condition for the R-union of two T-external
(resp., I-external, F-external) neutrosophic cubic sets to be T-
external (resp., I-external, F-external).

Theorem 2.2: Let A = (A, Λ) and B = (B, Ψ) be I-external
neutrosophic cubic sets in X such that
\max \{ \min \{ A_T^I(x), B_T^I(x) \}, \min \{ A_T^I(x), B_T^I(x) \} \}
\leq (\Lambda \land \Psi)(x)
< \min \{ \max \{ A_T^I(x), B_T^I(x) \}, \max \{ A_T^I(x), B_T^I(x) \} \}
for all x ∈ X. Then the R-union A ∪R B = (A ∪ B, Λ ∪ Ψ) is an I-
external neutrosophic cubic set in X.

Proof. For any x ∈ X, let
a_x := \max \{ \min \{ A_T^I(x), B_T^I(x) \}, \min \{ A_T^I(x), B_T^I(x) \} \}
and
b_x := \min \{ \max \{ A_T^I(x), B_T^I(x) \}, \max \{ A_T^I(x), B_T^I(x) \} \}.
Then b_x = A_T^I(x), b_x = B_T^I(x), or
It is possible to consider the cases \( b_x = B^+_I(x) \) and \( b_x = B^-_I(x) \) only because the remaining cases are similar to these cases. If \( b_x = B^-_I(x) \), then \( A^+_I(x) \leq B^-_I(x) \) and so
\[
A^-_I(x) \leq A^+_I(x) \leq B^-_I(x) \leq B^+_I(x).
\]
Thus \( a_x = A^+_I(x) \), and so
\[
(A_I \cup B_I)^- = B^-_I(x) = b_x > (\lambda_I \wedge \psi_I)(x).
\]
Hence
\[
(\lambda_I \wedge \psi_I)(x) \notin \left( (A_I \cup B_I)^-, (A_I \cup B_I)^+ \right).
\]
If \( b_x = B^+_I(x) \), then \( A^-_I(x) \leq B^+_I(x) \leq A^+_I(x) \) and thus \( a_x = \max\{A^-_I(x), B^+_I(x)\} \). Suppose that \( a_x = A^+_I(x) \). Then
\[
B_I(x) \leq A^-_I(x) = a_x \leq (\lambda_I \wedge \psi_I)(x)
\]
\[
< b_x = B^+_I(x) \leq A^+_I(x).
\]
It follows that
\[
B^-_I(x) \leq A^-_I(x) < (\lambda_I \wedge \psi_I)(x) < B^+_I(x) \leq A^+_I(x)
\]
or
\[
B^-_I(x) = (\lambda_I \wedge \psi_I)(x) < B^+_I(x) \leq A^+_I(x).
\]
The case (1) induces a contradiction. The case (2) implies that
\[
(\lambda_I \wedge \psi_I)(x) \notin \left( (A_I \cup B_I)^-, (A_I \cup B_I)^+ \right)
\]
since \( (\lambda_I \wedge \psi_I)(x) = A^-_I(x) = (A_I \cup B_I)^- \). Now, if \( a_x = B^+_I(x) \), then
\[
A^-_I(x) \leq B^+_I(x) = a_x \leq (\lambda_I \wedge \psi_I)(x)
\]
\[
< b_x = B^+_I(x) \leq A^+_I(x).
\]
Hence we have
\[
A^-_I(x) \leq B^-_I(x) < (\lambda_I \wedge \psi_I)(x) < B^+_I(x) \leq A^+_I(x)
\]
or
\[
A^-_I(x) \leq B^-_I(x) = (\lambda_I \wedge \psi_I)(x) < B^+_I(x) \leq A^+_I(x).
\]
The case (3) induces a contradiction. The case (4) implies that
\[
(\lambda_I \wedge \psi_I)(x) \notin \left( (A_I \cup B_I)^-, (A_I \cup B_I)^+ \right)
\]
Therefore the R-union \( A \cup_R B = (A \cup B) \wedge \Lambda \wedge \psi \) is an I-

**Corollary 2.5:** Let \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) be external neutrosophic cubic sets in \( X \). Then the R-union of \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) is an external neutrosophic cubic set in \( X \) when the conditions in Theorems 2.2, 2.3 and 2.4 are valid.

The following examples show that the R-intersection of two T-external (resp., I-external, F-external) neutrosophic cubic sets may not be T-external (resp., I-external, F-external).

**Example 2.6:** Let \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) be neutrosophic cubic sets in \( X = [0, 1] \) where
\[
A = \{ x: [0, 0.2, 0.4], [0.5, 0.7], [0.3, 0.5] \} \quad \text{for} \quad x \in [0, 1],
\]
\[
\Lambda = \{ x: 0.1, 0.4, 0.8 \} \quad \text{for} \quad x \in [0, 1],
\]
\[
B = \{ x: [0.6, 0.8], [0.4, 0.7], [0.7, 0.9] \} \quad \text{for} \quad x \in [0, 1],
\]
\[
\Psi = \{ x: 0.3, 0.3, 0.8 \} \quad \text{for} \quad x \in [0, 1],
\]
Then \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) are T-external neutrosophic cubic sets in \( X = [0, 1] \). The R-intersection \( A \cap_R B = (A \cap B, \Lambda \wedge \Psi) \) of \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) is given as follows:
\[
A \cap B = \{ x: [0, 2, 0.4], [0.4, 0.7], [0.3, 0.5] \} \quad \text{for} \quad x \in [0, 1],
\]
\[
\Lambda \wedge \Psi = \{ x: 0.3, 0.4, 0.8 \} \quad \text{for} \quad x \in [0, 1],
\]
and it is not a T-external neutrosophic cubic set in \( X = [0, 1] \).

We provide a condition for the R-intersection of two T-external (resp., I-external, F-external) neutrosophic cubic sets to be T-external (resp., I-external, F-external).

**Theorem 2.7:** Let \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) be T-

external neutrosophic cubic sets in \( X \) such that
\[
\max \left\{ \min\{A^+_I(x), B^-_I(x)\}, \min\{A^-_I(x), B^+_I(x)\} \right\}
\]
\[
\leq \left( \lambda_I \wedge \psi_I \right)(x)
\]
\[
\leq \min \left\{ \max\{A^+_I(x), B^-_I(x)\}, \max\{A^-_I(x), B^+_I(x)\} \right\}
\]
for all \( x \in X \). Then the R-intersection \( A \cap_R B = (A \cap B, \Lambda \wedge \Psi) \) is a T-external neutrosophic cubic set in \( X \).

**Proof:** For any \( x \in X \), let
\[
c_x := \max \left\{ \min\{A^+_I(x), B^-_I(x)\}, \min\{A^-_I(x), B^+_I(x)\} \right\}
\]
and
\[
d_x := \min \left\{ \max\{A^+_I(x), B^-_I(x)\}, \max\{A^-_I(x), B^+_I(x)\} \right\}.
\]
Then \( d_x = A^-_I(x) \), \( d_x = B^-_I(x) \), \( d_x = A^+_I(x) \), or \( d_x = B^+_I(x) \). It is possible to consider the cases \( d_x = A^-_I(x) \) and \( d_x = A^+_I(x) \) only because the remaining cases are similar to these cases. If \( d_x = A^-_I(x) \), then
\[
B^+_I(x) \leq A^-_I(x) \leq A^+_I(x) \leq A^+_I(x).
\]
Thus \( c_x = B^+_I(x) \), and so
\[
B^+_I(x) = (A \cap B)^- \leq (A \cap B)^+ \quad \text{and} \quad B^+_I(x) = c_x < (\lambda_I \vee \psi_I)(x)
\]
Hence \( (\lambda_I \vee \psi_I)(x) \notin \left( (A \cap B)^-, (A \cap B)^+ \right) \).

If \( d_x = A^+_I(x) \), then \( B^+_I(x) \leq A^+_I(x) \leq B^+_I(x) \) and thus \( c_x = \max\{A^+_I(x), B^-_I(x)\} \). Suppose that \( c_x = A^+_I(x) \). Then
\[
B^-_I(x) \leq A^-_I(x) = c_x < (\lambda_I \vee \psi_I)(x)
\]
\[
< d_x = A^+_I(x) \leq A^+_I(x) \leq B^+_I(x).
\]
It follows that
\[
B^+_I(x) \leq A^+_I(x) < (\lambda_I \vee \psi_I)(x) \leq B^+_I(x)
\]
(5)
The case (7) induces a contradiction. The case (8) induces 

\[ A_T^+(x) \leq B_T^-(x) < (\lambda T \lor \psi T)(x) = A_T^+(x) \leq B_T^+ (x). \]  

(6)

Hence we have

\[ A_T^-(x) \leq B_T^-(x) < (\lambda T \lor \psi T)(x) < A_T^+(x) \leq B_T^+ (x). \]  

(7)

or

\[ A_T^-(x) \leq B_T^-(x) < (\lambda T \lor \psi T)(x) = A_T^+(x) \leq B_T^+ (x). \]  

(8)

The case (7) induces a contradiction. The case (8) induces 

\[ (\lambda T \lor \psi T)(x) \notin ((A_T \cap B_T)^-(x), (A_T \cap B_T)^+(x)). \]

Therefore the R-intersection \( A \cap_R B = (A \cap B, \Lambda \lor \Psi) \) is a T-external neutrosophic cubic set in \( X \).

Similarly, we have the following theorems.

**Theorem 2.8:** Let \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) be I-external neutrosophic cubic sets in \( X \) such that

\[ \max \{ \min\{A_T^+(x), B_T^+(x)\}, \min\{A_T^-(x), B_T^-(x)\}\} \leq (\lambda \lor \psi)(x) \]

\[ \leq \min \{ \max\{A_T^+(x), B_T^+(x)\}, \max\{A_T^-(x), B_T^-(x)\}\} \]

for all \( x \in X \). Then the R-intersection \( A \cap_R B = (A \cap B, \Lambda \lor \Psi) \) is an I-external neutrosophic cubic set in \( X \).

**Theorem 2.9:** Let \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) be F-external neutrosophic cubic sets in \( X \) such that

\[ \max \{ \min\{A_F^+(x), B_F^+(x)\}, \min\{A_F^-(x), B_F^-(x)\}\} \leq (\lambda F \lor \psi F)(x) \]

\[ \leq \min \{ \max\{A_F^+(x), B_F^+(x)\}, \max\{A_F^-(x), B_F^-(x)\}\} \]

for all \( x \in X \). Then the R-intersection \( A \cap_R B = (A \cap B, \Lambda \lor \Psi) \) is an F-external neutrosophic cubic set in \( X \).

**Corollary 2.10:** Let \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) be external neutrosophic cubic sets in \( X \). Then the R-intersection \( A = (A, \Lambda) \) and \( B = (B, \Psi) \) is an external neutrosophic cubic set in \( X \) when the conditions in Theorems 2.7, 2.8 and 2.9 are valid.

### III. Conclusion

We have considered the R-union and R-intersection of T-external (I-external, F-external) neutrosophic cubic sets. We have provided examples to show that the R-intersection and R-union of T-external (resp. I-external and F-external) neutrosophic cubic sets may not be a T-external (resp. I-external and F-external) neutrosophic cubic set. We have discussed conditions for the R-union and R-intersection of T-external (resp. I-external and F-external) neutrosophic cubic sets to be a T-external (resp. I-external and F-external) neutrosophic cubic set. Based on this paper, we will study conditions for the R-intersection of two neutrosophic cubic sets to be both an \( \alpha \)-external neutrosophic cubic set and an \( \alpha \)-internal neutrosophic cubic set for \( \alpha \in \{ T, I, F \} \). Also, we will consider conditions for the R-union and R-intersection of two \( \alpha \)-internal neutrosophic cubic sets to be an \( \alpha \)-external neutrosophic cubic set for \( \alpha \in \{ T, I, F \} \).

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### REFERENCES


